Conformal Description of Starobinsky Model and Beyond

Shi Pi
Asia Pacific Center for Theoretical Physics
Based on arXiv: 1404.2560 with Yifu Cai and Jinn-ouk Gong

Outline

- Introduction: Starobinsky model and predictions
- Starobinsky-like model 1: decimal index
- Starobinsky-like model 2: T-models
- Beyond Staobinsky: dynamical index
- Summary

Set up

- We begin with (Kallosh et al 1306.5220)

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{\chi^2}{12}R(g) - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\phi^2}{12}R(g) - \frac{\lambda}{4}(\phi^2-\chi^2)^2\right].$$

- This Lagrangian bears two symmetries:
  - 1) local conformal symmetry under

$$g_{\mu\nu} \to e^{-2\sigma(x)}g_{\mu\nu}\,, \chi \to e^{\sigma(x)}\chi\,, \phi \to e^{\sigma(x)}\phi$$

  - 2) additional SO(1,1) symmetry in field space.

Fix the gauge

- The field χ is called conformon. It is not a real d.o.f. but can be removed by gauge fixing.
  - A gauge also respect the SO(1,1) symmetry is a hyperbola asymptotes to “light cone”.

$$\chi^2 - \phi^2 = 6\,.$$

  - Or parameterisation

$$\chi = \sqrt{6}\cosh\left(\frac{\varphi}{\sqrt{6}}\right)\,, \quad \phi = \sqrt{6}\sinh\left(\frac{\varphi}{\sqrt{6}}\right)\,.$$

- The Higgs-like potential turn to be a pure constant 9λ.

Starobinsky model

- If we revise the potential a little bit, Starobinsky model is recovered.

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{12}\left(\chi^2 - \phi^2\right) + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{\lambda}{4}\phi^2\left(\chi - \phi\right)^2\right].$$

- After we take the gauge fixing, it becomes

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{9}{4}\lambda e^{-4\varphi/\sqrt{6}}\left(1 - e^{2\varphi/\sqrt{6}}\right)^2\right].$$

Starobinsky model

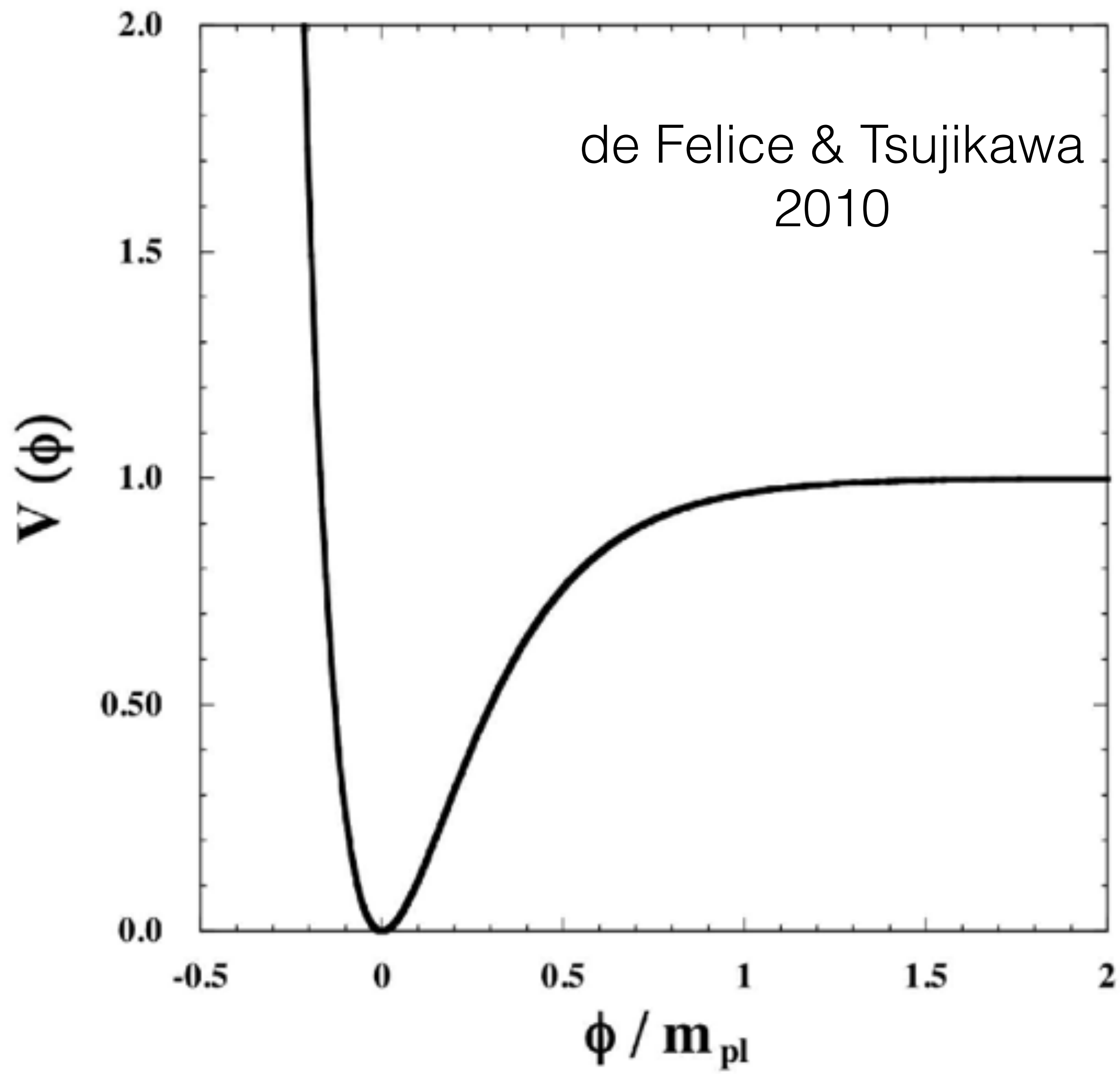
- Starobinsky 1980

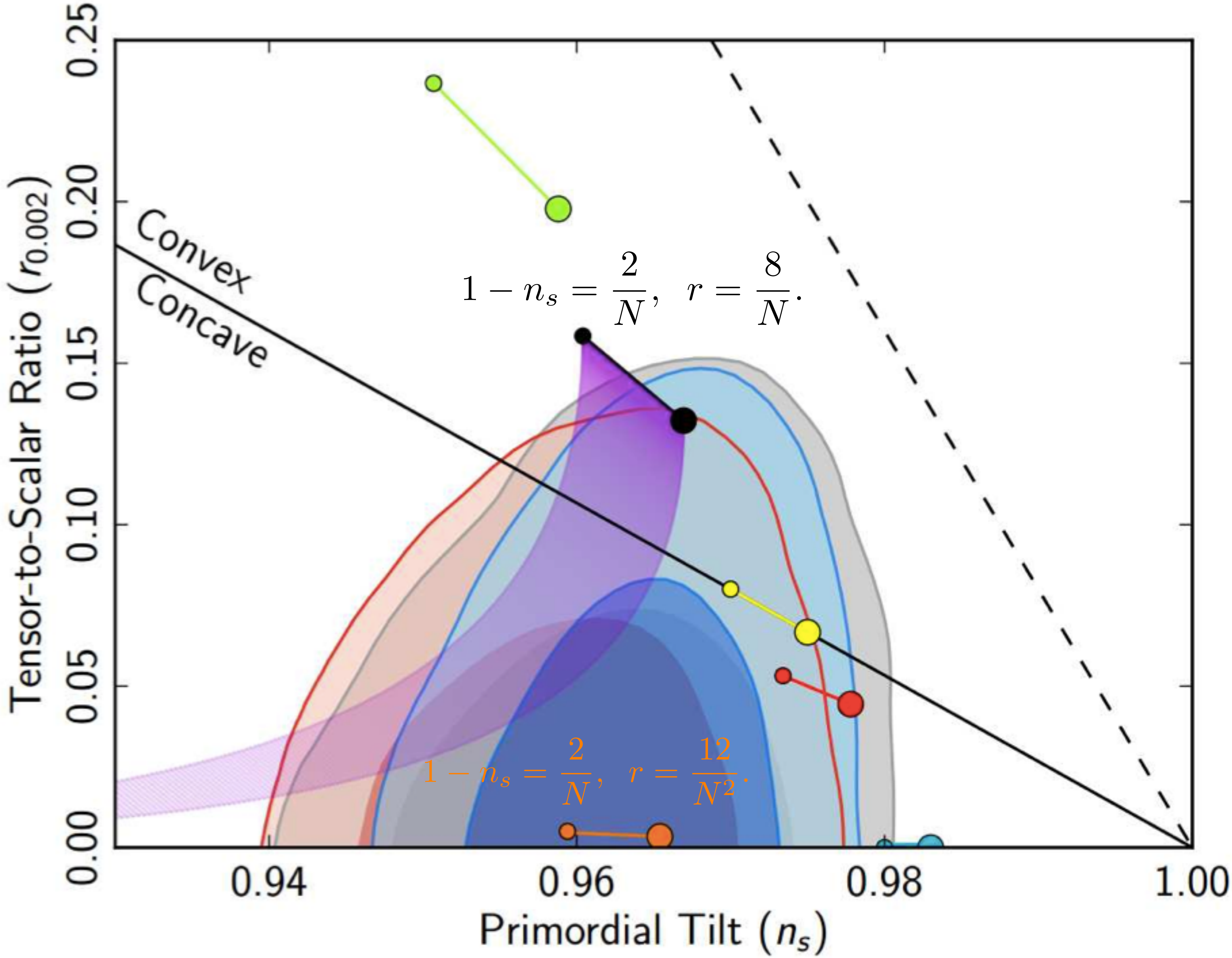
$$\mathcal{L} = \sqrt{-g}\left(R + \frac{R^2}{6M^2}\right).$$

- After the conformal transformation it goes in the Einstein frame with potential (Barrow et. al. 1988)

$$V(\phi) = \frac{3M^2}{4}\left(1 - e^{-\sqrt{2/3}\phi}\right)^2.$$

- This can be described by the conformal inv two-field theory.





Tensor-to-Scalar Ratio ($r_{0.002}$)
Primordial Tilt (n_s)
0.00
0.05
0.10
0.15
0.20
0.25
0.94
0.96
0.98
1.00
Convex
Concave
$1-n_s=\frac{2}{N}, \;\; r=\frac{8}{N}.$
$1-n_s=\frac{2}{N}, \;\; r=\frac{12}{N^2}.$

Outline

- Introduction: Starobinsky model and predictions
- Starobinsky-like model 1: decimal index
- Starobinsky-like model 2: T-models
- Beyond Staobinsky: dynamical index
- Summary

Extension

- We can see that the essential part of the conformal description is
  - 1. Preserve the conformal symmetry;
  - 2. Inflation happens near the SO(1,1) symmetry.
- Try to preserve these properties, and see the possible extension of Starobinsky-like model.

Extension

- The most general theory that respect the conformal symmetry is

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{12}\left(\chi^2 - \phi^2\right) + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{36}\chi^4 f(\phi/\chi)\right] .$$

- The potential does not respect the SO(1,1) symmetry. But it is invariant under the local conformal transformation.

- Define $z=\phi/\chi$ for future convenience.

Slow-roll parameter

- The first slow-roll parameter

$$\epsilon_1 = \frac{1}{2}\left(\frac{V_{,\varphi}}{V}\right)^2 = \frac{1}{12}\left[4z + (1-z^2)\frac{f'}{f}\right]$$

- Inflation occurs at
  - z->0. Then |f’/f| must be small.

$$\left|\frac{f'}{f}\right| \ll 1$$

  - z->1. Then f’/f has a first order pole at z=1:

$$\frac{f'}{f} = \frac{\beta}{1-z} + \text{bounded function.}$$

Residue

- How to interpret the residue β?
- After integration we have

$$f(z) = (1 - z)^{-\beta}$$

  - When β=-2, it is equivalent to Starobinsky model.

$$V(\phi) = \frac{3M^2}{4}\left(1 - e^{-\sqrt{2/3}\phi}\right)^2 .$$

Residue

- When β!=-2, it is (recover the conformon)

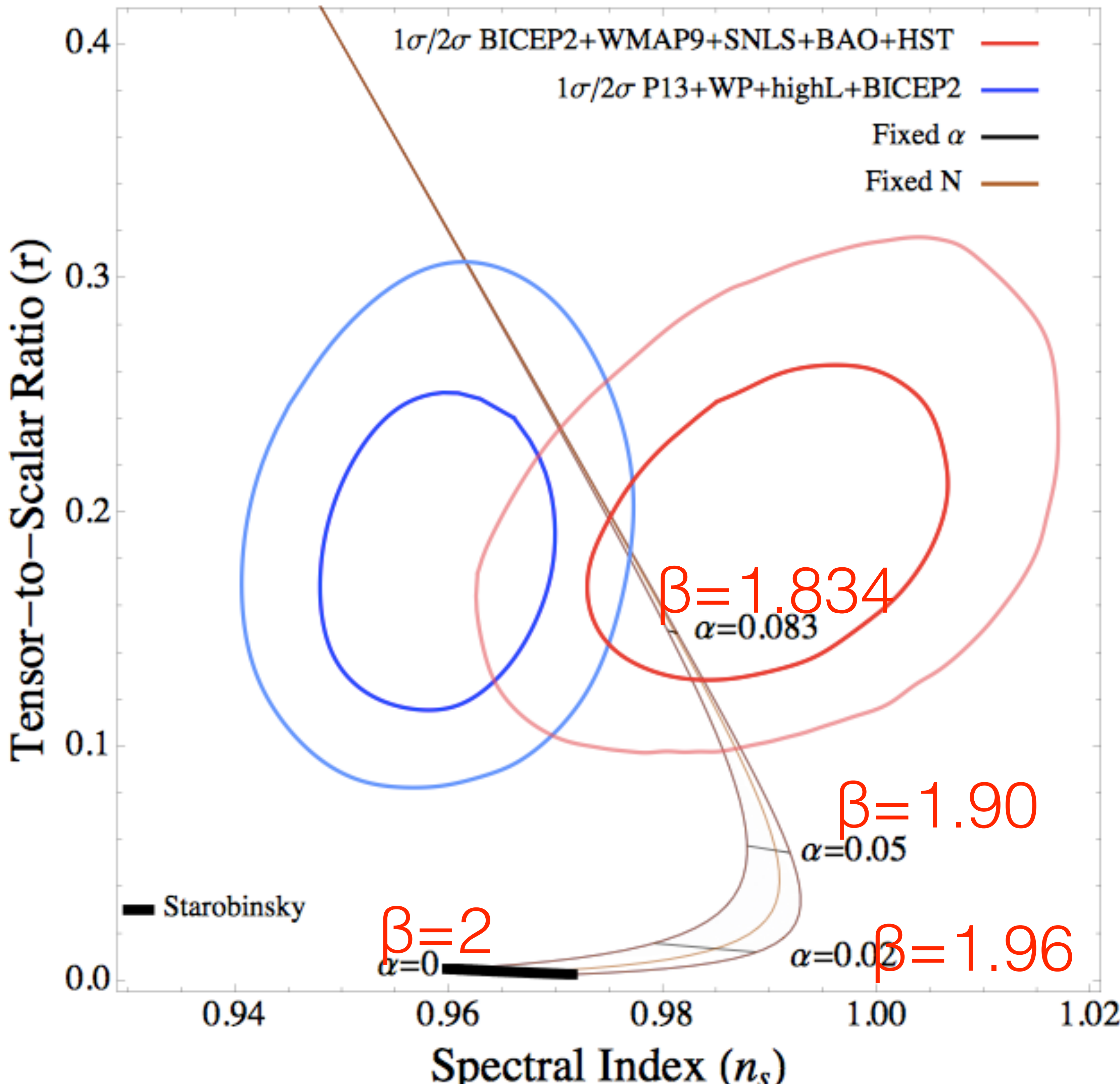
$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{12}\left(\chi^2-\phi^2\right)+\frac{1}{2}\partial^\mu\chi\partial_\mu\chi-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi-\frac{\lambda}{4}\phi^{4+\beta}(\chi-\phi)^{-\beta}\right],$$

  - equivalent to a fractal index of Ricci scalar (Cai, Gong and SP, 1404.2560).

$$\begin{aligned}\mathcal{L} &= \sqrt{-g}\left[R+\alpha R^n\right],\\ n &= \frac{4+\beta}{3+\beta}\end{aligned}$$

Residue

- There are some constraints based on recent observations on primordial B-modes.
- 2-sigma Planck data (without running): 1.988<β<2.055
- 2-sigma BICEP2 data: 1.806<β<1.848 (Codello et al 1404.3558; Chakravaty et al 1405.1321)



Outline

- Introduction: Starobinsky model and predictions
- Starobinsky-like model 1: decimal index
- Starobinsky-like model 2: T-models
- Beyond Staobinsky: dynamical index
- Summary

Other Extensions

- Other poles (relatively far from z=1) are possible.

$$\frac{f'}{f} = \frac{\beta}{1-z} + \sum_i \frac{c_i}{z-z_i} + \text{smooth function.}$$

- A typical example is the T-model.

$$\frac{f'}{f} = \frac{2}{z-1} + \frac{2}{z+1} - \frac{2n}{z}.$$

  - The poles at z=-1 and z=0 can never be reached.

T-model

- In the conformal description, T-model is

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{12}\left(\chi^2 - \phi^2\right) + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{36}F\left(\chi/\phi\right)\left(\chi^2 - \phi^2\right)^2\right]$$

  - F is an arbitrary function.

- F(χ/ϕ) breaks the SO(1,1) symmetry unless F(χ/ϕ)=constant.

T-model

- After choosing the gauge, it reduces to

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - F\left(\tanh\frac{\varphi}{\sqrt{6}}\right)\right].$$

- The hyper tangent can stretch the potential to make almost any potential flat enough around $\varphi \to \infty$

- Inflation happens when

$$\varphi \to \infty,$$
$$\tanh\frac{\varphi}{\sqrt{6}} \to 1^-.$$

T-model

- A typical (but not necessary) choice of F function is

$$F = \lambda_n \tanh^{2n} \frac{\varphi}{\sqrt{6}}.$$

  - λ is a coupling constant and n is an (integer?) index.

- Inflation happens when ϕ goes to infinity.

T-model

- There is a plateau around ϕ>>1

$$V(\varphi) \sim \lambda_n(1 - 4ne^{-\sqrt{2/3}\varphi} + \cdots).$$

- It is similar to Starobinsky plateau when n=1/2.

$$V(\phi) = \frac{3M^2}{4}\left(1 - e^{-\sqrt{2/3}\phi}\right)^2.$$

T-model

- The slow-roll e.o.m. in the large field case is

$$\frac{d\varphi}{dN} = \frac{V'}{V} = 4n\sqrt{\frac{2}{3}}e^{-\sqrt{\frac{2}{3}}\varphi} = 2n\sqrt{\frac{2}{3}}\xi,$$

$$\xi \equiv 2e^{-\sqrt{\frac{2}{3}}\varphi}.$$

- Integrate above we have

$$\xi = \frac{3}{4nN}.$$

T-model

- The physical meaning of new variable

$$\xi = 1 - \frac{\phi}{\chi} = 1 - \tanh\frac{\varphi}{\sqrt{6}} \approx 2e^{-\sqrt{2/3}\varphi}.$$

- Now the potential becomes

$$V(\xi) = \lambda(1 - 2n\xi + \mathcal{O}(\xi^2)).$$

T-model

$$\epsilon_1 \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{4}{3}n^2\xi^2 = \frac{3}{4N^2},$$

$$\epsilon_2 = \frac{8}{3}n\xi = \frac{2}{N}.$$

$$r = 16\epsilon_1 = \frac{12}{N^2},$$

$$n_s = 1 - 2\epsilon_1 - \epsilon_2 = 1 - \frac{2}{N} + \mathcal{O}(N^{-2}).$$

r is suppressed by 1/N independent the power of n, the same as Starobinsky model.

Beyond

- The above models are

$$\frac{f'}{f} = \frac{\beta}{1-z} + \text{bounded function.}$$

- What if it is bounded but not smooth?

Outline

- Introduction: Starobinsky model and predictions
- Starobinsky-like model 1: decimal index
- Starobinsky-like model 2: T-models
- Beyond Staobinsky: dynamical index
- Summary

A toy model

- Suppose β=-2, but the other part is a function which has a removable singularity at z=1.

$$\frac{f'}{f} = \frac{2}{z-1} - 2\lambda(1-z)\log(1-z).$$

- In this case the function f is

$$f(z) = (1-z)^{2+\lambda(1-z)} e^{-\lambda(1-z)}.$$

- Again, define ξ=1-z.

Slow-roll parameters

- The slow-roll parameter

$$\epsilon = \frac{\lambda^2}{3}\xi^2 \left(\log \xi\right)^2 ,$$

- We can solve

$$\xi = -\frac{\sqrt{3\epsilon}}{\lambda W_{-1}\left(-\sqrt{3\epsilon}/\lambda\right)} = \exp\left[W_{-1}\left(-\frac{\sqrt{3\epsilon}}{\lambda}\right)\right] .$$

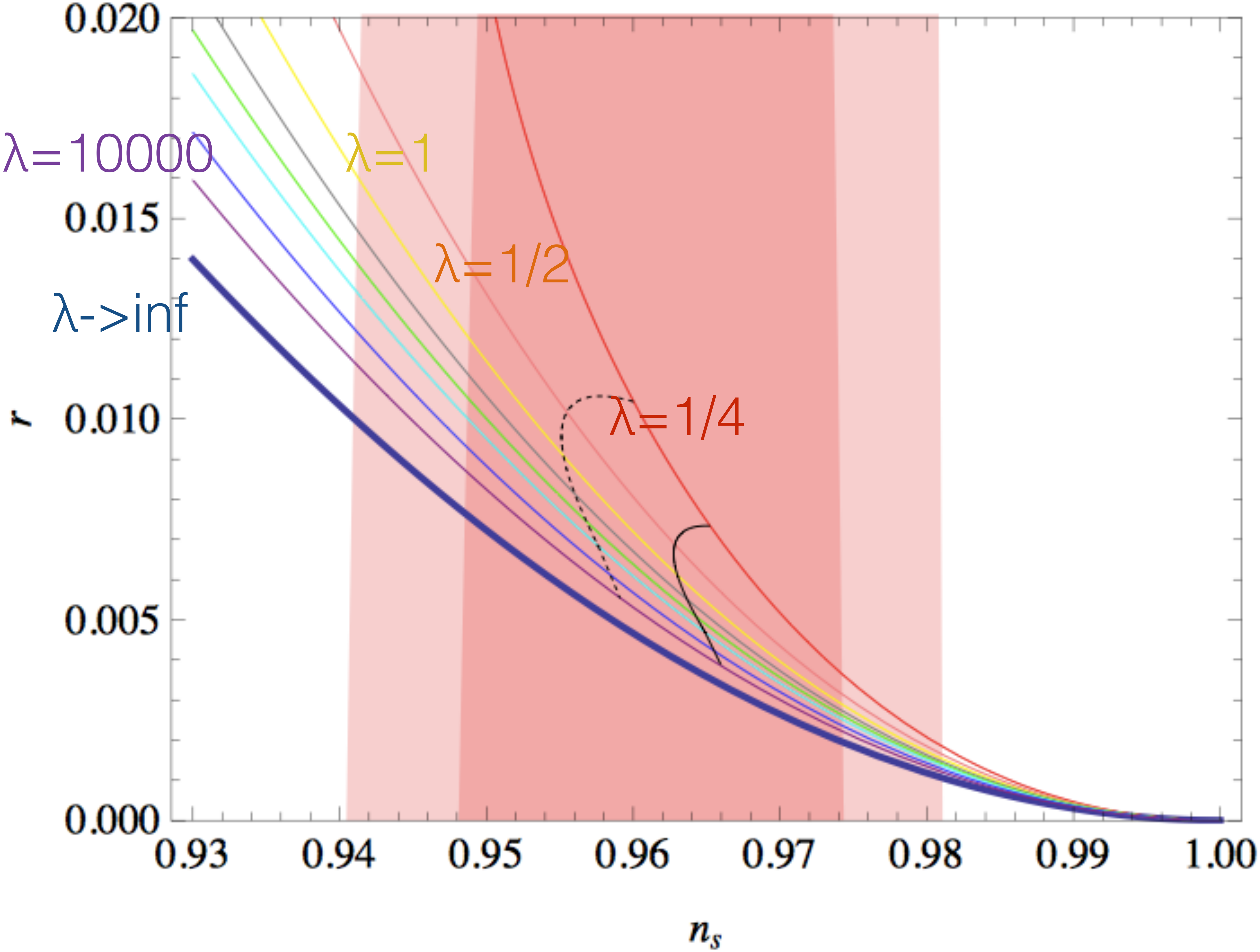
- W is the Lambert function (lower branch).

- tensor-to-scalar ratio is

$$n_s = 1 - \sqrt{\frac{r}{3}}\left[1 + \frac{1}{W_{-1}\left(-\sqrt{3r}/\lambda\right)}\right]\left[1 + \frac{\sqrt{3r}}{8\lambda W_{-1}\left(-\sqrt{3r}/\lambda\right)}\right] - \frac{r}{8}\,.$$

- The e-folding dependence is

$$\begin{aligned} N &= \frac{3}{\lambda}\int \frac{d\xi}{\xi^2(2+\xi)|\log\xi|} \approx \frac{3}{2\lambda}\int \frac{d\xi}{\xi^2|\log\xi|}, \\ &= \frac{3}{2\lambda}\mathrm{li}\ \frac{1}{\xi}. \\ \xi &= \left(\mathrm{li}^{(-1)}\frac{\lambda N}{3}\right)^{-1}. \end{aligned}$$


λ=10000
λ=1
λ=1/2
λ->inf
λ=1/4
r
0.020
0.015
0.010
0.005
0.000
0.93
0.94
0.95
0.96
0.97
0.98
0.99
1.00
n_s

Outline

- Introduction: Starobinsky model and predictions
- Starobinsky-like model 1: decimal index
- Starobinsky-like model 2: T-models
- Beyond Staobinsky: dynamical index
- Summary

Can we recover power-law?

- Yes. Inflation happens elsewhere.
  - We construct reversely from power-law chaotic inflation to get the form of f-function.
  - Or add another parameter to control the form of effective potential. Kallosh et al 1311.0472, 1405.3646.

Recover power-law

- Take a new parameter in the gauge fixing condition, we can have

$$\chi = \sqrt{6}\cosh\frac{\varphi}{\sqrt{6\alpha}}, \quad \phi = \sqrt{6}\sinh\frac{\varphi}{\sqrt{6\alpha}}$$

  - Here once α is of order 1, we go all back to the argument above.

  - Once α is large and makes inflation happens elsewhere

$$\varphi \ll \sqrt{6\alpha}$$

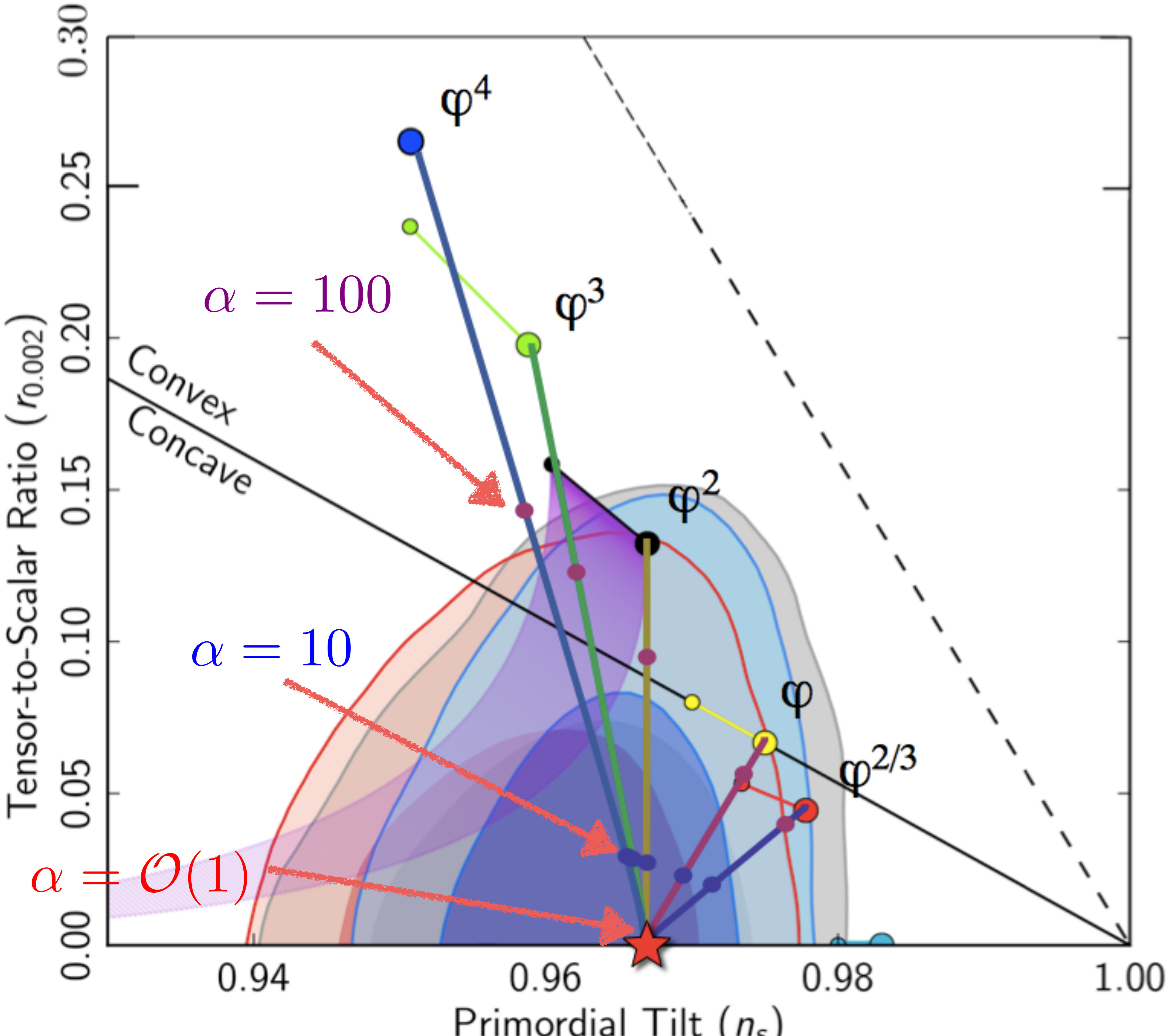
Recover power-law

- And still take a typical T-model potential

$$
\begin{aligned}
V &= V_0 \left(1 - 2ne^{-\sqrt{2/3\alpha}\varphi}\right)^2, \\
&\approx \frac{1}{2}\frac{4V_0}{3\alpha}\varphi^2.
\end{aligned}
$$

- This is a standard quadratic potential which predicts

$$
n_s = 1 - \frac{2}{N}, \quad r = \frac{8}{N}.
$$



0.30
0.25
0.20
0.15
0.10
0.05
0.00
Tensor-to-Scalar Ratio ($r_{0.002}$)
0.94
0.96
0.98
1.00
Primordial Tilt (n_s)
Convex
Concave
φ^4
φ^3
φ^2
φ
$\varphi^{2/3}$
$\alpha = 100$
$\alpha = 10$
$\alpha = \mathcal{O}(1)$

Summary

- Conformal description is a good mechanism to generate a class of Starobinsky-like and similar models.
- The “stretch” effect flatten the potential even if it is steep in the two-field case.
- They can also produce large tensor-to-scalor ratio as we introduce a new parameter.

Thank you!