# Topological inflation with large tensor-to-scalar ratio

Chunshan Lin
Kavli IPMU, University of Tokyo

# Outline

- Introduction: what’s topological inflation
- 3 types of topological inflation models:
  - Double-well
  - Single-well
  - No-well
- Conclusion

# Introduction

- What’s Topological inflation?
  - Inflation occurs at the core of topological defect, where the scalar field is forced to stay near the maximum of potential;
  - It can happen if the size of topological defect is larger than the Hubble radius;
  - Doesn’t require the fine-tuning of initial condition;

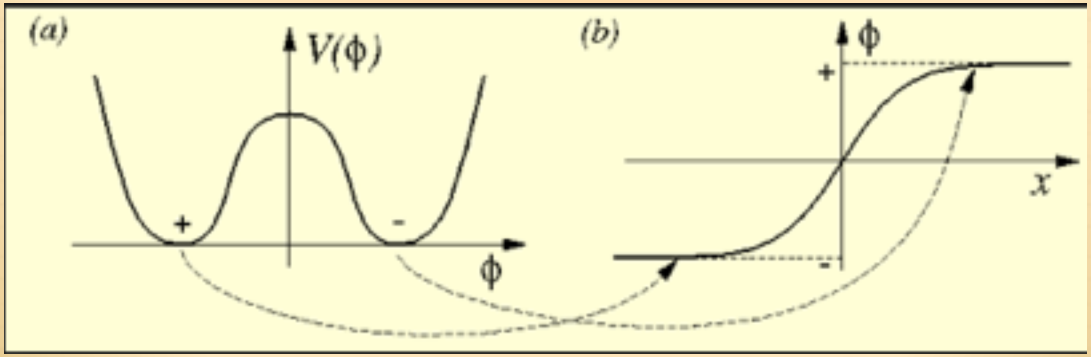
A. D. Linde, (1994) [astro-ph/9402031].

A. Vilenkin, (1994) [hep-th/9402085].

# Introduction

- For example, a Higgs-type double well potential

$$V(\phi) = \frac{1}{4}\lambda\left(\phi^2 - v^2\right)^2$$



**It requires energy!**

$$\phi = -v \quad \rightarrow \quad x = -\infty$$
$$\phi = +v \quad \rightarrow \quad x = +\infty$$

# Introduction

- Energy is minimized by balance of gradient and potential energy; on the other hand, it fixes the size of thickness of domain wall (flat space time);

| | |
|---|---|
| Minimize energy | $\left(\frac{v}{L}\right)^2 \sim \lambda v^4$ |
| Large size | $L > H^{-1}$ |
| Freedmann equation | $\lambda v^4 \sim M_p^2 H^2$ |

$$\Longrightarrow \quad v > M_p$$

With gravity take into account, symmetry breaking scale $v > M_p$ implies that no domain wall solution with fixed thickness. Instead, domain wall inflates!

# Introduction

- Slow-roll

$$\epsilon = \frac{1}{2}\left(\frac{M_p V'}{V}\right)^2 = \frac{8\phi^2 M_p^2}{\left(\upsilon^2 - \phi^2\right)^2} ,$$

$$\eta = \frac{M_p^2 V''}{V} = -\frac{4M_p^2\left(\upsilon^2 - 3\phi^2\right)}{\left(\upsilon^2 - \phi^2\right)^2},$$

The necessary condition for inflation to occur inside of domain wall is compatible with slow roll condition

$\upsilon > M_p$ → $\epsilon \ll 1, \quad \eta \ll 1$

# Introduction

- Gradient terms get diluted away, and eom reads

$$3H\dot{\phi} \simeq -V'(\phi)$$

Choose coordinates so that surface $\phi(\mathbf{x}, t_0) = 0$ lies locally in the x-y plane, expand the scalar field and its potential

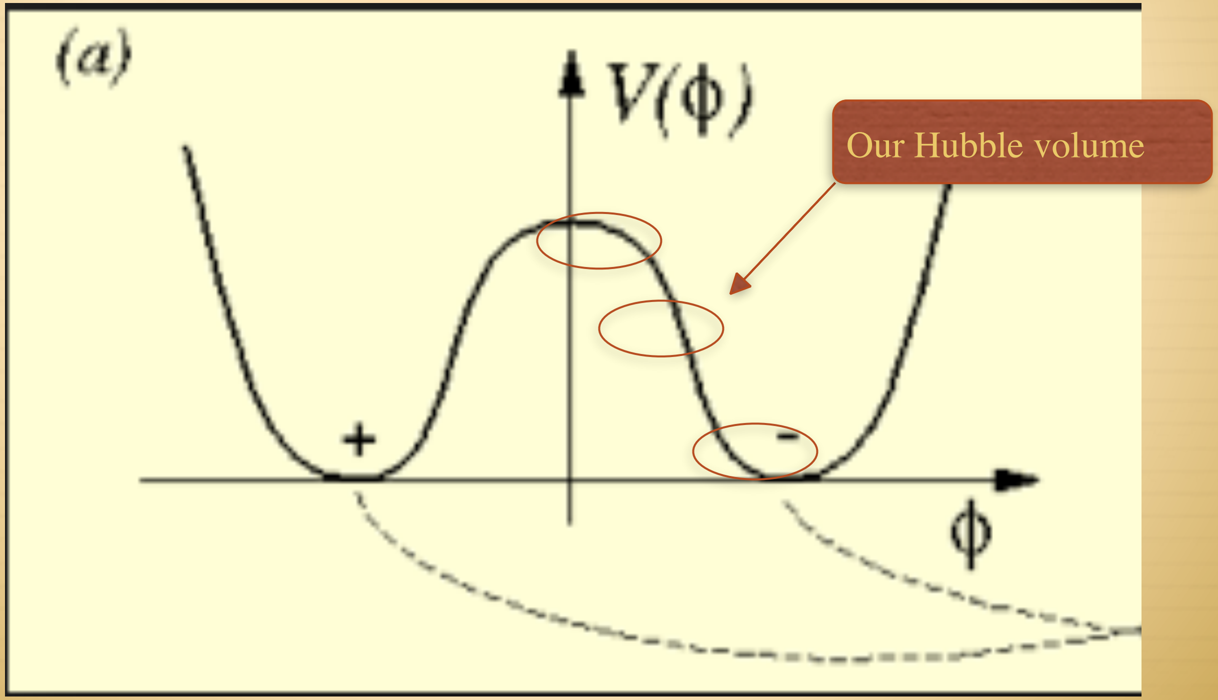
$$\begin{aligned} \phi &\simeq z \\ V(\phi) &\simeq V_0 - \frac{1}{2}\mu^2\phi^2 \end{aligned}$$

$$\begin{aligned} \phi &\simeq \phi_0 \exp\left[\frac{\mu^2}{3H_0}(t - t_0)\right] \\ a &\simeq \exp\left[H_0(t - t_0)\right] \end{aligned}$$

The size of domain wall inflates as well, L~a

# Introduction

- Global picture



Our Hubble volume

# Observable

- Higgs type double well potential, we have constraints on scalar tilt and tensor-to-scalar ratio,

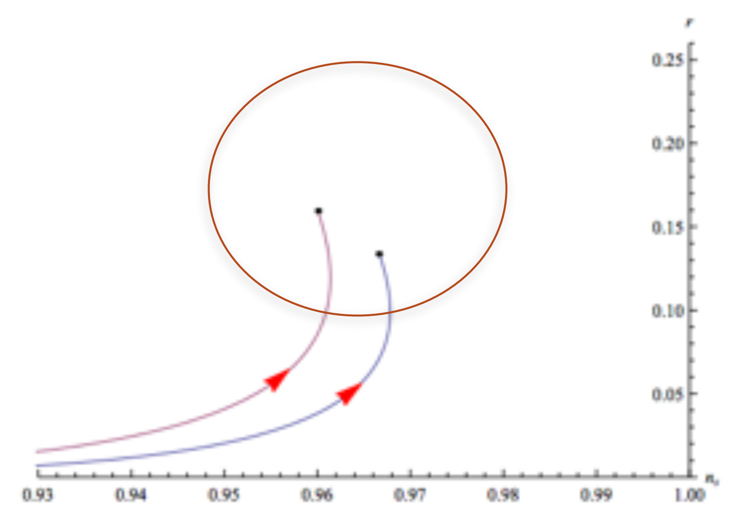
$$\begin{aligned}
\epsilon &= \frac{1}{2}\left(\frac{M_p V'}{V}\right)^2 = \frac{8\phi^2 M_p^2}{\left(v^2-\phi^2\right)^2}\,, \\
\eta &= \frac{M_p^2 V''}{V} = -\frac{4M_p^2\left(v^2-3\phi^2\right)}{\left(v^2-\phi^2\right)^2}, \\
n_s - 1 &= -\frac{8M_p^2\left(v^2+3\phi^2\right)}{\left(v^2-\phi^2\right)^2}\,, \\
r &= 16\epsilon\,.
\end{aligned}$$

In addition, we have another constraint of e-folding number

$$N = \int_{\phi_e}^{\phi_i}\left(\frac{V}{M_p^2 V'}\right) d\phi\,,$$

# Observables

- $n_s$ - r Plot



Horizontal axis: ns
Vertical axis: r
red curve N=50
Blue curve N=60
arrows denotes the direction of moving away from the center of domain wall.
The ends of curves corresponds to true vacuum.

Inflation of our Hubble volume has to happened somewhere near the edge of domain wall, to be consistent with BICEP2 observation, which requires $\upsilon > 50M_P$.....

# Observables

- Axion type of double well potential

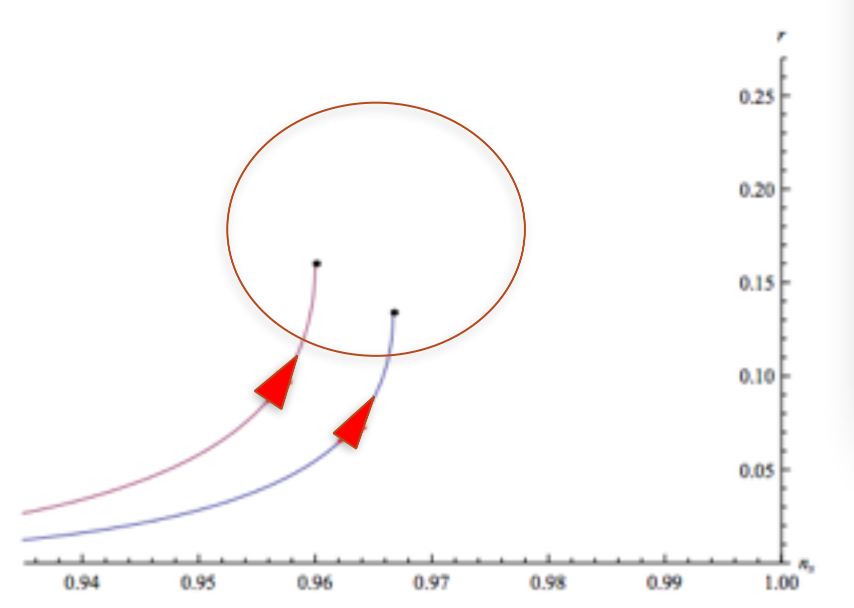
$$V(\phi) = \frac{m^2 v^2}{n^2}\left[1 - \cos(n\phi/v)\right] \ ,$$

Scalar tilt and tensor-to-scalar ratio read

$$\begin{aligned}
\epsilon &= \frac{n^2 M_p^2}{2v^2} \cdot \cot^2\left(\frac{n\phi}{2v}\right) \ , \\
\eta &= \frac{n^2 M_p^2}{2v^2} \cdot \cos\left(\frac{n\phi}{v}\right) \csc^2\left(\frac{n\phi}{2v}\right) , \\
n_s - 1 &= \frac{n^2 M_p^2}{v^2} \cdot \left[1 - 2\csc^2\left(\frac{n\phi}{2v}\right)\right] , \\
r &= 16\epsilon \ .
\end{aligned}$$

# Observables

- $n_s$ - r Plot



Horizontal axis: ns
Vertical axis: r
red curve N=50
Blue curve N=60
arrows denotes the direction of moving away from the center of domain wall.
The ends of curves corresponds to true vacuum.

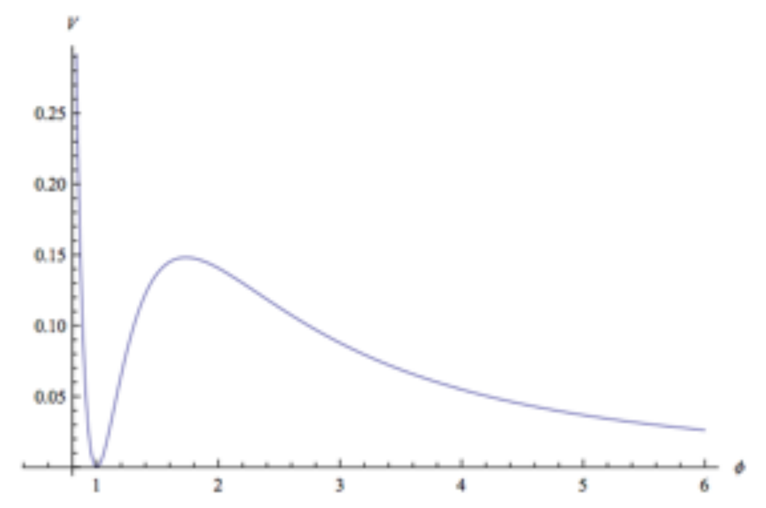
Again, inflation of our Hubble volume has to happened somewhere near the edge of domain wall, to be consistent with BICEP2 observation, which requires $\upsilon > 15M_P$.....

# Observables

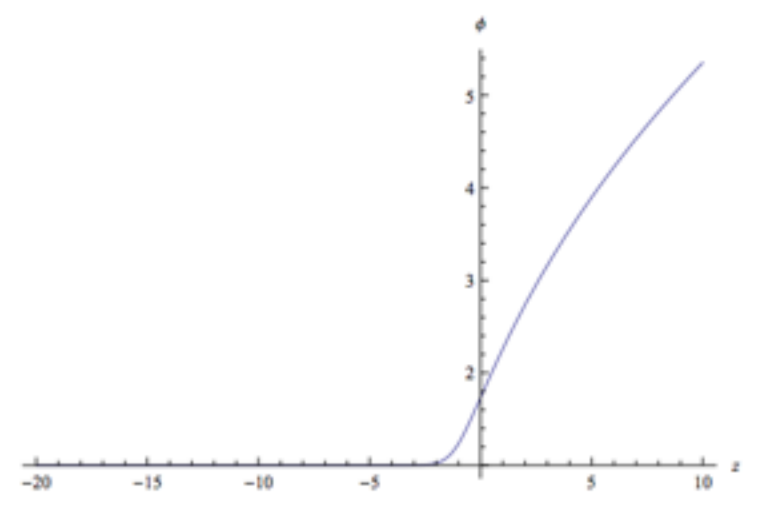
- Single-well potential

$$V(\phi) = \xi \frac{v^6}{\phi^6}\left(\phi^2 - v^2\right)^2 \ ,$$

$$\frac{\partial \phi}{\partial z} = \sqrt{2V} \ ,$$



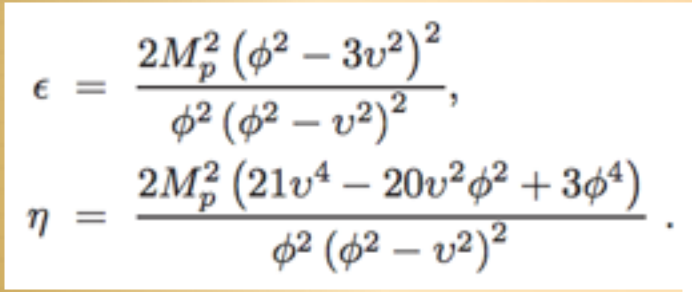
Scalar potential



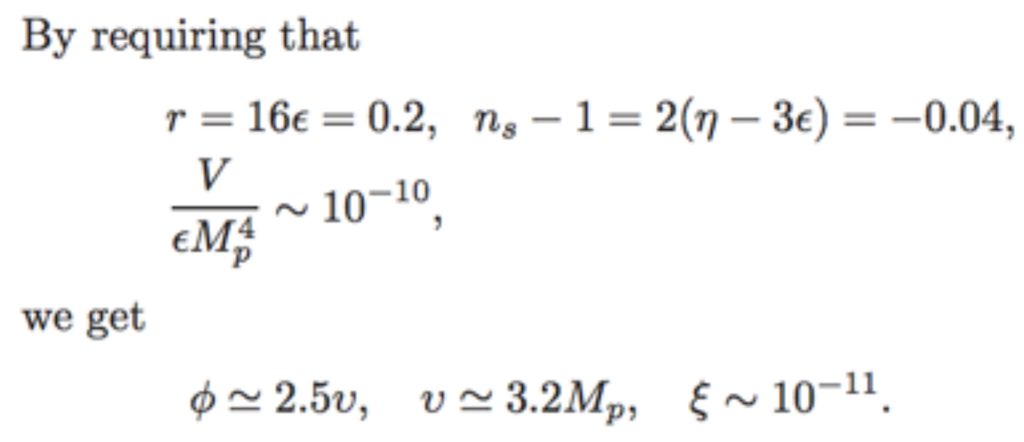
Scalar field configuration

# Observables

- The same as double well case, $\upsilon > M_p$ is required to inflate inside of topological defect;
- Slow roll parameters are calculated as



$$\begin{aligned}\epsilon &= \frac{2M_p^2\left(\phi^2-3\upsilon^2\right)^2}{\phi^2\left(\phi^2-\upsilon^2\right)^2},\\ \eta &= \frac{2M_p^2\left(21\upsilon^4-20\upsilon^2\phi^2+3\phi^4\right)}{\phi^2\left(\phi^2-\upsilon^2\right)^2}.\end{aligned}$$



By requiring that

$$r = 16\epsilon = 0.2,\quad n_s - 1 = 2(\eta - 3\epsilon) = -0.04,$$

$$\frac{V}{\epsilon M_p^4} \sim 10^{-10},$$

we get

$$\phi \simeq 2.5\upsilon,\quad \upsilon \simeq 3.2M_p,\quad \xi \sim 10^{-11}.$$

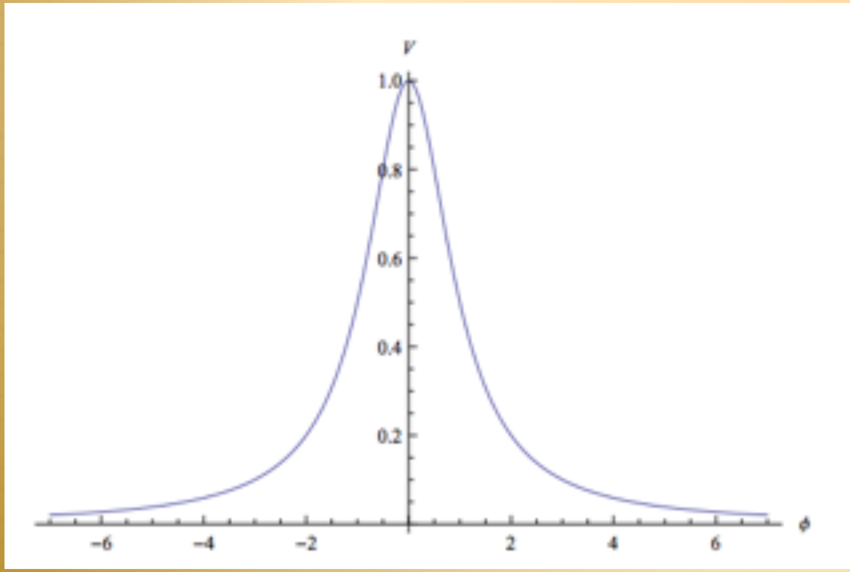
Reheating could be triggered by a waterfall scalar field, just like what people did in hybrid inflation.

# Observables

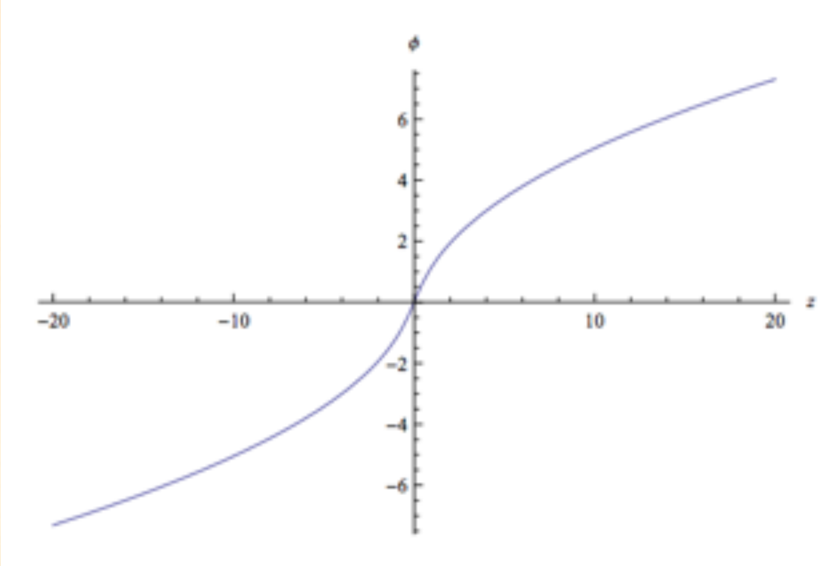
- No-well potential

$$V(\phi)=\frac{\xi\upsilon^4}{1+\phi^2\upsilon^{-2}}\ .$$

$$\frac{\partial\phi}{\partial z}=\sqrt{2V}\ ,$$



Scalar potential

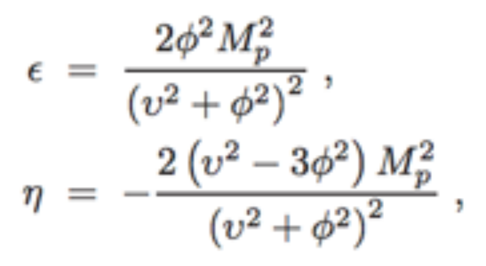


Scalar configuration

# Observables

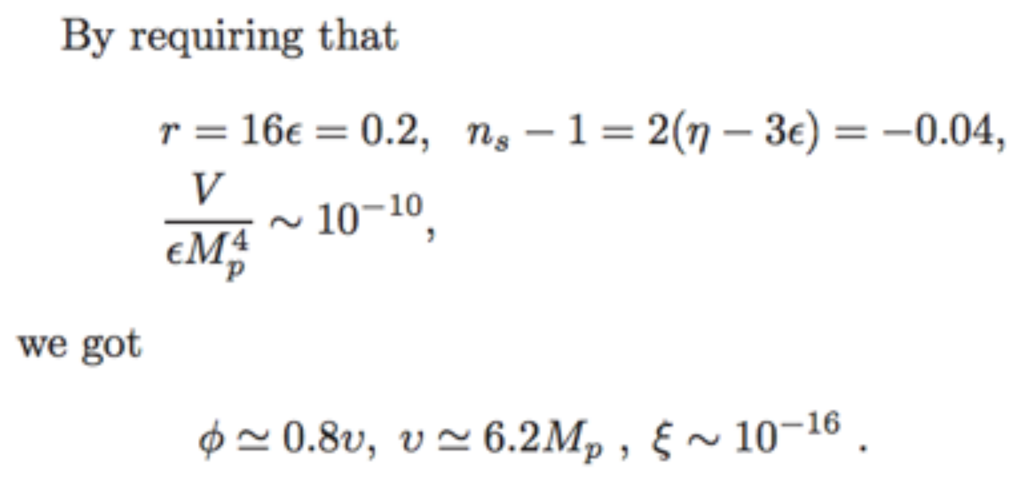
- The same, $v > M_p$ is required to inflate inside of topological defect;

- Slow roll parameters



$$\begin{aligned}\epsilon &= \frac{2\phi^2 M_p^2}{(v^2+\phi^2)^2}\,, \\ \eta &= -\frac{2\left(v^2-3\phi^2\right)M_p^2}{(v^2+\phi^2)^2}\,,\end{aligned}$$

Reheating could be triggered by a waterfall scalar field, just like what people did in hybrid inflation.



By requiring that

$$r = 16\epsilon = 0.2, \quad n_s - 1 = 2(\eta - 3\epsilon) = -0.04,$$
$$\frac{V}{\epsilon M_p^4} \sim 10^{-10},$$

we got

$$\phi \simeq 0.8v, \;\; v \simeq 6.2M_p\;, \;\; \xi \sim 10^{-16}\;.$$

# Conclusion

- Introduce what's topological inflation;
- Compare 3 types of topological inflation models with observation,
  - All this 3 type of models could be consistent with observations;
  - For Higgs and Axion type of potential, inflation of our Hubble volume has to happen near the edge of domain wall;