Naturalness and Higgs Inflation

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at IAS HKUST

Hikaru Kawai

Outline

SM without SUSY is very good.

From string theory point of view, SUSY breaking scale can be anywhere.

So the simplest guess is that SUSY breaks at the Planck scale.

In other words, SM physics may be directly connected to the Planck scale physics.

One concrete example is the Higgs inflation.

However we need a new principle by which Higgs mass and the cosmological constant are explained.

The naturalness problem

Suppose the underlying fundamental theory, such as string theory, has the momentum scale m_S and the coupling constant g_S .

Then, by dimensional analysis and the power counting of the couplings, the parameters of the low energy effective theory are given as follows:

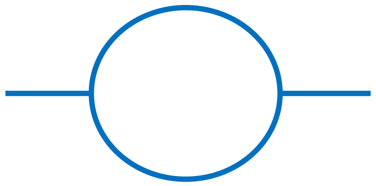
naturalness problem (cont.'d)

dimension -2 (Newton constant) $$G_N \sim \frac{g_S{}^2}{m_S{}^2}.$$

dimension 0

(gauge and Higgs couplings) $$g_1, g_2, g_3 \sim g_S,$$ $$\lambda_H \sim g_S{}^2.$$

dimension 2 (Higgs mass)



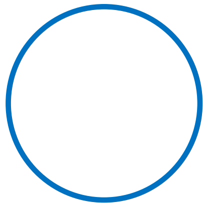
tree

$$m_H{}^2 \sim 0 + g_S{}^2 m_S{}^2.$$

unnatural ! $\rightarrow$ $m_H{}^2 \sim (100\,\mathrm{GeV})^2 \ll g_S{}^2 m_S{}^2 \sim (10^{18}\,\mathrm{GeV})^2$

dimension 4

(vacuum energy or cosmological constant)

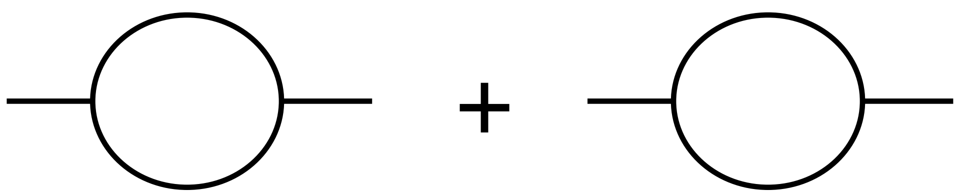


$$\lambda \sim 0 \cdot g_s{}^{-2} + m_S{}^4.$$

unnatural ! ! $\rightarrow$ $\lambda \sim (2 \sim 3\,\mathrm{meV})^4 \ll m_S{}^4 \sim (10^{18}\,\mathrm{GeV})^4$

SUSY as a solution to the naturalness problem

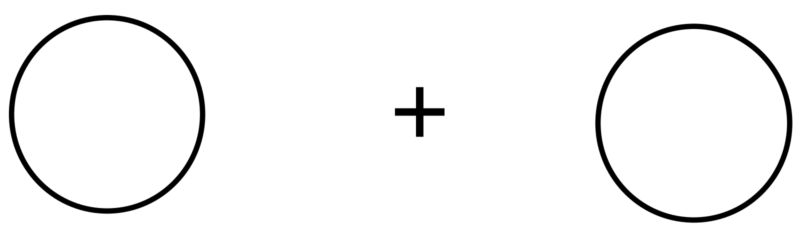
Bosons and fermions cancel the UV divergences:



bosons fermions

$$\Rightarrow \quad m_H{}^2 = 0.$$

$$\Rightarrow \quad m_H{}^2 \sim M_{\mathrm{SUSY}}{}^2.$$



$$\Rightarrow \quad \lambda = 0.$$

$$\Rightarrow \quad \lambda \sim M_{\mathrm{SUSY}}{}^4.$$

However, SUSY must be spontaneously broken at some momentum scale M_{SUSY}, below which the cancellation does not work.

(cont'd)

Therefore, if M_{SUSY} is close to m_H , the Higgs mass is naturally understood, although the cosmological constant is still a big problem.

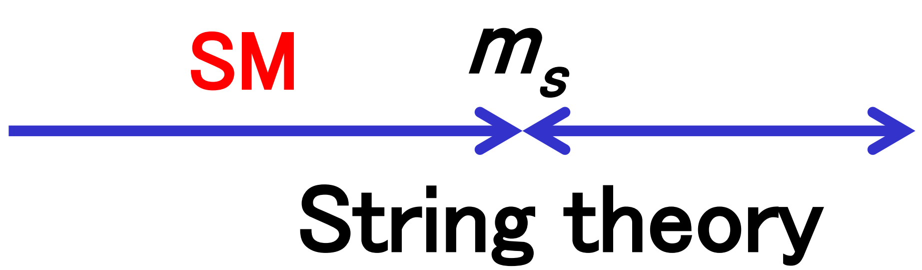
However, no signal of new particles is observed in the LHC below 1 TeV.

It is better to think about the other possibilities.

Possibility of desert

The first thing we should know is whether the SM is valid to the string/Planck scale or some new physics should come in.

If it is the former case, there is a possibility that the SM is directly connected to the Planck scale physics.



Is the SM valid to the Planck/string scale?

Y. Hamada, K. Oda and HK: arXiv:1210.2358 ,
arXiv:1305.7055 , 1308.6651

In order to answer the question, we consider the SM Lagrangian with cutoff momentum Λ,

$$\mathcal{L} = (D_\mu \phi_B)^\dagger (D^\mu \phi_B) - m_B^2 \phi_B^\dagger \phi_B - \lambda_B (\phi_B^\dagger \phi_B)^2 + \cdots .$$

and determine the bare parameters in such a way that the observed parameters are recovered.

If no inconsistency arises, it means that the SM can be valid to the energy scale Λ.

The bare couplings λ_B

The bare couplings can be approximated by the running couplings at momentum scale Λ in a mass independent scheme such as MS bar.
(The error can be evaluated once the cutoff scheme is specified, and is turns out to be as small as the two-loop corrections.)

$$\lambda^i{}_B \simeq \lambda^i{}_{\overline{MS}}(\Lambda).$$

$\lambda^i{}_B$: dimensionless couplings

(gauge, Yukawa, Higgs self couplings)

The bare mass $m_B{}^2$

- **In general, the bare mass consists of quadratically divergent part and logarithmically divergent part:**

$$m_B{}^2 = a\Lambda^2 + m_{phys}{}^2\left(b_1 \log\left(\frac{\Lambda^2}{m_{phys}{}^2}\right) + \cdots\right).$$

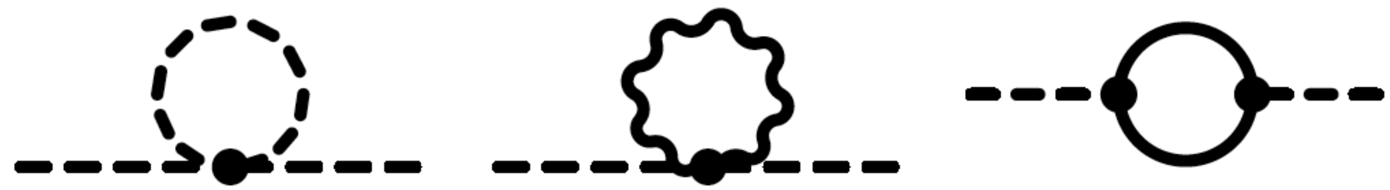
- **Here we consider only the first part, or we simply assume**

$$m_{phys}{}^2 = 0.$$

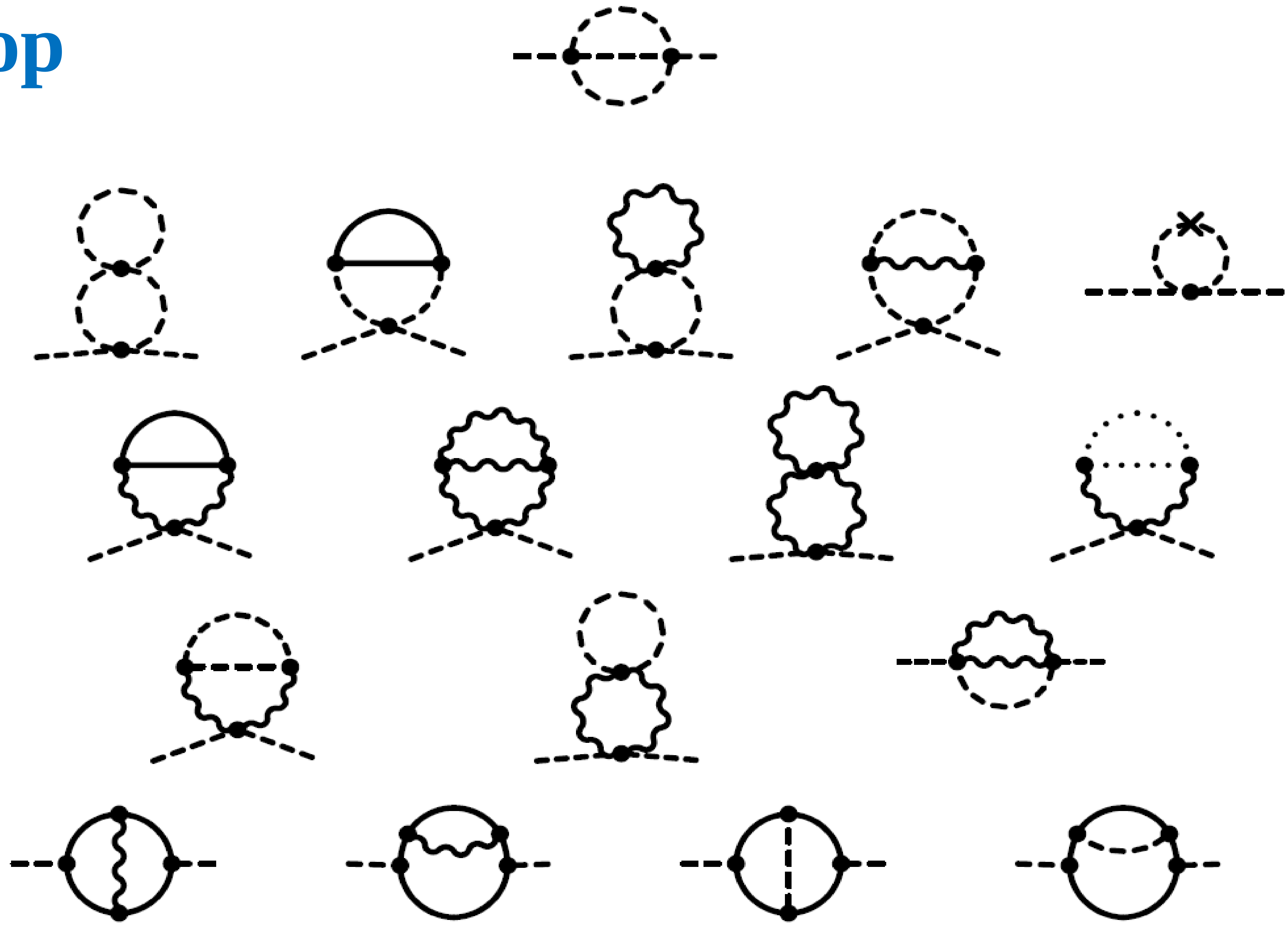
- $m_B{}^2$ **is determined by an order by order perturbative calculation in the *bare* couplings by demanding** $m_{phys}{}^2 = 0:$

$$m_B^2 = m_{B,\,\text{0-loop}}^2 + m_{B,\,\text{1-loop}}^2 + m_{B,\,\text{2-loop}}^2 + \cdots$$

1-loop



2-loop



$$m^2_{B,\,\text{1-loop}} = -\left(6\lambda_B + \frac{3}{4}g^2_{YB} + \frac{9}{4}g^2_{2B} - 6y^2_{tB}\right) I_1$$

$$m^2_{B,\,\text{2-loop}} = -\left\{9y^4_{tB} + y^2_{tB}\left(-\frac{7}{12}g^2_{YB} + \frac{9}{4}g^2_{2B} - 16g^2_{3B}\right)\right.$$

$$-\frac{87}{16}g^4_{YB} - \frac{63}{16}g^4_{2B} - \frac{15}{8}g^2_{YB}g^2_{2B}$$

$$\left. + \lambda_B\left(-18y^2_{tB} + 3g^2_{YB} + 9g^2_{2B}\right) - 12\lambda^2_B\right\} I_2$$

$$I_1 \simeq \frac{1}{16\pi^2}\Lambda^2, \quad I_2 \simeq \frac{1}{200}I_1.$$

It turns out that 2-loop contribution is small.

Renormalization group equation

$$\frac{dg_Y}{dt} = \frac{1}{16\pi^2}\frac{41}{6}g_Y^3 + \frac{g_Y^3}{(16\pi^2)^2}\left(\frac{199}{18}g_Y^2 + \frac{9}{2}g_2^2 + \frac{44}{3}g_3^2 - \frac{17}{6}y_t^2\right),$$

$$\frac{dg_2}{dt} = -\frac{1}{16\pi^2}\frac{19}{6}g_2^3 + \frac{g_2^3}{(16\pi^2)^2}\left(\frac{3}{2}g_Y^2 + \frac{35}{6}g_2^2 + 12g_3^2 - \frac{3}{2}y_t^2\right),$$

$$\frac{dg_3}{dt} = -\frac{7}{16\pi^2}g_3^3 + \frac{g_3^3}{(16\pi^2)^2}\left(\frac{11}{6}g_Y^2 + \frac{9}{2}g_2^2 - 26g_3^2 - 2y_t^2\right),$$

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 - \frac{17}{12}g_Y^2 - \frac{9}{4}g_2^2 - 8g_3^2\right) + \frac{y_t}{(16\pi^2)^2}\left(-12y_t^2 + 6\lambda^2 - 12\lambda y_t^2 + \frac{131}{16}g_Y^2y_t^2 + \frac{225}{16}g_2^2y_t^2 + 36g_3^2y_t^2 + \frac{1187}{216}g_Y^4 - \frac{23}{4}g_2^4 - 108g_3^4 - \frac{3}{4}g_Y^2g_2^2 + 9g_2^2g_3^2 + \frac{19}{9}g_3^2g_Y^2\right),$$

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left(24\lambda^2 - 3g_Y^2\lambda - 9g_2^2\lambda + \frac{3}{8}g_Y^4 + \frac{3}{4}g_Y^2g_2^2 + \frac{9}{8}g_2^4 + 12\lambda y_t^2 - 6y_t^4\right) + \frac{1}{(16\pi^2)^2}\left\{-312\lambda^3 + 36\lambda^2(g_Y^2 + 3g_2^2) - \lambda\left(\frac{629}{24}g_Y^4 - \frac{39}{4}g_Y^2g_2^2 + \frac{73}{8}g_2^4\right) + \frac{305}{16}g_2^6 - \frac{289}{48}g_Y^2g_2^4 - \frac{559}{48}g_Y^4g_2^2 - \frac{379}{48}g_Y^6 - 32g_3^2y_t^4 - \frac{8}{3}g_Y^2y_t^4 - \frac{9}{4}g_2^4y_t^2 + \lambda y_t^2\left(\frac{85}{6}g_Y^2 + \frac{45}{2}g_2^2 + 80g_3^2\right) + g_Y^2y_t^2\left(-\frac{19}{4}g_Y^2 + \frac{21}{2}g_2^2\right) - 144\lambda^2y_t^2 - 3\lambda y_t^4 + 30y_t^6\right\},$$

Initial values

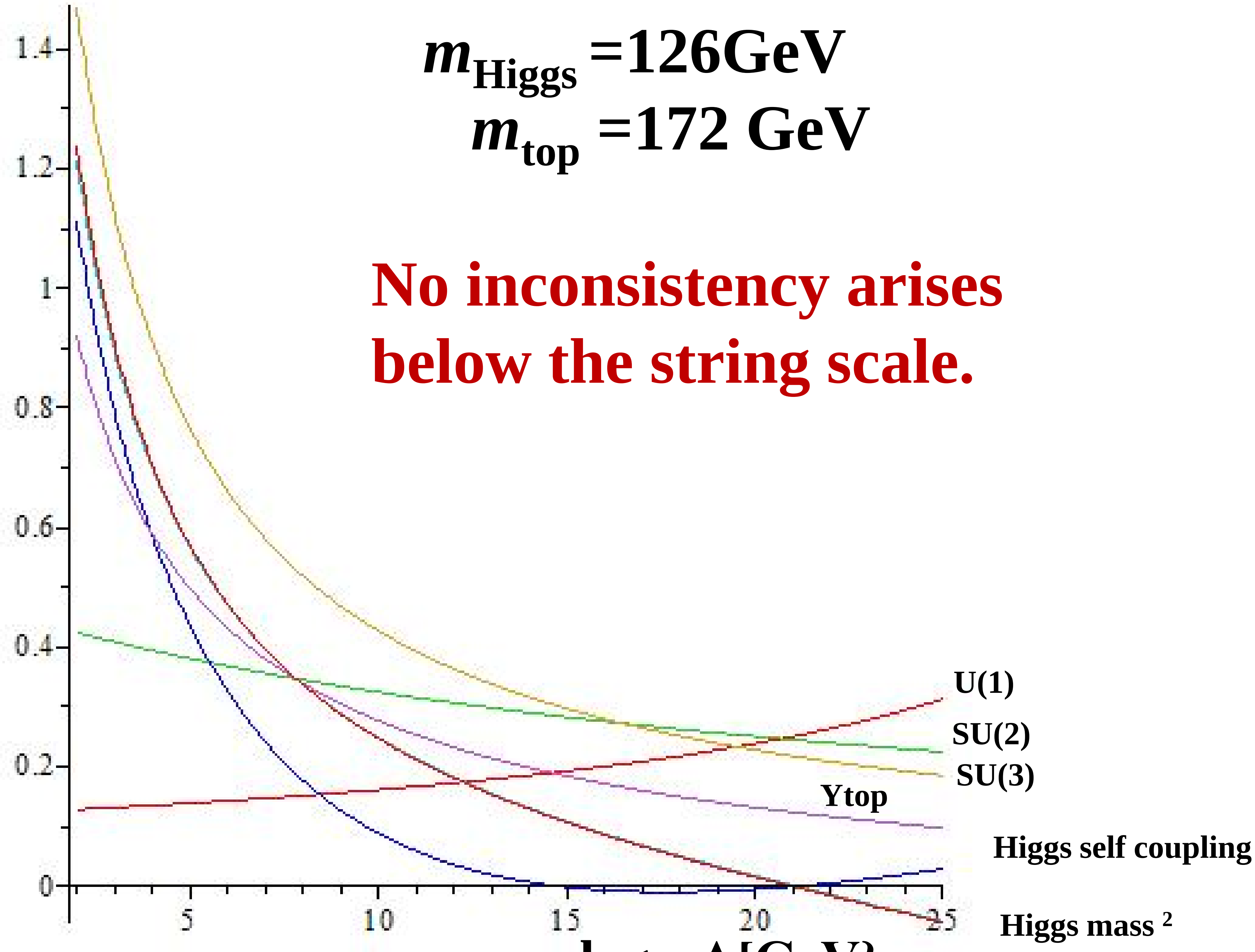
$$g_s(m_t^{\rm pole}) = 1.1645 + 0.0031\left(\frac{\alpha_s(m_Z) - 0.1184}{0.0007}\right) - 0.00046\left(\frac{m_t^{\rm pole}}{\rm GeV} - 173.15\right),$$

$$\lambda(m_t^{\rm pole}) = 0.12577 + 0.00205\left(\frac{m_H}{\rm GeV} - 125\right) - 0.00004\left(\frac{m_t^{\rm pole}}{\rm GeV} - 173.15\right) \pm 0.00140_{\rm th},$$

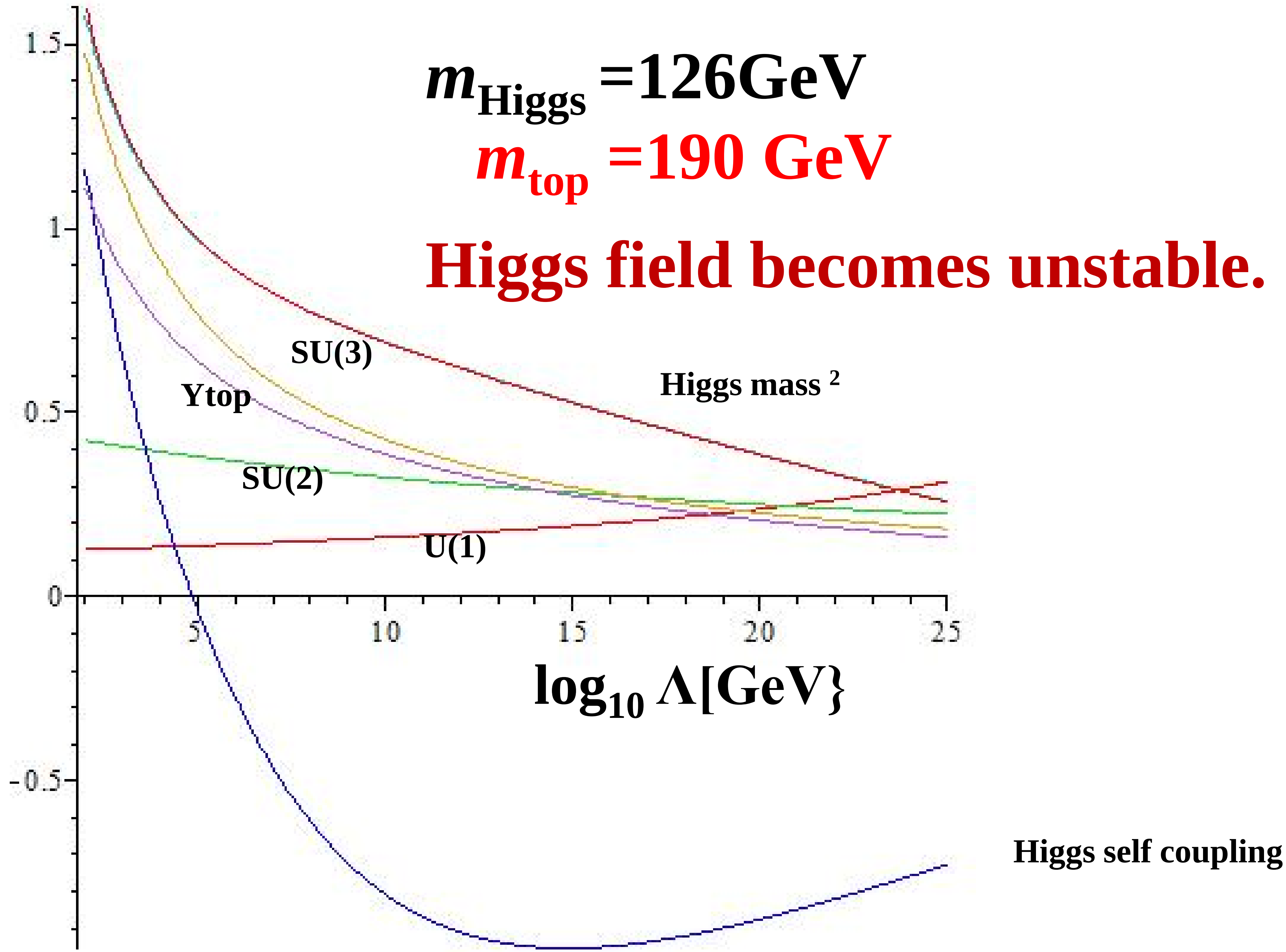
$$y_t(m_t^{\rm pole}) = 0.93587 + 0.00557\left(\frac{m_t^{\rm pole}}{\rm GeV} - 173.15\right) - 0.00003\left(\frac{m_H}{\rm GeV} - 125\right) - 0.00041\left(\frac{\alpha_s(m_Z) - 0.1184}{0.0007}\right) \pm 0.00200_{\rm th}.$$

G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori and A. Strumia,
Higgs mass and vacuum stability in the Standard Model at NNLO," JHEP 1208 (2012) 098 [arXiv:1205.6497 [hep-ph]].

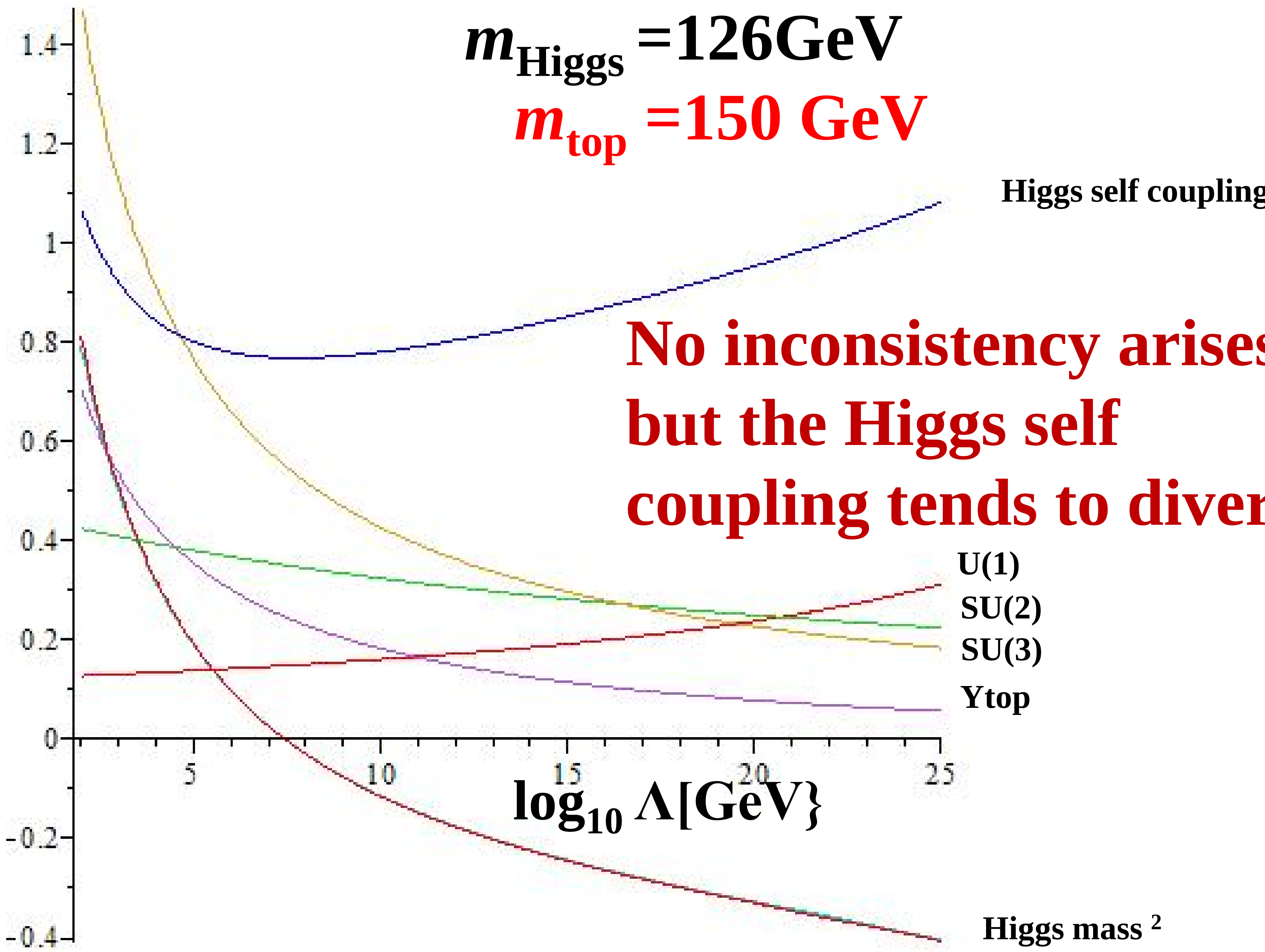
Bare parameters of the cutoff theory (1)



Bare parameters of the cutoff theory (2)



Bare parameters of the cutoff theory (3)



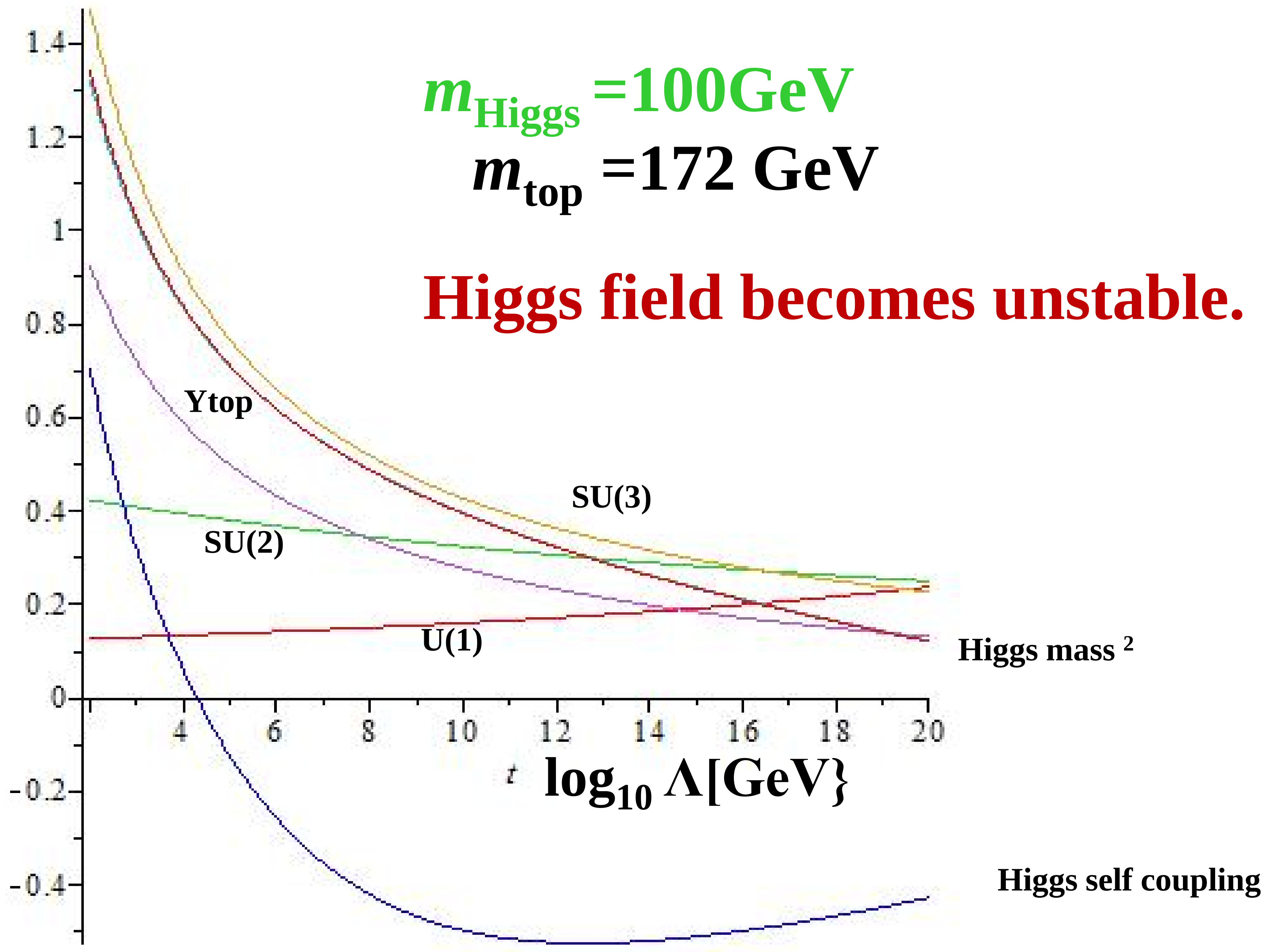
No inconsistency arises, but the Higgs self coupling tends to diverge.

Bare parameters of the cutoff theory (4)

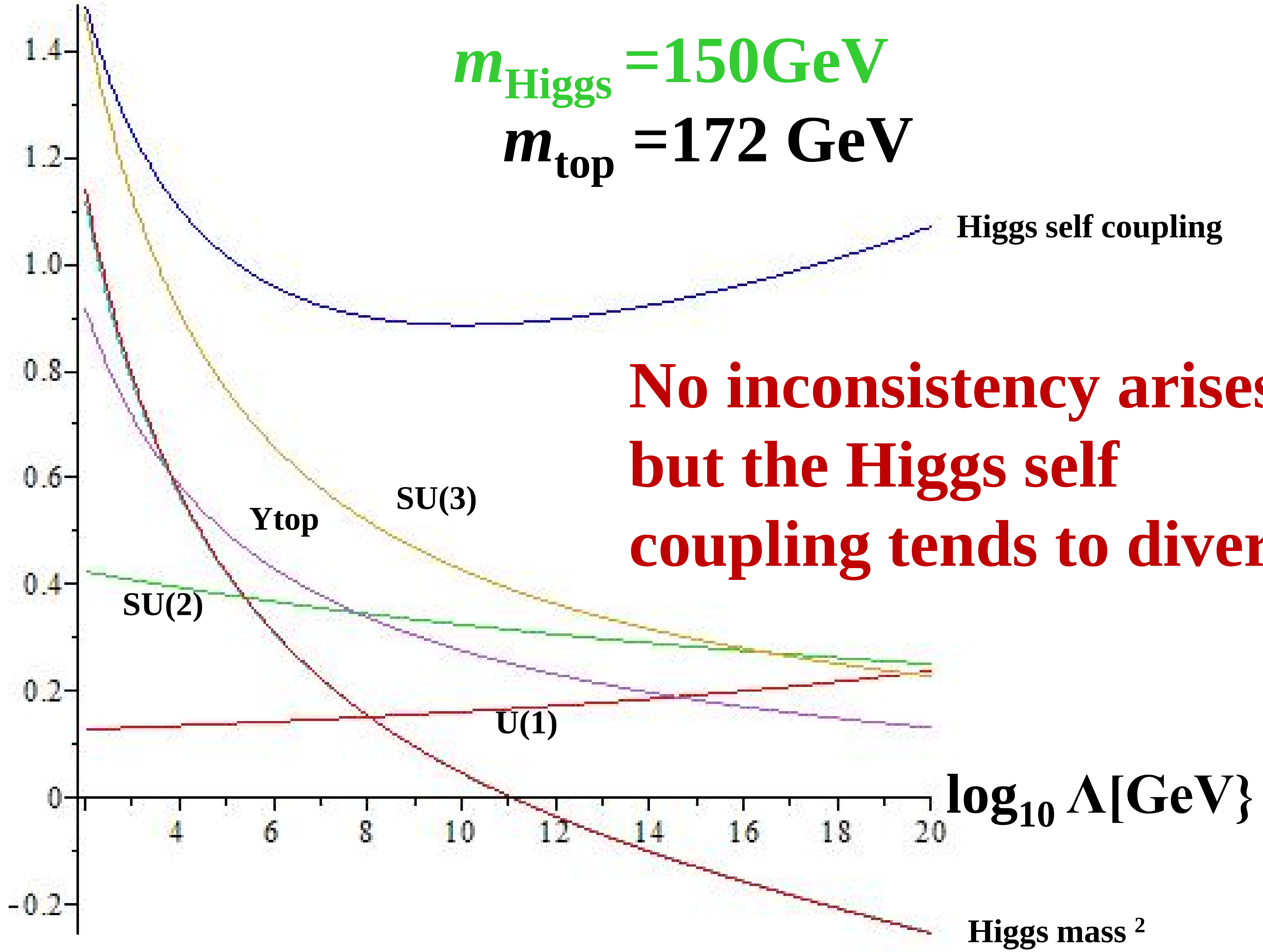
m_{Higgs} =100GeV

m_{top} =172 GeV

Higgs field becomes unstable.

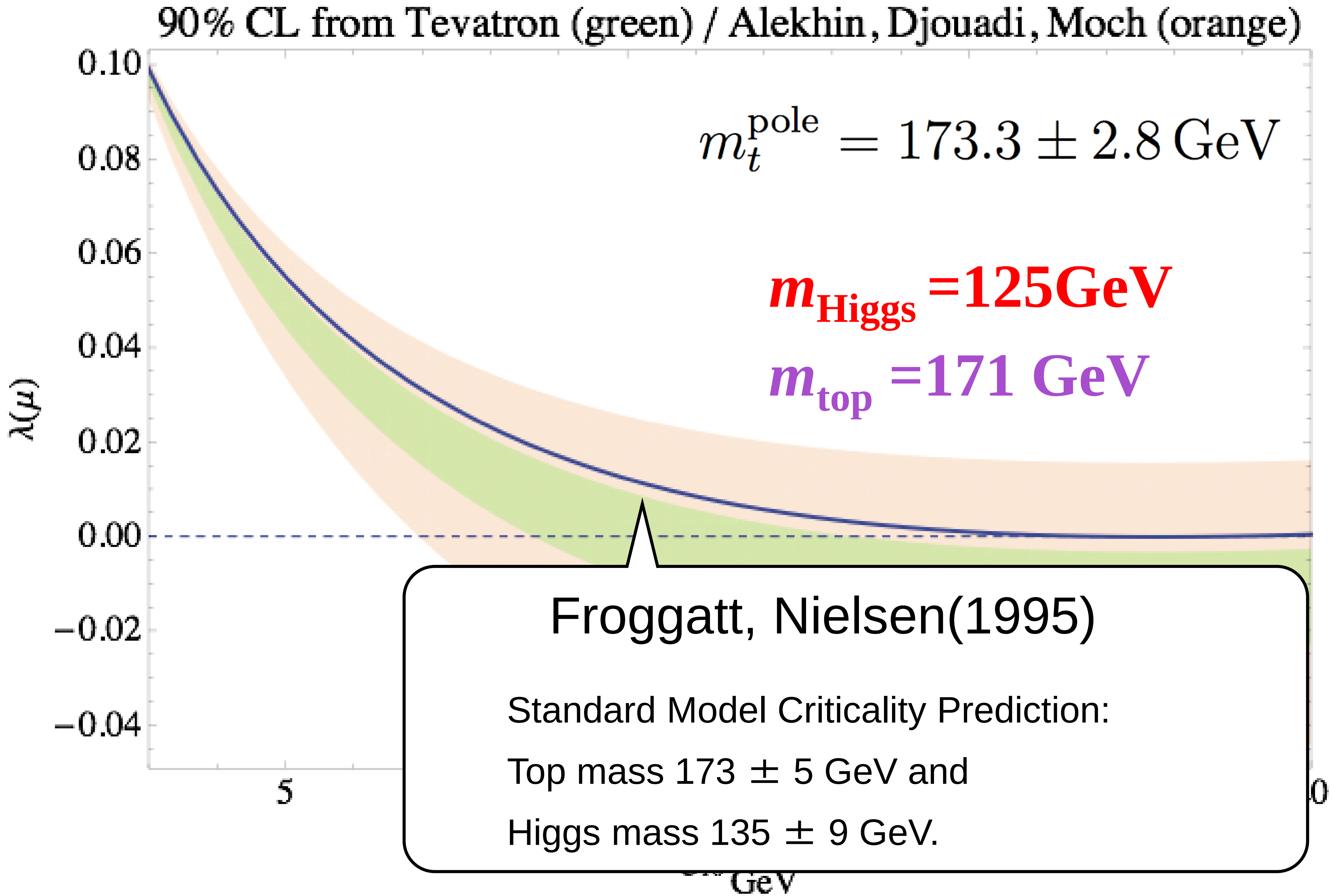


Bare parameters of the cutoff theory (5)



No inconsistency arises, but the Higgs self coupling tends to diverge.

Stability of the potential



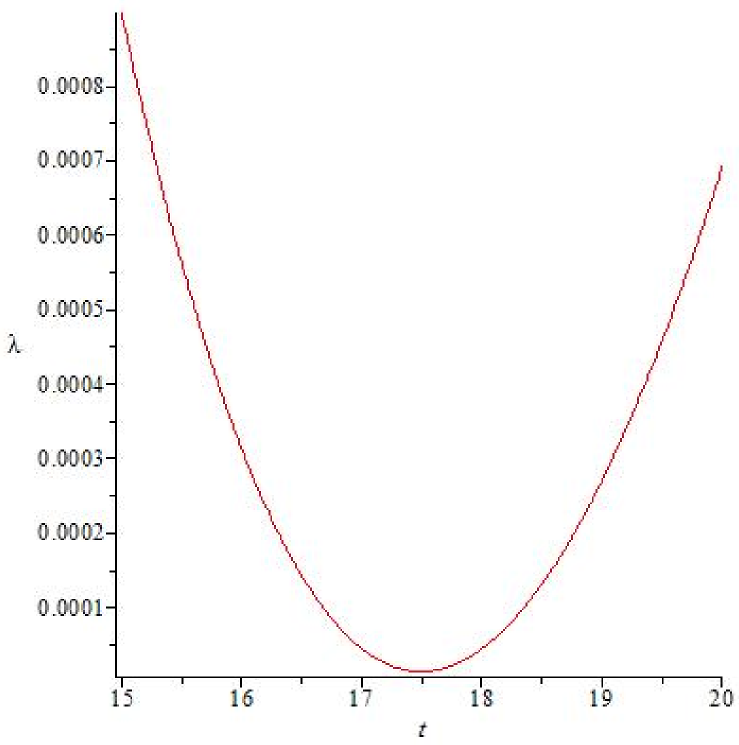
Froggatt, Nielsen(1995)

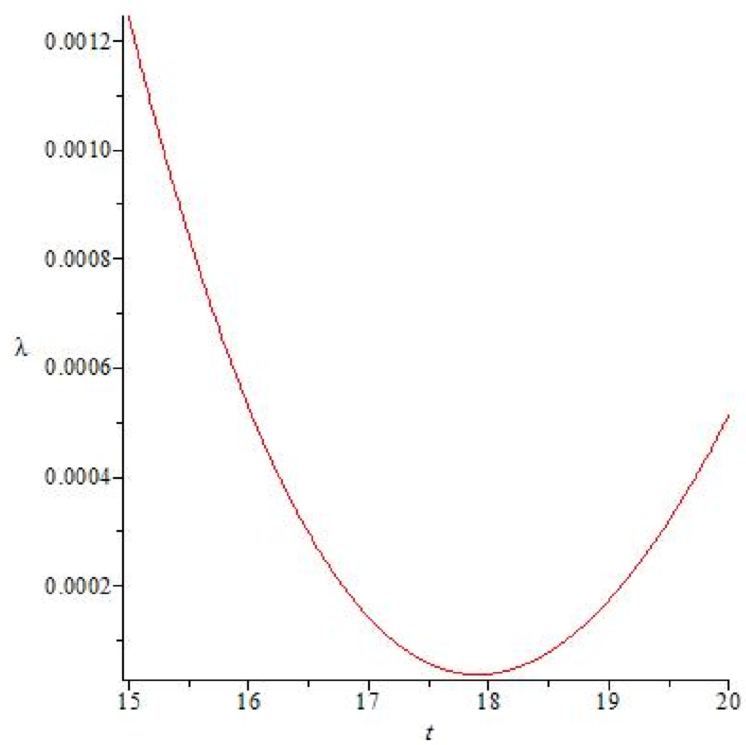
Standard Model Criticality Prediction:

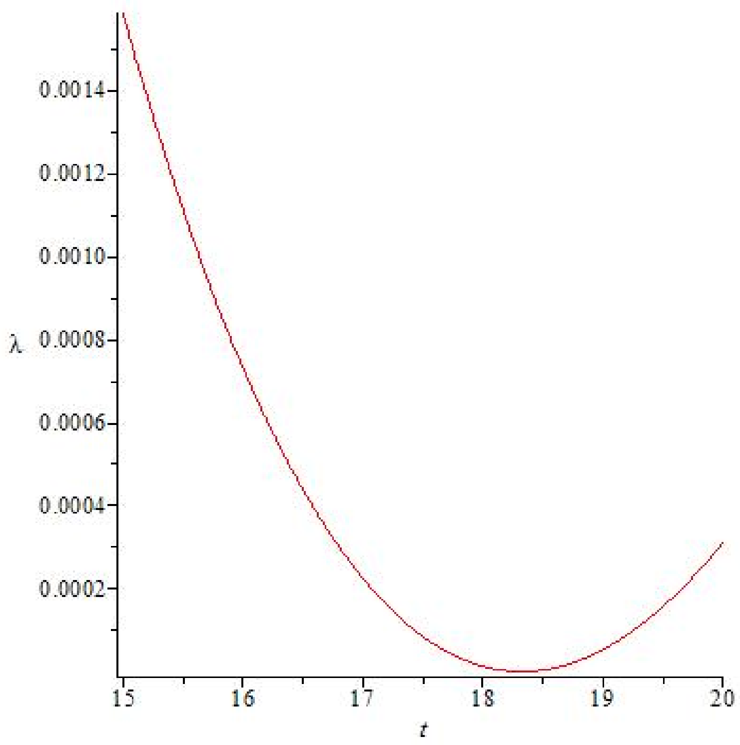
Top mass 173 ± 5 GeV and

Higgs mass 135 ± 9 GeV.

Froggatt Nielsen by the recent values







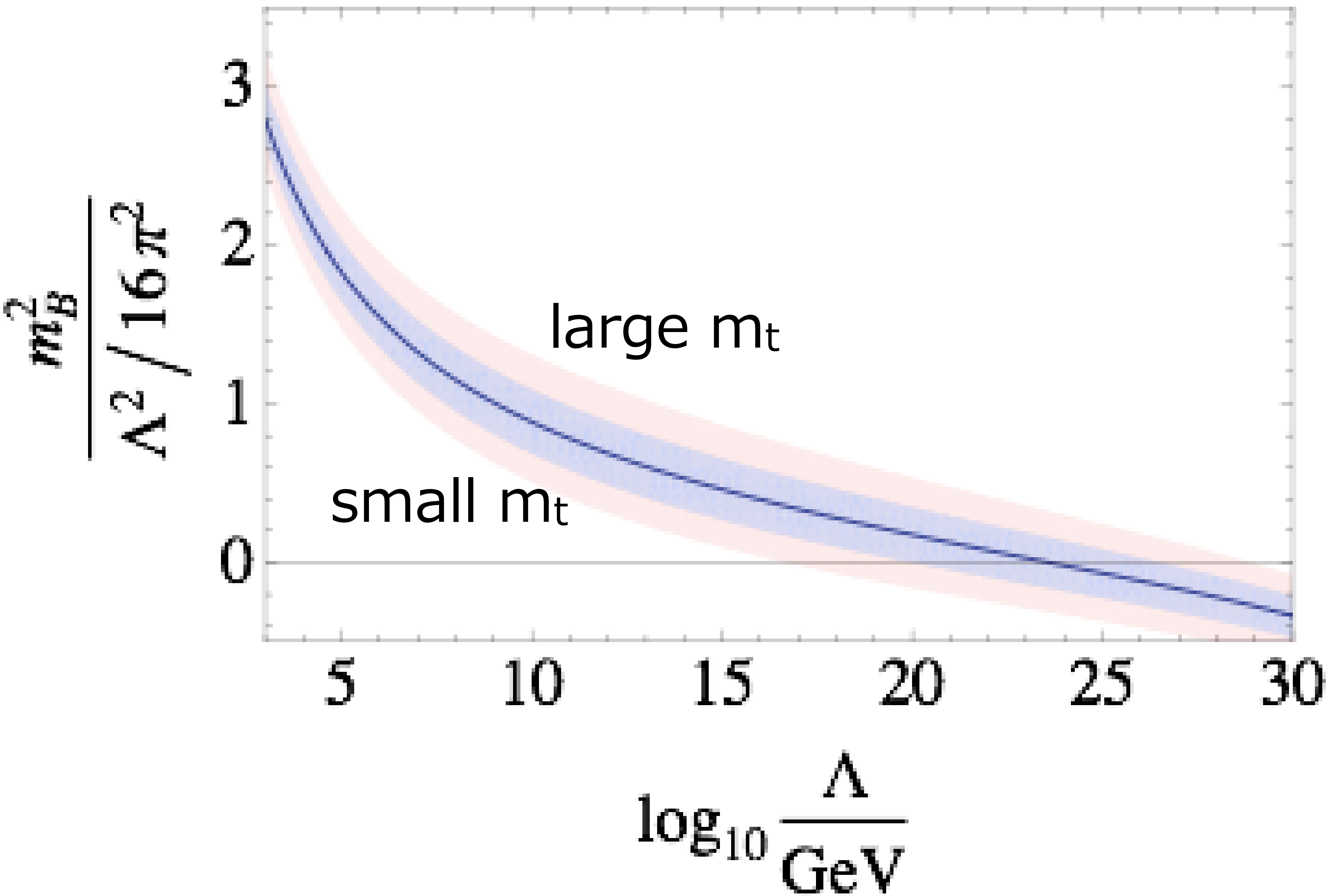
m_{Higgs} =125GeV
m_{top} =171.31 GeV

m_{Higgs} =127GeV
m_{top} =172.29 GeV

m_{Higgs} =129GeV
m_{top} =173.26 GeV

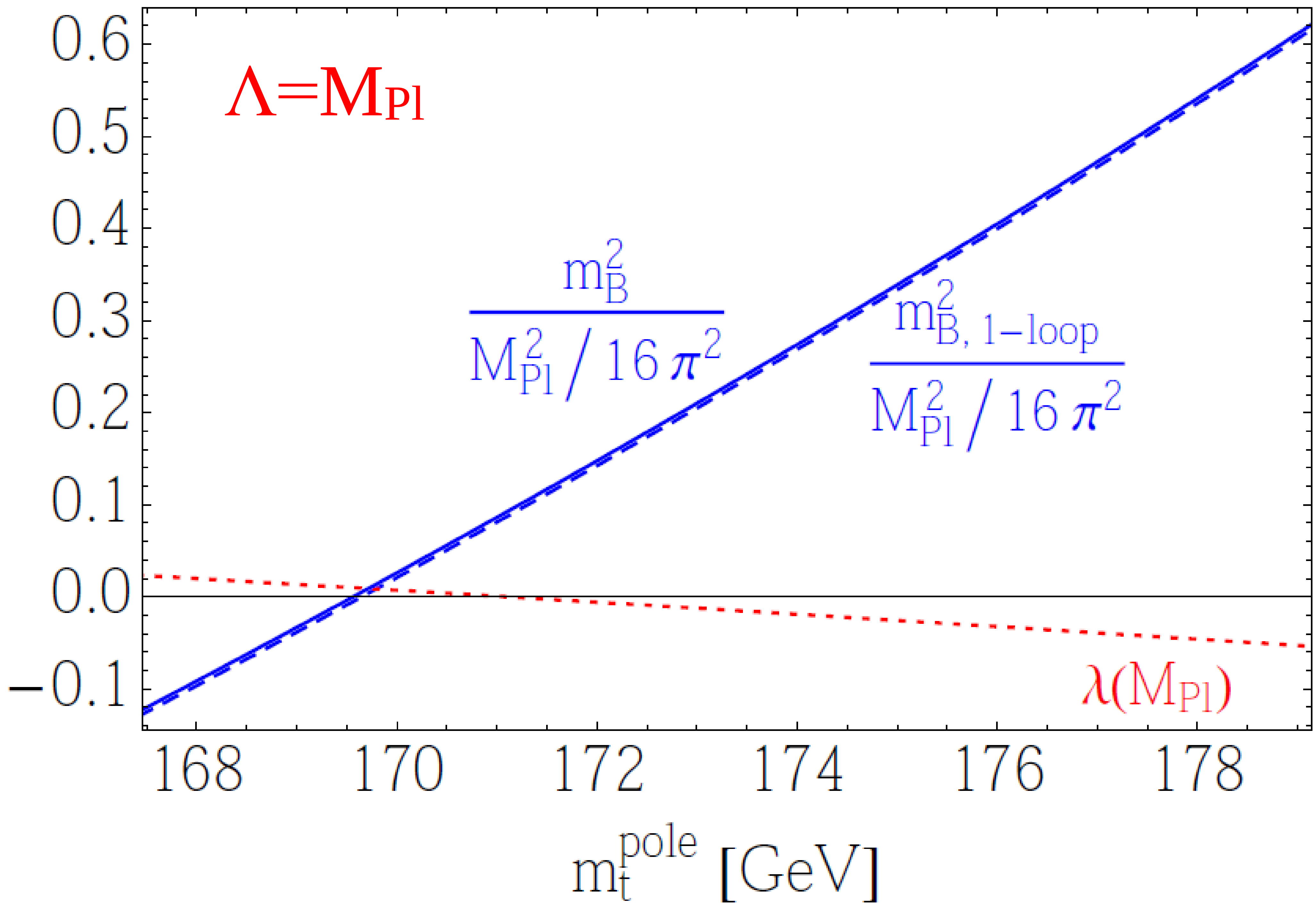
Cut off dependence of the bare mass

$$m_t^{\text{pole}} = 173.3 \pm 2.8\,\text{GeV}$$



Both m_B^2 and λ vanish around the Planck scale

Bare Higgs mass becomes zero if mt=170GeV.
Quadratic coupling vanishes if mt=171GeV.



Triple coincidence

Three quantities,

$$\lambda_B,\ \beta_\lambda\left(\lambda_B\right),\ m_B$$

become close to zero around the Planck/string scale.

Summary of the Higgs bare parameters

- The SM can be valid to the string scale.
- The Higgs mass seems to be just on the stability bound.

  Nature likes the marginal stability.
- The bare Higgs mass becomes close to zero at the string scale. It implies that Higgs particle comes from a massless state of string, which does not receive a large stringy loop correction.
- The Higgs self coupling and the beta function also become close to zero at the string scale.

  Higgs potential becomes almost flat around the string scale.

  Iso's talk   origin of the electro-weak scale

  Higgs field can play the roll of inflaton.

Higgs inflation

(1) conservative approach

We trust the effective potential only below the cutoff scale, and try to make bound on the parameters.

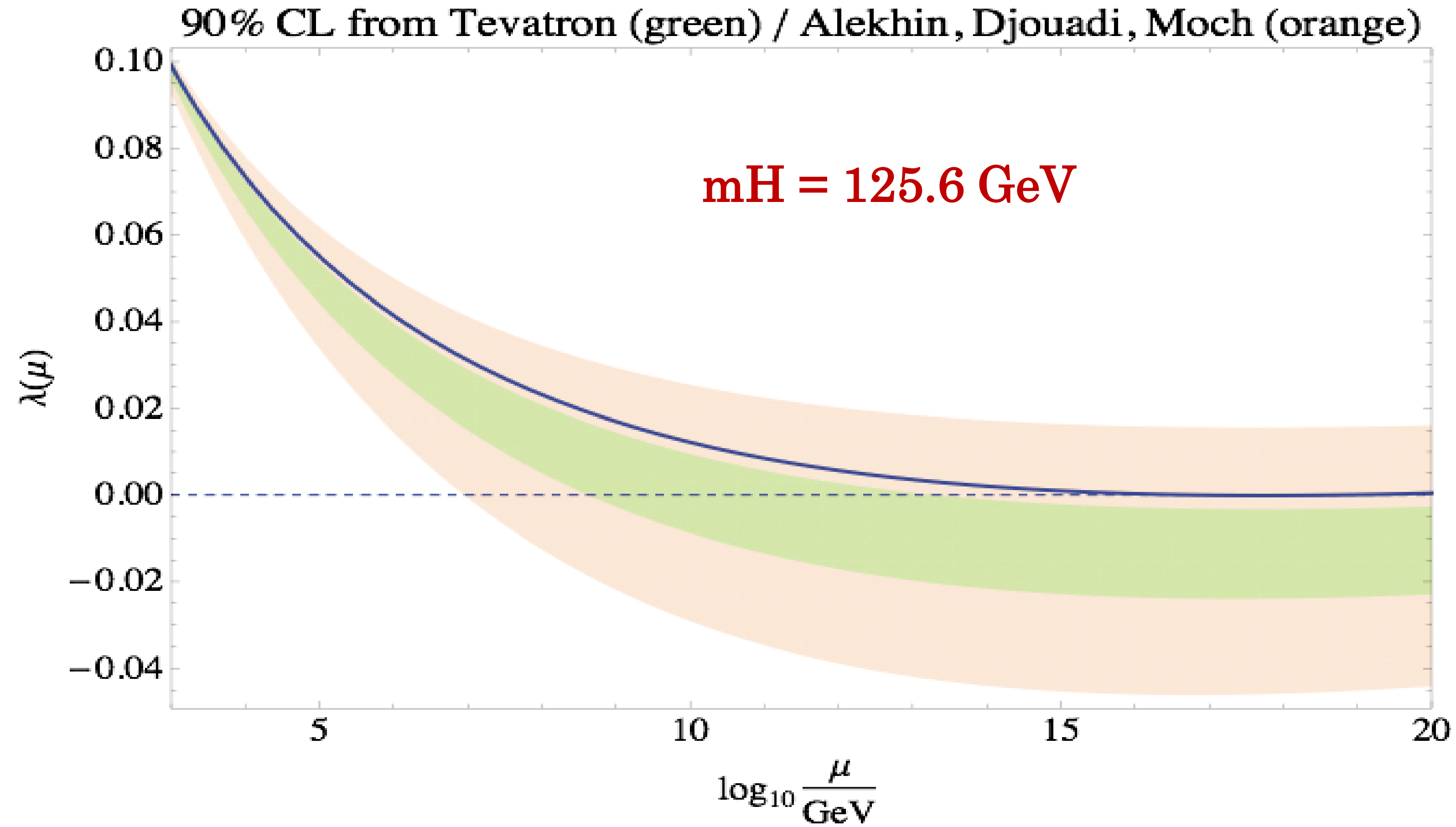
Hamada, Oda and HK: arXiv: 1308.6651

(2) radical approach

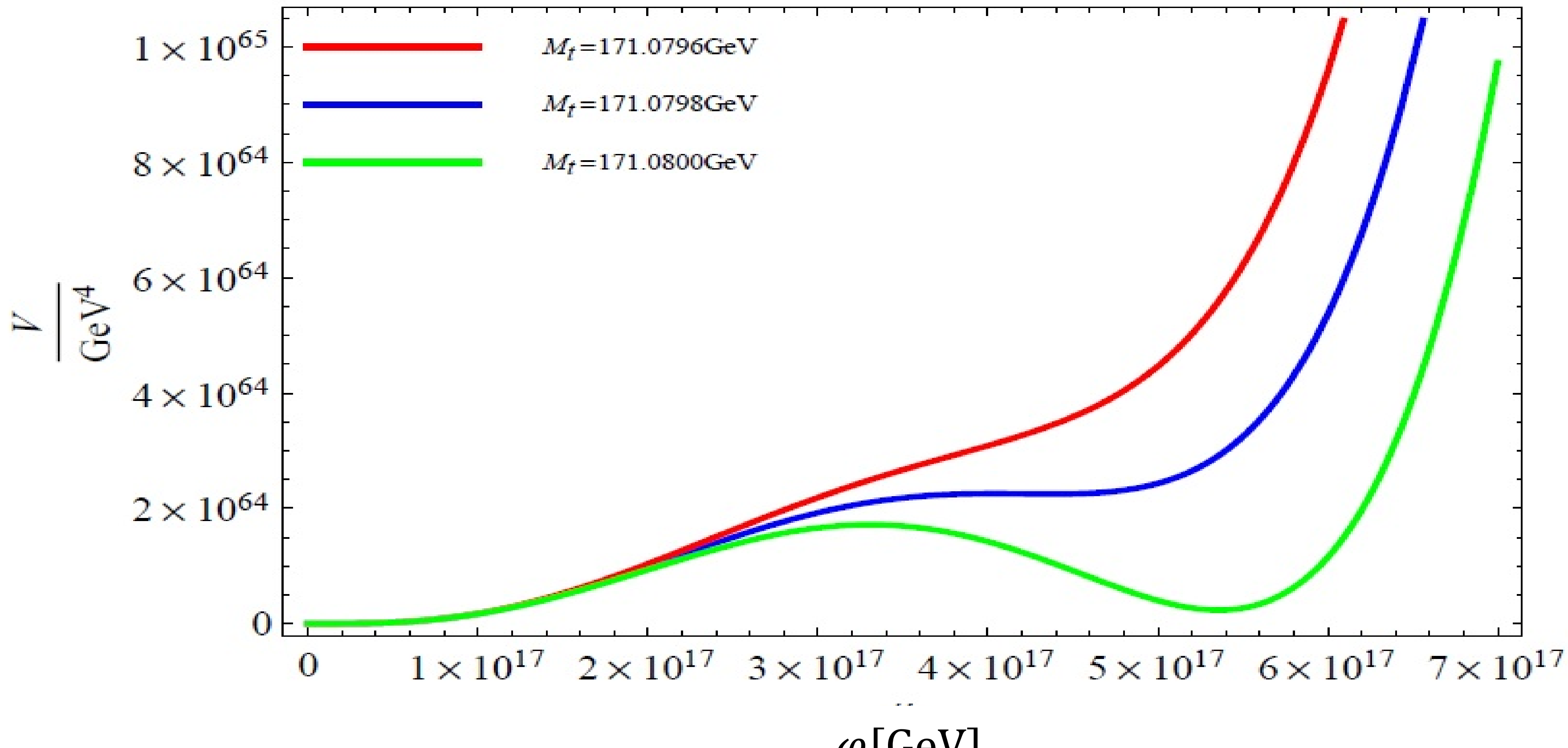
We trust the flat potential including the inflection point.
We assume that nature does fine tunings if they are necessary.
We introduce a non-minimal coupling.

Hamada, Oda, Park and HK, arXiv: 1403.5043
"Higgs inflation still alive"
Cook. Krauss, Lawrence , Long and Sabharwal, arXiv: 1403.4971
"Is Higgs Inflation Dead? "
Bezrukov and Shaposhnikov, arXiv: 1403.6078
"Higgs inflation at the critical point"

Higgs potential



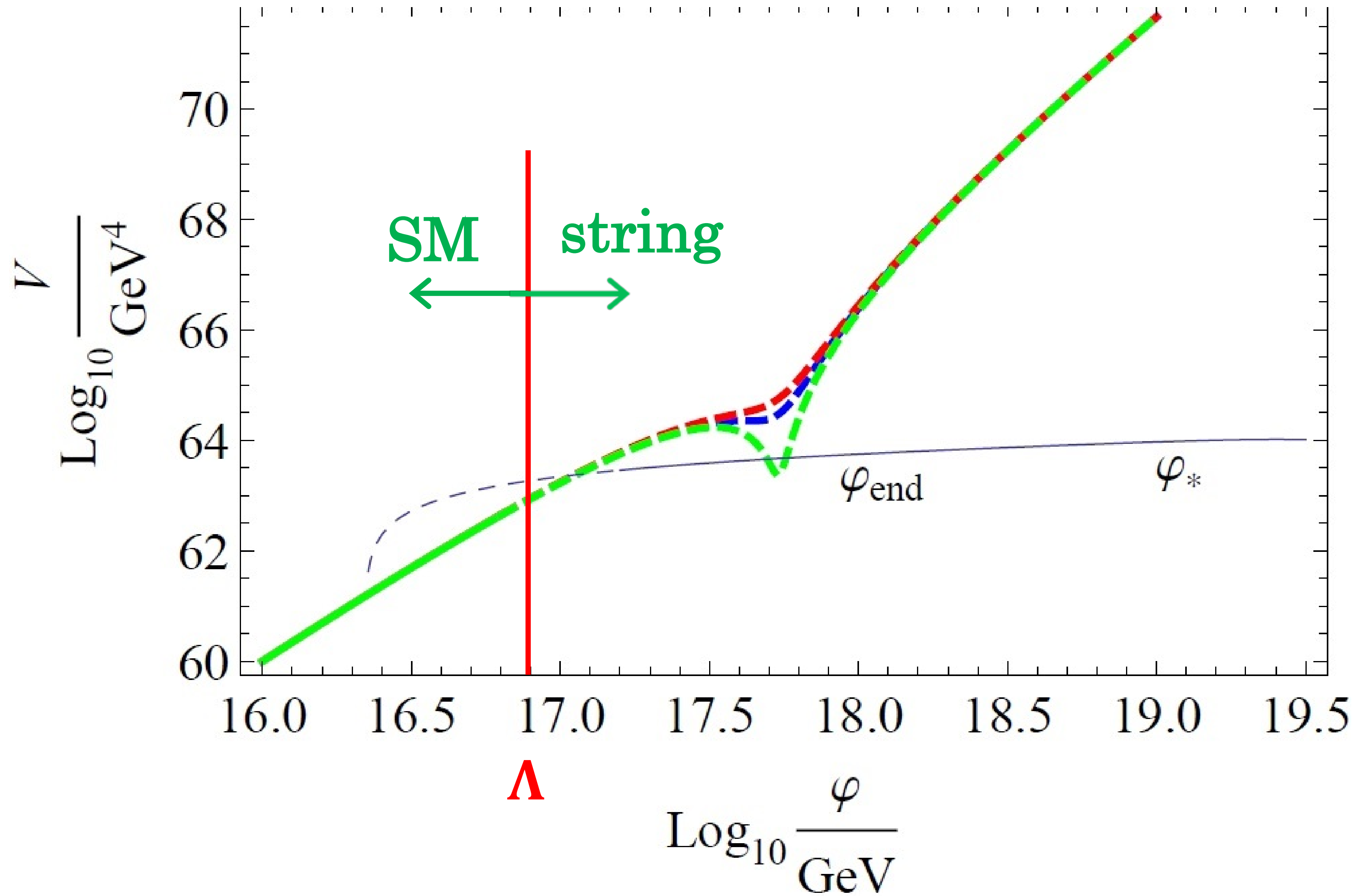
$$V(\varphi)=\frac{\lambda(\varphi)}{4}\varphi^4$$



Conservative approach

The effective potential we have seen can be trusted only below the cutoff scale Λ.

Above Λ it is not described by SM but we need string theory.



Necessary conditions

At present we do not have enough knowledge above Λ.

But there is a possibility that the Higgs potential in the stringy region is almost constant, and the inflation occurs in this region.

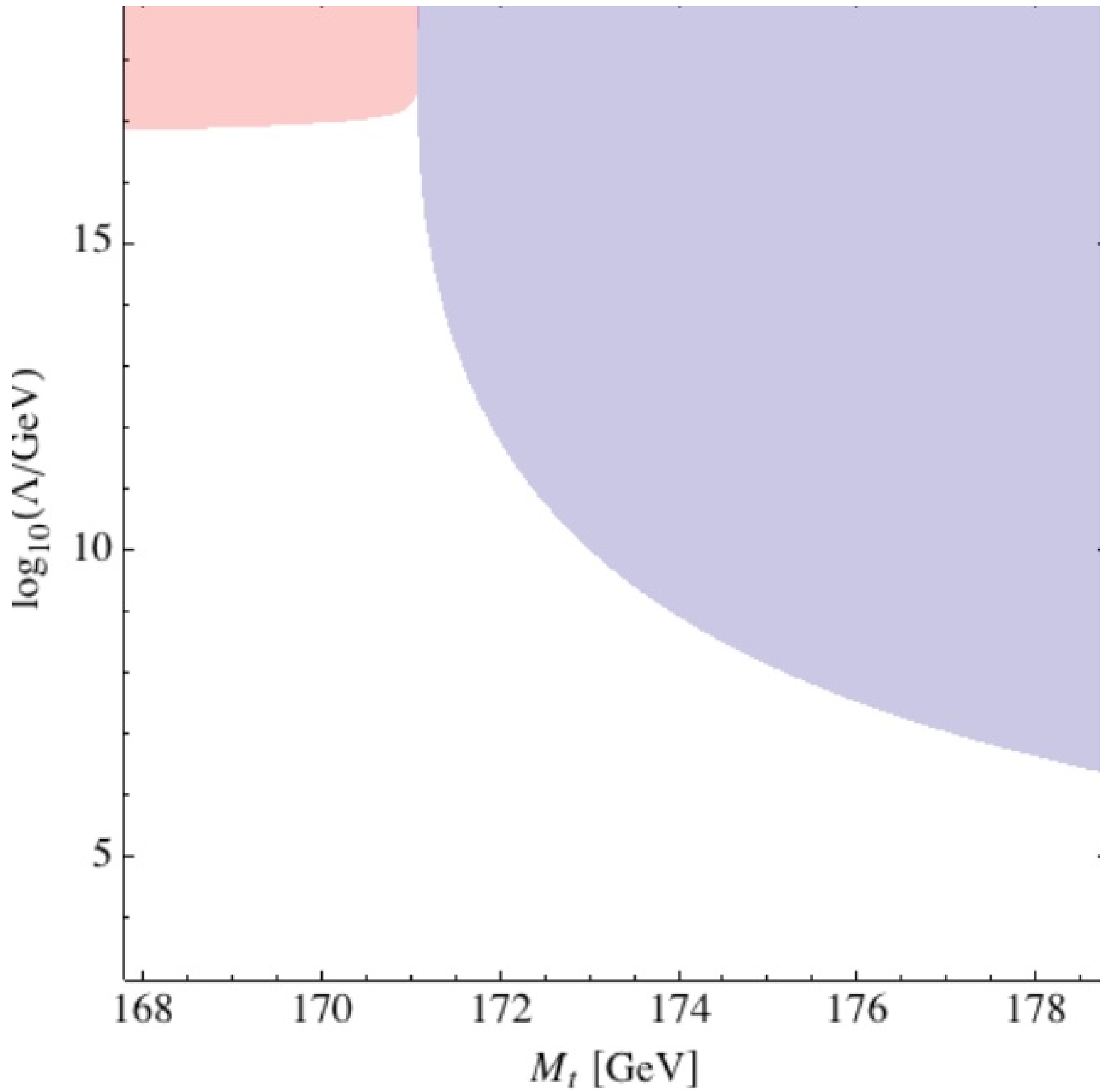
In order for this scenario to work, the necessary conditions are

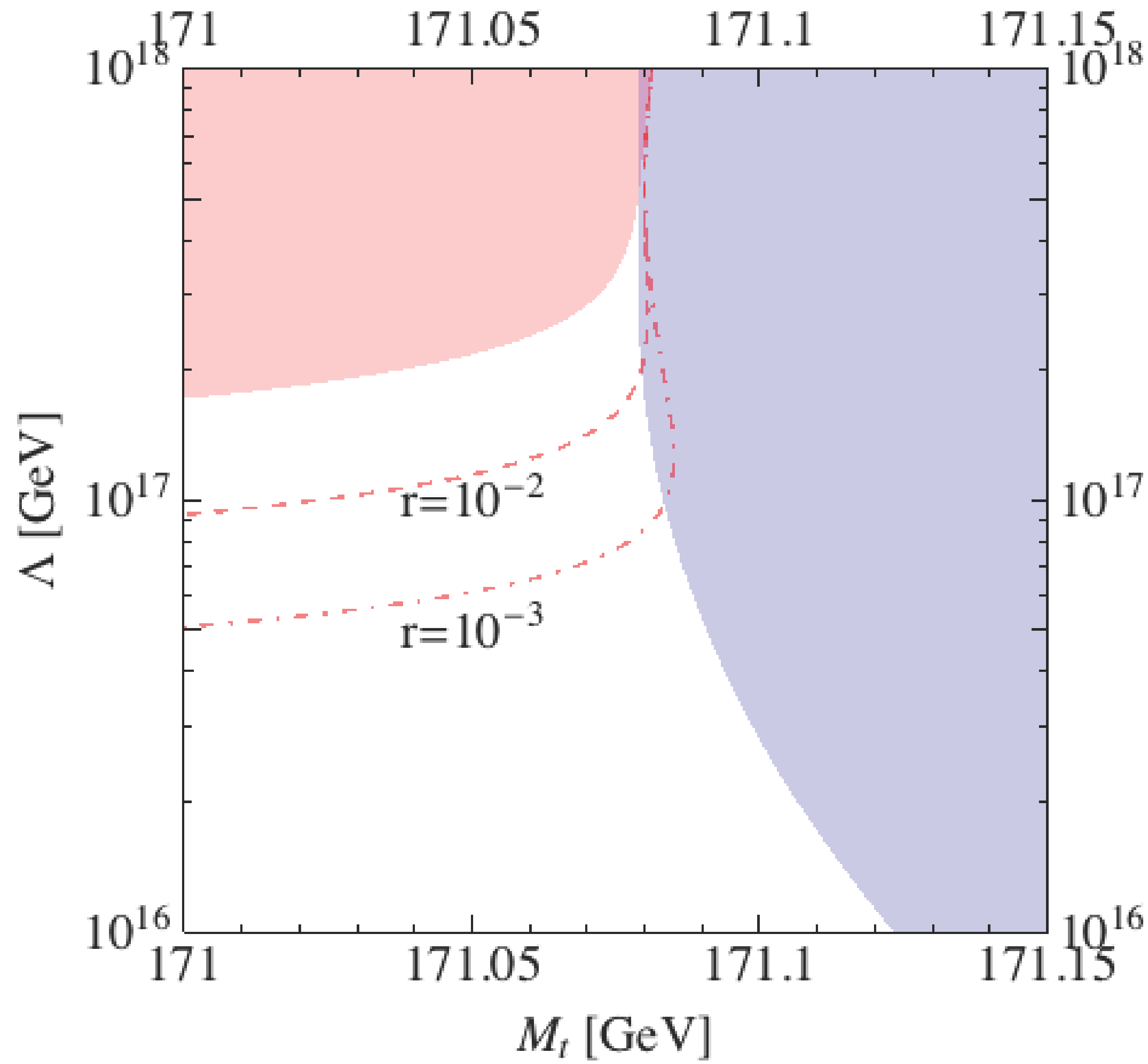
$$\frac{d}{d\varphi} V_{SM} > 0 \ \text{ for } \varphi < \Lambda \,,$$

$$V_{SM}\left(\Lambda\right) < V_{*}.$$

where

$$V_{*} = \frac{3\pi^{2} A_{S}}{2} r_{*} M_{P}{}^{4} = 1.3 \times 10^{65} \times \left(\frac{r_{*}}{0.11}\right) \mathrm{GeV}^{4}.$$





$$m_H = 125.6\,\mathrm{GeV}$$

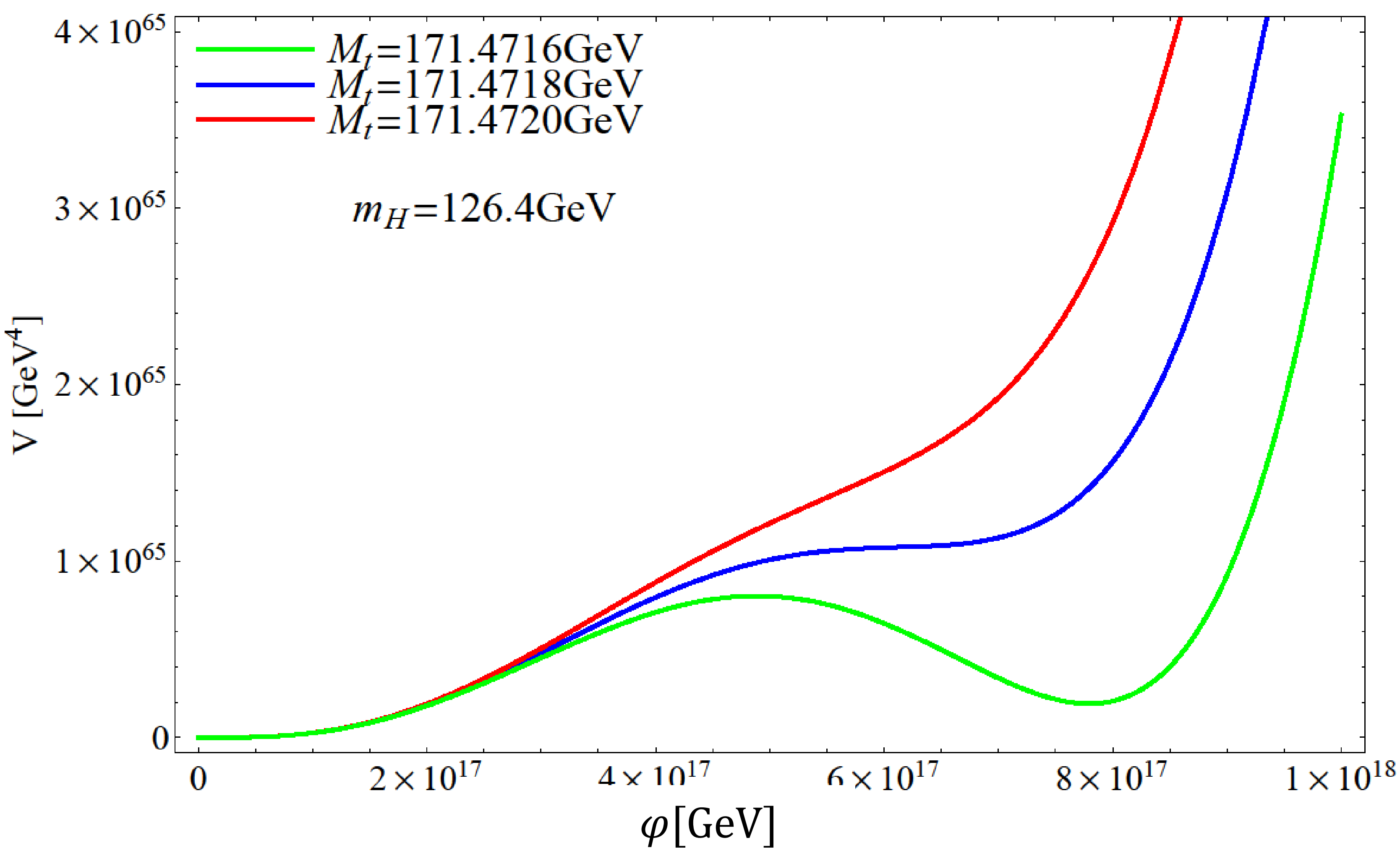
$$r = 0.1$$

Higgs inflation is possible if the cutoff scale is $\sim 10^{17}\,\mathrm{GeV}$.

Radical approach

We assume that the Higgs potential can be trusted up to the inflection point region.

The naïve guess is that inflation occurs if the parameters are tuned such that the inflection point almost becomes a saddle point.

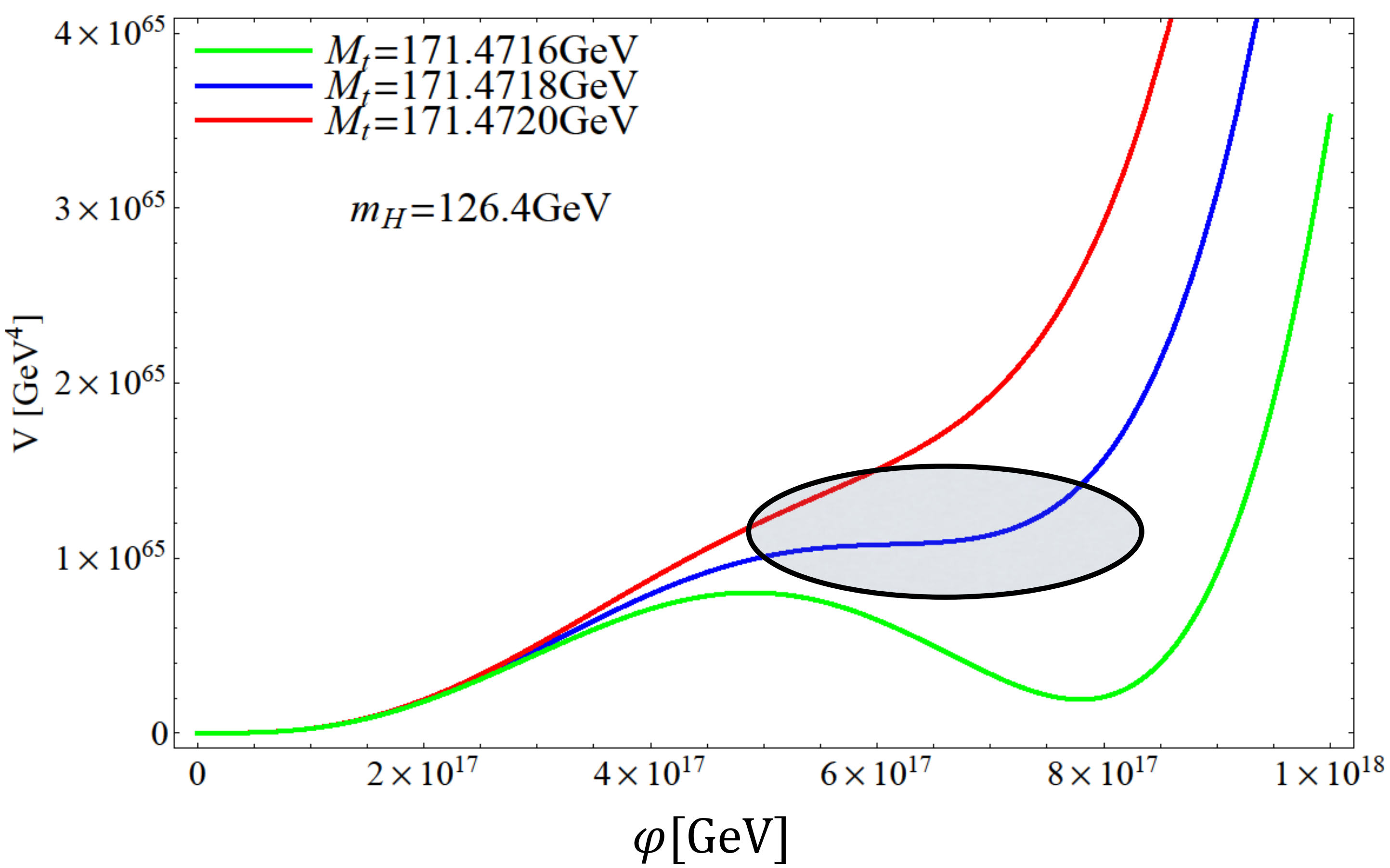


However, this inflection point can not produce a realistic inflation:

$$|\eta| < 1 \quad \Rightarrow \quad \varphi \sim \varphi_{\text{inflection}}$$

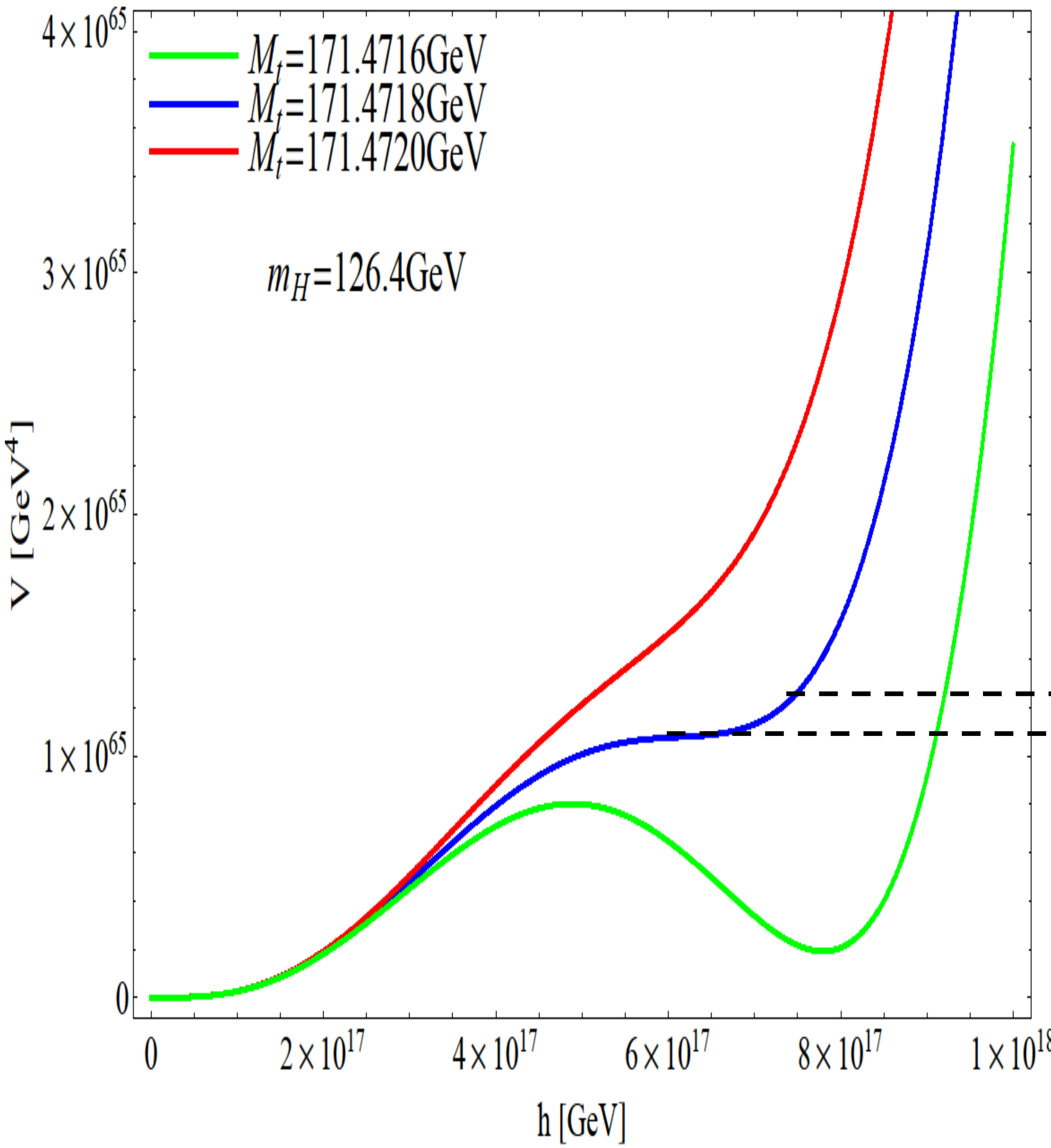
$$\text{sufficient } N_* \text{ in this region } \Rightarrow \text{ small } V'(\varphi)$$

$$\Rightarrow \text{ too large } A_S \sim V_* / \varepsilon$$



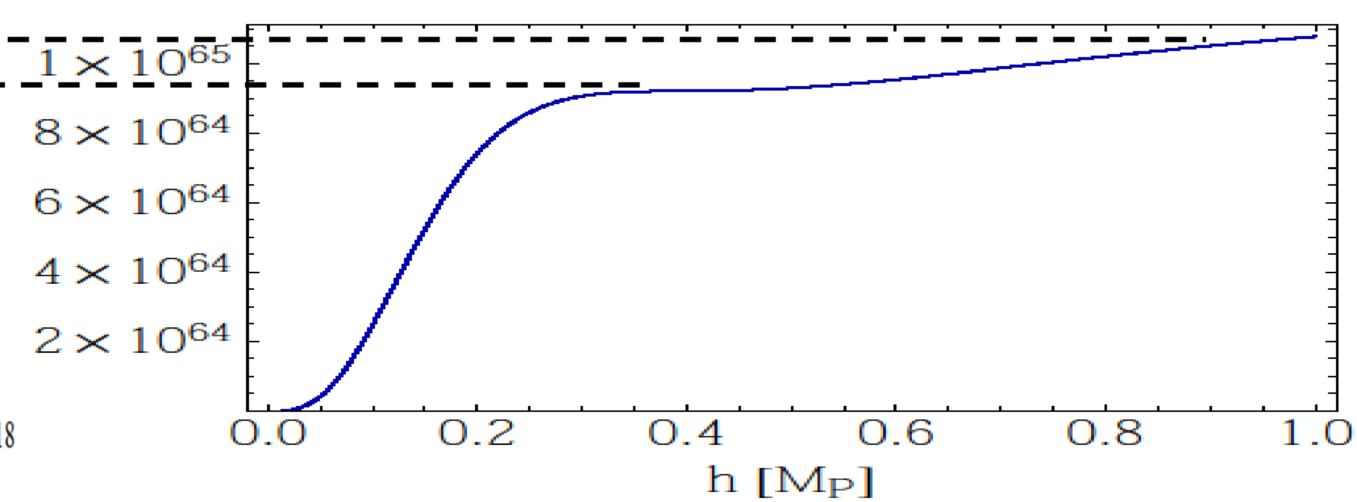
Non minimal Higgs inflation with flat potential

We introduce a non-minimal coupling $\xi R\varphi^2$
as the original Bezrukov-Shaposhnikov's.
Then a realistic Higgs inflation is possible.



In the Einstein frame the effective potential becomes

$$V\left(\varphi_h\right), \quad \varphi_h = \frac{h}{\sqrt{1+\xi h^2 / M_P{}^2}}.$$



Important difference from the original B-S

Higgs potential is small and flat around the string scale from the beginning.

We do not have to lower the potential by the coupling.

The only role of the coupling is to flatten the potential for the large field region.

- ξ need not be so large, and can be as small as 10.
- r is no longer $1/N_*^2$, and can be as large as 0.2.

ξ need not be very large

$$V_* = 1.3 \times 10^{65} \times \left(\frac{r_*}{0.11} \right) \mathrm{GeV}^4.$$

Maximum value of V is $V \sim \lambda(\varphi_h)\varphi_h^4 \sim \lambda(\varphi_h) M_P^4/\xi^2$.

The situation so far is the same as the original BS.

What is new here is that $\lambda(\varphi)$ becomes small around the string scale: $\lambda(\varphi_{h_*}) \sim 10^{-6}$.

This allows rather small values for $\xi \sim 10$.

$\boldsymbol{r}$ can be any value in $\mathbf{0.03 \leq \boldsymbol{r} \leq 0.3}$.

In this case r is no longer $1/N_*^2$.

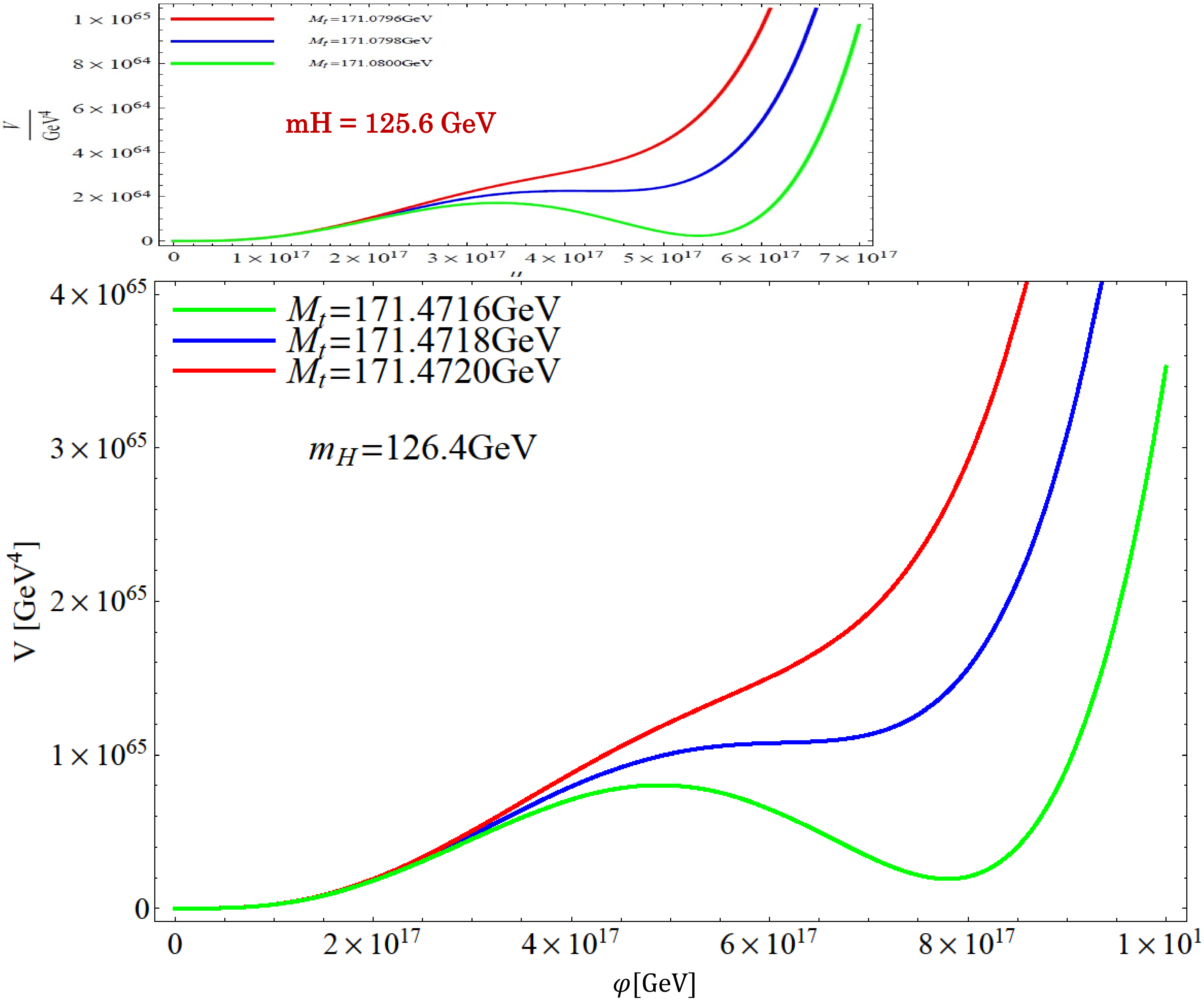
Instead possible values of r is governed by the height of the potential:

$$V_* = 1.3 \times 10^{65} \times \left(\frac{r_*}{0.11} \right) \mathrm{GeV}^4.$$

V_* varies from $\sim 2 \times 10^{64}\ \mathrm{GeV}^4$ to $\sim 2 \times 10^{65}\ \mathrm{GeV}^4$,

when m_H varies from $125.5\ \mathrm{GeV}$ to $126.5\ \mathrm{GeV}$. **(next slide)**

Higgs inflation is still alive even if $\boldsymbol{r} \sim \mathbf{0.03}$.



M_t=171.0796GeV
M_t=171.0798GeV
M_t=171.0800GeV
mH = 125.6 GeV
$\frac{V}{\mathrm{GeV}^4}$
1×10^{65}
8×10^{64}
6×10^{64}
4×10^{64}
2×10^{64}
0
1×10^{17}
2×10^{17}
3×10^{17}
4×10^{17}
5×10^{17}
6×10^{17}
7×10^{17}
M_t=171.4716GeV
M_t=171.4718GeV
M_t=171.4720GeV
m_H=126.4GeV
V [GeV^4]
4×10^{65}
3×10^{65}
2×10^{65}
1×10^{65}
2×10^{17}
4×10^{17}
6×10^{17}
8×10^{17}
1×10^{18}
φ[GeV]

example

$m_H = 126.4\ \mathrm{GeV}$

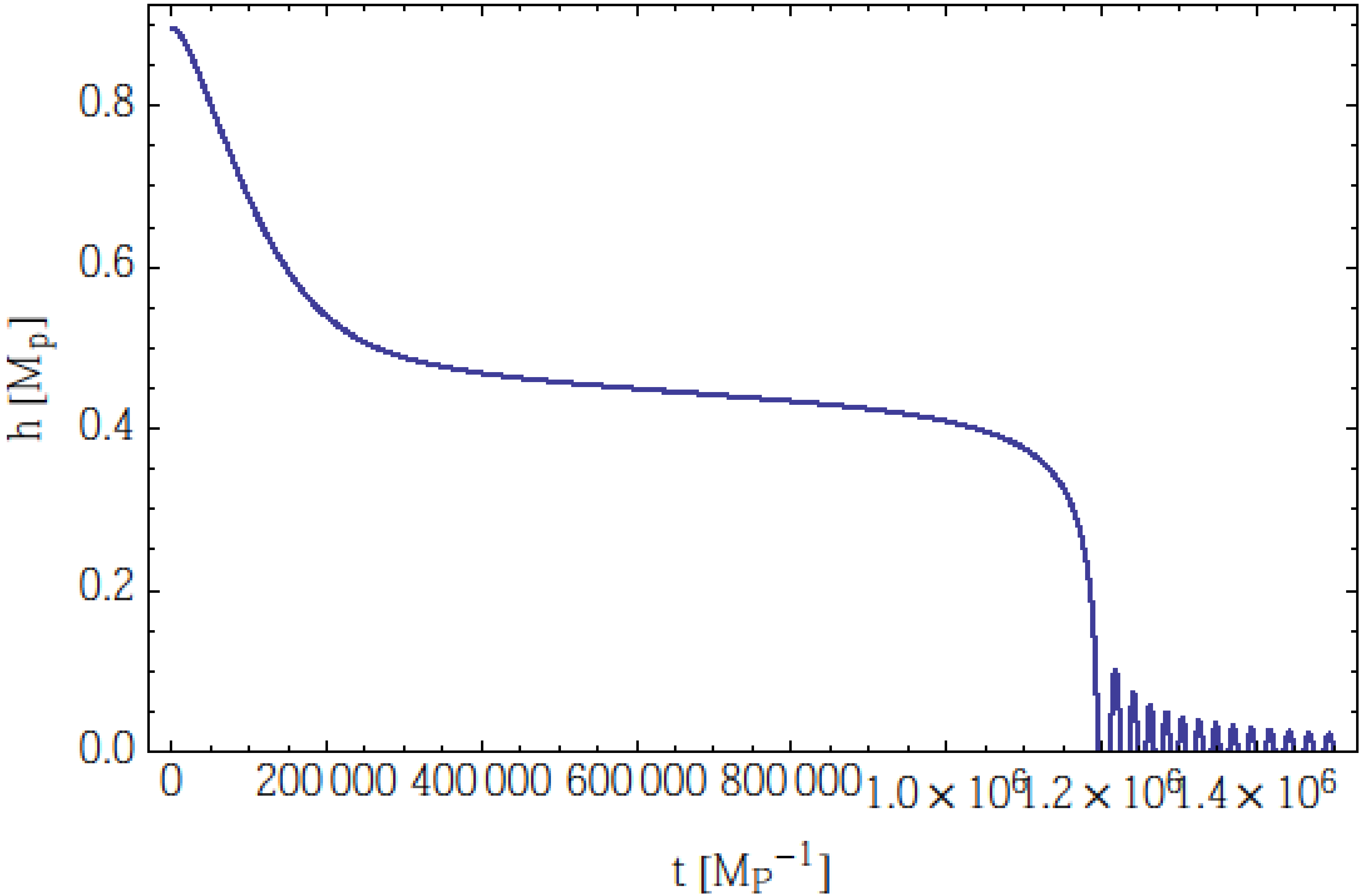
m_t : tuned around $\sim 171.5\ \mathrm{GeV}$ such that the height of the potential is 1 percent larger than that of the saddle point case.

$\xi = 7$
$h_* = 0.90\ \mathrm{M_P}$

Then we have

$r_* = 0.19, \quad N_* = 58\,, \quad \frac{\mathrm{V}_*}{\epsilon_*} = 5.0 \times 10^{-7}\,, \quad n_{S_*} = 0.955\ .$

time evolution of the example



Naturalness and Big Fix

There is a possibility that SM parameters are tuned such that the Higgs potential becomes almost flat around the string scale.

Such tuning seems unnatural in the ordinary local field theory , but it may be understood by slightly going beyond the local field theory.

There are several attempts to slightly extend the framework of the local field theory in order to explain such fine tunings.

- multiple point principle
  Froggatt, Nielsen, Takanishi
- baby universe and Big Fix
  Coleman
  Okada, Hamada, Kawana, HK
- classical conformality Iso's talk
  Meissner, Nicolai,
  Foot, Kobakhidze, McDonald, Volkas
  Iso, Okada, Orikasa
- asymptotic safety
  Shaposhnikov, Wetterich

1. Multiple point principle

"PREdicted the Higgs Mass"
H.B.Nielsen, arXiv:1212/5716

Why canonical ensemble?

In the ordinary quantum theory, the path integral of the form

$$\int [d\varphi] \exp(-S[\varphi])$$

is the most fundamental concept.

On the other hand in the statistical mechanics, the most fundamental concept is the micro canonical ensemble

$$Z = \int [d\varphi] \delta(H[\varphi] - E),$$

and the canonical ensemble follows from it in the thermodynamic limit:

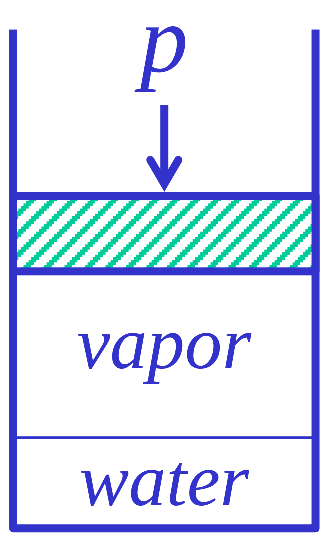
$$\int [d\varphi] \delta(H[\varphi] - E) \Rightarrow \int [d\varphi] \exp(-H[\varphi]/T).$$

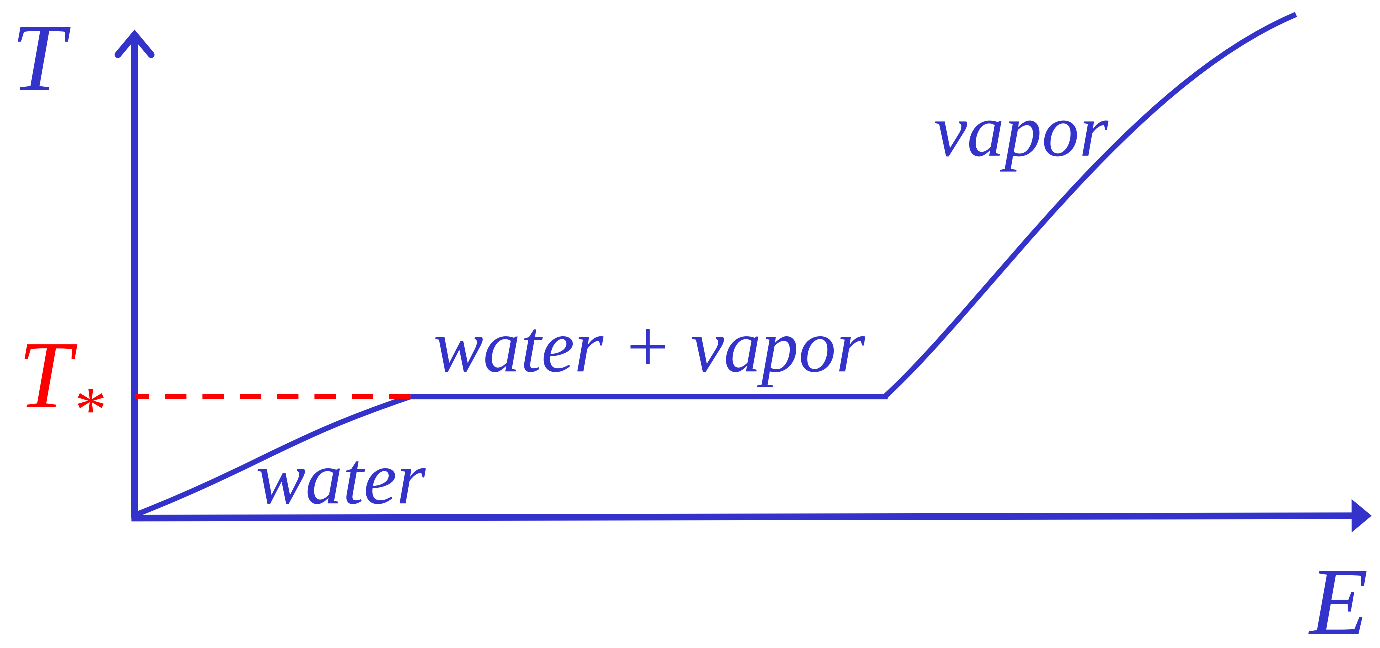
micro canonical canonical

$$\int [d\varphi]\,\delta\big(H[\varphi]-E\big) \Rightarrow \int [d\varphi]\exp\big(-H[\varphi]/T\big)$$

The total energy is given first, and the temperature is determined as a result.

Example: Water molecules in a cylinder with a fixed pressure.





T is automatically tuned to T_* for wide range of E.

T corresponds to coupling constants in field theory.

Question:

What happens if the quantum theory is defined by micro canonical like path integral?

$$Z = \int [d\varphi]\,\delta\left(S[\varphi] - E\right).$$

Here let's see what happens if we start with

$$Z = \int [d\phi]\,\delta\left(\int d^4x\,\phi^\dagger\phi - I_0\right)\exp\left(i\,S[\phi]\right)$$
$$= \int dm^2 \int [d\phi]\exp\left(i\left(S[\phi] - m^2\int d^4x\,\phi^\dagger\phi + m^2 I_0\right)\right).$$

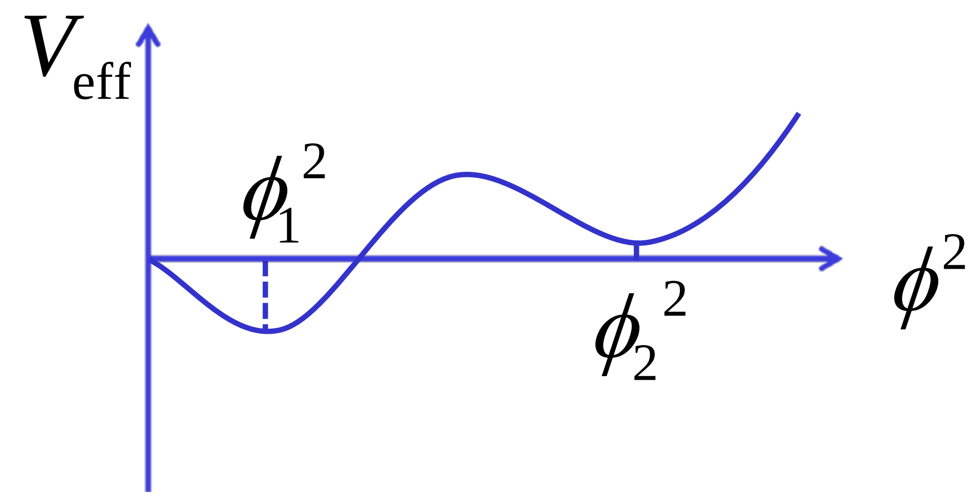
Then the path integral over ϕ becomes

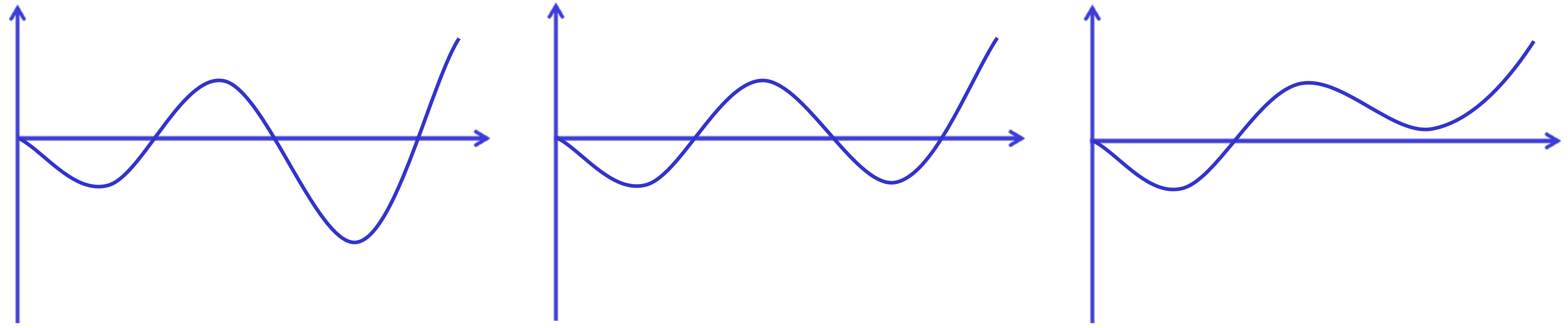
$$Z = \int dm^2 \exp\left(-iVF\left(m^2\right)\right).$$

One value of m^2 dominates, and the mass is effectively fixed to this value.

$$Z = \int dm^2 \int [d\phi] \exp\left(i \left(S[\phi] - m^2 \int d^4x\, \phi^\dagger \phi + m^2 I_0 \right) \right)$$

Assume that the effective potential for S has two minima.
$\Rightarrow$ There is some critical value for m^2.





$m^2 < m_c^2$	$m^2 = m_c^2$	$m^2 > m_c^2$
$\langle \phi^2 \rangle \sim \phi_2^2$	$\phi_1^2 \le \langle \phi^2 \rangle \le \phi_2^2$	$\langle \phi^2 \rangle \sim \phi_1^2$

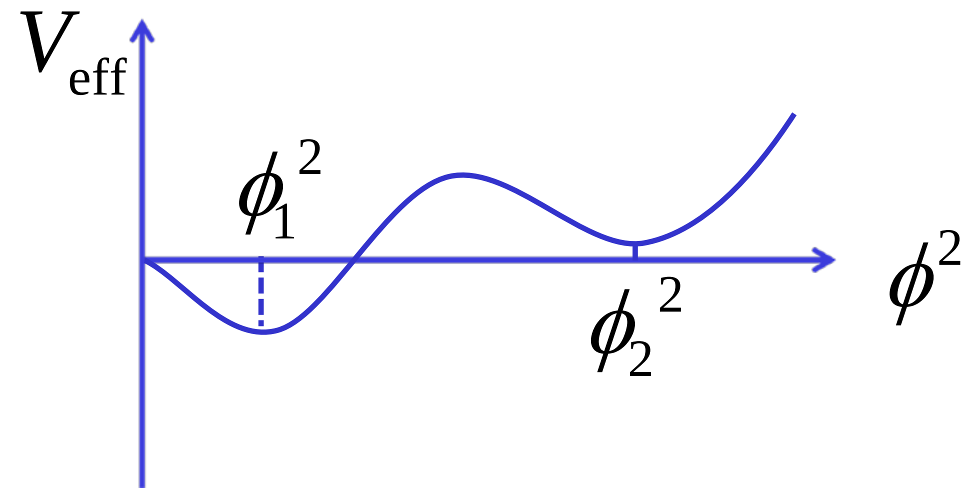
If $(\phi_1)^2 \leq I_0/V \leq (\phi_2)^2$, the constraint

$$\langle \int d^4x\, \phi^\dagger \phi \rangle = I_0$$

can be satisfied by making a mixture of the two phases.

This means that m^2 is automatically fixed to the critical value m_c^2.

What is the most probable value for $\phi_2^{\,2}$?

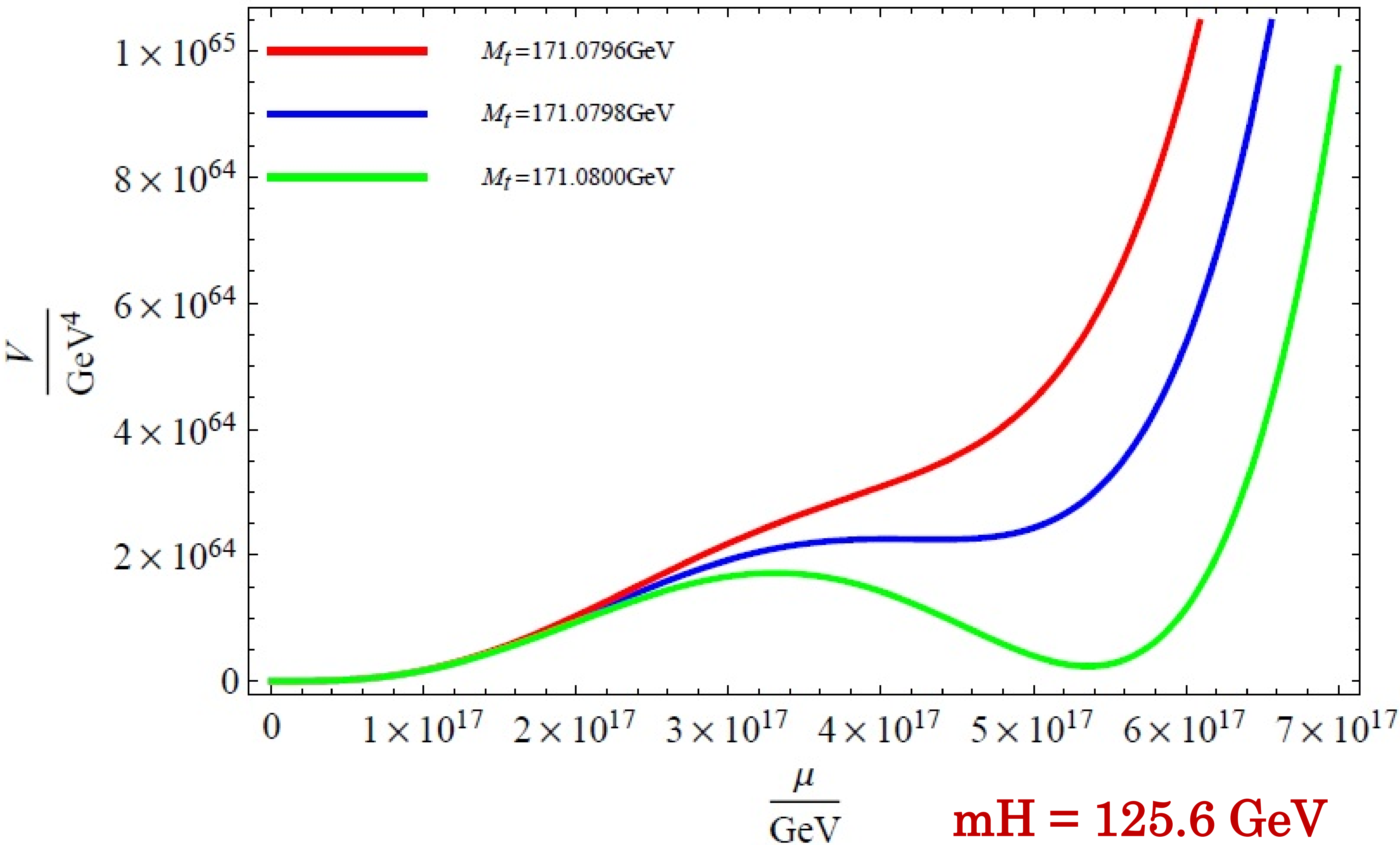


If there is no special reason, it is natural to expect

$$\phi_2^{\,2} \sim m_P^{\,2}\,.$$

Then $\phi_1^{\,2} \le I_0/V \le \phi_2^{\,2}$ can be naturally satisfied, because I_0/V is expected to be of order $m_P^{\,2}$.

The Higgs potential should have a degenerate minimum at a large value of the field.



mH = 125.6 GeV

generalization

The micro canonical like path integral can be generalized to

$$Z=\int[d\phi]\,\rho\left(\int d^4x\left(\phi^\dagger\phi-M^2\right)\right)\exp\left(i\,S[\phi]\right)$$

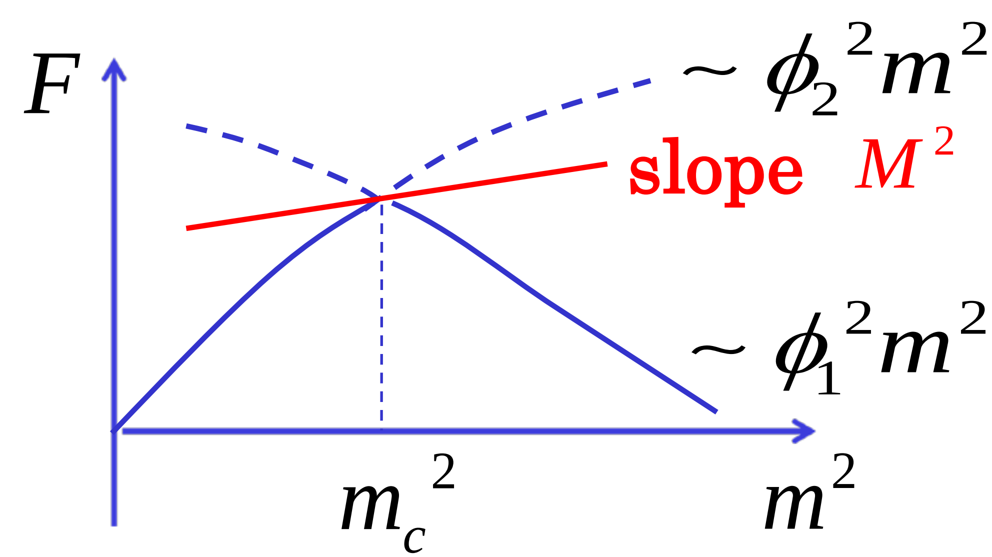
$$=\int dm^2 w\left(m^2\right)\int[d\phi]\exp\left(i\left(S[\phi]-m^2\int d^4x\left(\phi^\dagger\phi-M^2\right)\right)\right).$$

$M\sim$ Planck scale **is natural.**

Again $m^2\simeq m_c^{\ 2}$ **dominates in the RHS**

$$Z=\int dm^2 w\left(m^2\right)\exp\left(-iV\left(F\left(m^2\right)-M^2m^2\right)\right),$$

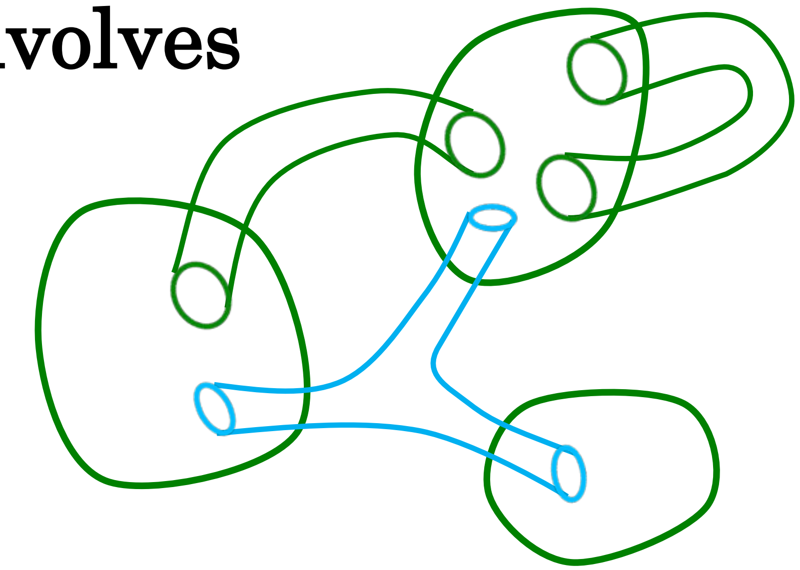
if $\phi_1^{\ 2}<M^2<\phi_2^{\ 2}$.



2-1 Baby universe and Big Fix

Coleman ('88) not completely consistent

Consider Euclidean path integral which involves the summation over topologies,



$$\sum_{\text{topology}} \int [dg] \exp(-S) \, .$$

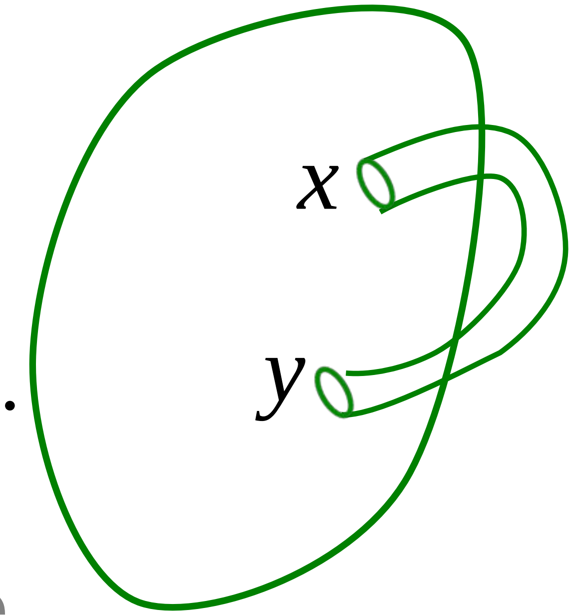
Then there should be a wormhole-like configuration in which a thin tube connects two points on the universe. Here, the two points may belong to either the same universe or the different universes.

If we see such configuration from the side of the large universe(s), it looks like two small punctures.

But the effect of a small puncture is equivalent to an insertion of a local operator.

Therefore, a wormhole contribute to the path integral as

$$\int [dg] \sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)} \sqrt{g(y)}\, O^i(x)\, O^j(y)\, \exp(-S)\,.$$



Summing over the number of wormholes, we have

$$\sum_{N=0}^{\infty} \frac{1}{n!} \left(\sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)} \sqrt{g(y)}\, O^i(x)\, O^j(y) \right)^n$$

$$= \exp\left(\sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)} \sqrt{g(y)}\, O^i(x)\, O^j(y) \right).$$

Thus wormholes contribute to the path integral as

$$\int [dg] \exp\left(-S + \sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)} \sqrt{g(y)}\, O^i(x)\, O^j(y) \right).$$

bifurcated wormholes

$\Rightarrow$ cubic terms, quartic terms, …

The effective action becomes a factorized form

$$S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

By introducing the Laplace transform

$$\exp\left(-S_{\text{eff}}(S_1, S_2, \cdots)\right) = \int d\lambda\, w(\lambda_1, \lambda_2, \cdots) \exp\left(-\sum_i \lambda_i S_i\right),$$

we can express the path integral as

$$Z = \int [d\phi] \exp(-S_{\text{eff}}) = \int d\vec{\lambda}\, w(\vec{\lambda}) \int [d\phi] \exp\left(-\sum_i \lambda_i S_i\right).$$

Coupling constants are not merely constant but to be integrated. ← **The same as MPP.**

Big Fix

Coleman pointed out the possibility that all the low energy coupling constants are fixed in such a way that the partition function is maximized.

However because of the inconsistency of Euclidean gravity, many confusions were made, and no concrete answer was obtained.

2-2 Lorentzian Path integral
and
maximum entropy principle

T. Okada, HK: arXiv:1110.2303, 1104.1764
HK: Int. J. of Mod. Phys. A vol. 28, nos. 3 & 4 (2013) 1340001
Hamada, Kawana, HK: arXiv:14051310

We consider the low energy effective action as before

$$S_{\text{eff}} = \sum_i c_i S_i + \sum_{i j} c_{i j} S_i S_j + \sum_{i j k} c_{i j k} S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

The path integral should be done in Minkovski time:

$$Z = \int [d\phi] \exp(i S_{\text{eff}}) = \int d\vec{\lambda} w(\vec{\lambda}) \int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right).$$

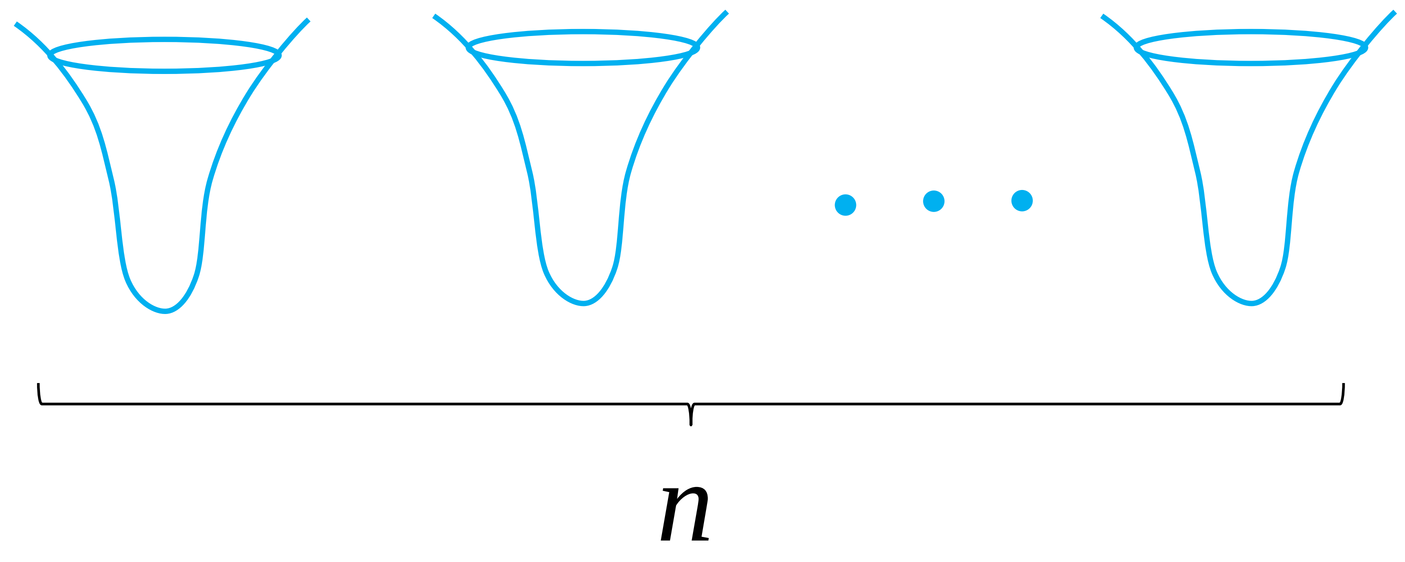
Coupling constants are not merely constant but to be integrated.

We also sum over the number of universes:

$$Z = \int d\vec{\lambda}\, w(\vec{\lambda}) \exp\left(Z_1(\vec{\lambda})\right).$$

$$\int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right) = \sum_{n=0}^{\infty} \frac{1}{n!} Z_1{}^n$$

$$Z_1 = \int [d\phi]_{\text{single universe}} \exp\left(i \sum_i \lambda_i S_i \right)$$



The path integral contains the integration over the time (lapse).
The universe spends most of its time in the late stages.
The late stages dominate in the path integral.

The partition function of a single universe is given by

$$Z_1(\lambda) \sim \mathrm{const}\sqrt[4]{C_{\mathrm{rad}}(\lambda)}.$$

$$\rho = \frac{1}{a^2} - \Lambda - \frac{C_{\mathrm{matt}}}{a^3} - \frac{C_{\mathrm{rad}}}{a^4}$$

maximum entropy principle

The coupling constants are determined in such a way that the entropy at the late stages of the universe is maximized.

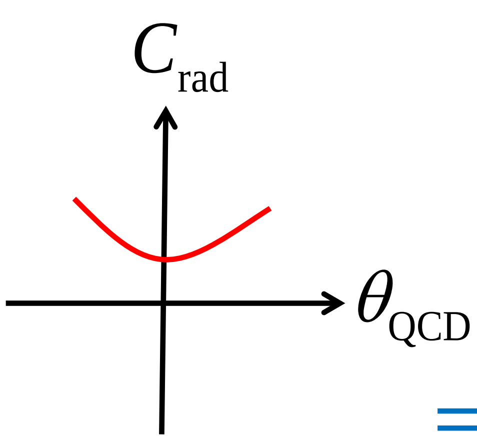
Examples of the Big Fix (1)

If the cosmological evolution is completely understood, we can calculate $C_{\mathrm{rad}}(\lambda)$ theoretically, and all of the renormalized couplings are in principle determined.

At present, we do not have enough knowledge about the very early and late stages of the universe, especially the origin of inflation, dark energy and dark matter.

However, some of the couplings can be determined without knowing the details of the cosmological evolution.

<u>case 1.</u> Symmetry   example θ_{QCD}

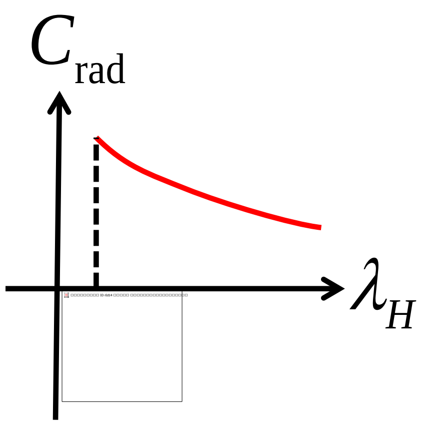


1. It becomes important only after the QCD phase transition.
2. Hadron mass spectrum is invariant under $\theta_{\mathrm{QCD}} \to -\theta_{\mathrm{QCD}}$.

$\Rightarrow C_{\mathrm{rad}}$ is minimum or maximum at $\theta_{\mathrm{QCD}} = 0$ at least locally.

Examples of the Big Fix (2)

case 2. End point example Higgs coupling λ_H



1. Some (renormalized) couplings are bounded.
2. $C_{\rm rad}$ can be monotonic in them.

$\Rightarrow$ $C_{\rm rad}$ is maximized at the end point.

Another explanation of Froggatt- Nielsen

A scenario for λ_H

Fix v_h to the observed value and vary λ_H.

assuming the leptogenesis

$\lambda_H \searrow$ $\Rightarrow$ sphaleron process $\nearrow$

$\Rightarrow$ baryon number $\nearrow$

$\Rightarrow$ radiation from baryon decay $\nearrow$

$\Rightarrow$ Higgs mass is at its lower bound.

Constraints on the Higgs portal DM

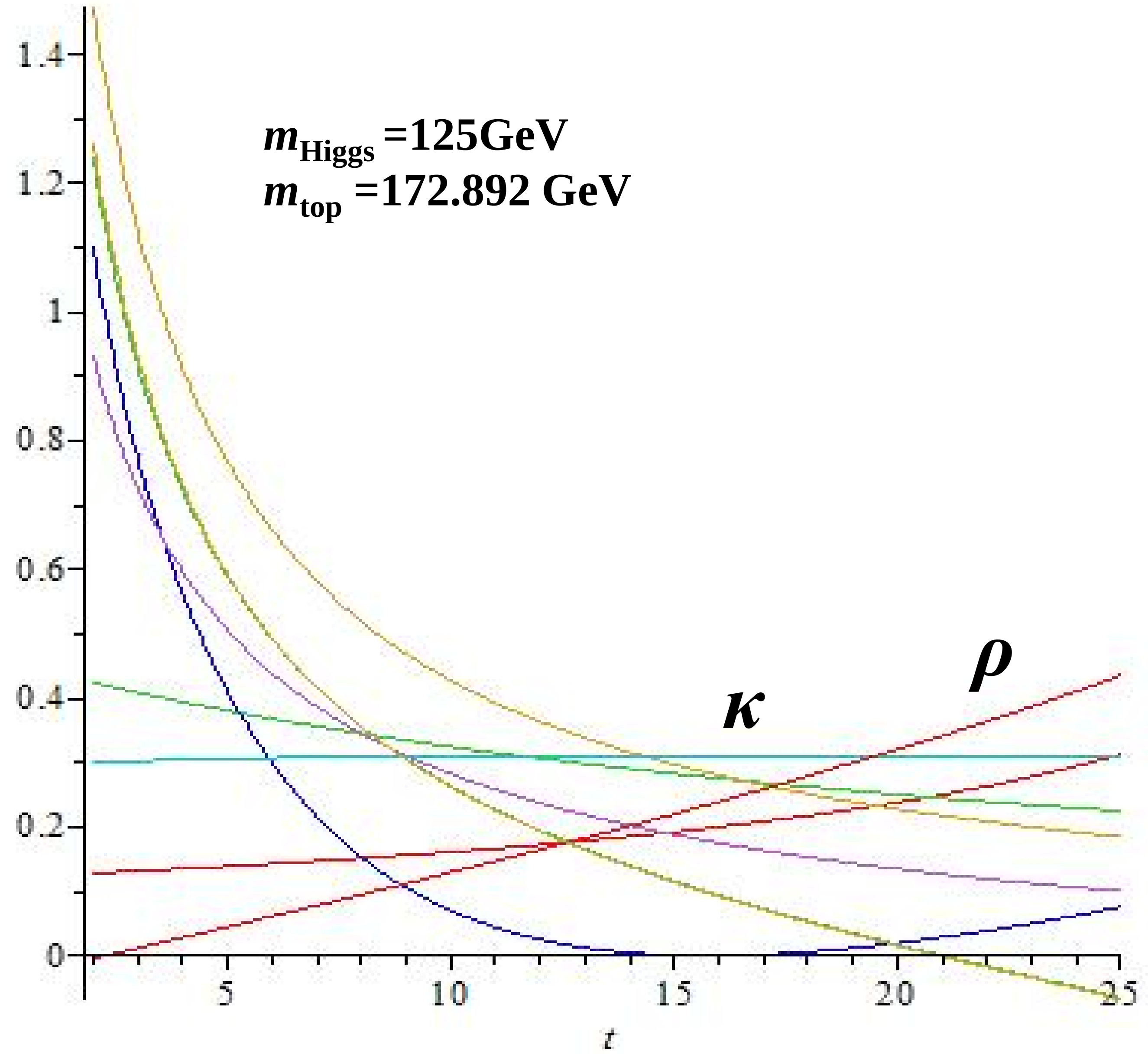
Flatness of the Higgs potential imposes constraints on the Higgs portal DM.

Hamada, Oda and HK: arXiv: 1404.6i41

Higgs portal dark matter

We consider Z_2 invariant Higgs portal DM:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\left(\partial_\mu S\right)^2 - \frac{1}{2}m_S^2 S^2 - \frac{\rho}{4!}S^4 - \frac{\kappa}{2}S^2 H^\dagger H$$



The effect of S on λ is opposite to that of top quark.

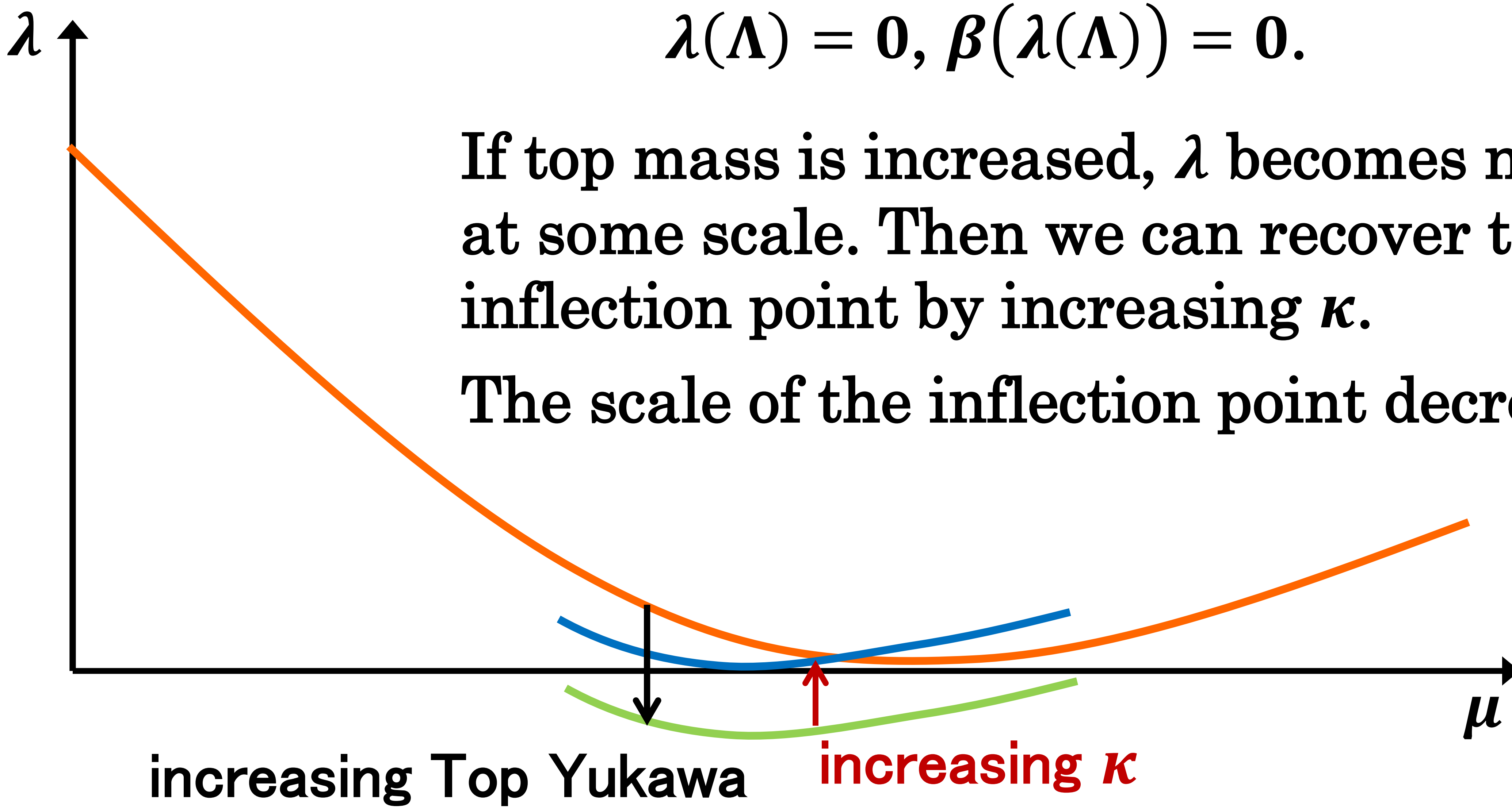
Top Yukawa lowers λ for smaller values of μ, while κ increases λ almost uniformly in μ.

Suppose we have an inflection point at some scale Λ:

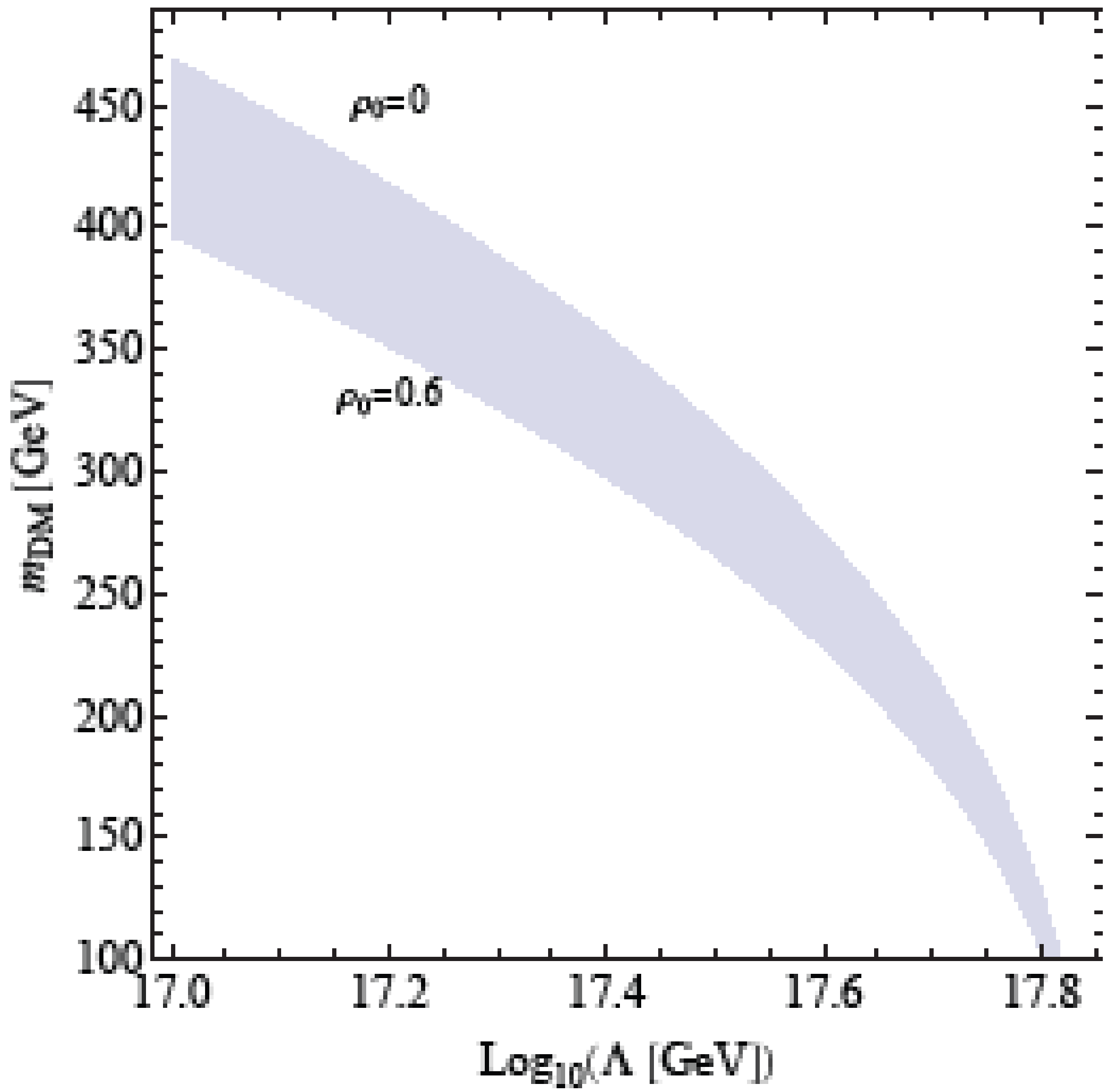
$$\lambda(\Lambda) = 0,\ \beta\big(\lambda(\Lambda)\big) = 0.$$

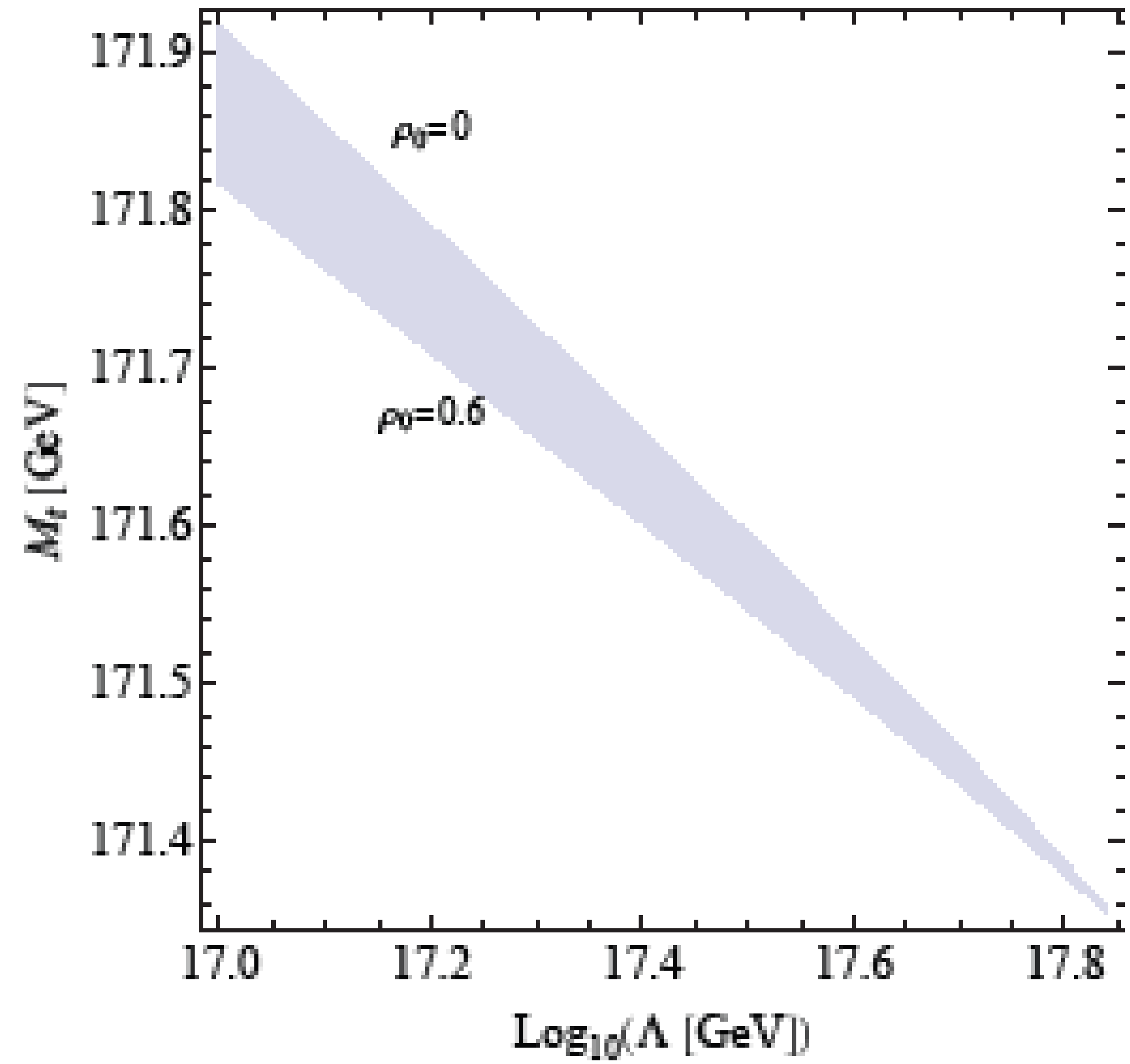
If top mass is increased, λ becomes negative at some scale. Then we can recover the inflection point by increasing κ.

The scale of the inflection point decrease.



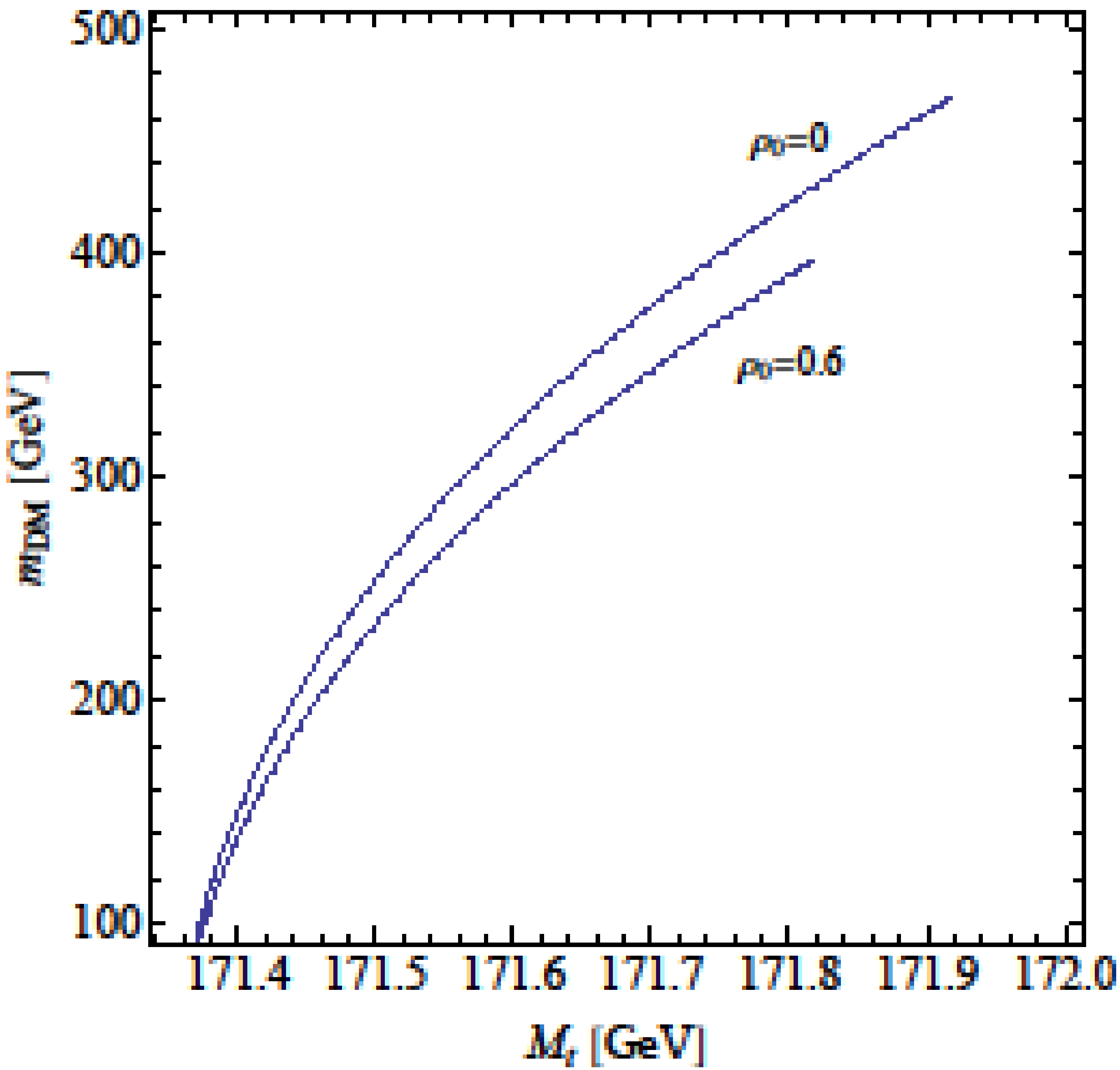
$m_H = 126\text{GeV}$





From the natural abundance of DM, the DM mass should be related to κ:

$$m_{DM} \sim 330\text{GeV} \times \frac{\kappa}{0.1}$$



500
400
300
200
100
$m_{\rm DM}$ [GeV]
ρ_0=0
ρ_0=0.6
171.4
171.5
171.6
171.7
171.8
171.9
172.0
M_t [GeV]

Naturalness and Higgs inflation

- It seems we have nothing other than a minor modification of SM below the string scale.
- It is possible that the fine tunings result from not the conventional local field theory but something (slightly) beyond.
- For example, we can consider the possibility that the couplings are fixed to maximize the entropy of the universe.
- From this point of view Higgs inflation is natural.
- If DM is the Higgs portal scalar, its mass is predictable.