5/21/2014

Probability of Vacuum Stability in Type IIB Multi-Kähler Moduli Models

Yoske Sumitomo
KEK Theory Center

- X. Chen, G. Shiu, YS, S.-H. H. Tye, JHEP 1204(2012)026, arXiv:1112.3338
- YS, M. Rummel, JHEP 1312(2013)003, arXiv:1310.4202

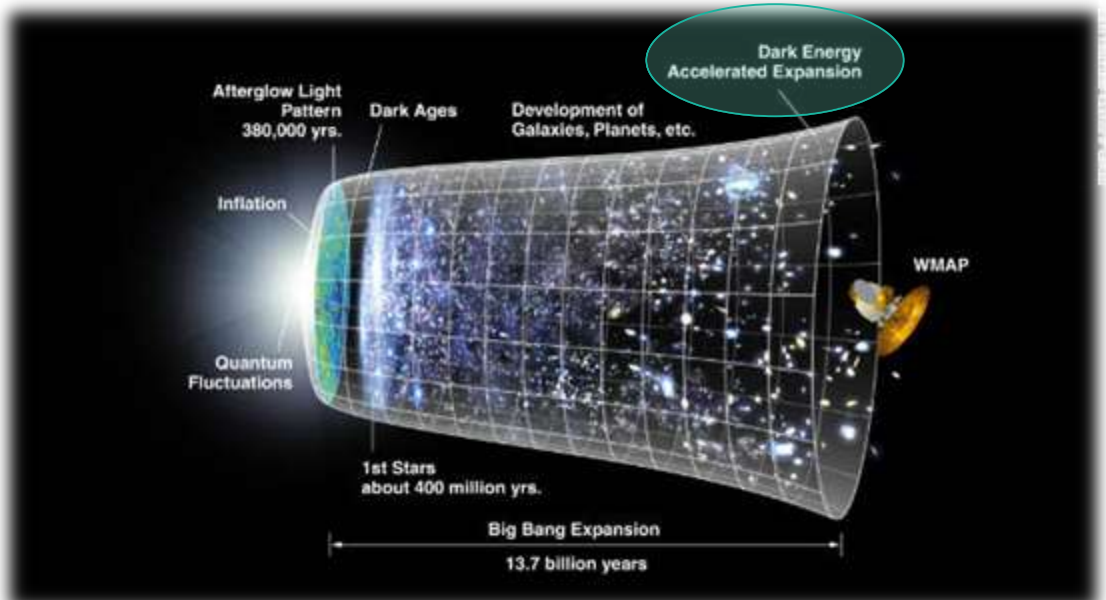
Contents

- Motivation
- Probability argument
- Moduli stabilization in IIB
- Probability: focusing on models

Motivation

Dark Energy

Late time expansion



Awarded Nobel Prize in 2011

String Theory, including quantum gravity can say something on this?

Recent observation

Planck Collaboration,
P. A. R. Ade et.al., arXiv 1303.5076

- **Cosmological Constant**

  Planck+WMAP+BAO

  pressure-density ratio

  $$w = p/\rho = -1.13^{+0.24}_{-0.25} \text{(95\% CL)}$$

- **Time-varying DE**

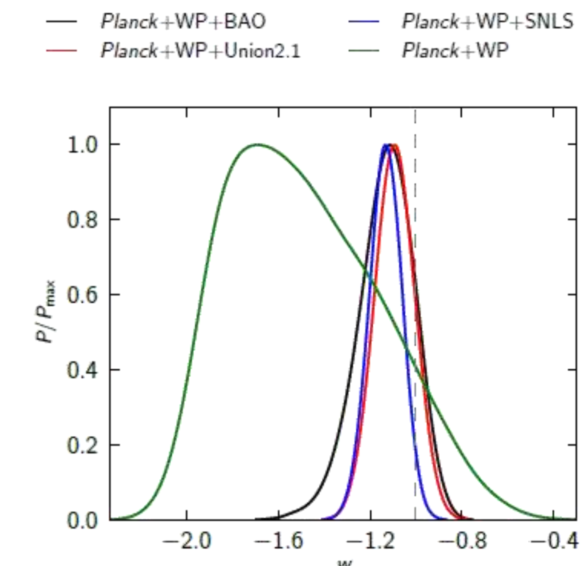
  Planck+WMAP+BAO

  $$w = w_0 + w_a(1 - a(t)) \qquad \text{(95\% CL)}$$

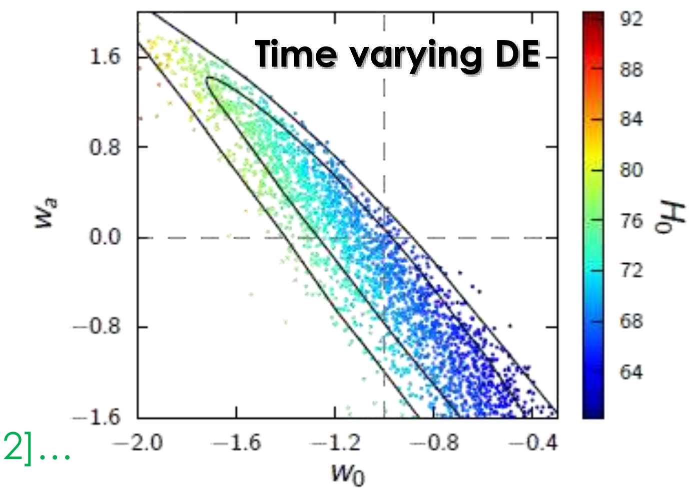
  $$w_0 = -1.04^{+0.72}_{-0.69}, \qquad \mathrm{w_a} < 1.32$$

  e.g. Stringy Quintessence models

[Choi, 99], [Svrcek, 06], [Kaloper, Sorbo, 08], [Panda, YS, Trivedi, 10], [Cicoli, Pedro, Tasinato, 12]…



Cosmological constant



Minima for stable life

Minima for stable life

Moduli Stabilization

Moduli stabilization → coupling constants in 4D

Cosmological moduli problem

$$T_r \sim \sqrt{M_P \Gamma_\phi}, \qquad \Gamma_\phi \sim \frac{m_{3/2}^3}{M_P^2} \sim \frac{m_\phi^3}{M_P^2}$$ in stringy model

↓ Reheating for BBN: $T_r \gtrsim 10$ MeV

$m_\phi \gtrsim \mathcal{O}(10)$ TeV

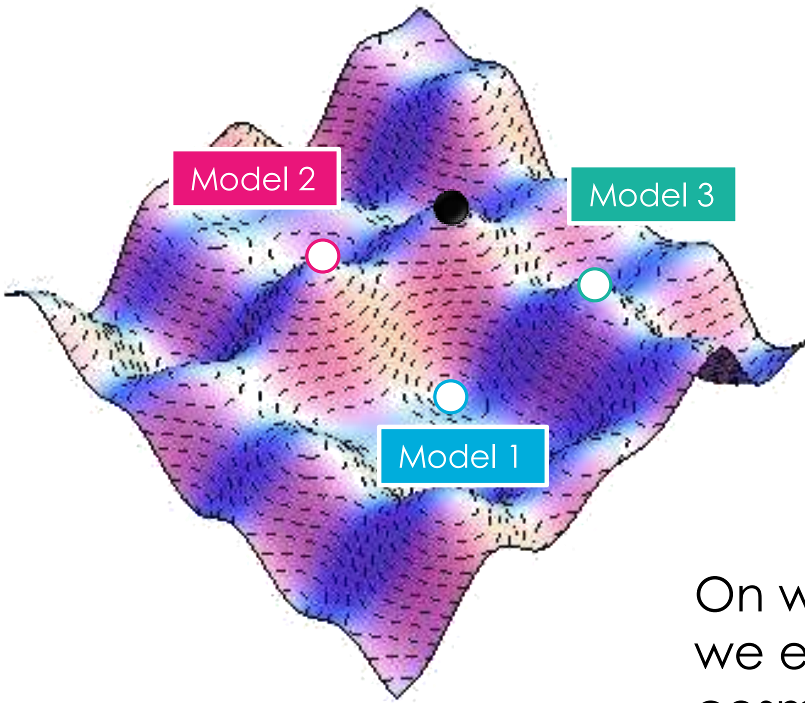
Stringy many moduli are stabilized at present.

→ Vacuum energy generated by moduli potential

Can we freeze a number of moduli fields at positive extrema?

Landscape

Metastable vacua in moduli space



- Inflation

  ⬇ rolling down (& tunneling)

- dS vacua

Low energy

Initial conditions?

On which directions (models), can we easy to achieve the tiny positive cosmological constant?

$$\Lambda \sim +10^{-123} M_P^4$$

Stringy Landscape: models

There are many types of vacua in string theory, as a result of a variety of (Calabi-Yau) compactification.

$$ds_{10}^2 = ds_4^2 + ds_6^2$$

A class of Calabi-Yau gives Swiss-cheese type of volume.

$$\mathcal{V}_6 = \gamma_1 (T_1 + \bar{T}_1)^{3/2} - \sum_{i=2} \gamma_i (T_i + \bar{T}_i)^{3/2} ,$$

Examples: [Denef, Douglas, Florea, 04]

- $\mathbb{P}^4_{[1,1,1,6,9]}$: $h^{1,1} = 2,\ h^{2,1} = 272$
- $\mathcal{F}_{11}$: $h^{1,1} = 3,\ h^{2,1} = 111$
- $\mathcal{F}_{18}$: $h^{1,1} = 5,\ h^{2,1} = 89$

All can be stabilized *(a la KKLT)*, with a variety of fluxes.

($h^{1,1}$: # of Kahler, $h^{2,1}$: # of c.s. moduli)

More recently, for $2 \le h^{1,1} \le 4$, 418 manifolds

Rich vacuum structures! [Gray, He, Jejjala, Jurke, Nelson, Simon, 12]

Stringy Landscape: fluxes

Many ways to include fluxes for complex moduli stabilization

Quantization (e.g. type IIB)

$$\frac{1}{2\pi\alpha'}\int F_3 \in 2\pi\mathbf{Z}, \qquad \frac{1}{2\pi\alpha'}\int H_3 \in 2\pi\mathbf{Z}$$

Gukov-Vafa-Witten superpotential

$$W_0 = \int_M \Omega \wedge (F_3 - SH_3) = W_0(S, U_i)$$

S, U_i: dilaton and complex structure moduli

Moduli stabilization of S, U_i

$D_{S,U_i} W_0 = 0$ give various values for $W_0|_{\min}$

Each value of W_0 determines different vacuum.

Probability argument

X. Chen, G. Shiu, YS, S.-H. H. Tye, JHEP 1204(2012)026, arXiv:1112.3338

Gaussian Ensemble

Many moduli → various vacua in string landscape

→ Mass matrix given randomly at extrema

Hessian $\partial_{\phi_i}\partial_{\phi_j}V|_{\min}$ via linear trans.

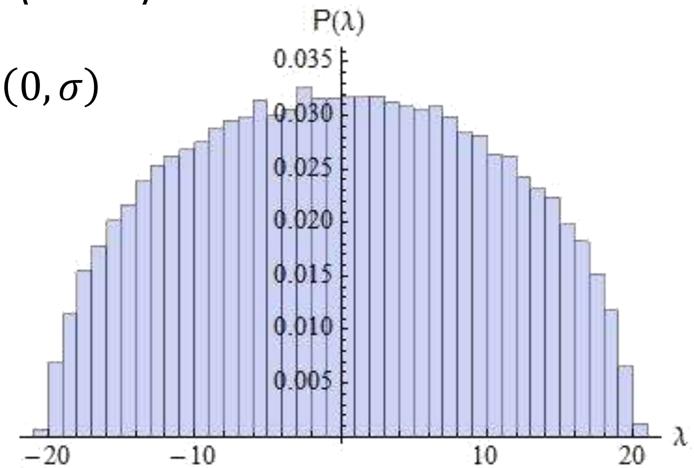
→ Real/complex symmetric matrix

E.g. Gaussian Orthogonal Ensemble (GOE)

$$Z \propto \int dM_{ij}\, e^{-\frac{1}{2\sigma^2}\mathrm{tr}\, M^2}, M = A + A^T, \quad A_{ij} \in \Omega(0,\sigma)$$

Wigner semi-circle law

$$\rho(\lambda) = \frac{1}{2\pi N\sigma^2}\sqrt{4N\sigma^2 - \lambda^2}$$



→ How likely stable minima exist?

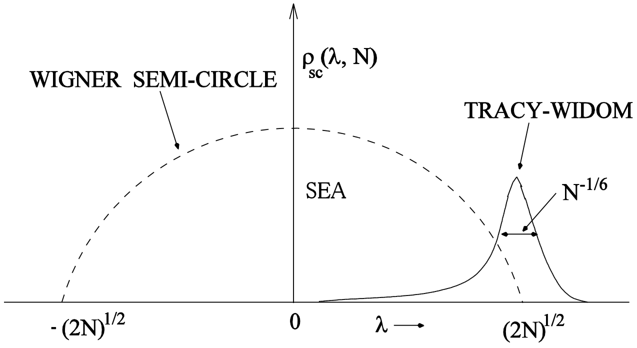
Positivity of mass matrix: all eigenvalues to be positive

Gaussian suppression on stability

Tracy-Widom fluctuation [Tracy, Widom, 94]

At finite N, the maximum eigenvalue fluctuates around the edge of Wigner semi-circle.

$$\lambda \sim 2\sqrt{N} + N^{-1/6}\chi$$



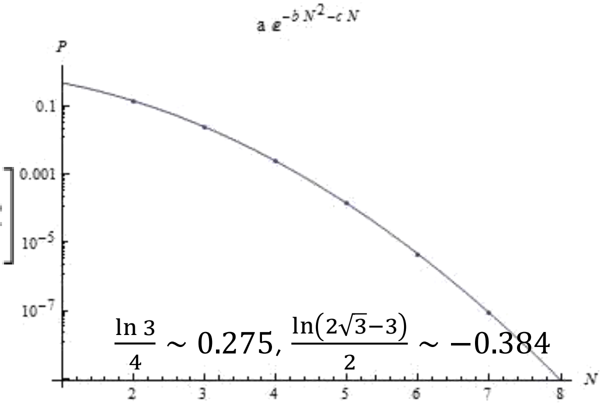
Calculated numerically, and then confirmed analytically

[Aazami, Easther, 05], [Dean, Majumdar, 08], [Borot, Eynard, Majumdar, Nadal, 10]

$$Z \propto \int dM_{ij}\, e^{-\frac{1}{2\sigma^2}\mathrm{tr}\, M^2}, M = A + A^T, \quad \text{(GOE)}$$

$$\mathcal{P} = \exp\left[-\underline{\frac{\ln 3}{4}N^2} + \frac{\ln(2\sqrt{3}-3)}{2}N - \frac{1}{24}\ln N - 0.0172\right]$$

Gaussian term dominates even at lower N.



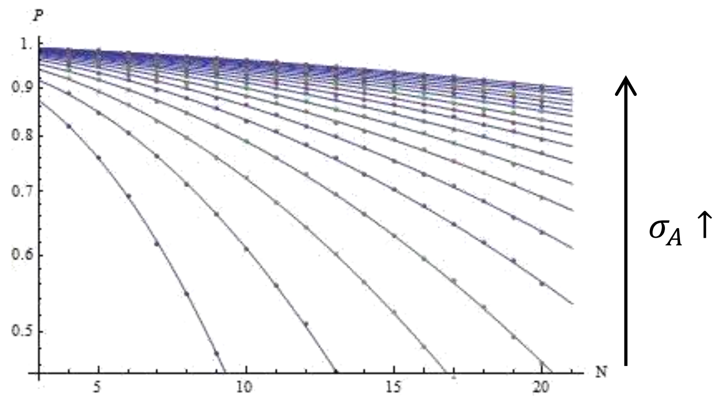
Hierarchical Matrix [X. Chen, Shiu, YS, Tye, 12]

Suppose a hierarchy between diagonal and off-diagonal entries. The uplift term does not dominate the entire matrix. Does the stability of AdS remain after uplift?

$M_{\text{total}} = M_{Ads} + M_{\text{uplift}}$

M_{Ads} : diagonal, half-normal with σ_A

M_{uplift} : GOE with σ_B $(= 1)$



Probability of the form:

$\mathcal{P} = a\, e^{-bN^2 - cN}$ between $N = 4 - 20$, varying $\sigma_A = 10 - 100$

- E.g. at $\sigma_A = 100$ ➡ $\mathcal{P} = 1.00\, e^{-0.000111\, N^2 - 0.00277\, N}$

➡ Gaussian suppression dominates when $N > \frac{0.00277}{0.000111} \sim 25.$

Random SUGRA

Introduce randomness at extremal points: [Denef, Douglas 04], [Marsh, McAllister, Wrase 11]

$$\left.\begin{aligned} &W \\ F_a &= D_a W \\ Z_{ab} &= D_a D_b W \\ U_{abc} &= D_a D_b D_c W \end{aligned}\right. \quad \in m_{susy}\,\Omega(0, 1/\sqrt{N}) \qquad \text{(Gaussian Ensemble)}$$

Also $K_{a\bar{b}1}, K_{a\bar{b}1\bar{1}} \in \Omega(0, 1/\sqrt{N})$ in the basis of $K_{a\bar{b}} = \delta_{a\bar{b}}$

Then, the Hessian can be estimated.

$$V = e^K (F_a \bar{F}^a - 3|W|^2), \qquad \mathrm{H} = \begin{pmatrix} \partial^2_{a\bar{b}} V & \partial^2_{ab} V \\ \partial^2_{\bar{a}b} V & \partial^2_{\bar{a}\bar{b}} V \end{pmatrix} = f_{2\mathrm{N}\times 2\mathrm{N}}(W, F, Z, U, \cdots)$$

Probability of positive Hessian at SUSY extrema: [Bachlechner, Marsh, McAllister, Wrase 12]

$$P = \exp\left(-\frac{2|W|^2}{m_{susy}^2} N^2 \right)$$ again Gaussianly suppressed

Moduli Stabilization in IIB

A bonus in type IIB

Moduli stabilization

- Fluxes → Complex structure & dilaton
- Non-perturbative effect, α'-correction, localized branes

→ Kahler [KKLT, 03], [Balasubramanian, Berglund, Conlon, Quevedo, 05], [Balasubramanian, Berglund, 04]…

$$V = V_{\text{Flux}} \quad + \quad \underline{V_{\text{NP}} + V_{\alpha'} + \cdots}$$

↳ Complex ↳ Kahler

E.g. $\mathbb{P}^4_{[1,1,1,6,9]}$: $h^{1,1} = 2, h^{2,1} = 272$

No scale structure → Hierarchy between Kahler and Complex sector

$$V_{\text{Flux}} = e^K \left| D_{S,U_i} W_0 \right|^2$$ convex downward

Hierarchical structure of mass matrix/potential helps to stabilize moduli at positive cosmological constant.

KKLT

[Kachru, Kallosh, Linde, Trivedi, 03]

Non-trivial potential for Kahler is generated by NP-corrections.

E.g. Gluino condensation on D7-branes

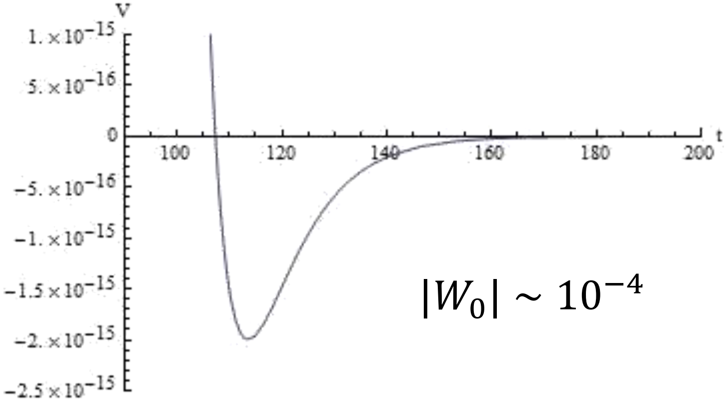
D7-branes wrapping the four cycle: $W_{NP} = A\, e^{-\tilde{a}\, 8\pi^2/g_{D7}} = A\, e^{-a\, T}$

Together with the superpotential from fluxes: $W = W_0 + W_{NP}$

Supersymmetric vacuum
$D_T W = 0$ existes.

But exponentially small W_0

$|W_0| \sim A\, e^{-a\, T}$



$|W_0| \sim 10^{-4}$

Large Volume Scenario

[Balasubramanian, Beglund, Conlon, Quevedo, 05]

α'-corrections can break no-scale structure too.

$\mathcal{O}(\alpha'^3)$-correction in type II action [Becker, Becker, Haack, Louis, 02]

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2}(S+\bar{S})^{3/2}\right) - \ln(S+\bar{S}) + \cdots$$

scales differently

S: dilaton

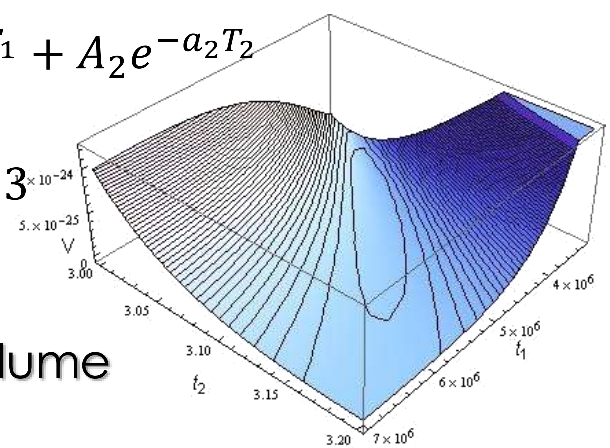
E.g. $\mathbb{P}^4_{[1,1,1,6,9]}$ model (assuming stabilized complex sector)

$$\mathcal{V} = \frac{1}{9\sqrt{2}}\left(t_1^{3/2} - t_2^{3/2}\right), \qquad W = W_0 + A_1 e^{-a_1 T_1} + A_2 e^{-a_2 T_2}$$

Solution: $W_0 \sim -20,\ A_1 \sim 1,\ t_1 \sim 10^6,\ t_2 \sim 3$

$V_{\min} \sim -10^{-25}$: AdS vacua

➡ $|W_0| \gg |W_{NP}|,\ \mathcal{V} \propto e^{a_2 t_2} \gg \xi$ larger volume

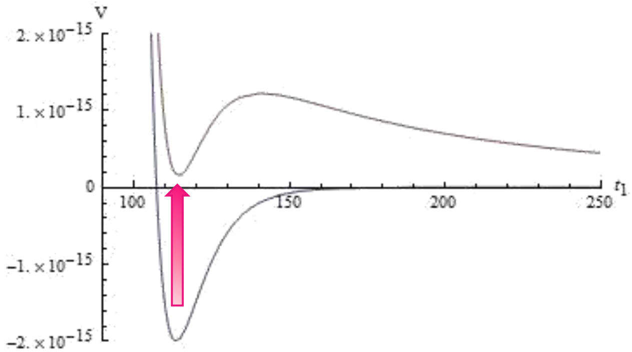


Uplift from AdS

Add an uplifting potential by hand

$$V = V_{SUGRA} + V_{D3-\overline{D3}}$$ [KKLT, 03]

$$V_{D3-\overline{D3}} = 2T_3 \int d^4x \sqrt{-g_4}$$



Backreaction of $\overline{D3}$? → a singularity exists, but finite action

Safe or not? [DeWolfe, Kachru, Mulligan, 08], [McGuirk, Shiu, YS, 09], [Bena, Giecold, Grana, Halmagyi, Massai, 09-12], [Dymarsky, 11],…

Many other ways:

- D-term uplift [Burgess, Kallosh, Quevedo, 03], …
- Complex structure uplift [Saltman, Silverstein, 04]
- Dilaton NP effects [Cicoli, Maharana, Quevedo, Burgess, 12]

Stability in multi-moduli space?

Probability: focusing on models

YS, M. Rummel, JHEP 1312(2013)003, arXiv:1310.4202

Comparison of models

Let's introduce some properties of dynamics and string compactification.

We focus on two models of stabilization in type IIB.

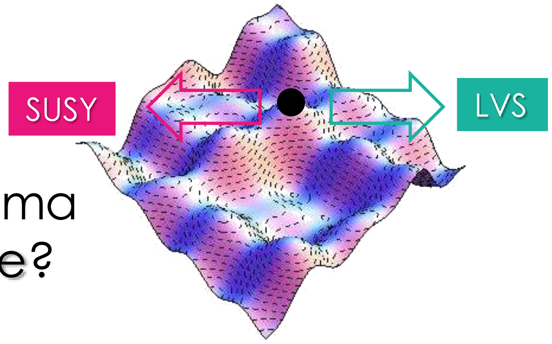
SUSY model (a la KKLT) ⟷ LVS model

$D_a W = 0$ $\partial_a V = 0$

Both minima stay AdS, but will be uplifted to dS.

→ Again interested in positive Hessian/mass matrix

The model might specify a direction to go.



On which direction, are positive stable minima likely to come by in multi-dimensional space?

SUSY model

Consider Swiss-Cheese type of compactification

$$K = -2\ln\mathcal{V}, \qquad \mathcal{V} = \gamma_1(T_1+\bar{T}_1)^{3/2} - \sum_{i=2}^{N}\gamma_i(T_i+\bar{T}_i)^{3/2}$$

Non-perturbative correction from Euclidean D3 or D7 gaugino condensation, negligible α'-correction

$$W = W_0 + \sum_{I=1}^{N} A_I e^{-a_I T_I} \qquad \text{then, } V = e^K\left(|D_T W|^2 - 3|W|^2\right)$$

Employ larger volume to justify the SUGRA approximation

$$D_{T_I}W = 0 \;\Longrightarrow\; \begin{cases} x_1 = a_1 \mathrm{Re}\, T_1 \simeq -\mathcal{W}_{-1}(3W_0/2A_1) \\ x_i = a_i \mathrm{Re}\, T_i \simeq \dfrac{1}{2}\mathcal{W}_0\left(-\dfrac{8A_i^2}{9W_0^2}\mathcal{W}_{-1}^3(3W_0/2A_1)\right) \\ \mathrm{Im}\, T_I = 0 \end{cases}$$

$z = \mathcal{W}e^{\mathcal{W}}$: Lambert-W function

Stability of SUSY model

Dangerous direction:

$$\partial^2_{x_i} V \Big|_{\text{ext}} \propto (2x_i + 1)(4x_i - 1) + \text{subleading}$$

Hence we need $(x_1 \gg)\ x_i = a_i \operatorname{Re} T_i > 1/4$ for positive Hessian.

We can check positivity of all eigenvalues using Sylvester criteria.

Introduce randomness

- $-10^3 \leq W_0, A_I \leq 10^3$: uniformly distributed
- $a_I = 2\pi/n_I$: possible gauge rank for compactification

Then, for $\mathcal{V} > 30,\ \gamma_I = \sqrt{2}/3\,,\ \operatorname{Re} T_I > 1$ similar for other cond.

N	2	3	4	5	6
$P = \#_{sta}/\#_{ext}$	0.997	0.892	0.668	0.381	0.178

LVS type

Swiss-Cheese, non-perturbative and leading α'-correction

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right), \qquad \mathcal{V} = \gamma_1 (T_1 + \bar{T}_1)^{3/2} - \sum_{i=2}^{N} \gamma_i (T_i + \bar{T}_i)^{3/2}$$

$$W = W_0 + A_I e^{-a_1 T_1} + \sum_{I=2}^{N} A_I e^{-a_I T_I}$$

negligible

EOM $\partial_i V = 0$ are simplified, but difficult for analytic solutions.

$$\frac{A_i}{W_0} = -e^{x_i} \frac{6\sqrt{2}\gamma_i x_i^{1/2}(x_i - 1)}{a_i^{3/2}\mathcal{V}(4x_i - 1)}, \qquad \xi = 64\sqrt{2} \sum_{i=2}^{N} \frac{\gamma_i x_i^{5/2}(x_i - 1)}{a_i^{3/2}(4x_i - 1)^2}$$

Stability condition

$$\partial_{t_i}^2 V \Big|_{\text{ext}} \propto \frac{(x_i - 1)\left(8x_i^3 - 6x_i^2 + 3x_i + 1\right)}{\mathcal{V}^3 \, x_i^{1/2}(4x_i - 1)^2} > 0$$

→ $x_i = a_i \,\mathrm{Re}\, T_i > 1$

All eigenvalues are positive.

Probability of LVS type

We impose a set of randomness (for $\mathcal{V} > 30,\ \gamma_I = \sqrt{2}/3,\ \mathrm{Re}\,T_I > 1$):

- $-10^3 \le W_0, A_i \le 10^3$: uniformly distributed
- $a_I = 2\pi/n_I$: possible gauge rank for compactification
- $\xi \sim 4.85 \times 10^{-3}(N_C - N)g_s^{-3/2}$ with uniform $1 \le N_C \le 300$

Case 1 uniformly distributed $1 < g_s^{-1} \le 100$, upper bound for ξ

N	2	3	4	5	6
$P = \#_{sta}/\#_{ext}$	1.00	0.676	0.230	0.0332	0.00458

Case 2 uniformly distributed $0 < g_s^{+1} < 1$, larger ξ is disfavored

N	2	3	4	5
$P = \#_{sta}/\#_{ext}$	1.00	0.0677	0.00978	0.000569

Comparison: SUSY vs LVS

SUSY $D_{T_I}V = 0$ for $\mathcal{V} > 30,\ \gamma_I = \sqrt{2}/3,\ \mathrm{Re}\,T_I > 1$

N	2	3	4	5	6
$P = \#_{sta}/\#_{ext}$	0.997	0.892	0.668	0.381	0.178

LVS Case 2 $\partial_{T_I}V = 0$, uniformly distributed $0 < g_s < 1$

N	2	3	4	5
$P = \#_{sta}/\#_{ext}$	1.00	0.0677	0.00978	0.000569

Positive Hessian in SUSY models is more likely.

Distribution of g_s?

Simple model for complex sector: $K_S = -\ln(S+\bar{S}), W_0 = C_1 + C_2\,S$

➡ $g_s = \frac{C_2}{C_1},\ W_0 = 2C_1$: uniformly distributed $W_0, 0 < g_s^{+1} < 1$ for SUGRA

Summary & Discussion

Summary & Discussion

Random Landscape in String Theory

Fluxes, varieties of compactification

→ Highly non-trivial potential in multi-moduli space

Chances to achieve stable de-Sitter vacua with the *positive* (and tiny) CC?

Probability analysis

Probability is a kind of universal quantity.

→ Quantify the property of models

In general, chances are suppressed as Gaussian function of N.

But the type IIB has special the feature, and the positive Hessian in SUSY model seems favored with reasonable inputs.

Related direction

Distribution for tiny CC [Sumitomo, Tye, Wong, 11-13]

Random analyses in inflationary physics $P(N_e, n_s, r)$