

Angular momentum – mass relation in dark matter haloes

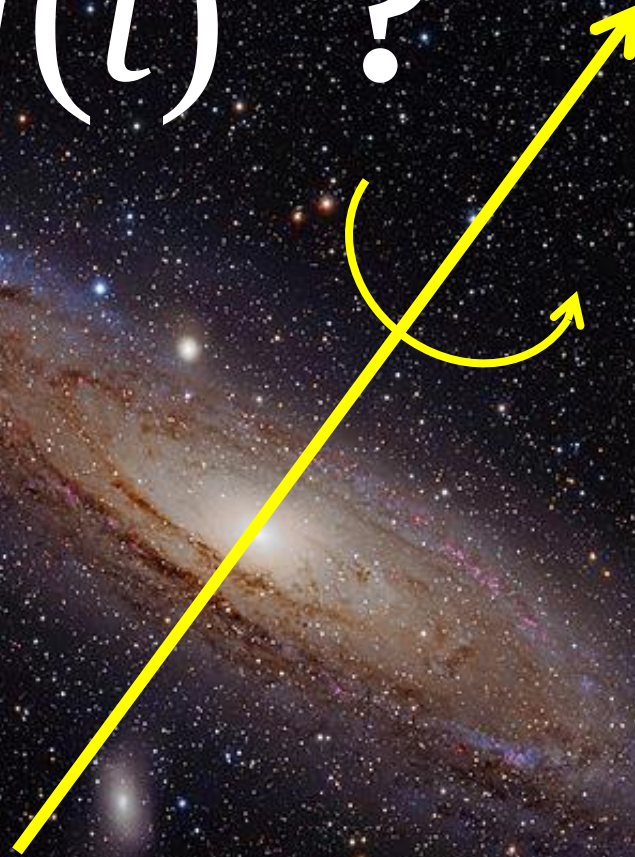
Shihong Liao



the Chinese University of Hong Kong

With Dalong Cheng, Jiayu Tang and Ming Chung Chu

$J(t)$?



M31 (by Adam Evans)

Contents

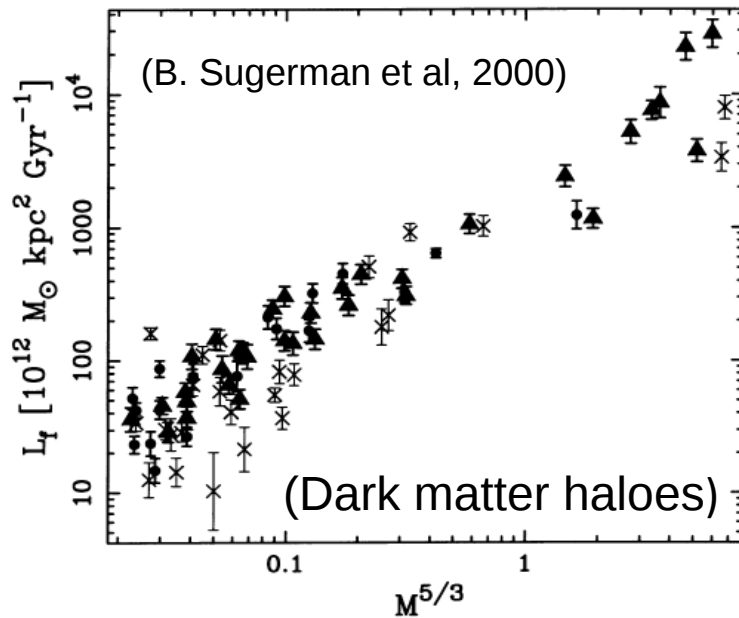
- Angular momentum – mass relation ($J - M$ relation)
- Time evolution of $J - M$ relation
- Summary

$J - M$ relation

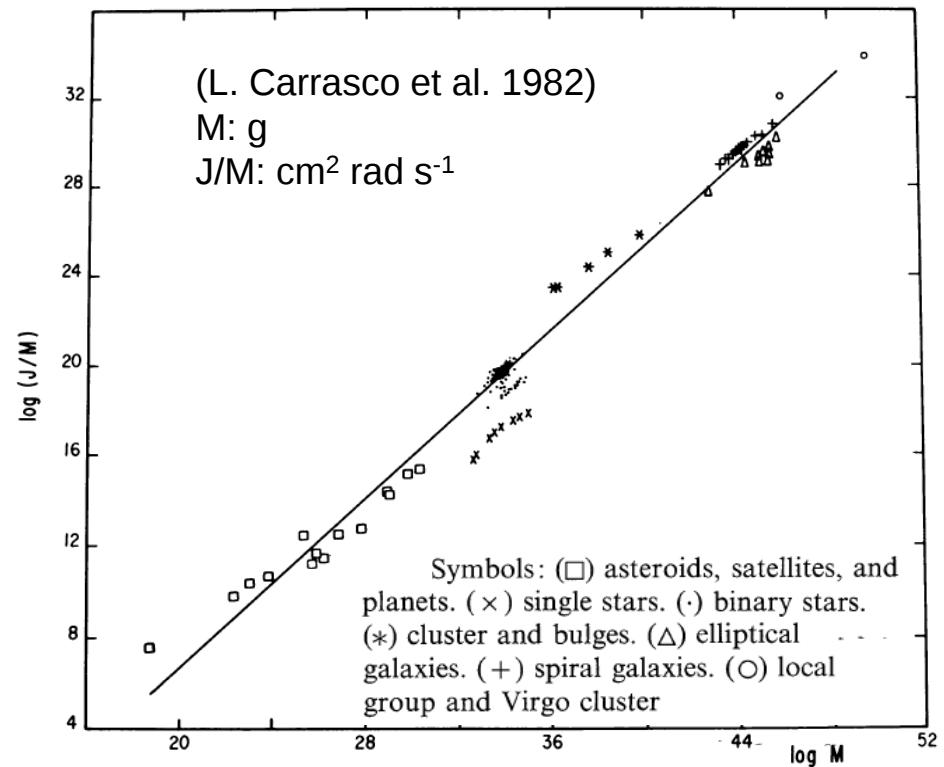
- An empirical and universal power law relation (Brosche, 1963)

$$J = \beta M^{\alpha \approx 5/3}$$

- Many orders of magnitude
- From asteroids to haloes



N-body simulation



Observation

Explanations

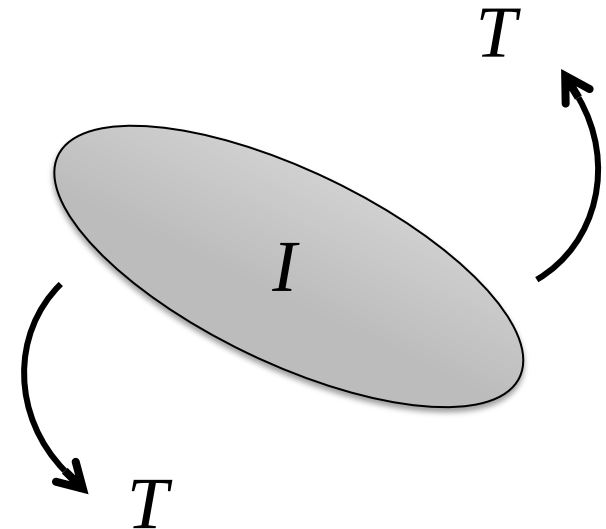
- Mechanical equilibrium (Ozernoy, 1967; Carrasco et al, 1982)

$$\text{Virial relation } 2K + U = 0 \Rightarrow J \propto MRV \propto M^{5/3}$$

- Simple argument from **tidal torque theory** (Peebles, 1969; White, 1994; Catelan & Theuns, 1996)

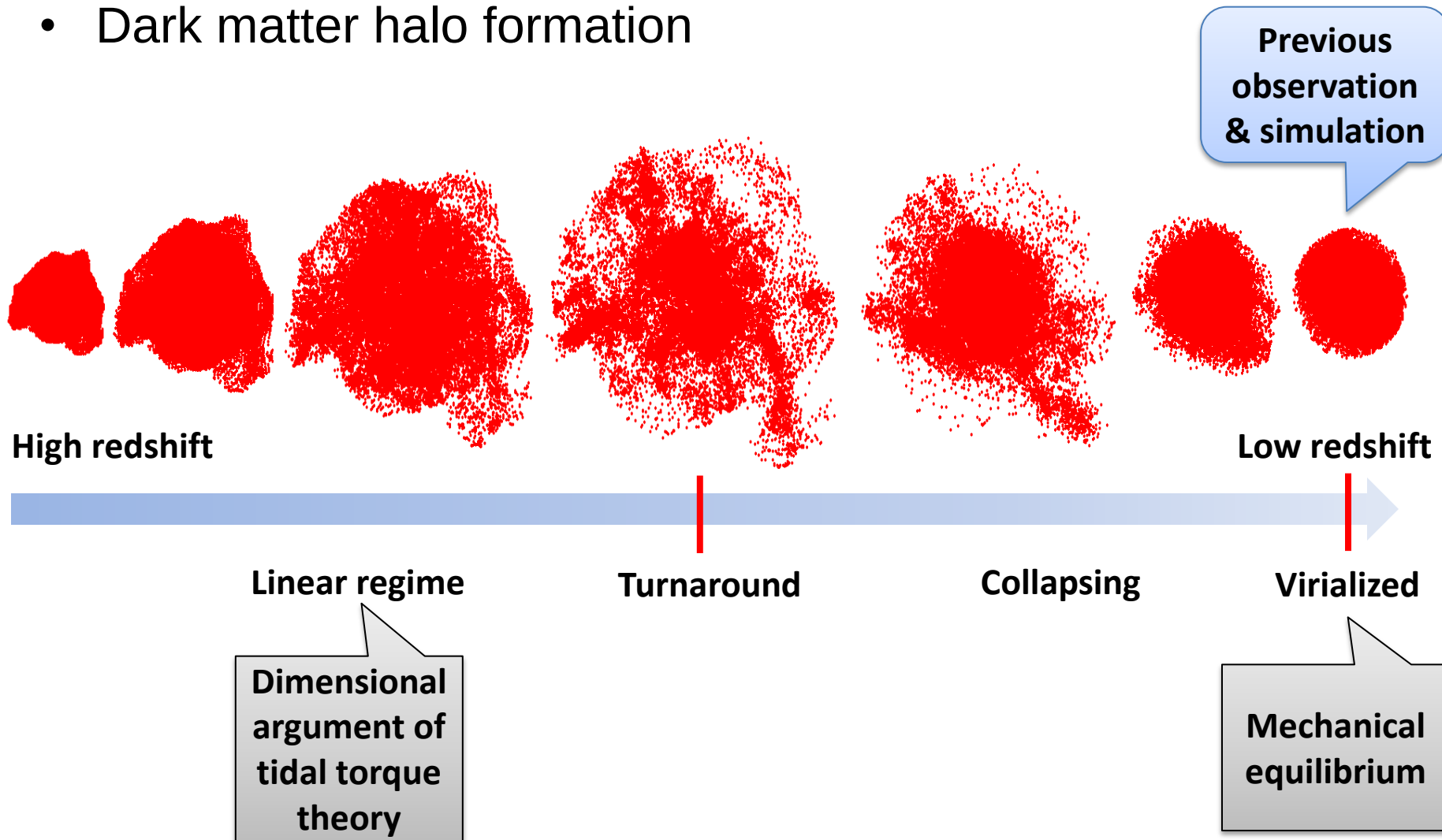
$$J_i = -a^2 \dot{D} \varepsilon_{ijk} T_{jl} I_{lk}$$

$$J \sim I \text{ and } T \Rightarrow J \propto I \propto MR^2 \propto M^{5/3} \\ (M \sim R^3)$$



Explanations

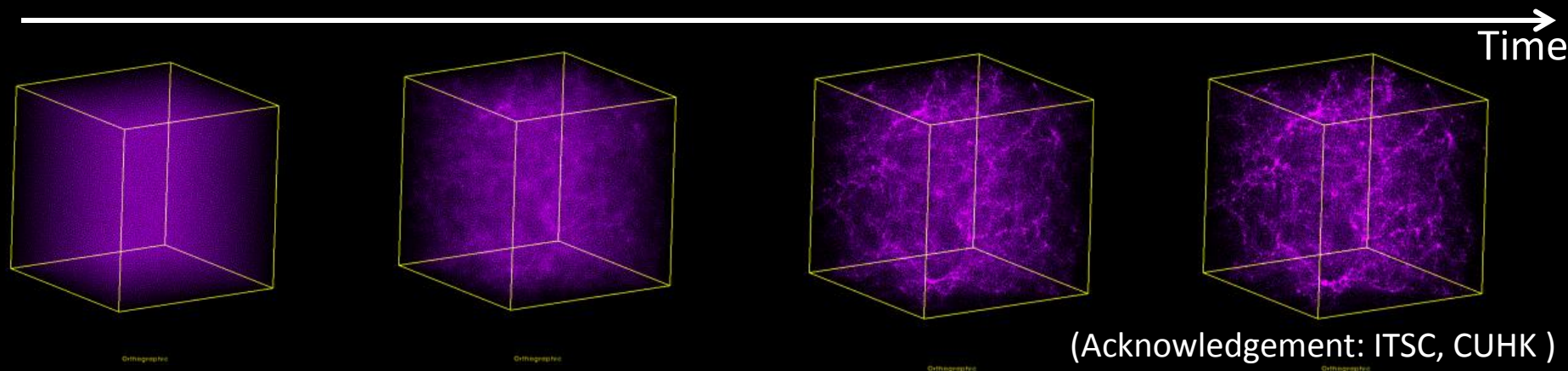
- Dark matter halo formation



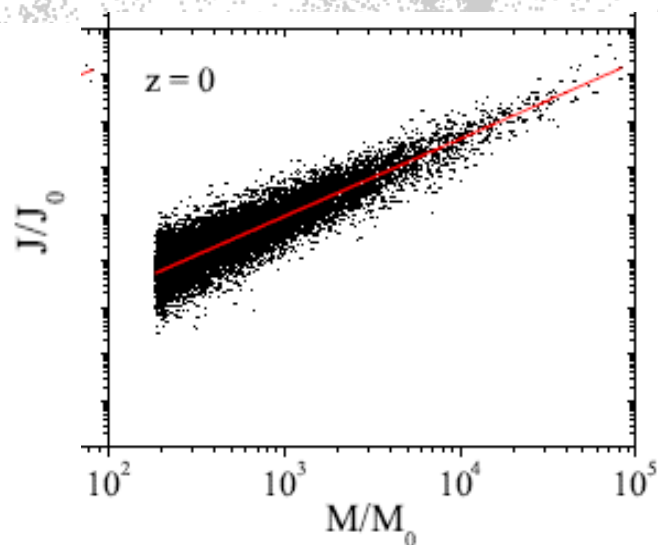
Question (motivation)

$$J(t) = \beta(t)M^{\alpha(t)}$$

- Time evolution of J - M relation?
- To get a more clear picture about the establishment of J - M relation
- To see the nonlinear effects in collapse stage on halos' spins



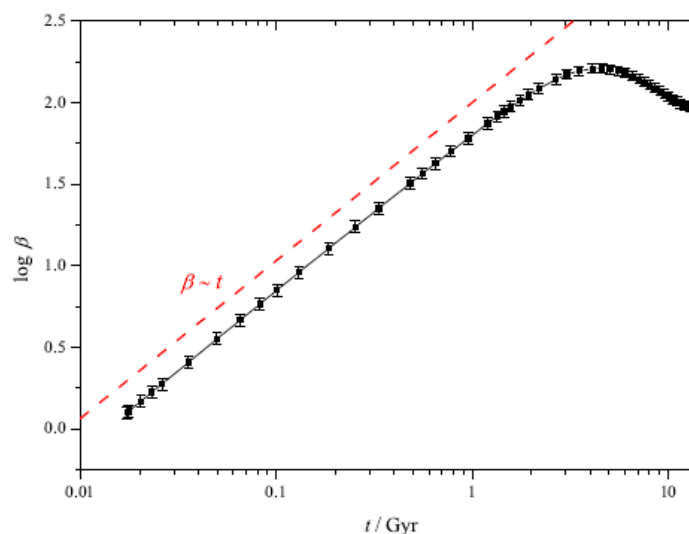
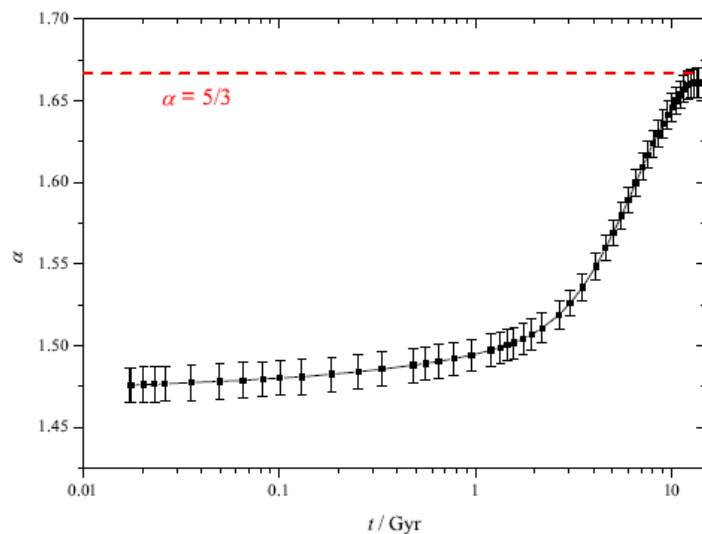
Results from N-body simulation



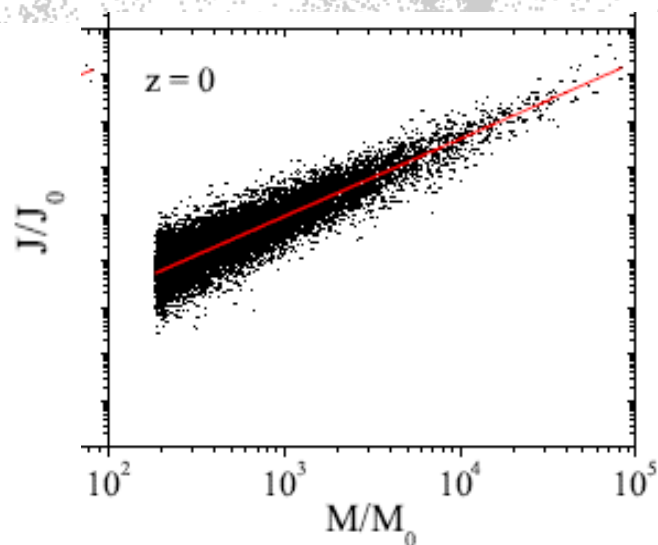
Λ CDM:

$$J(t) = \beta(t)M^{\alpha(t)}$$

- J - M relation holds from high redshift ($z=100$) to today
- α starts from ~ 1.5 and reaches $5/3$ recently
- β evolves linearly with time in the beginning



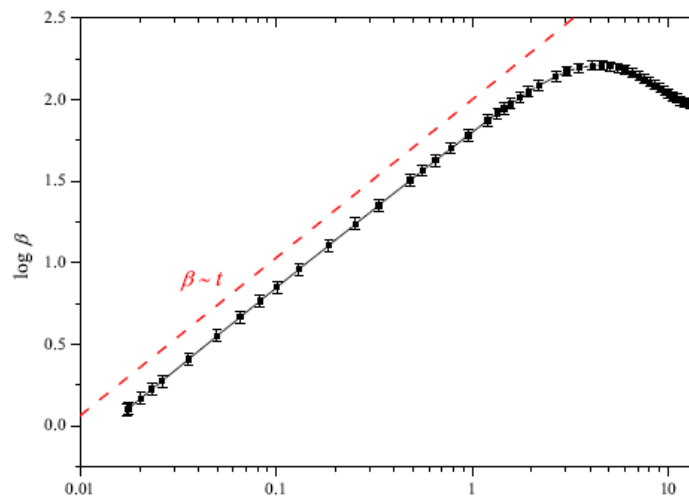
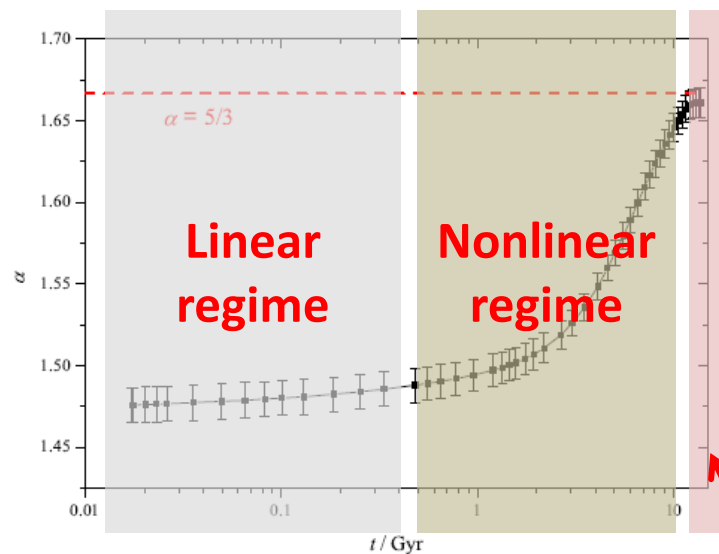
Results from N-body simulation



Λ CDM:

$$J(t) = \beta(t)M^{\alpha(t)}$$

- J - M relation holds from high redshift ($z=100$) to today
- α starts from ~ 1.5 and reaches $5/3$ recently
- β evolves linearly with time in the beginning



Equilibrium regime

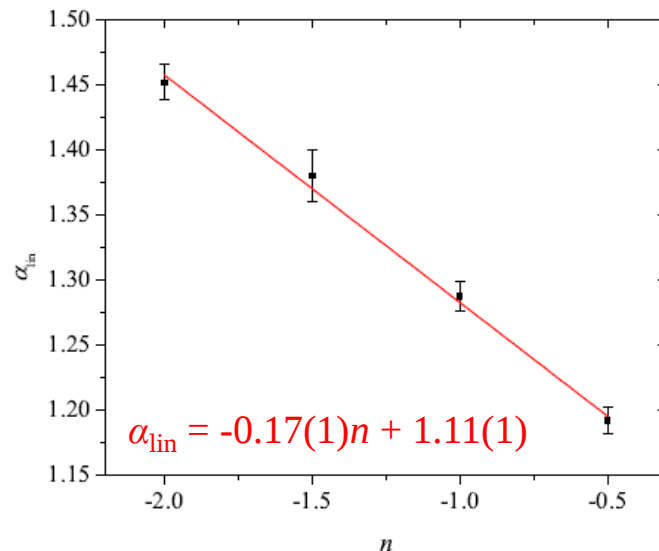
Results from N-body simulation

- Linear regime:

if $P(k) = Ak^n$,

$$\text{TTT} \rightarrow \alpha_{\text{lin}}(t) = -\frac{n}{6} + \frac{7}{6} \approx -0.167n + 1.167$$

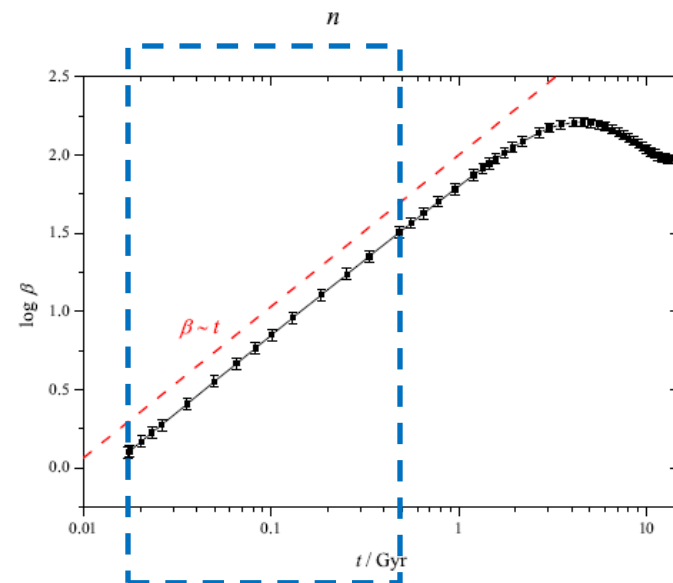
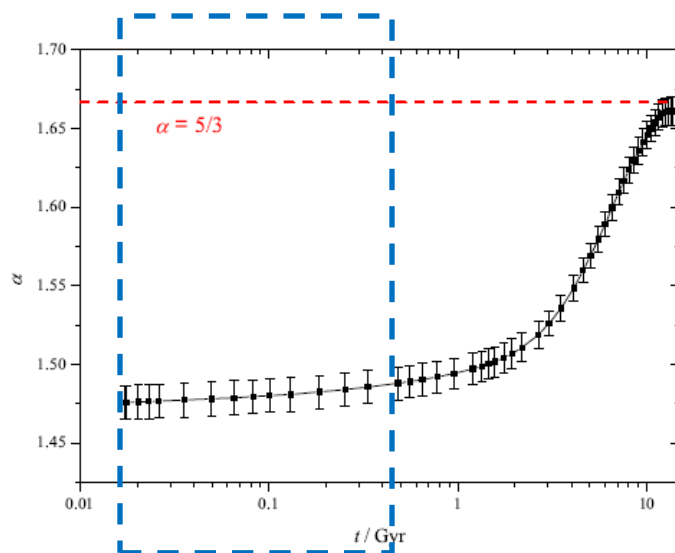
$$\beta(t) \propto t$$



$$n_{\text{eff}}(k) = \frac{d \ln P(k)}{d \ln k}$$

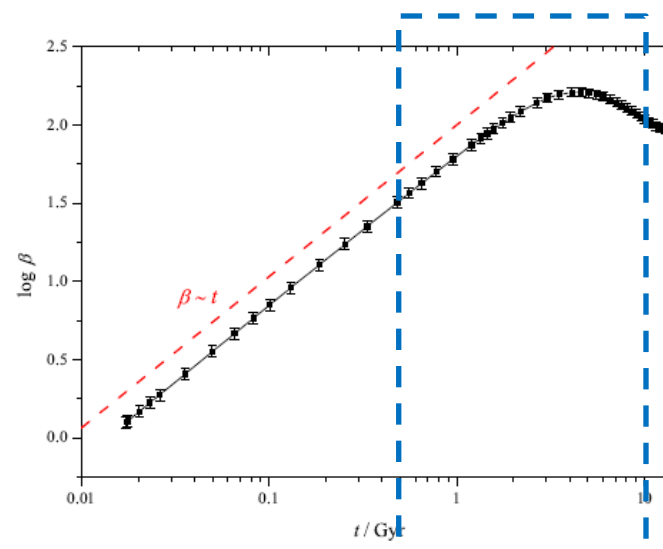
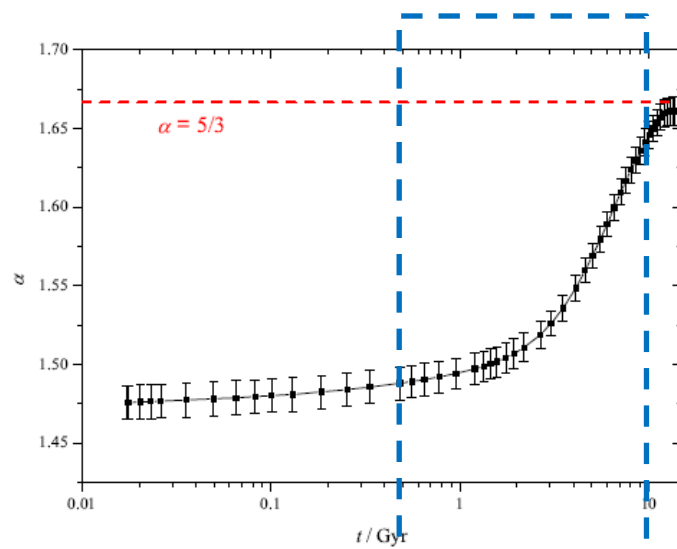
$$n_{\text{eff}} \approx -2$$

$$\alpha_{\text{lin}} \approx 1.5$$



Results from N-body simulation

- Nonlinear regime:
Nonlinear effects affect J - M relation significantly.
Drive α to $5/3$.



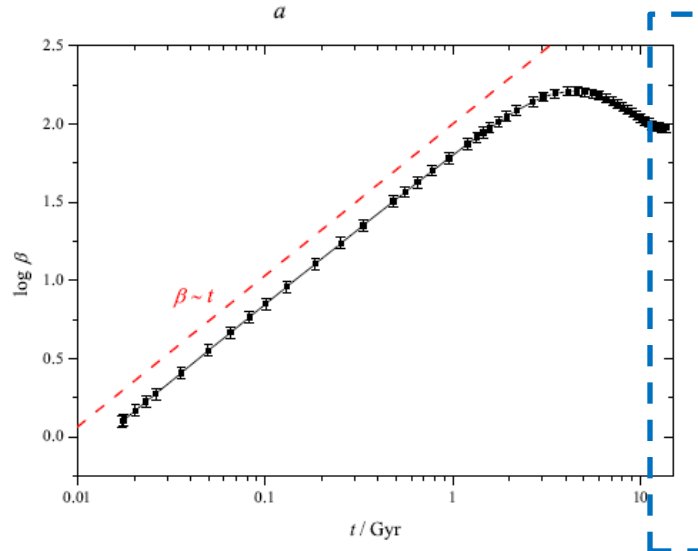
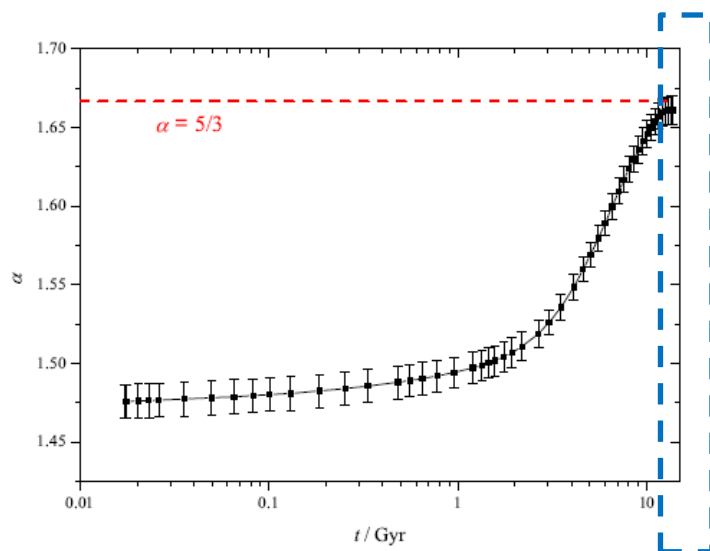
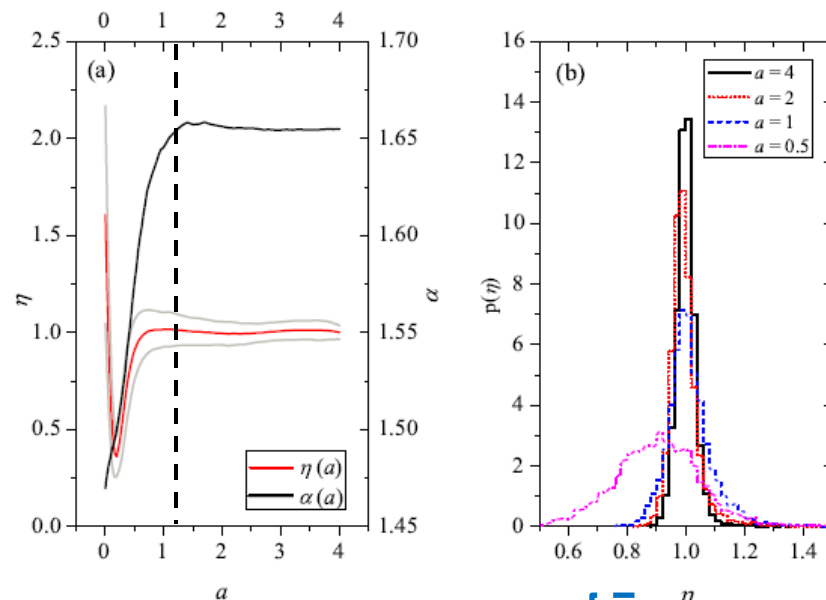
Results from N-body simulation

- Equilibrium regime:

virial equilibrium $2K + U = 0$

virial parameter $\eta = \frac{2K}{|U|}$

$$\eta = 1 \rightarrow \alpha = 5/3$$



Summary

$$J(t) = \beta(t)M^{\alpha(t)}$$

- A new, detailed picture of evolution and establishment for J - M relation is given, based on simulation and theoretical calculation.
- Three regimes' scheme: (1) the value of power index in linear regime depends on initial $P(k)$; $\beta(t)$ evolves linearly with time; (2) Nonlinear process has a significant effect on the evolution of J - M relation; (3) Virialization makes $\alpha = 5/3$ and leads to a “memory loss” of initial $P(k)$; β also becomes a stable value.
- Detailed calculation of tidal torque theory (TTT) can explain J - M relation in linear regime well. This supports the validity of TTT in linear regime.