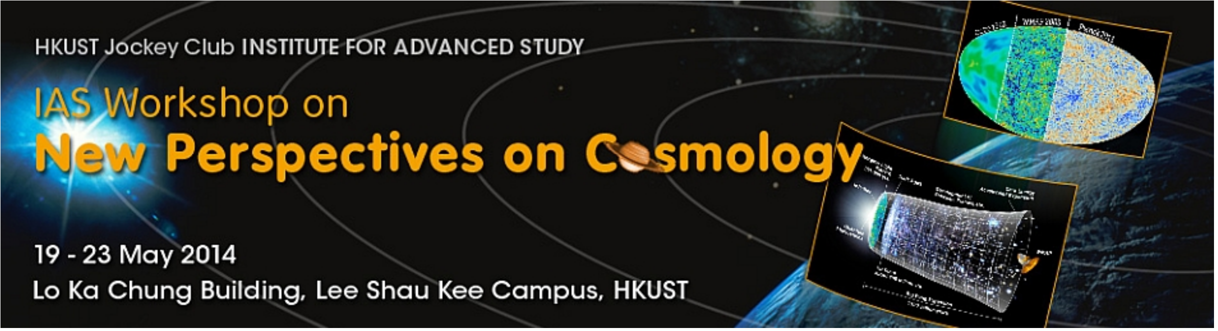

HKUST, Hongkong
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New Bubbles in Cosmology

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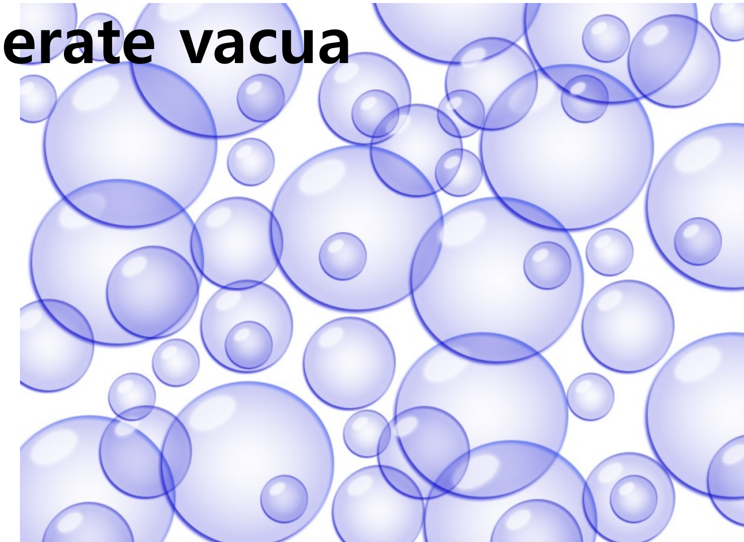
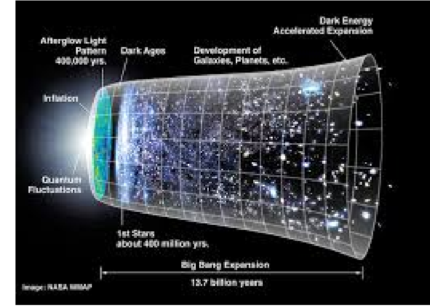
Contents

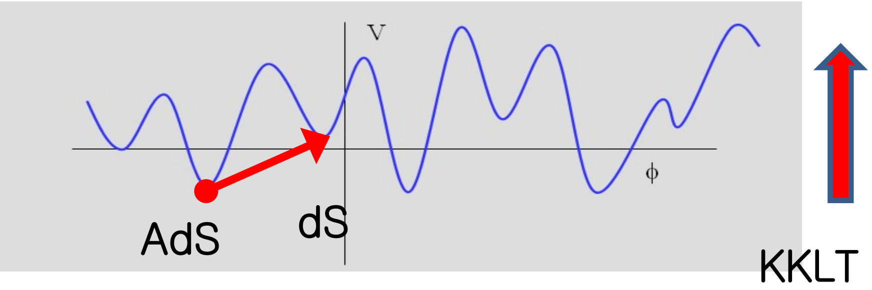


1. Motivation & Basics



- ◆ Observation : Universe is expanding with acceleration
  - needs positive cosmological constant

◆ The string theory landscape provides a vast number of metastable vacua.



◆ How can we be in the vacuum with positive cosmol const ?

- KKLT(Kachru,Kallosh,Linde,Trivedi, PRD 2003)) : “lifting” the potential
- an alternative way? : Revisit the gravity effect in cosmol. phase trans.

◆ Bubbles

(*) For Phase Tran. in the early universe with or w/o cosmol. constants,
Can we obtain the mechanism for the nucleation of a false vac bubble?
Can a false vacuum bubble expand within the true vacuum background?

(*) Other possible roles of the Euclidean Bubbles and Walls?
Ex) quantum cosmology? Domain Wall Universe?

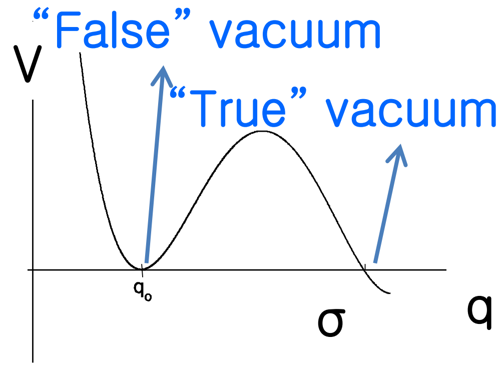
We will revisit and analyse vacuum bubbles in various setup.

Basics : Bubble formation

– Nonperturbative Quantum Tunneling

Vacuum-to-vacuum phase transition rate

$$\Gamma / V = A e^{-B/\hbar}[1 + O(\hbar)]$$



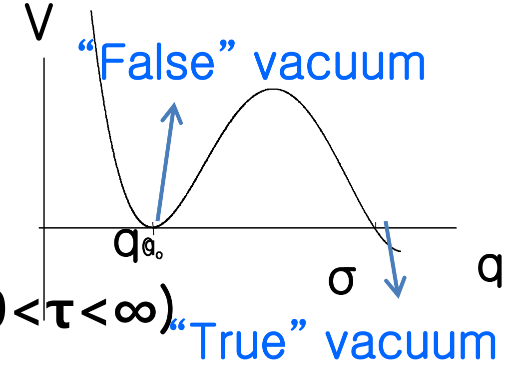
B : Euclidean Action (semiclassical approx.)
S. Coleman, PRD 15, 2929 (1977)
S. Coleman and F. De Luccia, PRD21, 3305 (1980)
S. Parke, PLB121, 313 (1983)

A : determinant factor from the quantum correction
C. G. Callan and S. Coleman, PRD 16, 1762 (1977)

(1) Tunneling in Quantum Mechanics

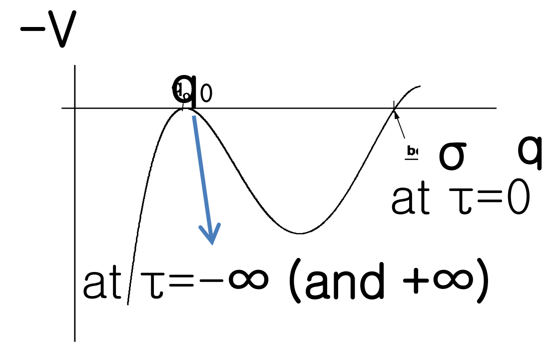
- particle in one dim. with unit mass -

Lagrangian $$L = \frac{1}{2}\left(\frac{dq}{dt}\right)^2 - V(q)$$



- Quantum Tunneling:(Euclidean time $-\infty < \tau < 0$ or $0 < \tau < \infty$)

The particle (at "false vacuum" q_0) penetrates the potential barrier and materializes at the escape point, σ , with zero kinetic energy,



Tunneling probability

$$\Gamma / V = A e^{-B/\hbar}[1 + O(\hbar)]$$

where $$\bar{S} = B = \int_{-\infty}^{\infty} d\tau L_E(q(\tau)) = \int d\tau \left[\frac{1}{2}\left(\frac{dq}{d\tau}\right)^2 + V(q)\right]$$ = Classical Euclidean action (difference)

Eq. of motion : $\tau = it$

$$\frac{d^2 q}{d\tau^2} + \left(-\frac{dV}{dq}\right) = 0$$

boundary conditions

$$x_i = x_f = 0 \qquad \lim_{\tau \to \mp\infty} q(\tau) = q_o, \; \frac{dq}{d\eta}\Big|_{\tau=0(\sigma)} = 0$$

→ The bounce solution is a particle moving in the potential –V in time τ is unstable (exists a mode w/ negative eigenvalue)

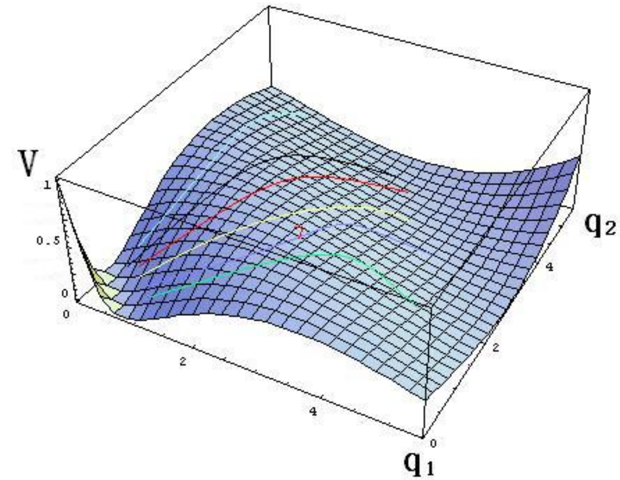
- Time evolution after tunneling : (back to Minkowski time, t > 0)

Classical Propagation after tunneling (at τ = 0=t)

(2) Tunneling in multidimension

Lagrangian

$$L=\frac{1}{2}\sum_{ij}\dot{q}_i\,\dot{q}_j-V(q) \qquad V(q_{oi})=V(\sigma_i)=0$$



The leading approx. to the tunneling rate is obtained from the path and endpoints σ_i that minimize the tunneling exponent B.

$$B=2\int_{S1}^{S2}ds(2V)^{1/2}=\int_{-\infty}^{\infty}d\tau L_E=S_E$$

$$\delta\int d\tau L_E^b=0 \qquad L_E=\frac{1}{2}\frac{d\vec{q}}{d\tau}\bullet\frac{d\vec{q}}{d\tau}+V \quad \Longrightarrow \quad \frac{d^2\vec{q}}{d\tau^2}=\frac{\partial V}{\partial\vec{q}}$$

Boundary conditions for the bounce

$$\lim_{\tau\to-\infty}q_i(\eta)=q_{oi},\ \frac{dq_i}{d\tau}|_{\tau=0(\sigma_i)}=0$$

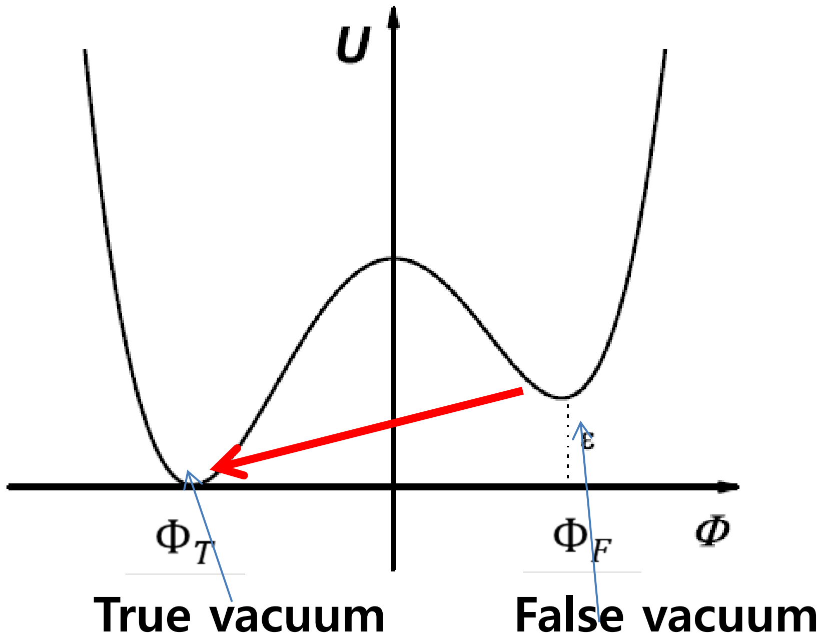
Time evolution after the tunneling is classical with the ordinary Minkowski time.

(3) Tunneling in field theory (in flat spacetime – no gravity)

Theory with single scalar field

$$S=\int\sqrt{-\eta}\,d^4x\left[-\frac{1}{2}\nabla^\alpha\Phi\nabla_\alpha\Phi-U(\Phi)\right]$$

where $\eta=\det\eta_{\mu\nu},\ (-,+,+,+)$

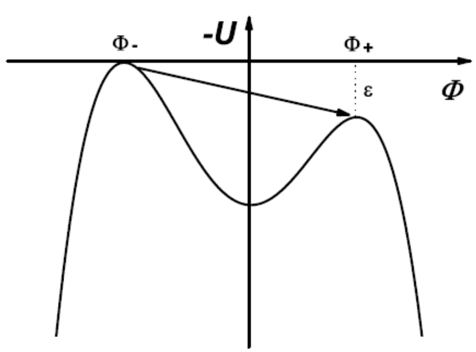


Equation for the bounce $\delta\int d\tau L_E^b=0$

$$\left(\frac{\partial^2}{\partial\tau^2}+\nabla^2\right)\Phi=U'(\Phi)$$

with boundary conditions (Finite size bubble)

$$\lim_{\tau\to\pm\infty}\Phi(\tau,\vec{x})=\Phi_F \qquad \frac{\partial\Phi}{\partial\tau}(0,\vec{x})=0 \qquad \lim_{|\vec{x}|\to\infty}\Phi(\tau,\vec{x})=\Phi$$



Tunneling rate :

$$\Gamma/V=Ae^{-B/\hbar}[1+O(\hbar)] \qquad B=S_E=\int d\tau d^3x\left[\frac{1}{2}\left(\frac{\partial\Phi}{\partial\tau}\right)^2+\underbrace{\frac{1}{2}(\vec{\nabla}\Phi)^2+U(\Phi)}_{V}\right]$$

O(4)-symmetry : Rotationally invariant Euclidean metric

$$ds^2 = d\eta^2 + \eta^2[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)] \quad (\eta^2 = \tau^2 + r^2)$$

Tunneling probability factor

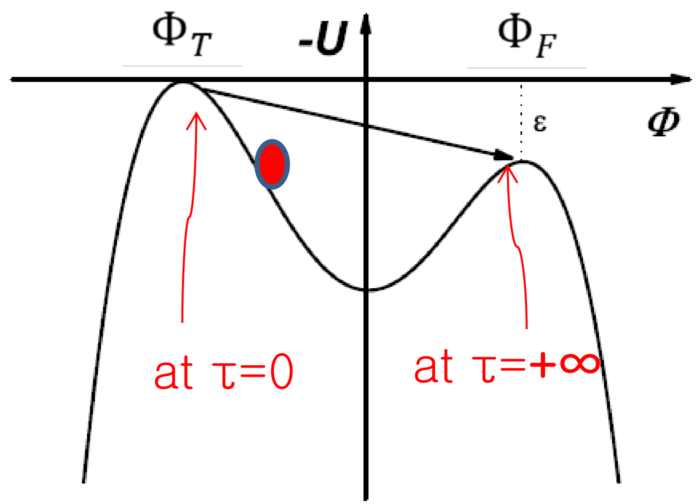
$$B = S_E = 2\pi^2 \int_0^\infty \eta^3 d\eta \left[\frac{1}{2}\left(\frac{\partial\Phi}{\partial\eta}\right)^2 + U(\Phi) \right]$$

The Euclidean field equations & boundary conditions $\left(\frac{\partial^2}{\partial\tau^2} + \nabla^2 = \frac{d^2}{d\eta^2} + \frac{3}{\eta}\frac{d}{d\eta} \right)$

$$\Phi'' + \frac{3}{\eta}\Phi' = \frac{dU}{d\Phi} \qquad \lim_{\eta\to\infty}\Phi(\eta) = \Phi_F, \frac{d\Phi}{d\eta}|_{\eta=0} = 0$$

"Particle" Analogy : (at position Φ at time η)

The motion of a particle
in the potential −U
with the damping force
proportional to 1/ η.

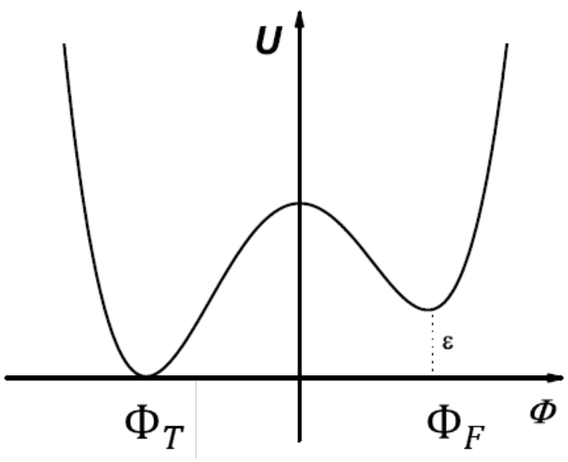


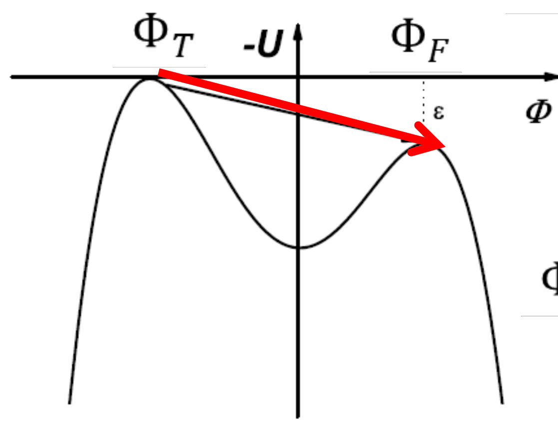
At time 0,
the particle is released at rest
(The initial position should be chosen such that)
at time infinity,
the particle will come to rest at Φ_F .

Thin-wall approximation

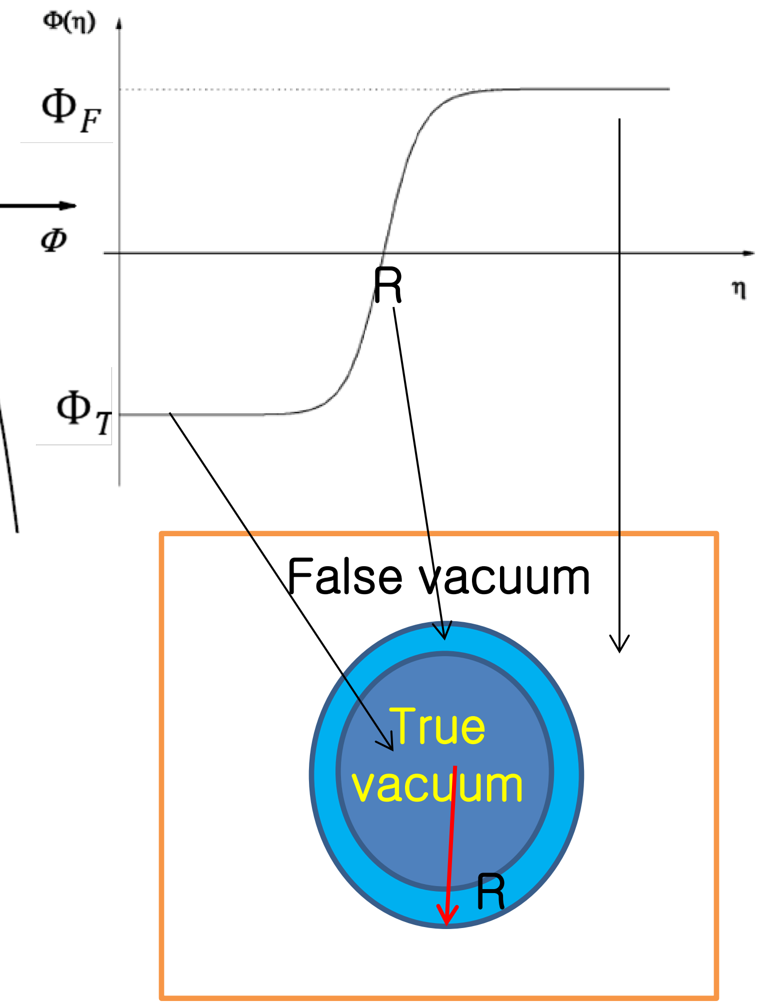
$$U(\Phi) = \frac{\lambda}{8}(\Phi^2 - b^2)^2 - \frac{\varepsilon}{2b}(\Phi - b)$$

(ε: small parameter)





Bubble solution



Large 4dim. spherical bubble with radius R and thin wall

B is the difference

$$B = S_E^b - S_E^F$$

In this approximation

$$B = B_{in} + B_{wall} + B_{out}$$

Inside the wall

$$B_{in} = -\frac{1}{2}\pi^2\eta^4\varepsilon$$

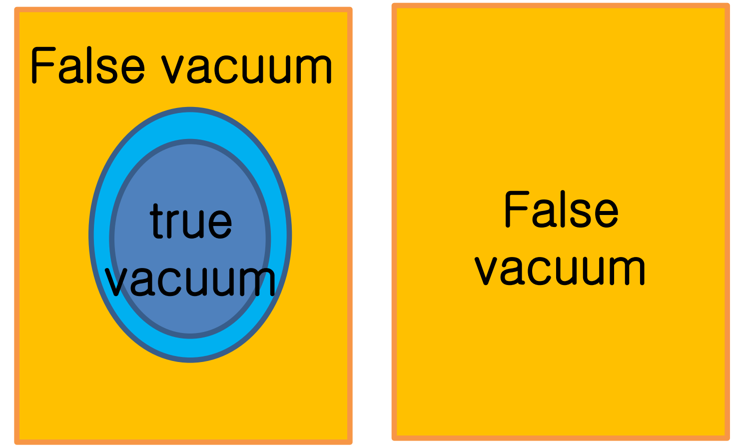
on the wall

$$B_{wall} = 2\pi^2\eta^3 S_o,$$

where $S_o = \int_{\Phi_T}^{\Phi_F} \sqrt{2[U(\Phi) - U(\Phi_T)]}\,d\Phi$

Outside the wall

$$B_{out} = S_E(\Phi_F) - S_E(\Phi_F) = 0$$



$$B = S_E^b - S_E^F$$

$$B = B_{in} + B_{wall} + B_{out}$$

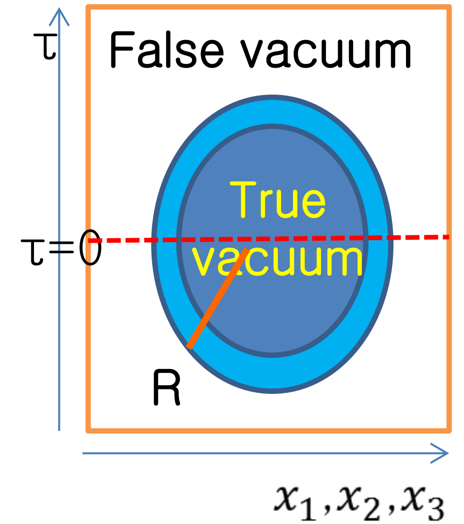
the radius of a true vacuum bubble

$$\bar{\eta} = 3S_o/\varepsilon \equiv R$$

the nucleation rate of a true vacuum bubble

$$B_o = S_E = \frac{27\pi^2 S_o^4}{2\varepsilon^3}$$

Evolution of the bubble



The false vacuum makes a quantum tunneling into a true vacuum bubble at time τ=t=0, such that

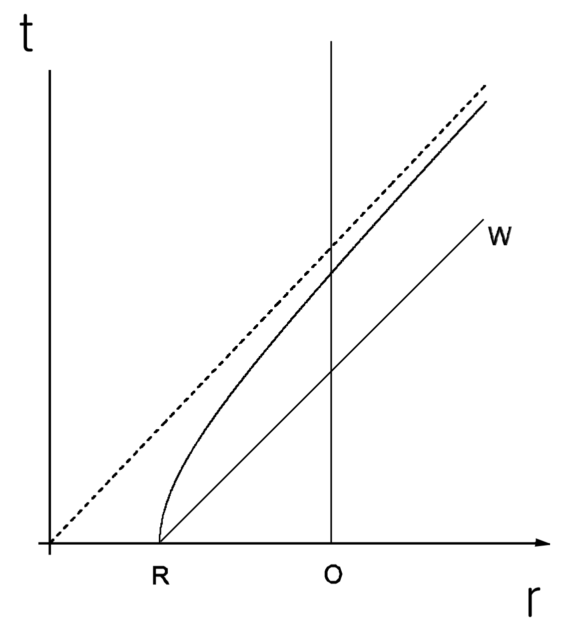
$$\Phi(t=0,\vec{x}) = \Phi(\tau=0,\vec{x})$$

$$\frac{\partial}{\partial t}\Phi(t=0,\vec{x}) = 0.$$

That is, the initial condition is given by the τ=t=0 slice at rest.

Afterwards, it evolves according to the classical equation of motion in Lorentzian spacetime.

$$-\frac{\partial^2 \Phi}{\partial^2 t} + \nabla^2 \Phi = U'(\Phi)$$



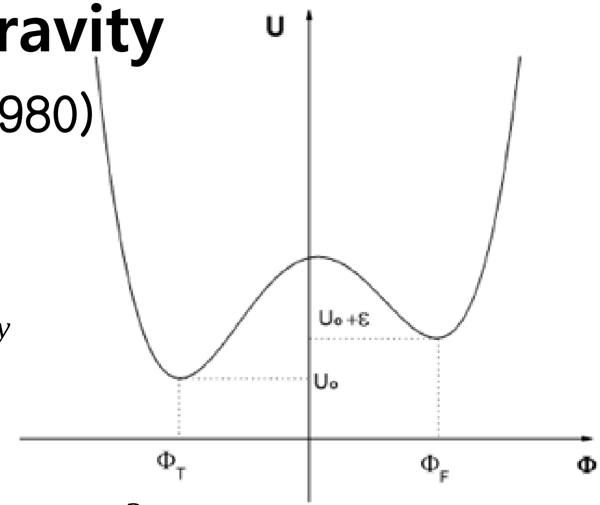
The solution (by analytic continuation)

$$\Phi(t,\vec{x}) = \Phi(\eta = (|\vec{x}|^2 - t^2)^{1/2})$$

(Note: Φ is a function of η) $(\eta^2 = \tau^2 + r^2)$

2. Bubble nucleation in the Einstein gravity

S. Coleman and F. De Luccia, PRD21, 3305 (1980)



Action

$$S = \int \sqrt{g} d^4 x \left[\frac{R}{2\kappa} - \frac{1}{2} \nabla^\alpha \Phi \nabla_\alpha \Phi - U(\Phi) \right] + S_{boundary}$$

O(4)-symmetric Euclidean metric Ansatz

$$ds^2 = d\eta^2 + \rho^2(\eta)[d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2)]$$

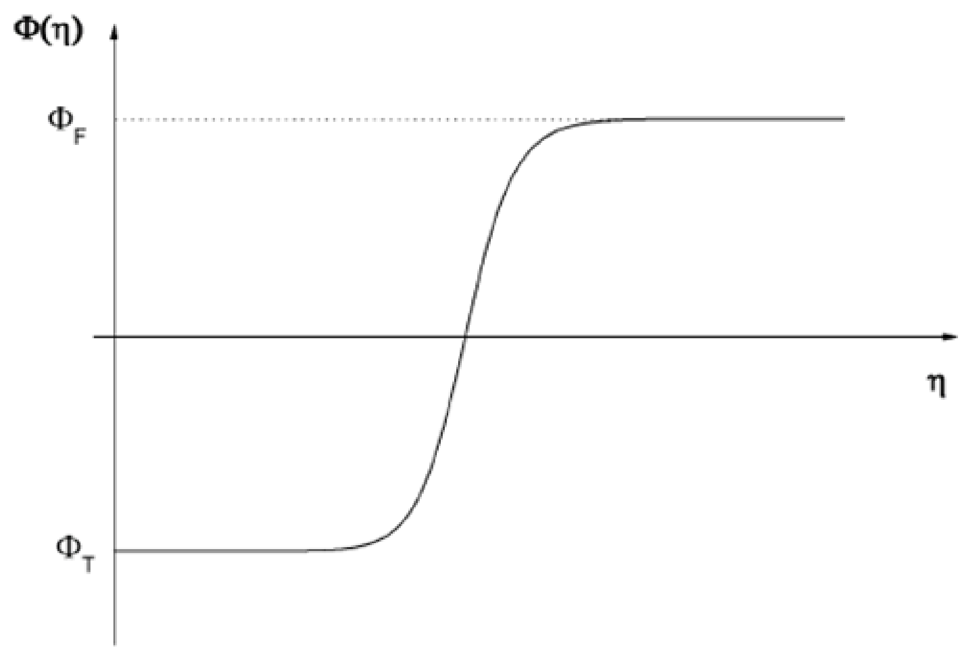
The Euclidean field equations (scalar eq. & Einstein eq.)

$$\Phi'' + \frac{3\rho'}{\rho} \Phi' = \frac{dU}{d\Phi}$$

$$\rho'^2 = 1 + \frac{\kappa \rho^2}{3} \left(\frac{1}{2} \Phi'^2 - U \right)$$

boundary conditions for bubbles

$$\lim_{\eta \to \eta(\max)} \Phi(\eta) = \Phi_F, \frac{d\Phi}{d\eta}\Big|_{\eta=0} = 0$$

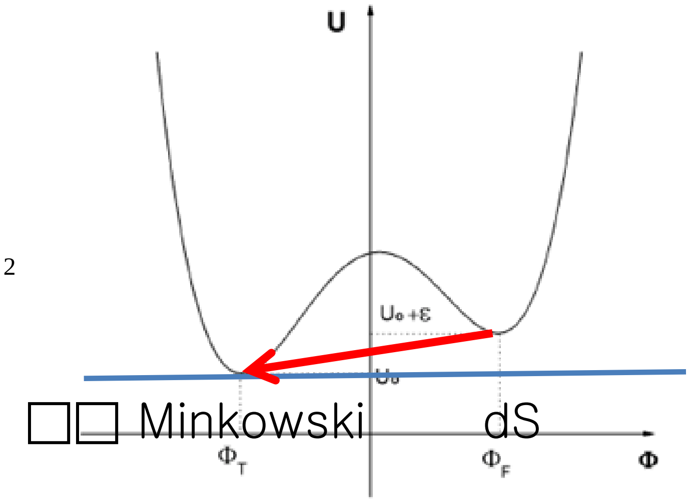


(i) From de Sitter to flat spacetime

the radius of the bubble $$\bar{\rho} = \frac{\bar{\eta}}{1+(\bar{\eta}/2\Lambda)^2}$$

where $\Lambda = (\kappa\varepsilon/3)^{-1/2}$

the nucleation rate $$B = \frac{B_o}{[1+(\bar{\eta}/2\Lambda)^2]^2}$$

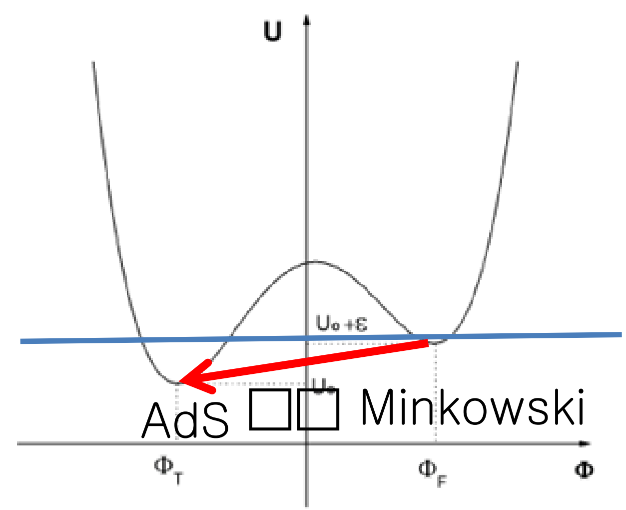


Note : 1) $\bar{\rho} < \bar{\eta}$ gravity makes the bubble smaller in dS
2) $B < B_0$ Transition probability increases

(ii) From flat to Anti-de Sitter spacetime

the radius of the bubble $$\bar{\rho} = \frac{\bar{\eta}}{1-(\bar{\eta}/2\Lambda)^2}$$

the nucleation rate $$B = \frac{B_o}{[1-(\bar{\eta}/2\Lambda)^2]^2}$$



Note : $\bar{\rho} > \bar{\eta}$ the bubble becomes larger
$B > B_0$ Transition probability decreases.
For small enough ϵ, false vacuum can be stable

(iii) the case of arbitrary vacuum energy

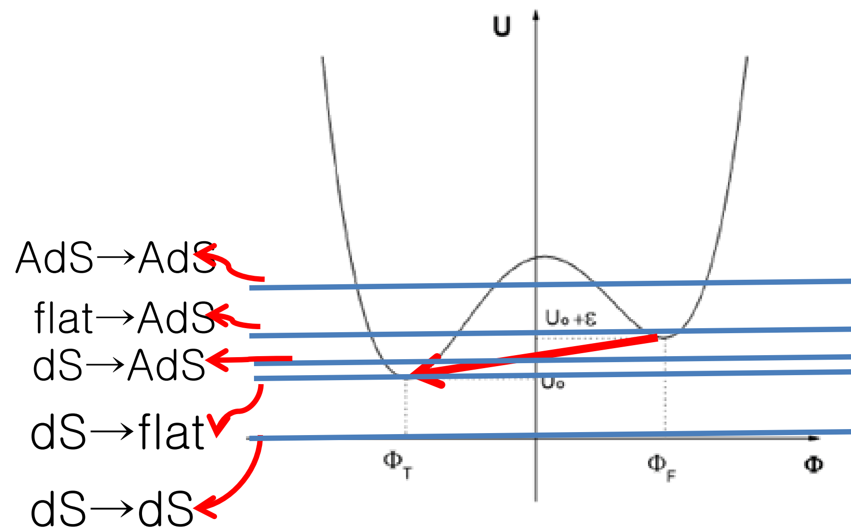
S. Parke, PLB121, 313 (1983)

the radius

$$\rho^2{}_p = \frac{\bar{\eta}^2}{\left[1 + 2\left(\frac{\bar{\eta}}{2\lambda_1}\right)^2 + \left(\frac{\bar{\eta}}{2\lambda_2}\right)^4\right]} \qquad \lambda_1^2 = [3/\kappa(U_F - U_T)] \quad \lambda_2^2 = [3/\kappa(U_F + U_T)]$$

the nucleation rate

$$B_p = \frac{2B_o\left[\left\{1 + \left(\frac{\bar{\eta}}{2\lambda_1}\right)^2\right\} - \left\{1 + 2\left(\frac{\bar{\eta}}{2\lambda_1}\right)^2 + \left(\frac{\bar{\eta}}{2\lambda_2}\right)^4\right\}^{1/2}\right]}{\left[\left(\frac{\bar{\eta}}{2\lambda_2}\right)^4\left\{\left(\frac{\lambda_2}{\lambda_1}\right)^2 - 1\right\}\left\{1 + 2\left(\frac{\bar{\eta}}{2\lambda_1}\right)^2 + \left(\frac{\bar{\eta}}{2\lambda_2}\right)^4\right\}^{1/2}\right]}$$



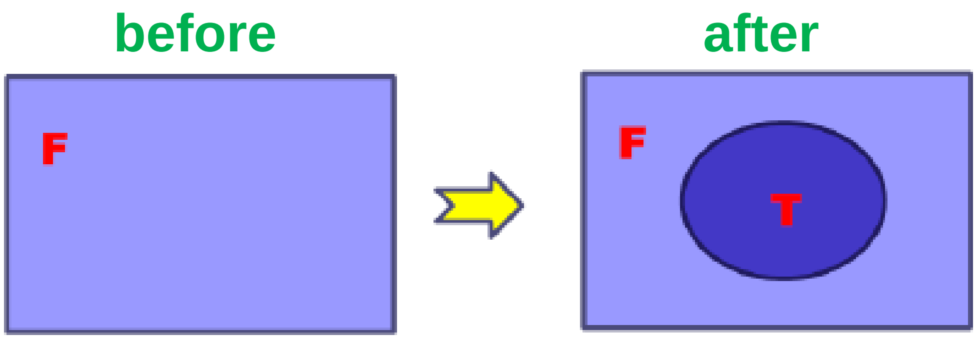
- Evolution of the bubble
  → via analytic continuation back to Lorentzian time

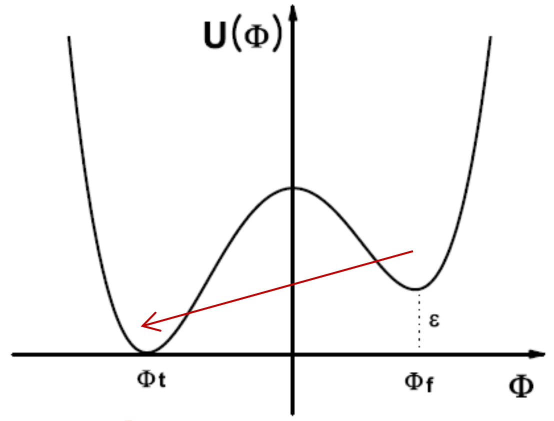
Note : Analytic continuation in the presence of gravity is nontrivial.

Ex) de Sitter → de Sitter : A. Brown & E. Weinberg, PRD 2007

Homogeneous tunneling

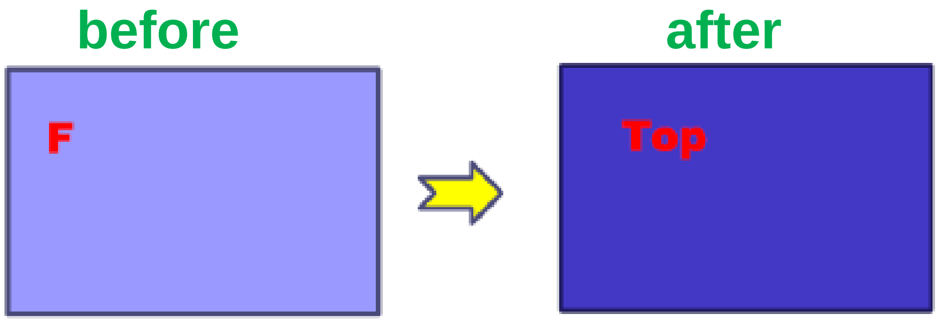
A channel of the inhomogeneous tunneling

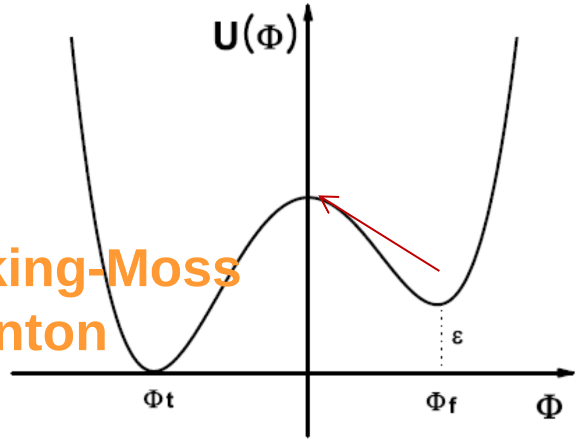




Ordinary bounce
(vacuum bubble) solutions

A channel of the homogeneous tunneling





Hawking-Moss
instanton

3. More Bubbles and Tunneling

3.1 False vacuum bubble nucleation

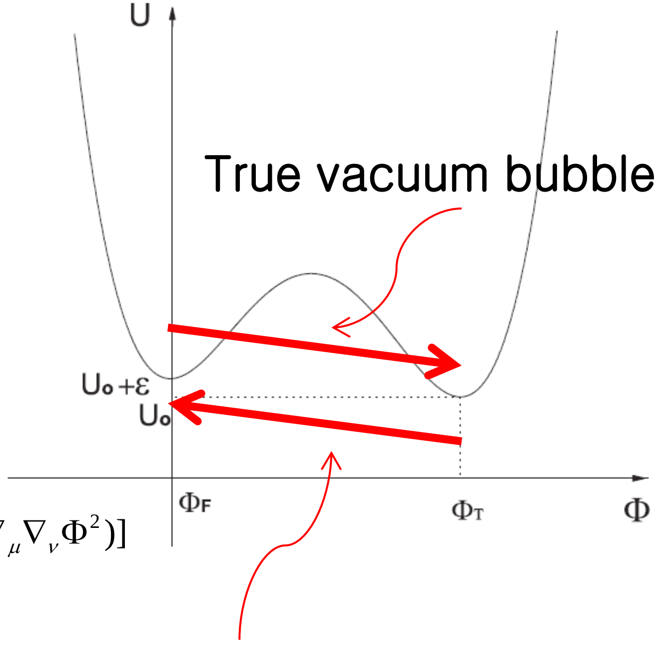
– The Einstein theory of gravity with a nonminimally coupled scalar field

Action

$$S = \int \sqrt{g}\, d^4x \left[\frac{R}{2\kappa} - \frac{1}{2}\nabla^\alpha \Phi \nabla_\alpha \Phi - \frac{1}{2}\xi R \Phi^2 - U(\Phi)\right] + S_{boundary}$$

Potential

$$U(\Phi) = \frac{\lambda}{8}\Phi^2(\Phi - 2b)^2 - \frac{\varepsilon}{2b}(\Phi - 2b) + U_o$$



Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$$

$$T_{\mu\nu} = \frac{1}{1-\xi\Phi^2\kappa}\left[\nabla_\mu \Phi \nabla_\nu \Phi - g_{\mu\nu}\left(\frac{1}{2}\nabla^\alpha \Phi \nabla_\alpha \Phi + U\right) + \xi\left(g_{\mu\nu}\nabla^\alpha \nabla_\alpha \Phi^2 - \nabla_\mu \nabla_\nu \Phi^2\right)\right]$$

curvature scalar

$$R = \frac{\kappa[4U(\Phi) + \nabla^\mu \Phi \nabla_\mu \Phi - 3\xi \nabla^\mu \nabla_\mu \Phi^2]}{1-\xi\Phi^2\kappa}$$

Rotationally invariant Euclidean metric ：O(4)−symmetry

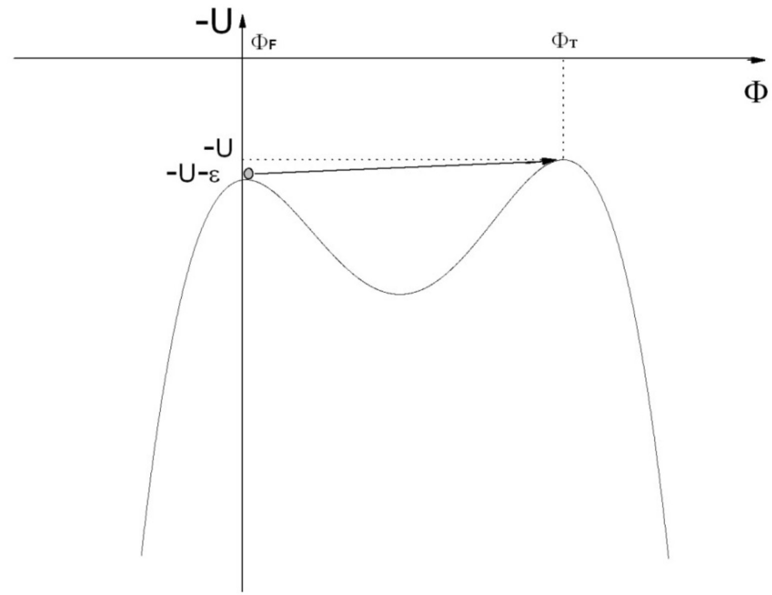
$$ds^2 = d\eta^2 + \rho^2(\eta)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)]$$

The Euclidean field equations

$$\Phi'' + \frac{3\rho'}{\rho}\Phi' - \xi R_E \Phi = \frac{dU}{d\Phi} \qquad \rho'^2 = 1 + \frac{\kappa\rho^2}{3(1-\xi\Phi^2\kappa)}\left(\frac{1}{2}\Phi'^2 - U\right)$$

boundary conditions

$$\lim_{\eta\to\eta(\max)} \Phi(\eta) = \Phi_T, \frac{d\Phi}{d\eta}\Big|_{\eta=0} = 0$$



Our main idea

$$\xi R_E \Phi > \frac{3\rho'}{\rho}\Phi'$$

(during the phase transition)

True & False Vacuum Bubbles

(*)Lee, Weinberg, PRD

	False–to–true (True vac. Bubble)	True–to–false (*) **(False vac. Bubble)**
De Sitter – de Sitter	O	O (*)
Flat – de Sitter	O	O
Anti de Sitter – de Sitter	O	X
Anti de Sitter – flat	O	O
Anti de Sitter – Anti de Sitter	O	O

(*) exists in

(1)non–minimally coupled gravity

(W.Lee, BHL, C.H.Lee,C.Park, PRD(2006))

or in

(2)Brans–Dicke type theory

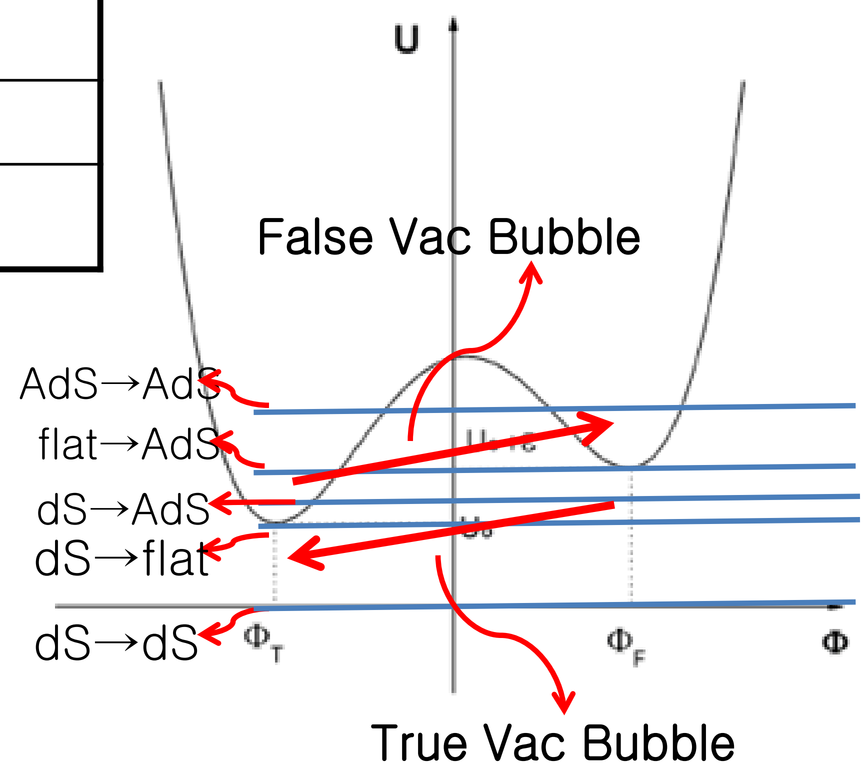
(H.Kim,BHL,W.Lee, Y.J. Lee, D.–H.Yoem, PRD(2011))

Dynamics of False Vacuum Bubble :

Can exist an expanding false vac bubble inside the true vacuum

BHL, C.H.Lee, W.Lee, S. Nam, C.Park, PRD(2008) (for nonminimal coupling)

BHL, W.Lee, D.–H. Yeom, JCAP(2011) (for Brans–Dicke)



3.2 vacuum bubbles with finite geometry

BHL, C.H. Lee, W.Lee & C.Oh,

dS-dS

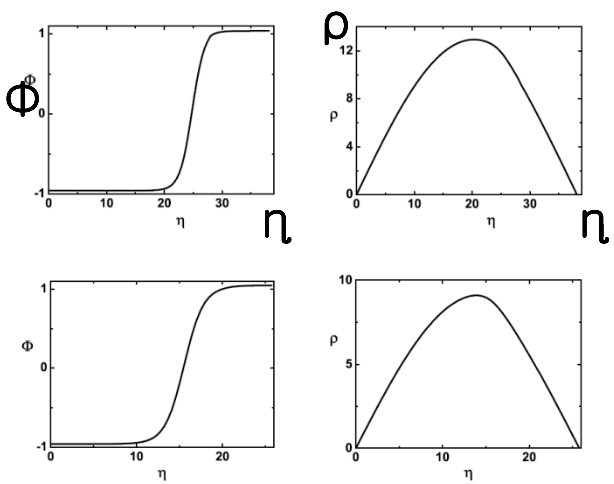


Figure 2: dS-dS cases. $\epsilon = 0.04$, $\kappa = 0.1$, and $U_0 = 0.1$ for for top figure. $\epsilon = 0.04$, $\kappa = 0.2$, and $U_0 = 0.1$ for for bottom figure.

dS-AdS

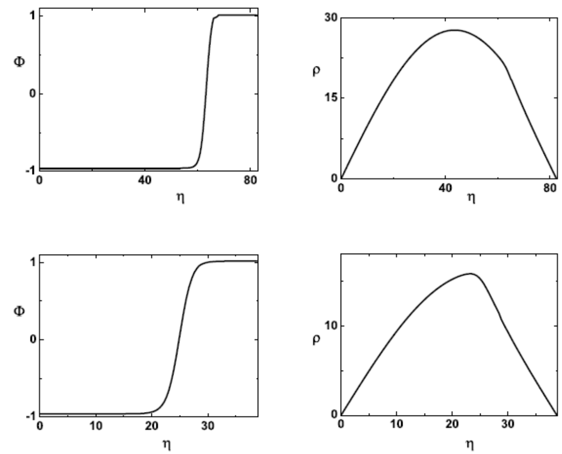


Figure 3: dS-AdS cases. $\epsilon = 0.04$, $\kappa = 0.1$, and $U_0 = -0.04$ for for top figure. $\epsilon = 0.04$, $\kappa = 0.3$, and $U_0 = -0.04$ for for bottom figure.

dS-flat

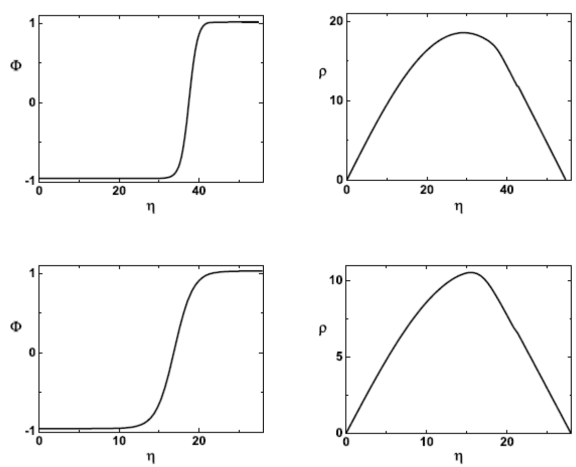


Figure 4: ds-flat cases. $\epsilon = 0.04$, $\kappa = 0.1$, and $U_0 = 0.0077$ for for top figure. $\epsilon = 0.04$, $\kappa = 0.3$, and $U_0 = 0.0077$ for for bottom figure.

flat-AdS and AdS-AdS

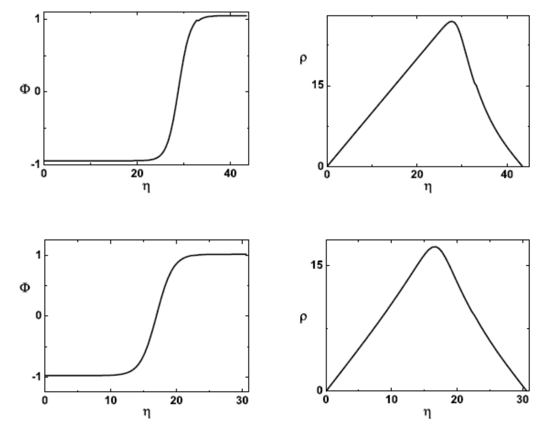
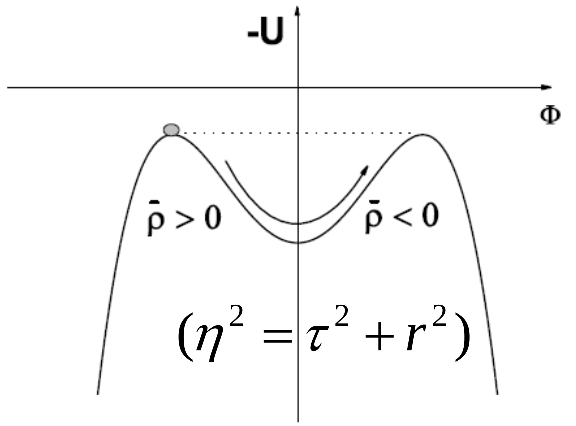


Figure 5: Flat-AdS and AdS-AdS cases. $\epsilon = 0.05$, $\kappa = 0.7$, and $U_0 = -0.09868$ for flat-AdS case. $\epsilon = 0.02$, $\kappa = 0.7$, and $U_0 = -0.05$ for AdS-AdS case.

3.3 Tunneling between the degenerate vacua

∃ Z2−symm. with finite geometry bubble



Potential $U(\Phi) = \frac{\lambda}{8}\left(\Phi^2 - \frac{\mu^2}{\lambda}\right)^2 + U_o.$

O(4)−symmetric Euclidean metric

$$ds^2 = d\eta^2 + \rho^2(\eta)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)]$$

Equations of motions

$$\Phi'' + \frac{3\rho'}{\rho}\Phi' = \frac{dU}{d\Phi} \qquad \rho'' = -\frac{\kappa}{3}\rho(\Phi'^2 + U),$$

Boundary condition (consistent with Z2−sym.)

$$\rho|_{\eta=0} = 0, \quad \rho|_{\eta=\eta_{max}} = 0, \quad \left.\frac{d\Phi}{d\eta}\right|_{\eta=0} = 0, \quad \text{and} \quad \left.\frac{d\Phi}{d\eta}\right|_{\eta=\eta_{max}} = 0.$$

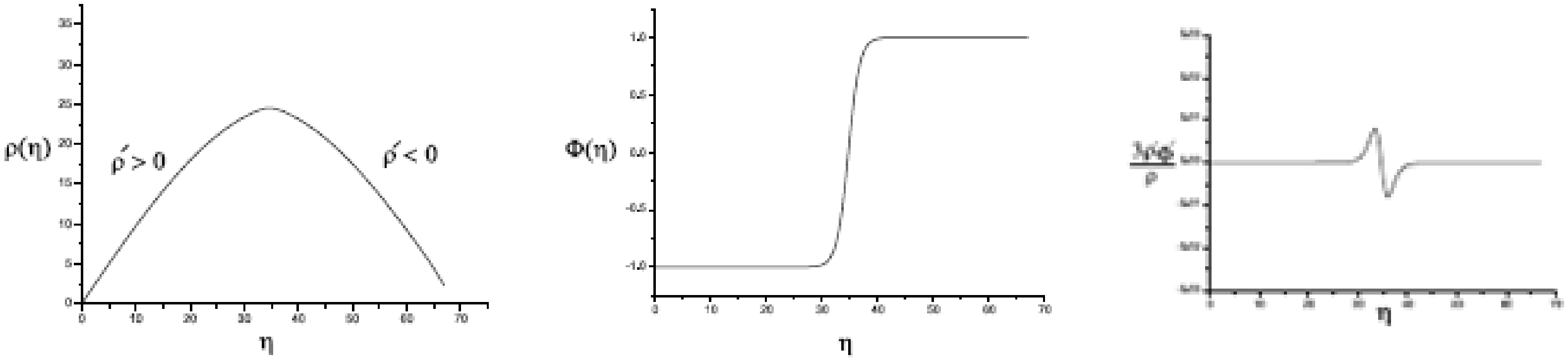
− in de Sitter space.

The numerical solution by Hackworth and Weinberg.

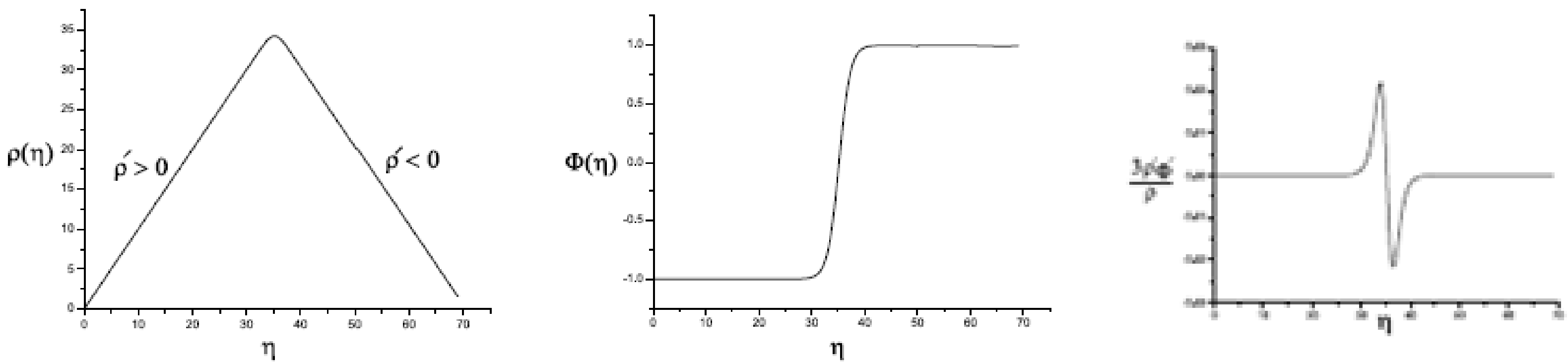
The analytic computation and interpretation :

(BHL & W. Lee, CQG (2009))

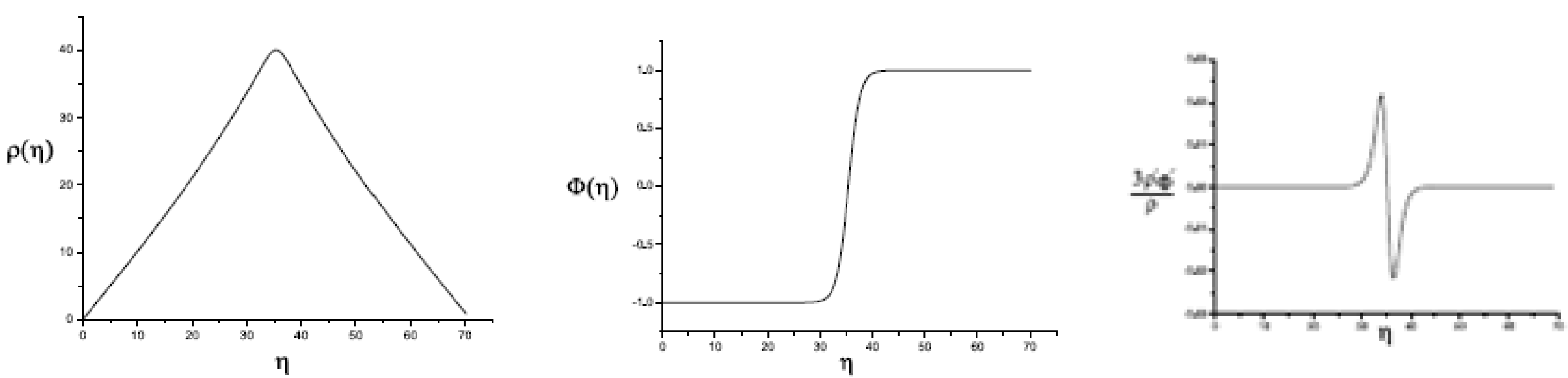
dS − dS



flat −flat



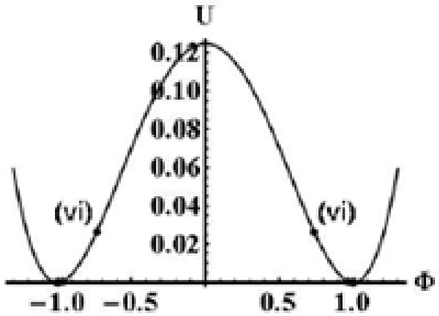
AdS−AdS



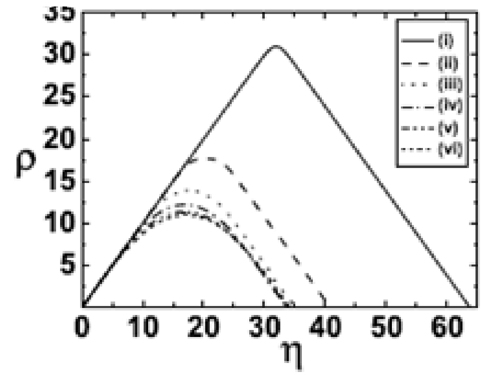
B.-H. L, C. H. Lee, W. Lee & C. Oh, PRD**82** (2010)

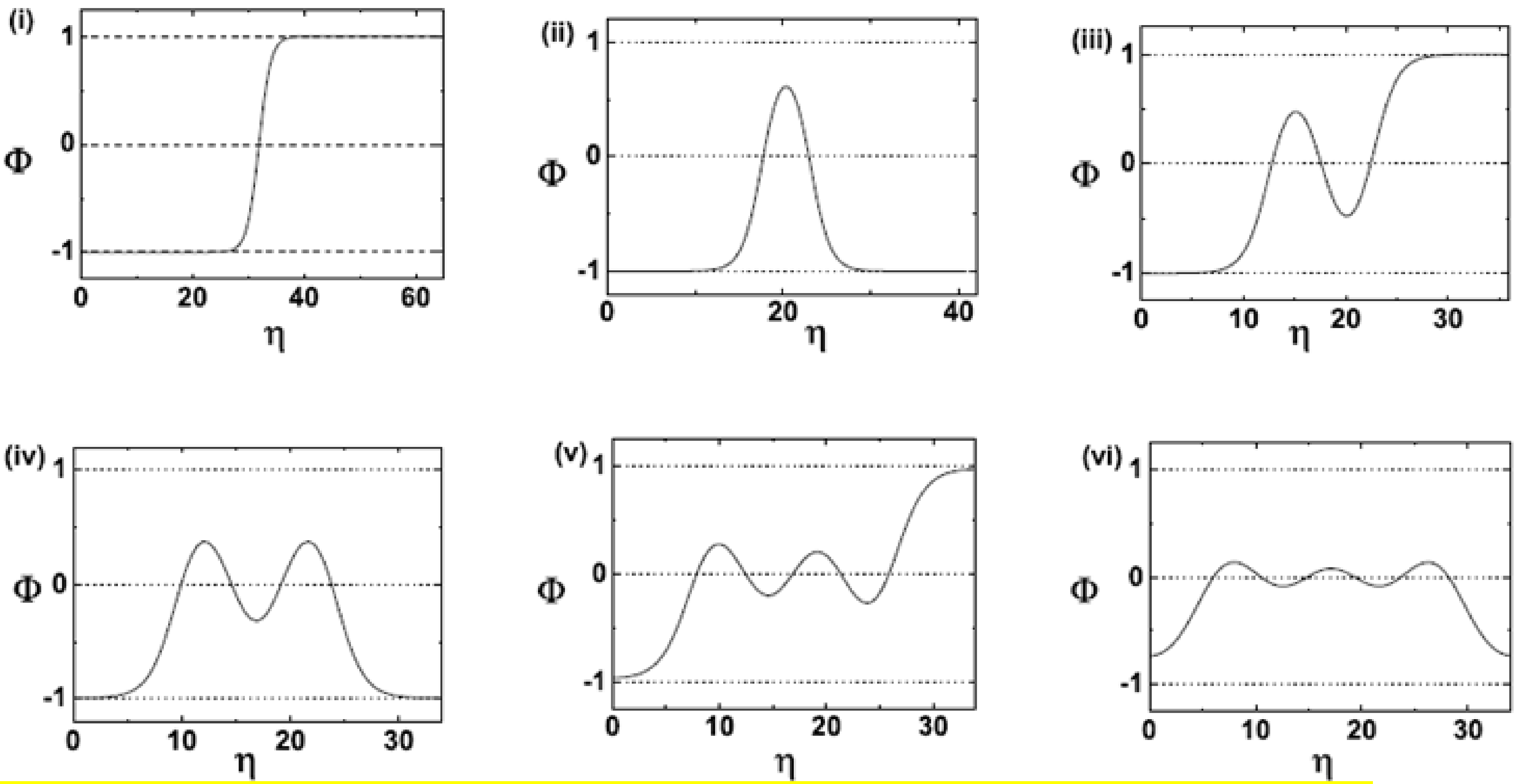
3.3 Oscillaing solutions : a) between flat-flat degenerate vacua $\tilde{\kappa} = 0.2$)

B.-H. Lee, C. H. Lee, W. Lee & C. Oh, arXiv:1106.5865



Number of Oscillation	Φ_0
1	-0.99999999999355985
2	-0.99999585754
3	-0.9995857315805
4	-0.994499
5	-0.9661682
6	-0.7348584

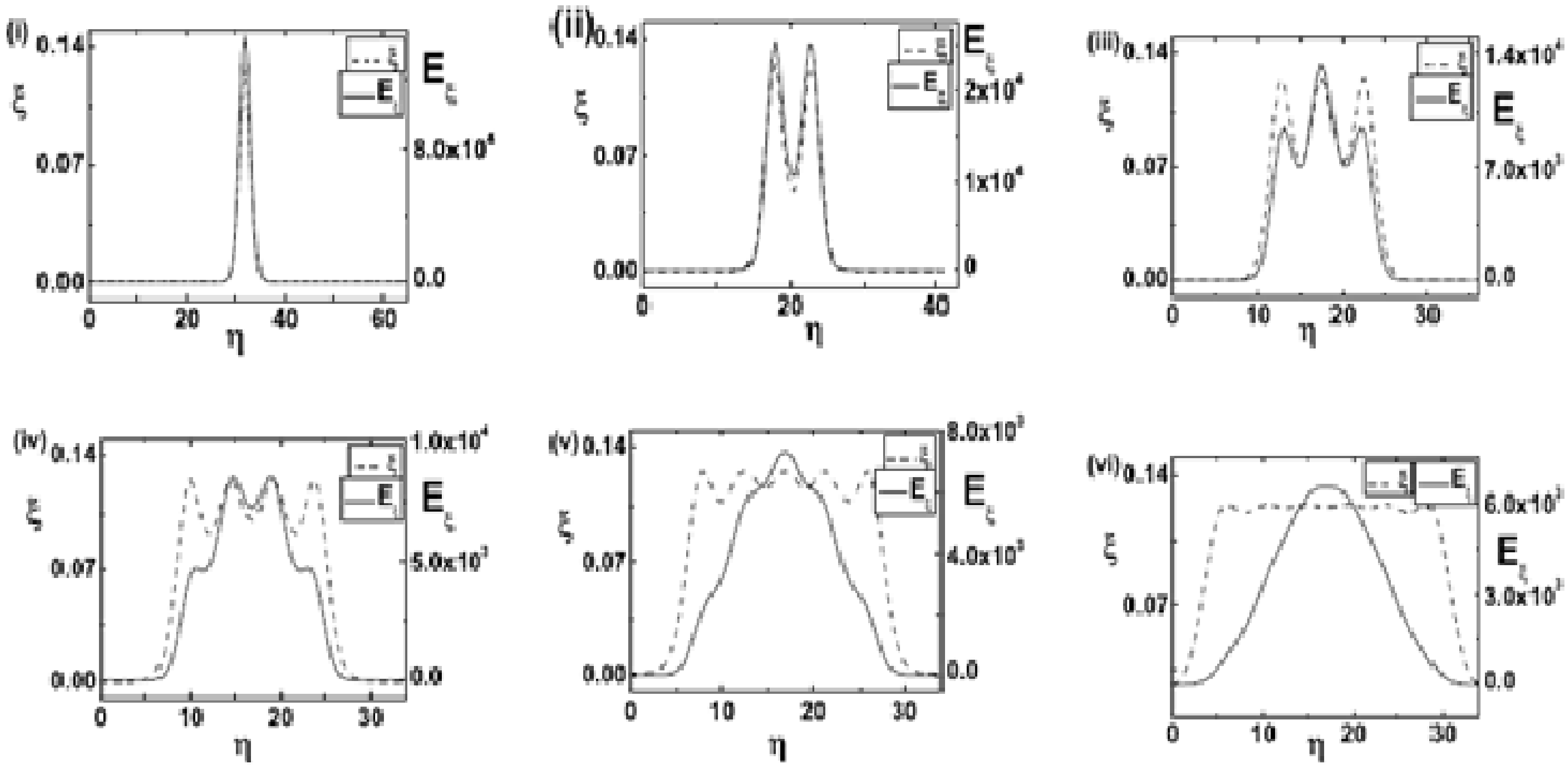




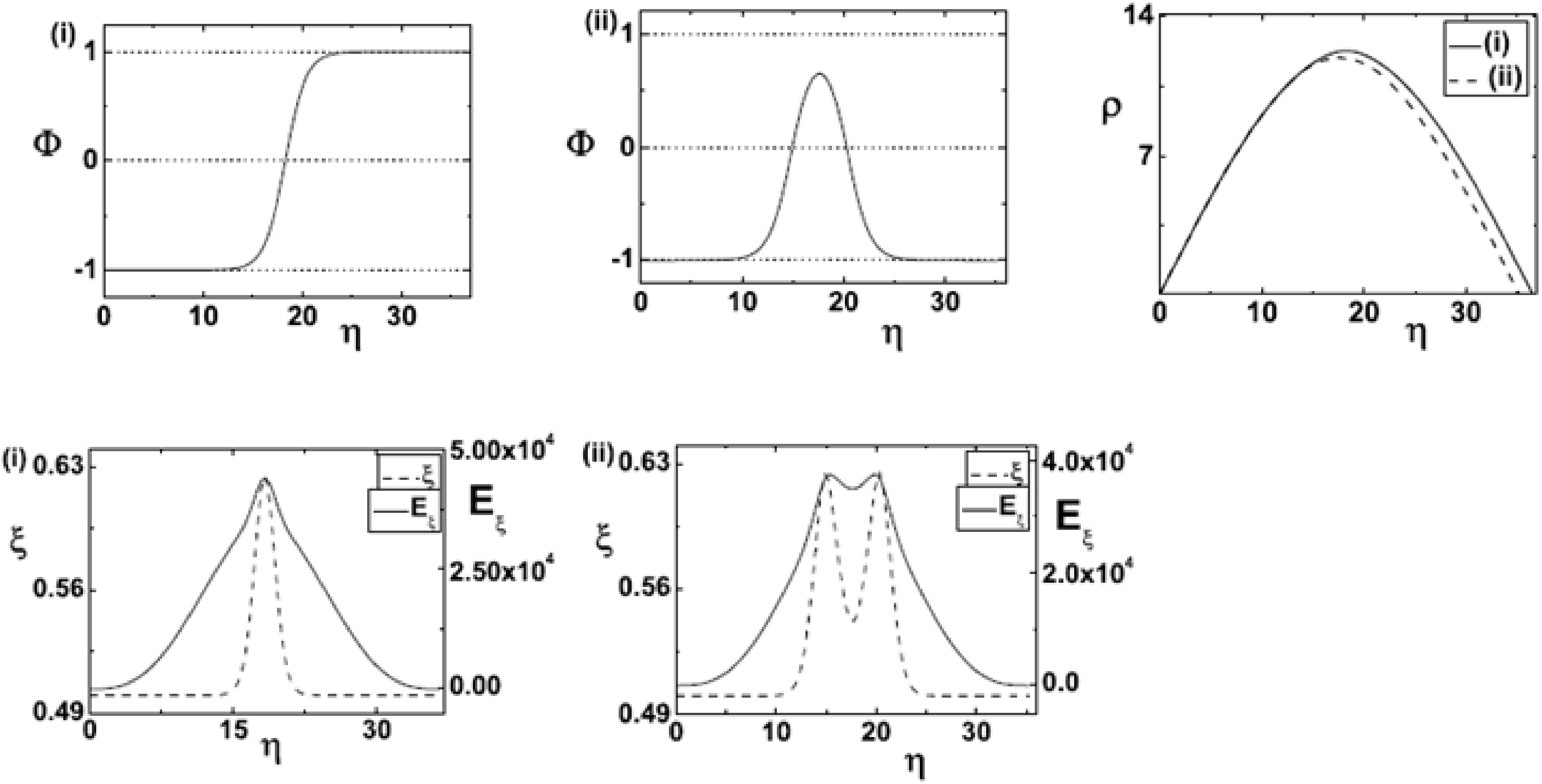
This type of solutions is possible only if gravity is taken into account.

energy density

$$\xi \equiv -\mathcal{H} = -\left[-\frac{R}{2\kappa} + \frac{1}{2}\Phi'^2 + U\right] = U$$



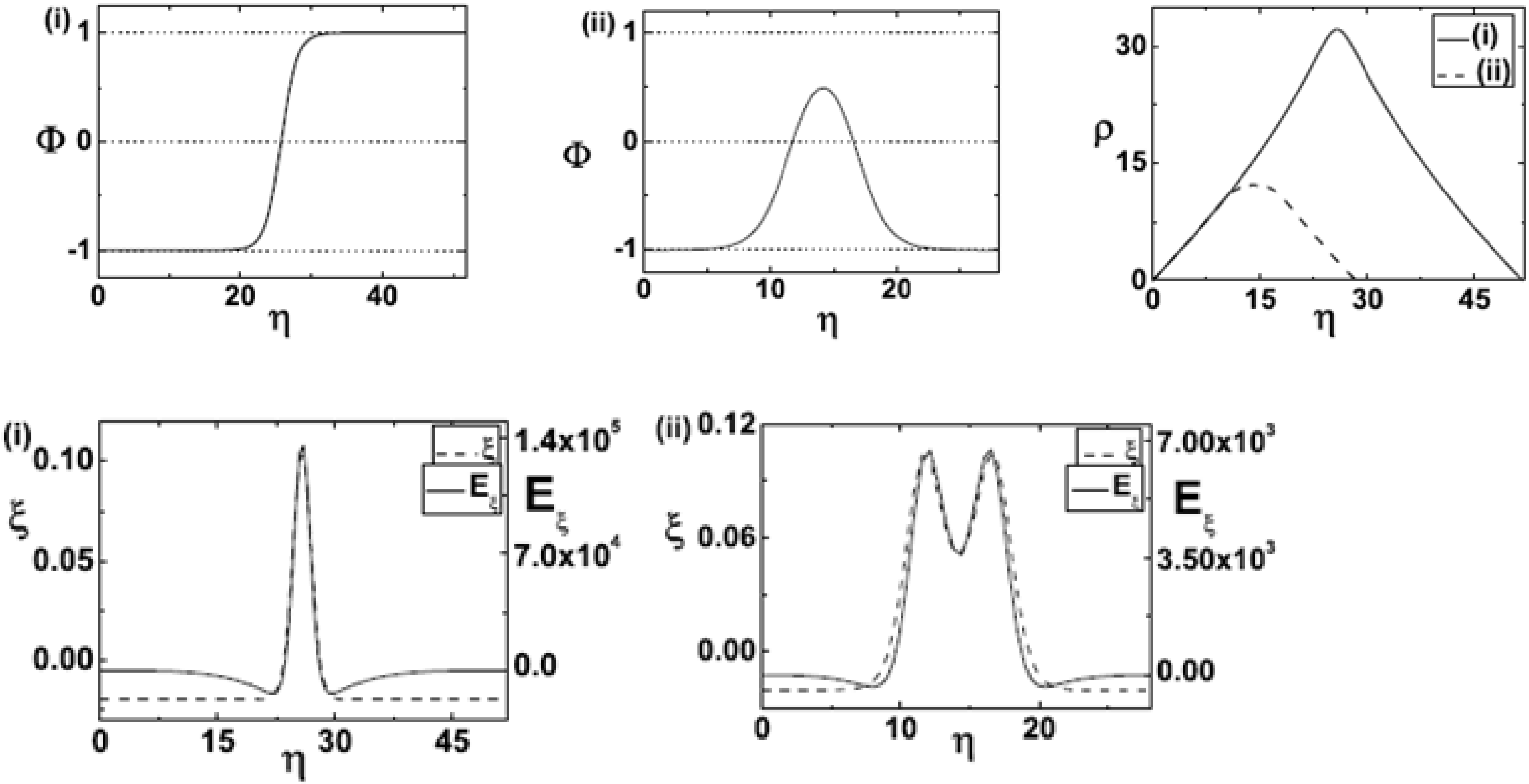
3.3 Oscillaing solutions - b) between dS-dS degenerate vacua



$\tilde{U}_o = 0.5$ and $\tilde{\kappa} = 0.04$

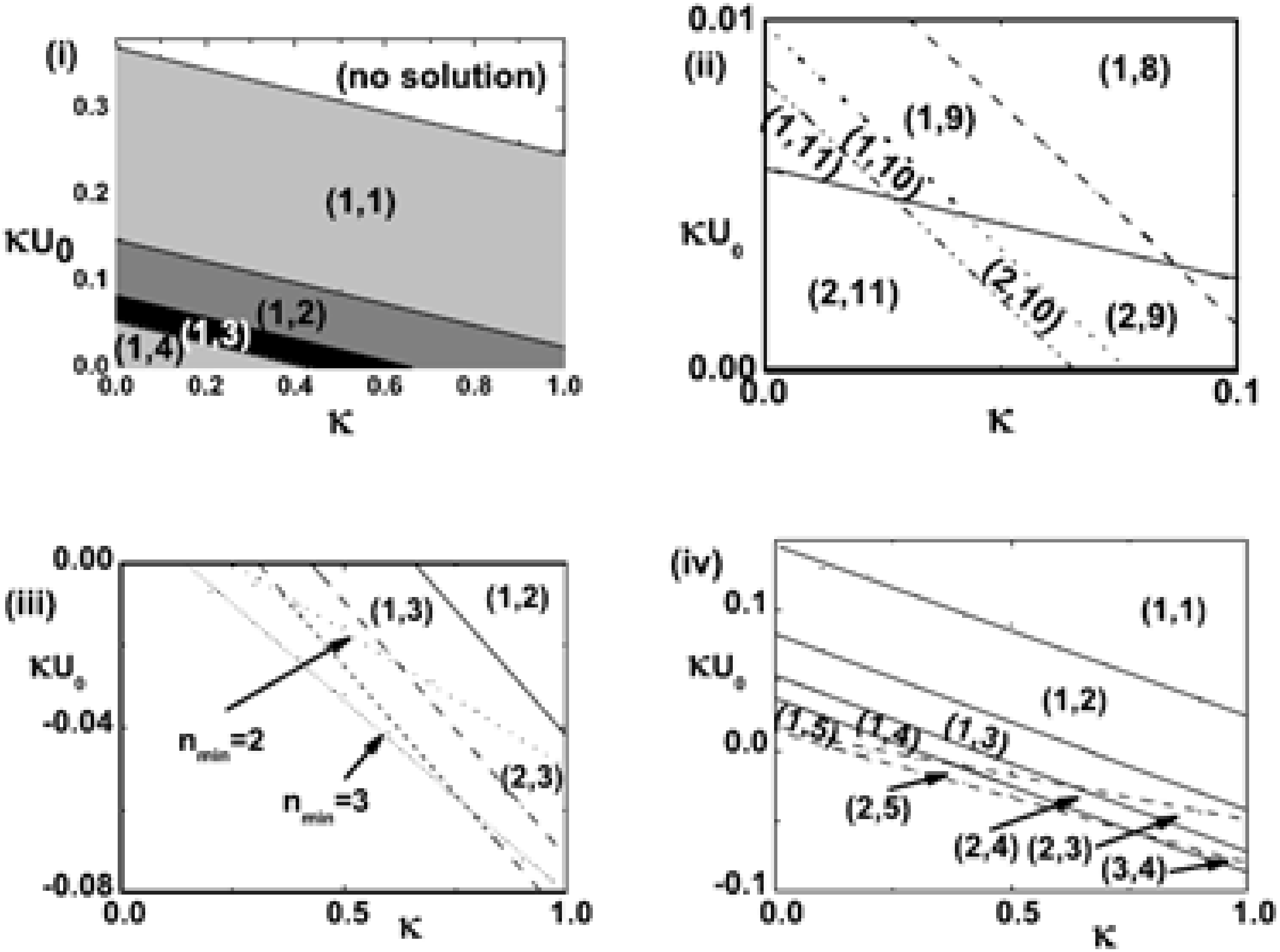
3.3 Oscillaing solutions – c) between AdS-AdS degenerate vacua

$\tilde{U}_o = -0$and $\tilde{\kappa} = 0.4$

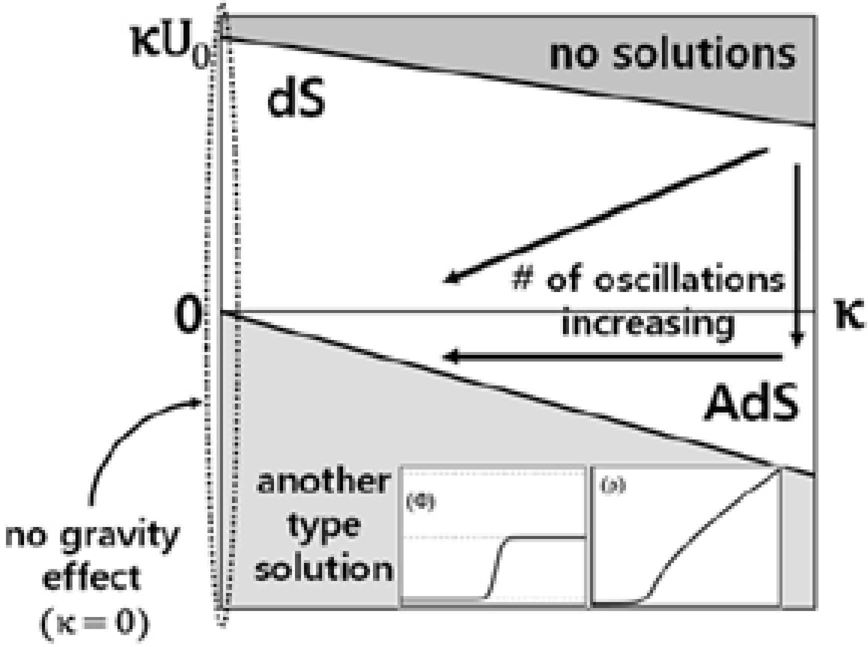


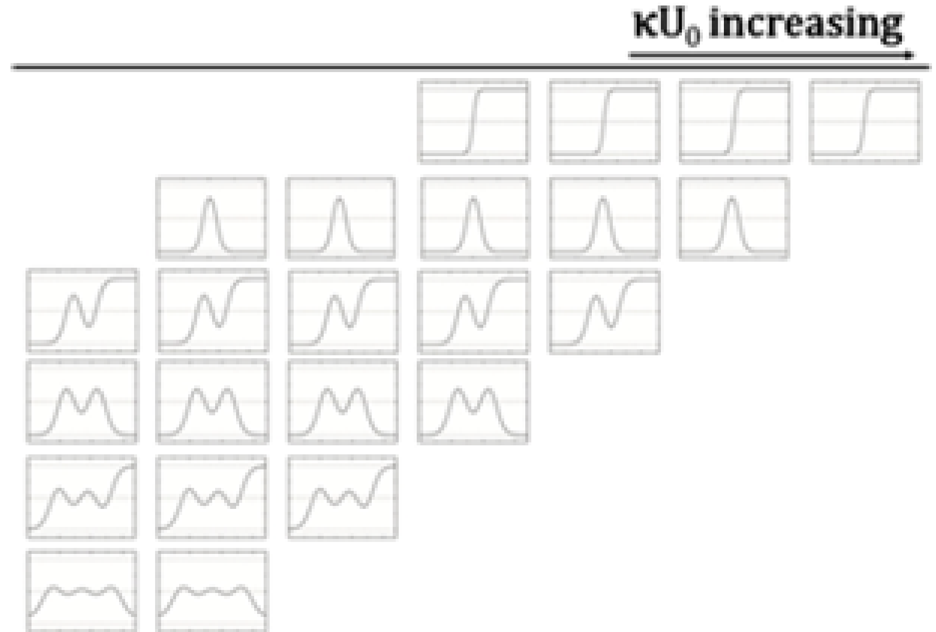
The phase space of solutions

$0 \leqq \tilde{\kappa} \leqq 1$



The y-axis represents no gravity.
the notation (n_min, n_max), where n_min means the minimum number of oscillations and n_max the maximum number of oscillation





the schematic diagram of the phase space of all solutions including another type solution and the number of oscillating solutions with different κs.

The left figure has κ = 0 line indicating no gravity effect. In the middle area including the flat case, n_min and n_max are increased as κ and κUo are decreased. The tendencies are indicated as the arrows. In the left lower region, there exist another type solution.

The right figure shows n_min and n_max are changed in terms of κUo and κ. As we can see from the figure, n_max and n_min are increased as κUo is decreased.

3.4 Fubini Instanton in Gravity

Review : In the Absence of Gravity

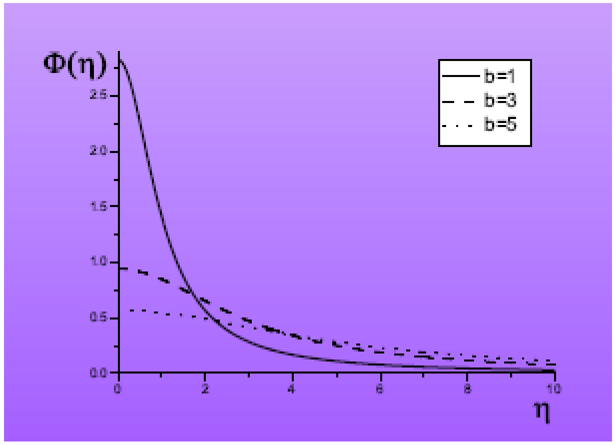
action
$$S = \int_{\mathcal{M}} \sqrt{-g}\, d^4x \left[-\frac{1}{2} \nabla^\alpha \Phi \nabla_\alpha \Phi - U(\Phi) \right]$$

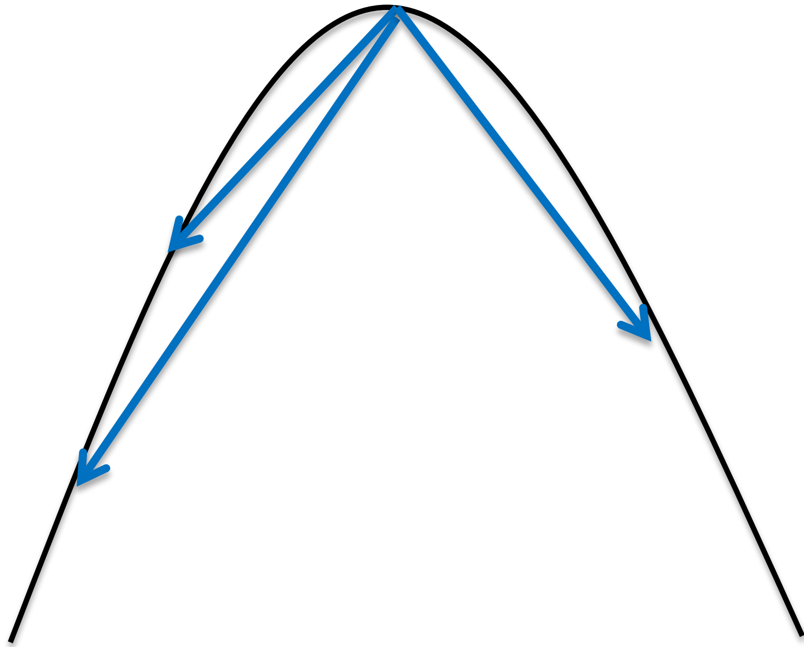
$$\boldsymbol{U(\Phi) = -\frac{\lambda}{4} \Phi^4}$$

Equation of motion
$$\Phi'' + \frac{3\rho'}{\rho} \Phi' = \frac{dU}{d\Phi}$$

Boundary conditions
$$\Phi|_{\eta=0} = \Phi_o \quad \text{and} \quad \left.\frac{d\Phi}{d\eta}\right|_{\eta=\infty} = 0$$

solution
$$\Phi(\eta) = \sqrt{\frac{8}{\lambda}} \frac{b}{\eta^2 + b^2}$$





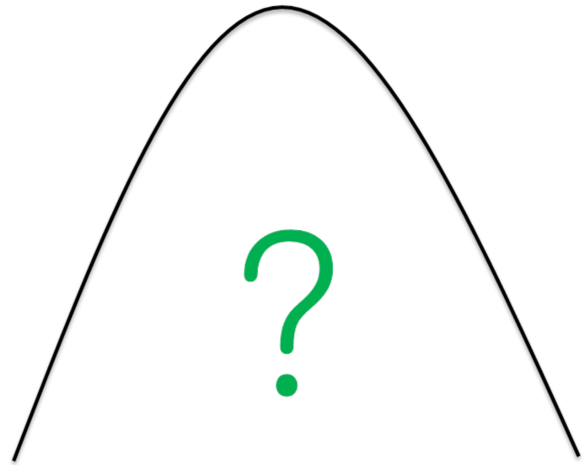
- Fubini, Nuovo Cimento 34A (1976)
- Lipatov – JETP45 (1977)

In the Presence of Gravity

$$U(\Phi) = \frac{\lambda}{4}\Phi^4 - \frac{m^2}{2}\Phi^2 + \epsilon\Phi$$

$$U(\Phi) = -\frac{\lambda}{4}\Phi^4$$

$$U(\Phi) = -\frac{\lambda}{4}\Phi^4 + \frac{m^2}{2}\Phi^2$$



Action

$$S = \int_{\mathcal{M}} \sqrt{-g}\, d^4x \left[-\frac{1}{2}\nabla^\alpha\Phi\nabla_\alpha\Phi - U(\Phi) \right]$$

O(4) symmetric metric

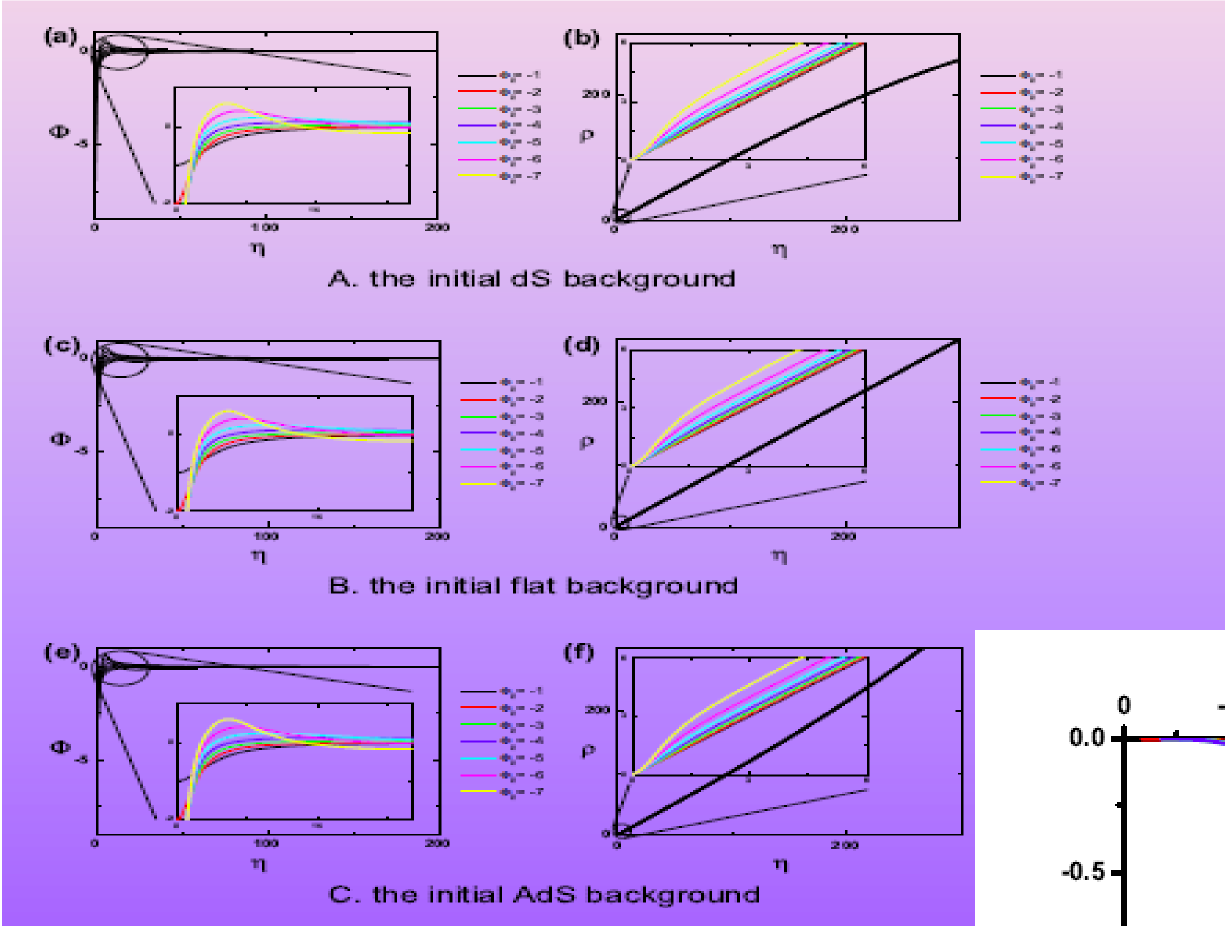
$$ds^2 \;=\; d\eta^2 + \rho(\eta)^2 \left[d\chi^2 + \sin^2\chi \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right) \right]$$

Equantions of motion

$$\Phi'' + \frac{3\rho'}{\rho}\Phi' = \frac{dU}{d\Phi} \qquad \rho'' = -\frac{\kappa}{3}\rho\left(\Phi'^2 + U\right) \qquad \rho'^2 - 1 - \frac{\kappa\rho^2}{3}\left(\frac{1}{2}\Phi'^2 - U\right) = 0,$$

Boundary Conditions

$$\rho|_{\eta=0} = 0, \quad \left.\frac{d\rho}{d\eta}\right|_{\eta=0} = 1, \quad \left.\frac{d\Phi}{d\eta}\right|_{\eta=0} = 0, \quad \text{and} \quad \Phi|_{\eta=\eta_{max}} = 0$$



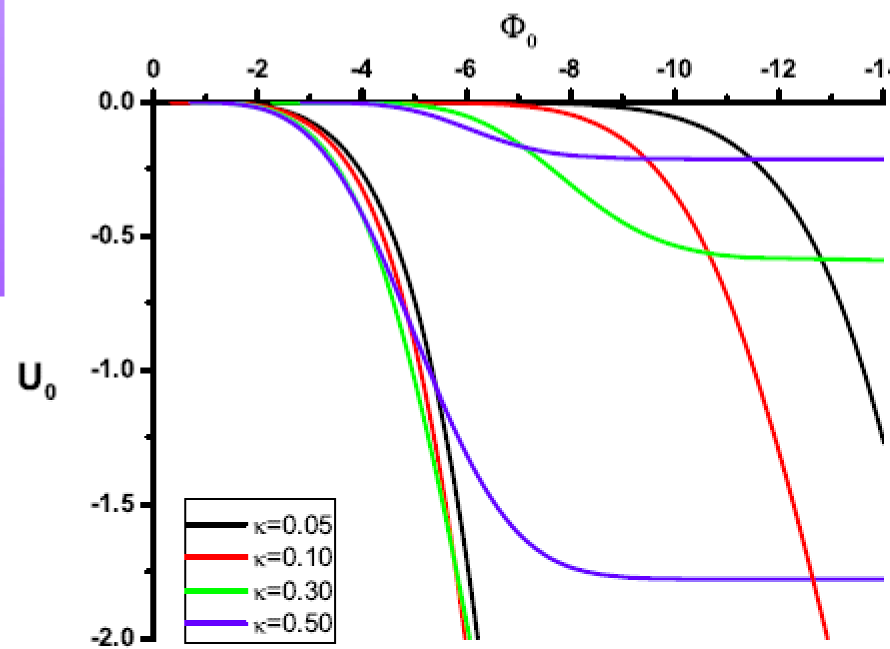


Figure 9: Phase diagram for $\Lambda \leq 0$ with several κ's.

B-HL, W. Lee, C.Oh, D. Yeom, D. Rho
JHEP 1306 (2013) 003;
in preparation

4. Possible Cosmological Implication

4.1 5Dim. Z2 symmetric Black hole with a domain wall solution.

After the nucleation, the domain wall (that may be interpreted as our braneworld universe) evolves in the radial direction of the bulk spacetime.

$$r = a(\tau), \quad \dot{a}^2 + V(a) = 0$$

The equation becomes

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{1}{3}\lambda + \frac{2m_*}{a^4} - \frac{q^2}{a^6},$$

$\lambda = 3\Lambda$: the effective cosmological constant.
mass term ~ the radiation in the universe
charge term ~ the stiff matter with a negative energy density.

Cosmological solutions

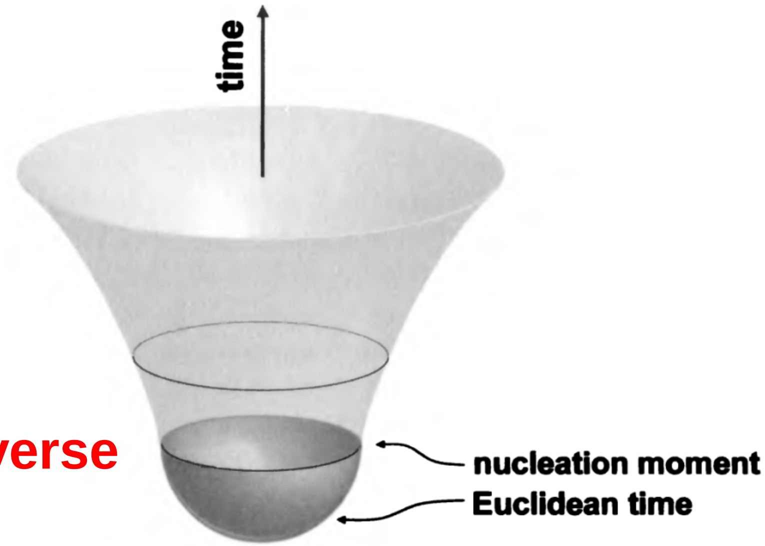
the expanding domain wall (universe) solution ($a > r*,+$).
approaching the de Sitter inflation with λ, since the contributions of the mass and charge terms are diluted.

contracting solution ($a < r*,+$) : the initially collapsing universe.
The domain wall does not run into the singularity & experiences a bounce since there is the barrier in $V(a)$ because of the charge q.

4.2 Application to the No-boundary

before → after

nothing ⇨ **Something, homogeneous**



to avoid the singularity problem of a Universe

the no-boundary proposal by Hartle and Hawking

cf) Vilenkin's tunneling boundary condition

the ground state wave function of the universe is given by the Euclidean path integral satisfying the WD equation

$$\Psi[h_{\mu\nu},\chi] = \int_{\partial g = h, \partial\phi = \chi} \mathcal{D}g\mathcal{D}\phi\, e^{-S_{\rm E}[g,\phi]}$$

Consider the Euclidean action $S_{\rm E} = -\int d^4x\sqrt{+g}\left(\frac{1}{16\pi}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right)$

the mini-superspace approximation => the scale factor as the only dof.

$$ds_{\rm E}^2 = d\eta^2 + \rho^2(\eta)\left(d\chi^2 + \sin^2\chi\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right)$$

The equations of motion

$$\ddot{\phi} = -3\frac{\dot{\rho}}{\rho}\dot{\phi} \pm V' \,, \qquad \ddot{\rho} = -\frac{8\pi}{3}\rho(\dot{\phi}^2 \pm V) \,,$$

the regular initial conditions at $\eta = 0$

$$\phi = \phi_0 \,, \quad \rho(0) = 0 \,, \quad \dot{\rho}(0) = 1 \,, \quad \dot{\phi}(0) = 0 \,,$$

We want to analytically continue the solution to the Lorentzian manifold using $d\eta = i dt.$ **Then at the tunneling point** $\eta = \eta_{\rm max},$ **we have to impose the followings**

$$\rho(t=0) = \rho(\eta = \eta_{\rm max}) \,, \quad \dot{\rho}(t=0) = i\dot{\rho}(\eta = \eta_{\rm max}) \,,$$

$$\phi(t=0) = \phi(\eta = \eta_{\rm max}) \,, \quad \dot{\phi}(t=0) = i\dot{\phi}(\eta = \eta_{\rm max}) \,,$$

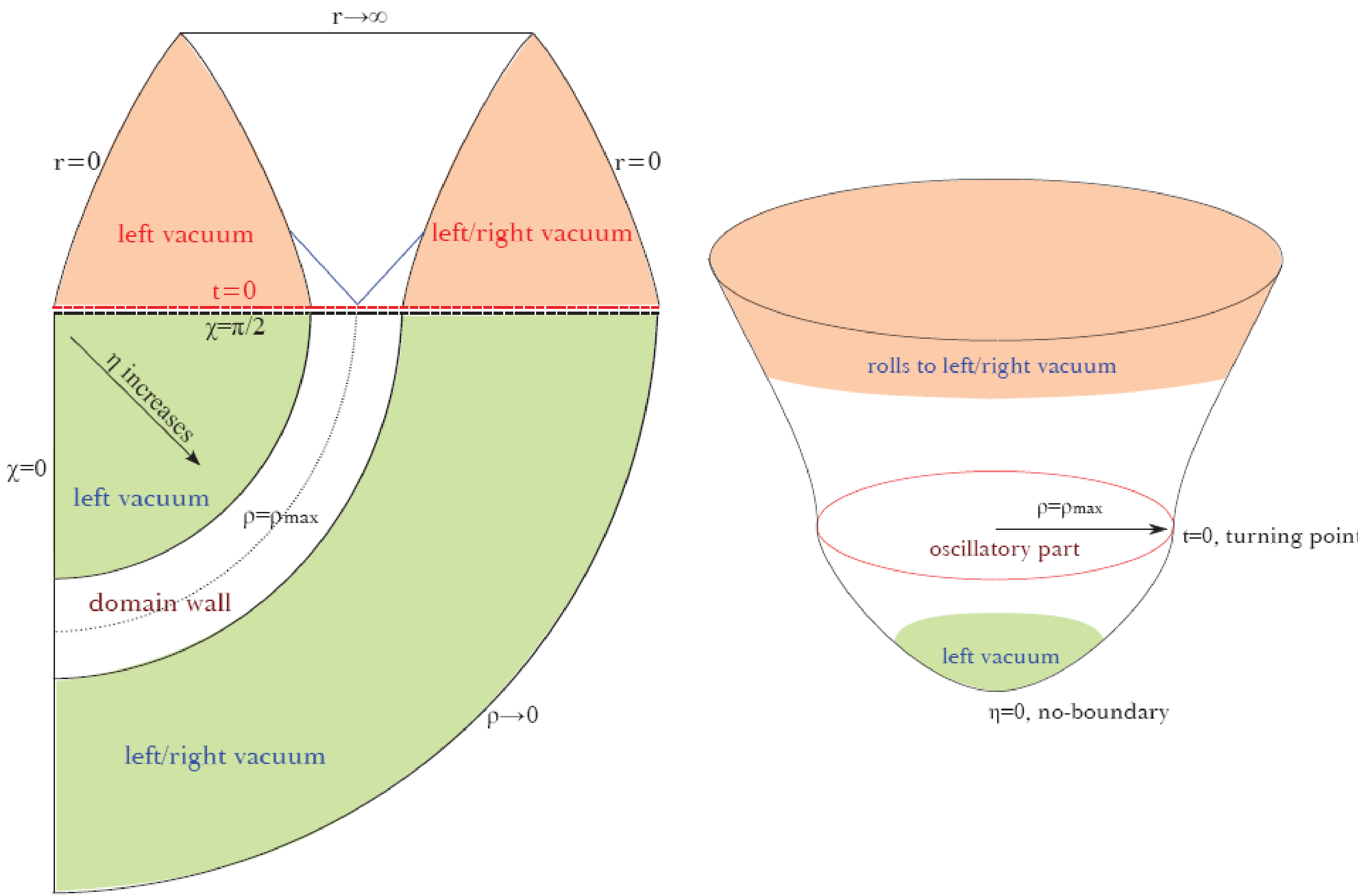


Fig. 2. Left: analytic continuation $\chi \to \pi/2 + it$. Right: analytic continuation $\eta \to \eta_{\rm max} + it$.

4.3 General vacuum decay problem

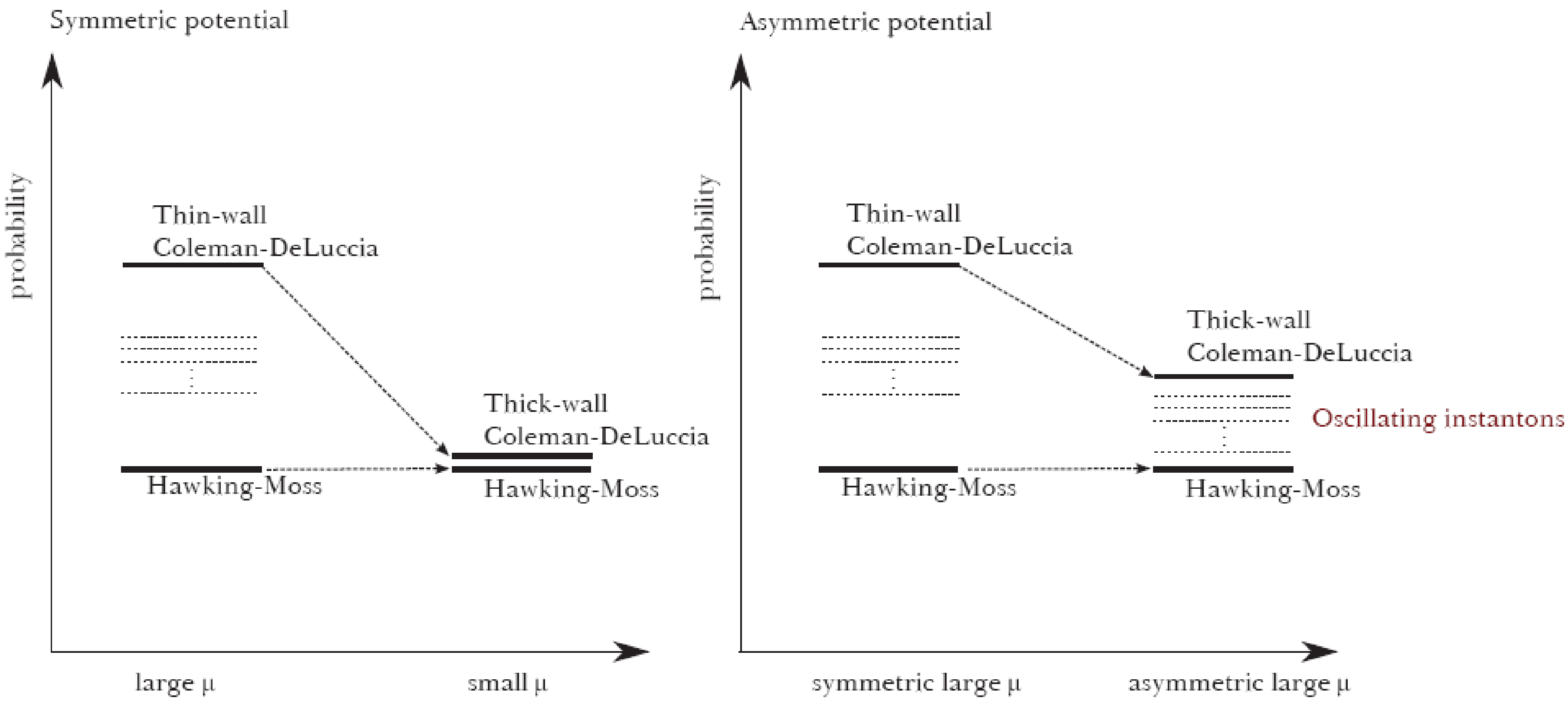


Fig. 9. Conceptual picture of probabilities. Left: When we decrease μ around the local maximum with symmetry, then the thin-wall Coleman–de Luccia solution approaches the thick-wall Coleman–de Luccia solution and this approaches the Hawking–Moss solution. Right: When we change the symmetry with a constant large μ, then the thin-wall Coleman–de Luccia solution approaches a think-wall Coleman–de Luccia solution and oscillating instantons do not disappear.

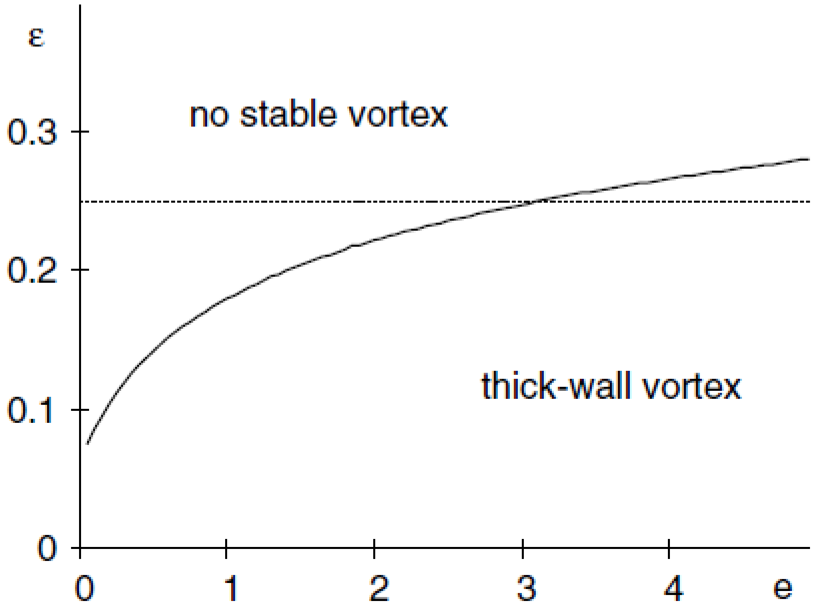
4.4 False Cosmic String and its Decay

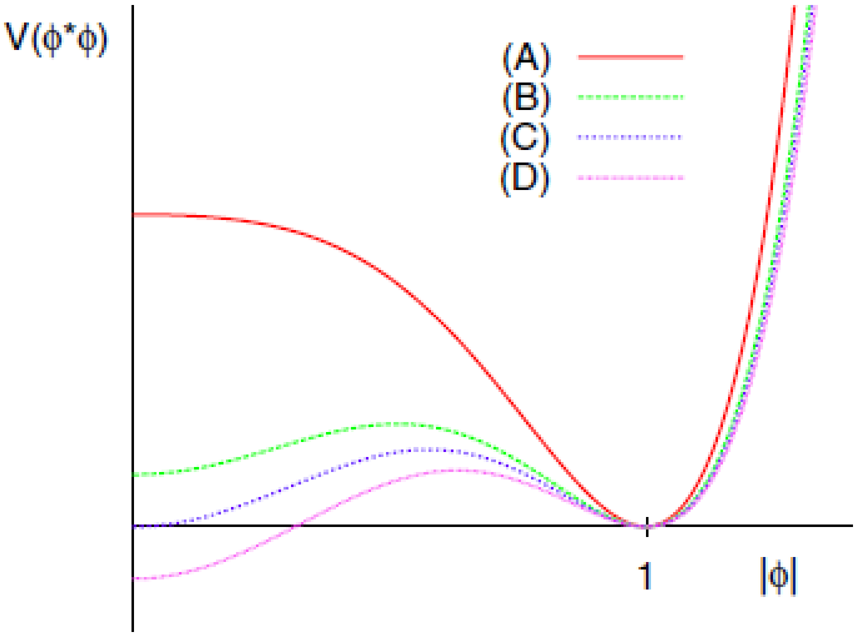
Acton

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu \phi)^*(D^\mu \phi) - V(\phi^*\phi)$$

$$V(\phi^*\phi) = \lambda(|\phi|^2 - \epsilon v^2)(|\phi|^2 - v^2)^2$$

Vortex Solution





B-HL, W. Lee, R. MacKenzie, M.Paranjape U. Yajnik, D-h Yeom.
PRD88 (2013) 085031 arXiv:1308.3501
PRD88 (2013) 105008 arXiv:1310.3005

Decay of the False String (thin wall approximation)

$$S_E = \frac{1}{\lambda v^2}\int d^2x \frac{1}{2} M(R(z,\tau))(\dot{R}^2 + R'^2) + E(R(z,\tau)) - E(R_0)$$

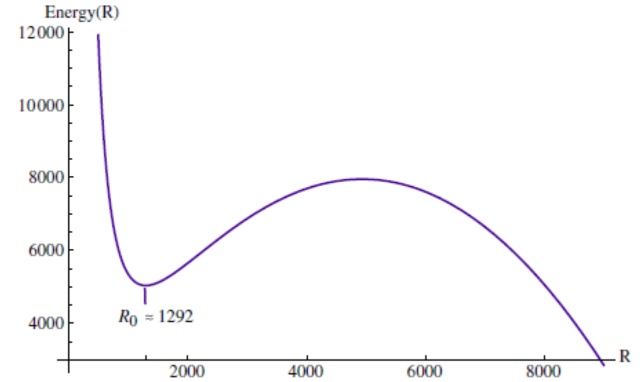


FIG. 1 (color online). The energy as a function of R, for $n = 100$, $e = 0.005$ and $\epsilon = 0.0001$.

Tunnelling Solution

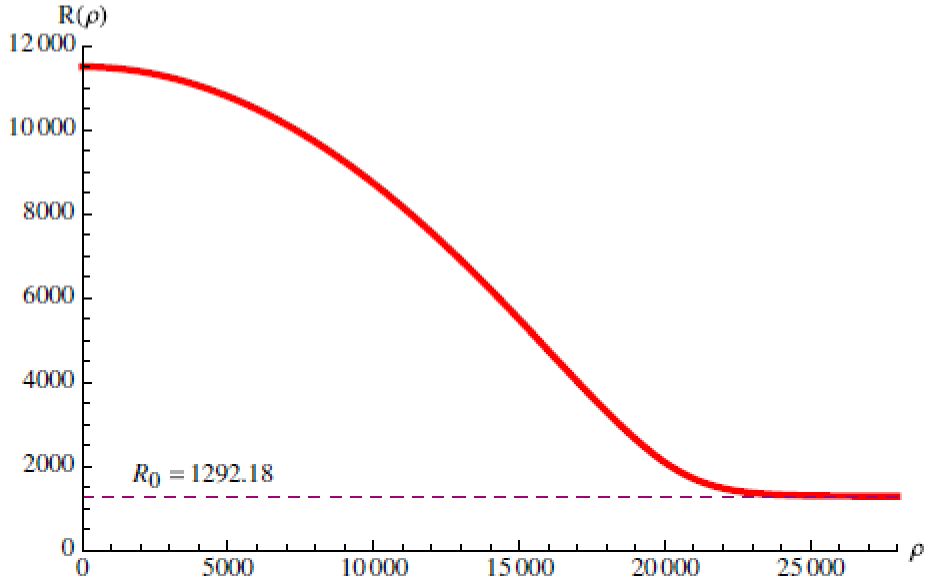
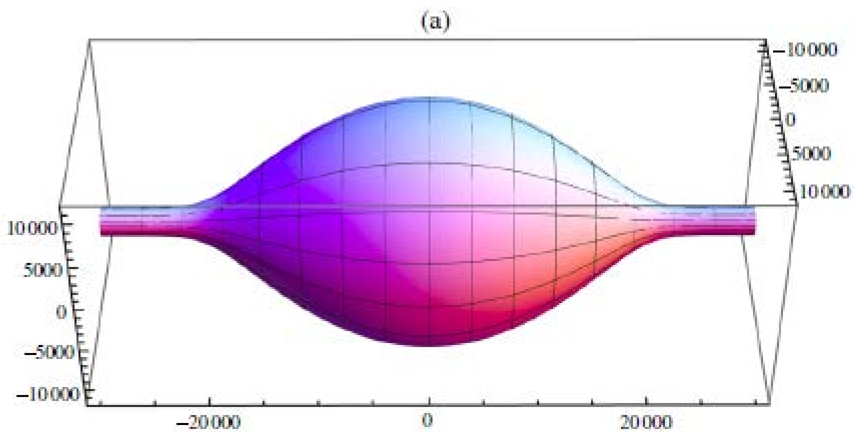
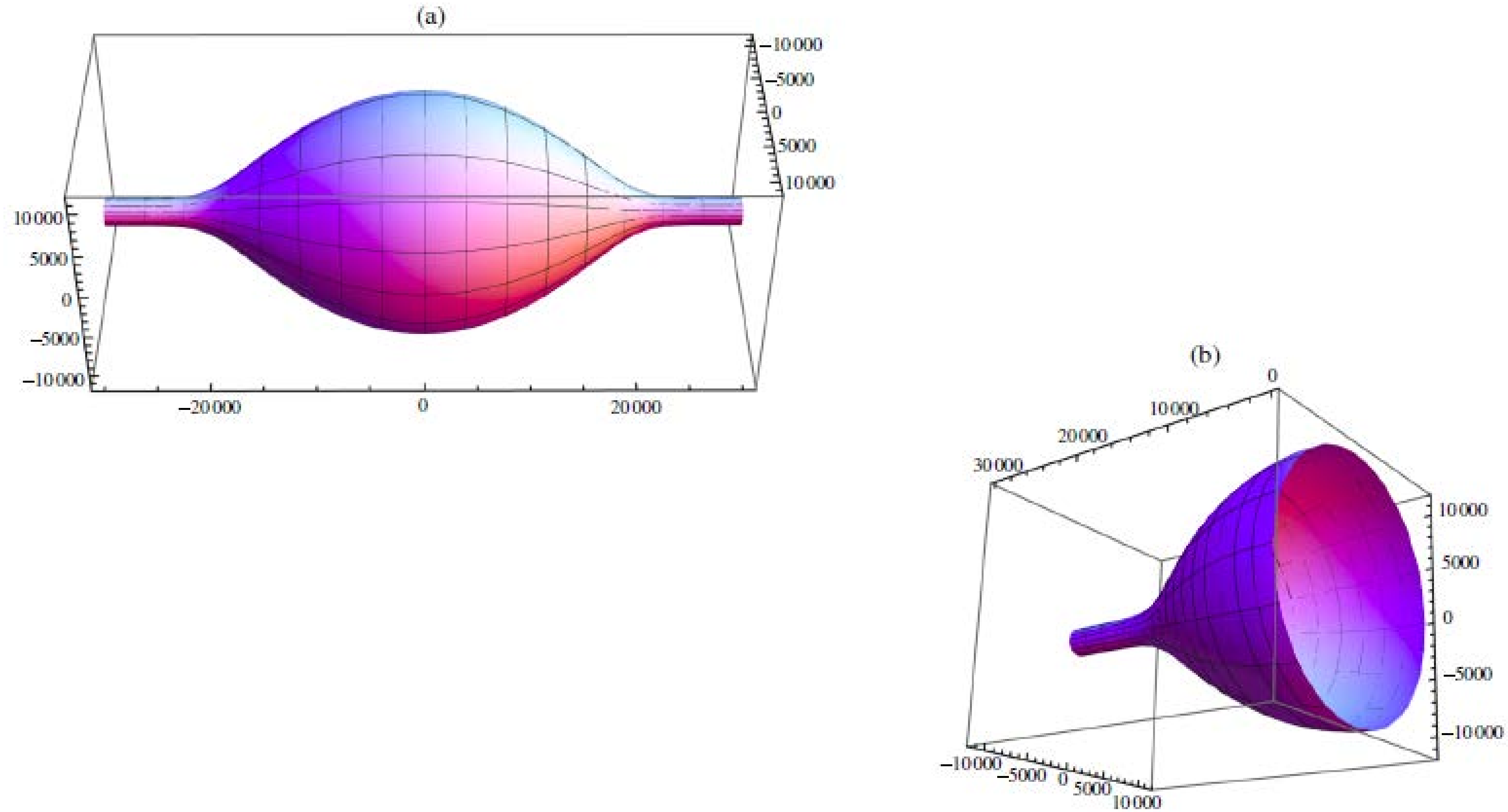


FIG. 2 (color online). The radius as a function of ρ.




(a)
-10000
-5000
0
5000
10000
10000
5000
0
-5000
-10000
-20000
0
20000
(b)
0
10000
20000
30000
10000
5000
0
-5000
-10000
-10000
-5000
0
5000
10000

5. Summary and Discussions

- We reviewed the formulation of the bubble.

- False vacua exist e.g., in non-minimally coupled theory.

- Vacuum bubbles with finite geometry, with the radius & nucleation rate

- New Type of the solutions :
  Ex) bubble with compact geometry,
  degenerate vacua in dS, flat, & AdS.
  Oscillating solutions; can make the thick domain wall.

- Similar analysis for the Fubini instanton

- Physical role and interpretation of many solutions are still not clear.

- The application to the braneworld cosmology has been discussed for the model of magnetically charged BH pairs separated by a domain wall in the 4 or 5-dim. spacetime with a cosmological constant.

- Can there be alternative model for the accelerating expanding universe?

Thank you !