QCD axions with high scale inflation

(QCD axions after BICEP2)

KC, K.S. Jeong and M.S. Seo, arXiv:1404.3880

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The IBS Center for Theoretical Physics of the Universe

Institute for Basic Science

Outline

* Introduction

* Cosmological constraints on the QCD axion
  (before & after BICEP2)

* Implications for string theoretic QCD axions

Strong CP problem

CP violation in the SM: $\frac{\theta_{\rm QCD}}{32\pi^2} G^{a\mu\nu}\tilde{G}^a_{\mu\nu} + (y_q H \bar{q}_L q_R + {\rm h.c})$

➔ Neutron EDM:

$$d_n \sim 10^{-16}(\theta_{\rm QCD} + {\rm Arg\,Det}(y_q))\, e\cdot {\rm cm} \quad < 10^{-26}\, e\cdot{\rm cm}$$

➔ $|\theta_{\rm QCD} + {\rm ArgDet}(y_q)| < 10^{-10}$

Why $|\theta_{QCD}+$ Arg $\mathrm{Det}(y_q)| < 10^{-10}$, while $\delta_{KM} \sim$ Arg $(y_q) \sim 1$?

Unlike the gauge hierarchy problem, anthrophic argument can not explain this puzzle. It is thus quite likely that there should be some physical explanation for this small parameter combination.

Axion solution to the strong CP problem

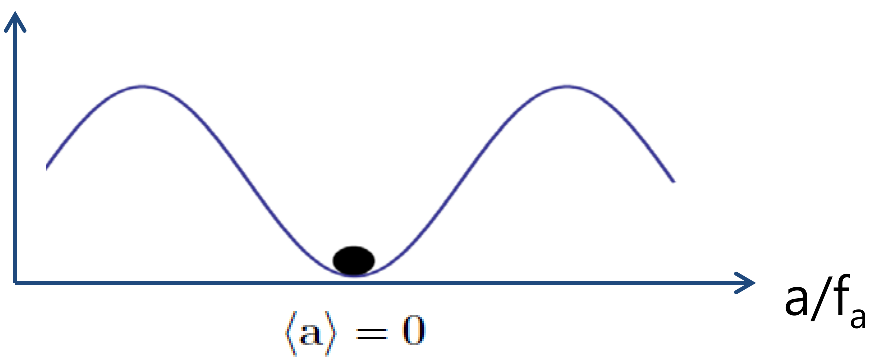
Introduce a spontaneously broken anomalous global U(1) symmetry (Peccei-Quinn symmetry), which makes θ_{QCD} a dynamical field "axion"

= Nambu-Goldstone boson of the spontaneously broken $U(1)_{PQ}$

$$\frac{1}{32\pi^2}(\theta_{QCD} + \mathrm{Arg\,Det}(y_q))G^{a\mu\nu}\tilde{G}^a_{\mu\nu} \quad \rightarrow \quad \frac{1}{32\pi^2}\frac{a}{f_a}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$$

(f_a = Axion scale = Scale of the spontaneous breaking of $U(1)_{PQ}$)

➔ Low energy QCD dynamics develops an axion potential minimized at $\langle a \rangle = 0$:



Dynamical relaxation of θ_{QCD} + Arg Det(y_q) = $\langle a \rangle / f_a$

Most of axion physics is determined by the axion scale f_a:

* Axion mass: $m_a \sim 5 \times 10^{-5} \left(\frac{10^{11}\,\mathrm{GeV}}{f_a} \right) \mathrm{eV}$

* Axion-photon couplings: $g_{a\gamma\gamma} \sim 10^{-14} \left(\frac{10^{11}\,\mathrm{GeV}}{f_a} \right) \mathrm{GeV}^{-1}$

(Astrophysical bound on $g_{a\gamma\gamma}$: $f_a > 10^9$ GeV)

* Relic abundance of axion dark matter:

$$\Omega_a \sim 0.02 \left(\theta_0^2 + \delta\theta^2 + R_{\mathrm{defect}} \right) \left(\frac{f_a}{10^{11}\,\mathrm{GeV}} \right)^{7/6}$$

θ_0 = Root-mean-square axion misalignment angle

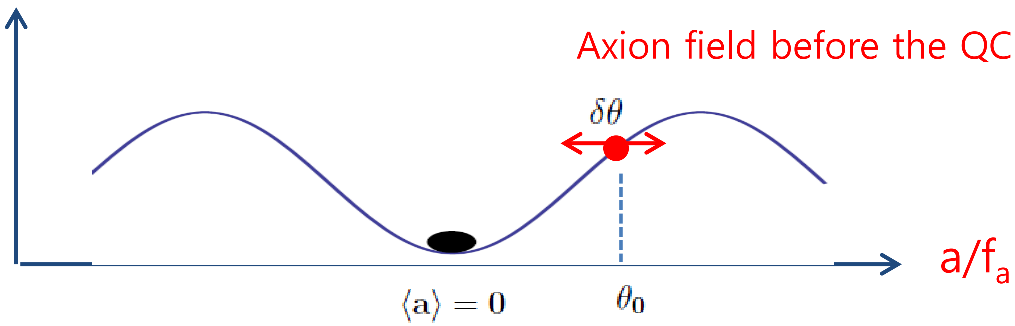
Preskill, Wise, Wilczek; Abbott, Sikivie; Dine, Fischler '81

$\delta\theta$ = Axion fluctuation produced during the inflation period

Fox, Pierce, Thomas '04

R_{defect} = Axions radiated from topological defects (strings, walls)

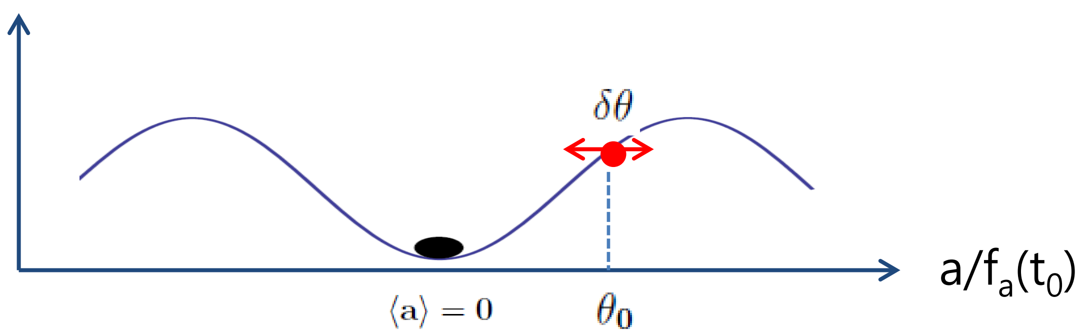
Davis '85; Harari, Sikivie '87; Davis, Shellard '89



Cosmological constraints on the QCD axions

A: If the PQ symmetry is spontaneously broken (non-linearly realized) during the inflation period, and never restored thereafter:

No axionic strings or domain walls within the horizon of the present universe, but the axion field could have a classical misalignment, as well as a fluctuation which originates from quantum fluctuations produced during inflation:



$f_a(t_0)$ = Axion scale in the present universe (also during the QCD phase transition)

θ_0^2 = Free parameters in the range $[0, \pi^2]$

$$\delta\theta \equiv \frac{\delta a(t_I)}{f_a(t_I)} = \frac{H(t_I)}{2\pi f_a(t_I)} \quad (\, f_a(t_I) = \text{Axion scale during the inflation period}\,)$$

$=$ Free parameter which could be as small as desired before BICEP2

Axion dark matter produced by the coherent oscillation of the misaligned axion field:

$$\Omega_a \sim 0.02 \left(\gamma\theta_0^2 + \delta\theta^2 + R_{\rm defect}\right)\left(\frac{f_a(t_0)}{10^{11}\,{\rm GeV}}\right)^{7/6}$$

$$\theta = \theta_0 + \delta\theta$$

$$\theta_0^2 = \text{Free parameters in the range } [0, \pi^2]$$

$$\gamma = \text{Anharmonic factor} = \mathcal{O}(1) \quad (\simeq 1 \text{ for } \theta_0^2 \ll 1)$$

$$\delta\theta^2 \equiv \langle\delta\theta^2\rangle = \left(\frac{H(t_I)}{2\pi f_a(t_I)}\right)^2$$

$$= \text{Free parameter which could be as small as desired before BICEP2}$$

$$R_{\rm defect} = 0$$

Axion isocurvature perturbation:

Adiabatic perturbations: $\delta\rho_{\rm TOT} \neq 0 \;\; \text{with} \;\; \delta\left(\frac{n_i}{s}\right) = 0$

Isocurvature perturbations: $\delta\rho_{\rm TOT} = 0 \;\; \text{but} \;\; \delta\left(\frac{n_i}{s}\right) \neq 0$

* Inflaton (= curvature) perturbation = Adiabatic perturbation

$$\Rightarrow \quad \delta\phi = \dot\phi(t)\delta t(\vec{x}, t) \quad \rightarrow \quad \delta n_i = \dot n_i \delta t = -3Hn_i\,\delta t = n_i\,\delta s/s$$

* Axion perturbation = Isocurvature perturbation

$$\frac{\delta\rho_{\rm TOT}}{\delta a(t_I)} = 0 \quad \text{but} \quad \frac{\delta(n_a/s)}{\delta a(t_I)} \neq 0$$

Power spectrum of the axion isocurvature perturbation:

Fox, Pierce, Thomas '04; Mack, Steinhardt '11

$$\mathcal{P}_{\rm iso} \propto \langle \delta T^2_{\rm iso}\rangle \propto \langle \delta n_a^2\rangle \propto \langle \left(\theta^2 - \langle\theta^2\rangle\right)^2\rangle f_a^{7/3}(t_0) = 2\delta\theta^2\left(2\theta_0^2 + \delta\theta^2\right) f_a^{7/3}(t_0)$$

$$n_a \propto \langle\theta^2\rangle\, f_a^{7/6}(t_0) = \left(\theta_0^2 + \delta\theta^2\right) f_a^{7/6}(t_0)$$

B: If the last spontaneous PQ breaking occurred after the reheating of the primordial inflation:

To avoid the domain wall problem, we first need

Axion domain-wall number = $N_{DW} = \sum_i q_i \mathrm{Tr}(T_c^2(\psi_i)) = 1$

Axion dark matters are produced by the collapsing axionic string & domain walls, as well as by the coherent oscillation of misaligned axion field:

$$\Omega_a \sim 0.02 \left(\gamma\theta_0^2 + \delta\theta^2 + R_{defect} \right) \left(\frac{f_a(t_0)}{10^{11}\,\mathrm{GeV}} \right)^{7/6} \leq \Omega_{DM}$$

Present horizon involves many different patches which were casually disconnected at the moment of PQ phase transition: $\gamma\theta_0^2 \simeq 1.85 \times \frac{\pi^2}{3}$

No axion isocurvature perturbation: $\delta\theta = 0$

Numerical simulation: $R_{defect} \simeq 40 - 120$ Hiramatsu et al, '12

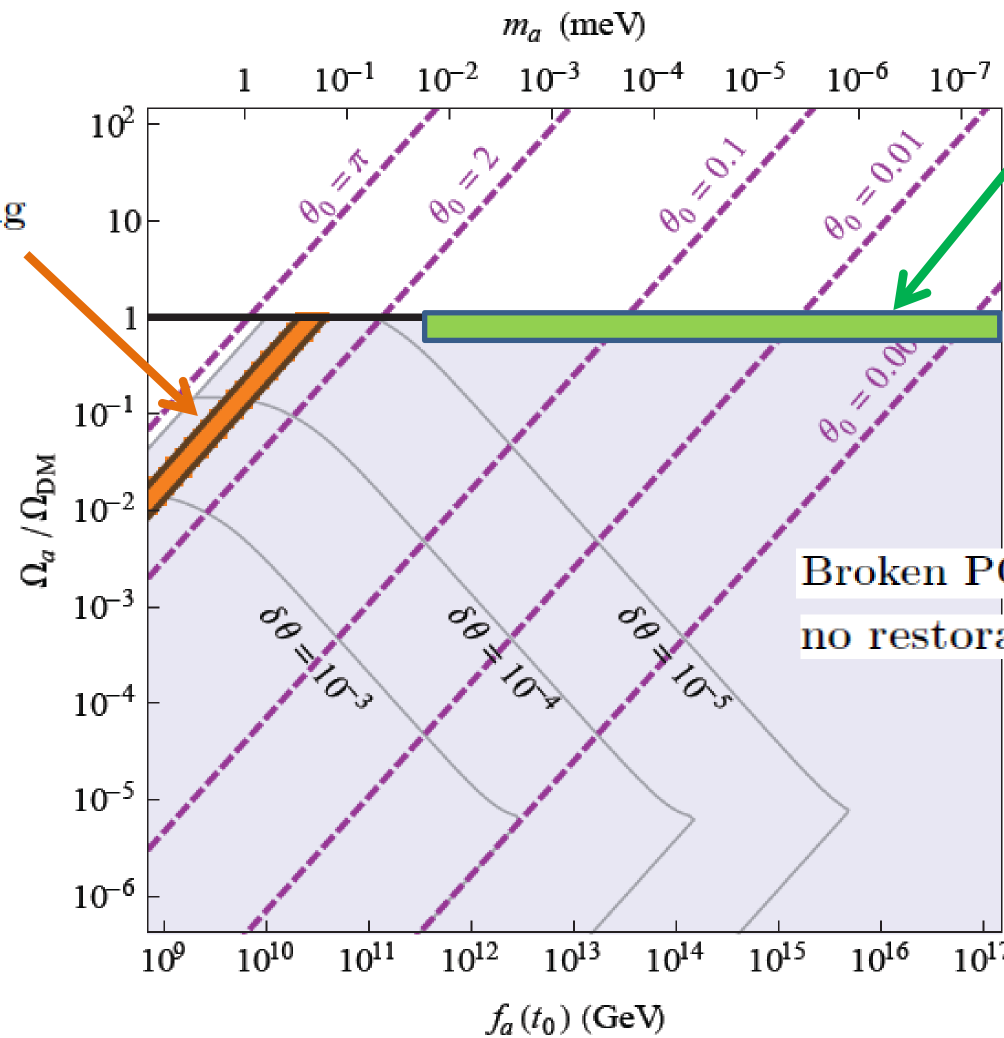
➔ $10^9\,\mathrm{GeV} < f_a < (2-4) \times 10^{10}\,\mathrm{GeV}$

Before BICEP2:

$$\Omega_a \leq \Omega_{DM} \quad \left(\Omega_a \propto \left(\theta_0^2 + \delta\theta^2\right) f_a^{7/6}(t_0)\right) \qquad \delta\theta^2 \equiv \langle\delta\theta^2\rangle = \left(\frac{H(t_I)}{2\pi f_a(t_I)}\right)^2$$

$$\mathcal{P}_{iso} < 0.04\,\mathcal{P}_{adi} \simeq 9\times 10^{-11} \quad \left(\mathcal{P}_{iso} \propto \delta\theta^2\left(2\theta_0^2 + \delta\theta^2\right) f_a^{7/3}(t_0)\right)$$

Last PQ breaking after the reheating



anthropic axion dark matter

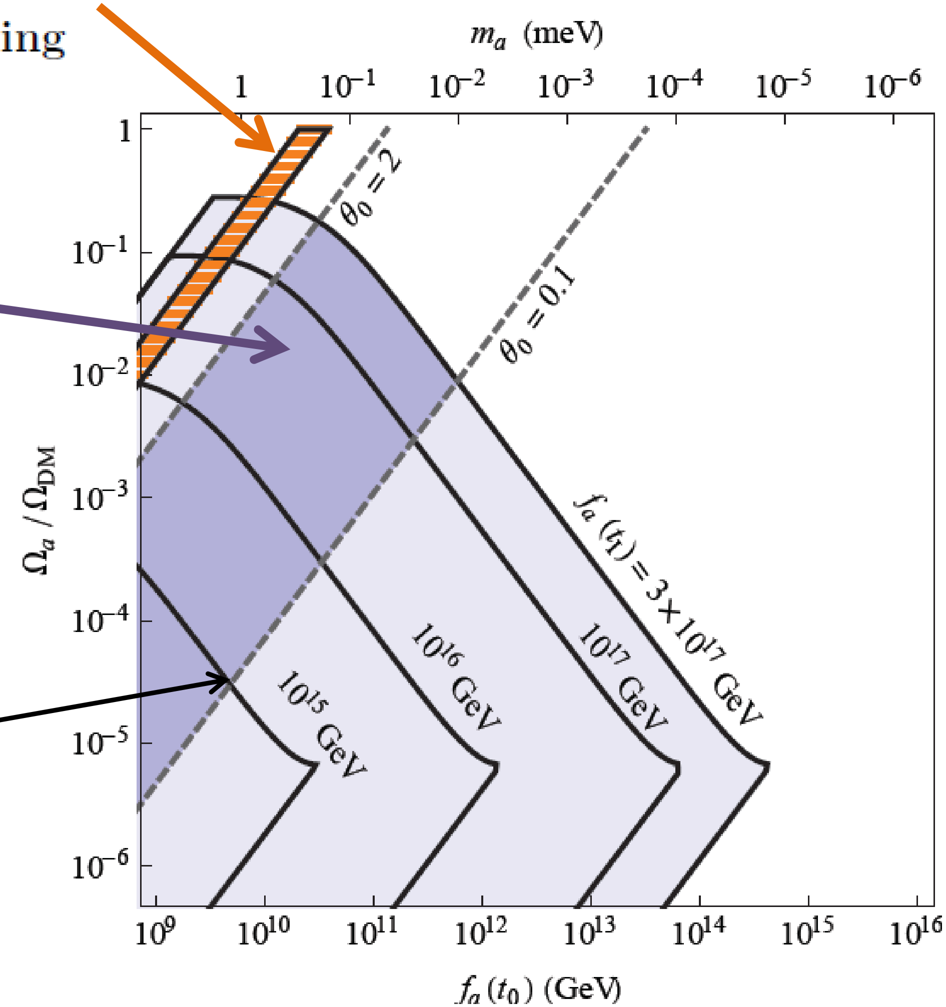
Broken PQ during inflation, no restoration thereafter

After BICEP2: $r \simeq 0.2 \quad \rightarrow \quad H(t_I) \simeq 10^{14}\,\text{GeV}$

Last PQ breaking after the reheating

Broken PQ during inflation with $\theta_0 = 0.1 - 2$, no restoration thereafter

$f_a(t_I)$

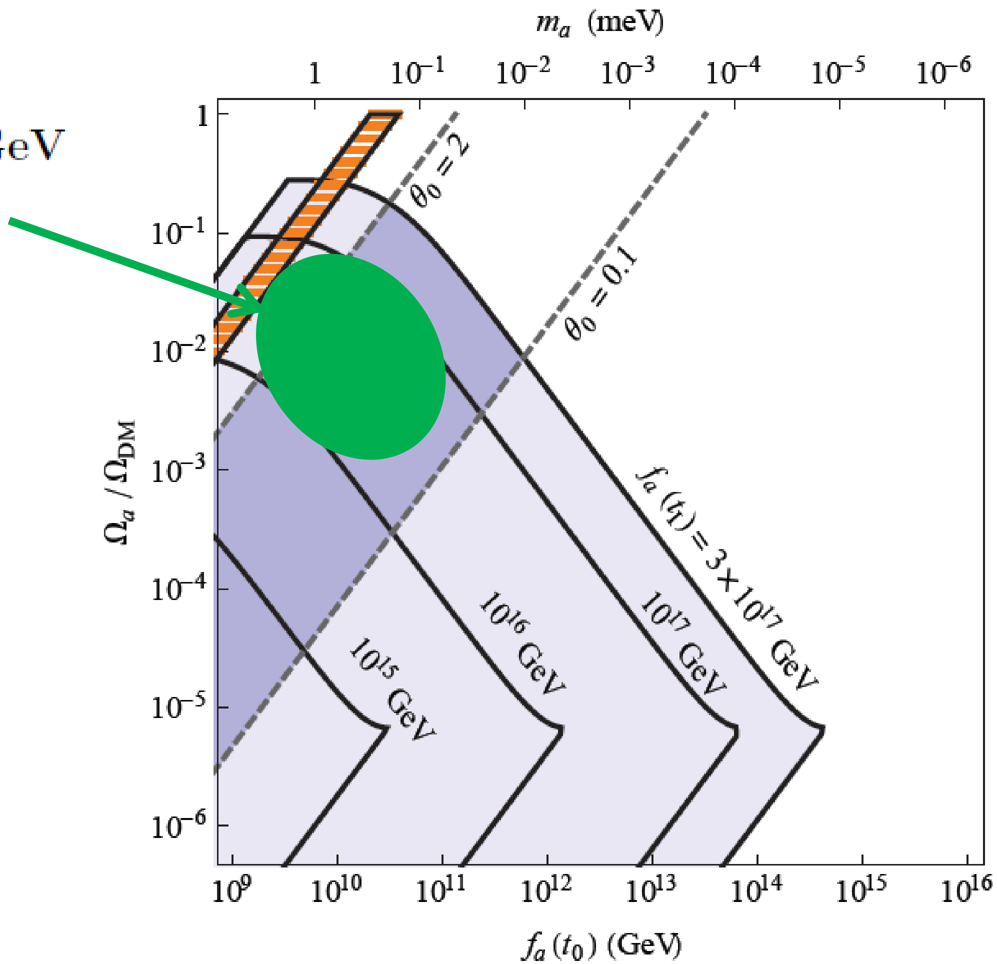


The PQ symmetry should be either restored, or broken at much higher scale during inflation: $f_a(t_I) = 0$, or $f_a(t_I) \gg f_a(t_0)$

Most plausible range of $f_a(t_0)$ and Ω_a in view of our results

$$f_a(t_I) = 10^{16} - 10^{17}\ \mathrm{GeV}$$

$$\theta_0 = \mathcal{O}(1)$$



$$f_a(t_0) \sim m_{SUSY} \quad \text{or} \quad \sqrt{m_{SUSY} M_{Pl}}$$

$$\sim 10^9 - 10^{11}\ \mathrm{GeV}$$

Implication of BICEP2 for string theoretic QCD axions

String theory is the right place to realize the axion solution of the strong CP problem.

4D effective theory of string compactification generically involves axion-like fields originating from higher-dim antisymmetric tensor gauge fields.

Witten '84

Gauge-axion unification (with extra dimension)

➔ 4-dim global PQ symmetry which is locally equivalent to a higher-dim gauge symmetry

This can explain why the PQ symmetry is well protected from quantum gravity.

Example:

Antisymmetric tensor gauge field C_{MN} on 2-sphere S_2 with radius R in the internal space, with a gauge symmetry:

$$G_\Lambda: \ C_{MN} \ \rightarrow \ C_{MN} + \partial_{[N}\Lambda_{N]} \quad \text{for globally well defined } \Lambda_N$$

Harmonic area 2-form on S_2: $\omega = R^2 d\phi \wedge d\cos\theta$

$$d\omega = 0, \quad \text{but} \ \int_{S_2} \omega = 4\pi R^2 \neq 0$$

$$\Rightarrow \quad \omega_{[56]} = \partial_{[5}\Lambda_{6]} \ \text{locally, but not globally on } S_2 \text{ for } (y^5, y^6) = (\theta, \phi)$$

Axion-like fluctuation: $C_{[56]}(x,y) = a_{st}(x)\omega_{[56]}(y)$

Global shift symmetry locally equivalent to the gauge symmetry G_Λ:

$$U(1)_{shift}: \ a_{st} \rightarrow a_{st} + \text{constant}$$

$$\Rightarrow \quad U(1)_{shift}: \ C_{[56]} \ \rightarrow \ C_{[56]} + \text{constant} \times \omega_{[56]} = C_{[56]} + \partial_{[5}\Lambda_{6]} \ \text{only locally}$$

In this scheme, $U(1)_{shift}$: $a_{st} \rightarrow a_{st}$ + constant can be broken only by non-local (in the extra dim) effects associated with $\int_S \omega = 4\pi R^2 \neq 0$.

* Stringy instantons wrapping S_2 :

Instanton amplitudes $\propto \exp\left(-\int_S \omega / l_{st}^2\right) = \exp\left(-4\pi R^2/l_{st}^2\right)$

* Fluxes or branes wrapping S_2 (or monodromy structure)

➔ Flux or brane-induced axion potential

* Axion couplings to the 4-dim gauge field instantons

$$\int_{M_4 \times I_6} C \wedge F \wedge F \wedge F \wedge F \quad \rightarrow \quad \int_{M_4} a_{st}(x) \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \int_S \omega \int_{I_4} \langle F \wedge F \rangle$$

With

* a proper choice of fluxes and branes to avoid the flux or brane-induced axion potential,

* a moderately large cycle radius R,

$U(1)_{shift}$ can be broken dominantly by the axion coupling to the QCD instantons.

Axion scales:

$U(1)_{shift}$ is non-linearly realized already at the string scale, and therefore the associated axion scale is around the GUT scale: KC & Kim '85

$$\Delta\mathcal{L} = \frac{g^2}{32\pi^2}\frac{a_{st}}{f_a}G^{a\mu\nu}\tilde{G}^a_{\mu\nu} \sim \frac{1}{4}\frac{a_{st}}{M_{Pl}}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$$

This simple estimate of the axion scale applies for the most of compactified string models in which the compactification scale is close to the reduced Planck scale M_{Pl} = 2.4 x 10^{18} GeV. Svrcek & Witten '06

(cf: BICEP2 → $E_{inflation} \simeq 10^{16}$ GeV → $M_{com} \gg 10^{16}$ GeV)

$$\rightarrow \quad f_a \sim \frac{g_{GUT}^2}{8\pi^2}M_{Pl} \sim 10^{16}\ \mathrm{GeV}$$

Generalization with anomalous $U(1)_A$ gauge symmetry:

Quite often, string theory allows a generalization of this scheme, in which $U(1)_{PQ}$ is still locally equivalent to a gauge symmetry, while the axion scale can be far below the GUT scale.

* Global shift symmetry which is locally equivalent to higher-dim gauge symmetry:

$$\mathrm{U(1)_{shift}} : \mathrm{a_{st}} \rightarrow \mathrm{a_{st}} + \mathrm{constant}$$

* Anomalous U(1) gauge symmetry under which the QCD axion is charged:

$$\mathrm{U(1)_A} : \mathrm{A}_\mu \rightarrow \mathrm{A}_\mu + \partial_\mu \alpha(\mathrm{x}), \quad \mathrm{a_{st}} \rightarrow \mathrm{a_{st}} + \delta_{\mathrm{GS}} \alpha(\mathrm{x}), \quad \phi_{\mathrm{i}} \rightarrow e^{\mathrm{i q_i} \alpha(\mathrm{x})} \phi_{\mathrm{i}}$$

$$\left(\delta_{\mathrm{GS}} = \frac{1}{8\pi^2} \sum_{\mathrm{i}} \mathrm{q_i} \mathrm{Tr}(\mathrm{T_a^2}(\phi_{\mathrm{i}})) = \mathcal{O}\left(\frac{1}{8\pi^2}\right) \right)$$

$U(1)_{PQ}$ = Combination of $U(1)_{shift}$ and $U(1)_A$

QCD axion = Combination of a_{st} and arg(ϕ)

$$\mathcal{L} = \frac{1}{2}M_{Pl}^2 \frac{\partial^2 K}{\partial \tau^2}(\partial_\mu a_{st} - \delta_{GS} A_\mu)^2 + \frac{1}{4}\frac{a_{st}}{M_{Pl}} G\tilde{G} \; - g_A^2 \left(\delta_{GS}\frac{\partial K}{\partial \tau} M_{Pl}^2 - \sum_i q_i |\phi_i|^2\right)^2 + \ldots$$

$(\tau = \text{ modulus partner of } a_{st})$

Mass parameters: Stuckelberg mass: $f_{st} = \delta_{GS}\sqrt{\frac{\partial^2 K}{\partial \tau^2}} M_{Pl} \sim \frac{g^2}{8\pi^2} M_{Pl}$

Fayet-Illiopoulos (FI) term: $\xi_{FI} = \delta_{GS}\frac{\partial K}{\partial \tau} M_{Pl}^2$

Matter field VEV: $v^2 = \sum_i q_i^2 |\phi_i|^2 = \mathcal{O}(\xi_{FI})$

QCD axion scale: $f_a = \dfrac{f_{st} v}{\sqrt{f_{st}^2 + v^2}}$

If $\xi_{FI} \ll f_{st}^2$, so that $v \ll f_{st}$, the axion scale can be far below the GUT scale, while preserving SUSY at M_{GUT} : KC, Jeong, Okumura, Yamaguchi '11

$$f_a \simeq v \ll \frac{g^2}{8\pi^2} M_{Pl}$$

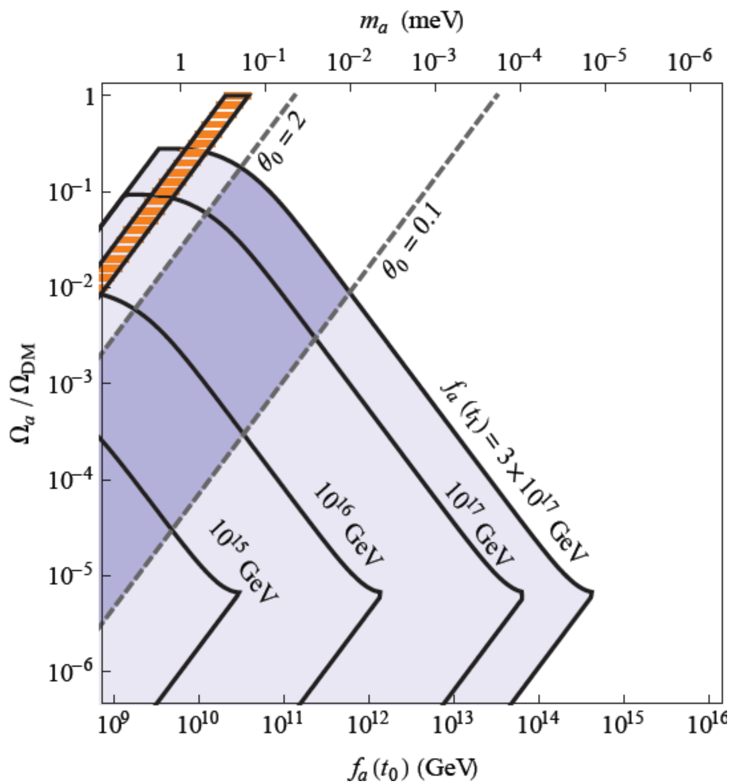
which can be realized in many string models including the Type II string theory with D-branes and the heterotc string theory with U(1) gauge bundles.

Implications of BICEP2 for string theoretic QCD axion

* String theoretic QCD axion which is not charged under an anomalous U(1) gauge symmetry, and therefore has

$$\mathrm{f_a(t_0)} \sim \mathrm{f_a(t_I)} \sim \frac{\mathrm{g^2}}{8\pi^2}\mathrm{M_{Pl}} \sim 10^{16}\ \mathrm{GeV}$$

is excluded by the BICEP2 result.



* Cosmological constraints from PLANCK and BICEP2 require that the PQ symmetry is either restored ($f_a(t_I) = 0$), or broken at a much higher scale ($f_a(t_I) \gg f_a(t_0)$), during inflation.

This suggests that the axion scale might be generated by SUSY-breaking effects, which can be successfully implemented for string theoretic QCD axion charged under an anomalous U(1) gauge symmetry with vanishing FI term in the SUSY limit.

$$f_a(t) = \frac{v(t) f_{st}}{\sqrt{v^2(t) + f_{st}^2}} \quad \left(v(t) = \langle \phi(t) \rangle, \ f_{st} \sim \frac{M_{Pl}}{8\pi^2} \right)$$

Present universe: $V(\phi; t_0) = -m_{SUSY}^2 |\phi|^2 + \frac{|\phi|^{4+2n}}{M_{Pl}^{2n}} + \dots \quad \left(W = \frac{\chi \phi^{n+2}}{M_{Pl}^n} \right)$

$\rightarrow \quad f_a(t_0) \simeq v(t_0) \sim (m_{SUSY} M_{Pl}^n)^{1/(n+1)}$

Inflation epoch: $V(\phi; t_I) = -c H_I^2 |\phi|^2 + \frac{|\phi|^{4+2n}}{M_{Pl}^{2n}} \quad (c H_I^2 \gg m_{SUSY}^2)$

$\rightarrow \quad c < 0 : \quad f_a(t_I) = v(t_I) = 0$

$c > 0 : \quad f_a(t_I) \simeq v(t_I) \sim \left(\sqrt{c} H_I M_{Pl}^n \right)^{1/(n+1)} \gg v(t_0)$

More careful analysis shows that the Hubble-induced $U(1)_A$ D-term plays an important role for the symmetry breaking during inflation.

For generic parameter region, the PQ symmetry is spontaneously broken during the inflation period with

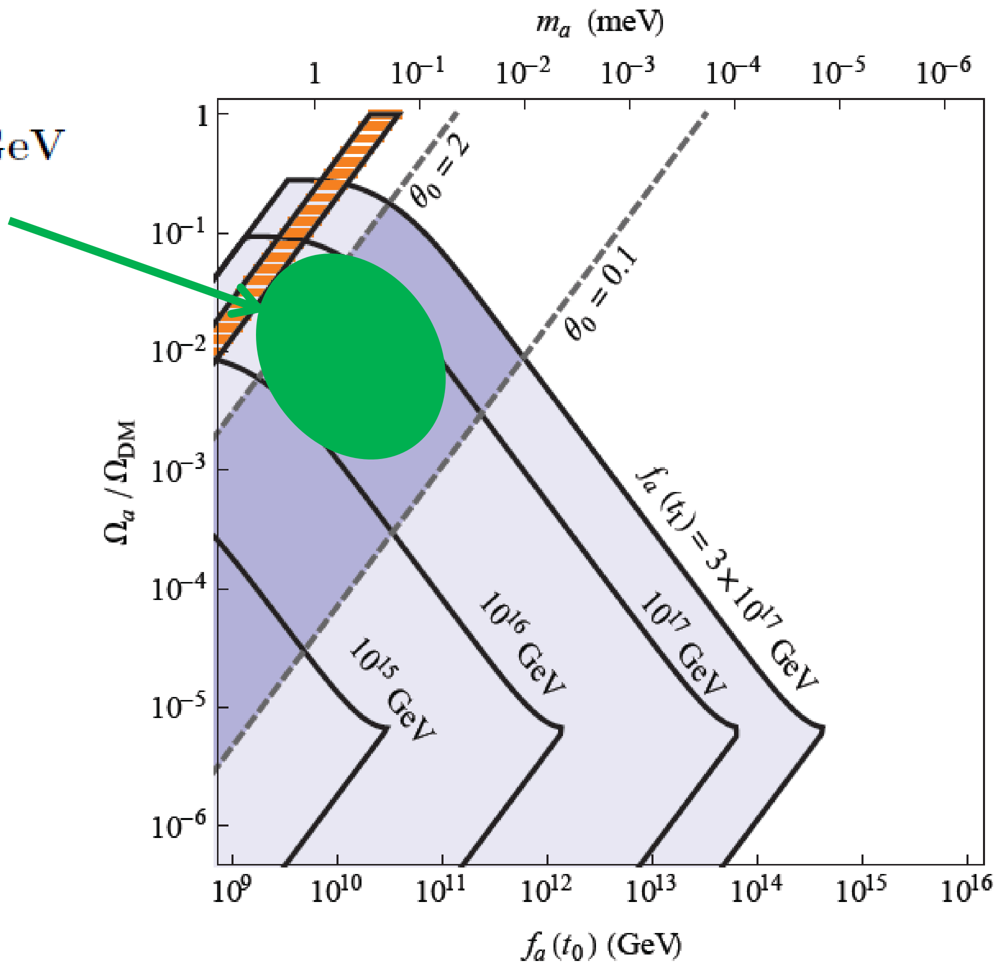
$$f_a(t_I) \sim (4\pi H(t_I) M_{Pl}^n)^{1/(n+1)} \sim 10^{16} - 10^{17}\,\mathrm{GeV} \gg f_a(t_0)$$

although there exists also an unnatural parameter range in which the PQ symmetry is restored during the inflation period.

Most plausible range of $f_a(t_0)$ and Ω_a in view of our results

$$f_a(t_I) = 10^{16} - 10^{17}\ \mathrm{GeV}$$

$$\theta_0 = \mathcal{O}(1)$$



$$f_a(t_0) \sim m_{SUSY} \quad \text{or} \quad \sqrt{m_{SUSY} M_{Pl}}$$

$$\sim 10^9 - 10^{11}\ \mathrm{GeV}$$

Conclusion

* Compactified string models involving an anomalous U(1) gauge symmetry with vanishing FI-term appears to be the best theoretical setup for the QCD axion, which explains the origin of the PQ symmetry, while giving an axion scale well below the GUT scale.

* PLANCK and BICEP2 results require that either the PQ symmetry is restored or broken at much higher scale during the inflation period.

For high scale inflation ($H(t_I) \simeq 10^{14}\,GeV$) suggested by the BICEP2 results, the allowed range of the axion scale and the relic axion abundance is greatly reduced.

The results fit well to the scenario that the PQ symmetry is spontaneously broken by SUSY-breaking effects, leading to a specific connection between the axion scale and the SUSY breaking mass as

$$f_a(t_0) \sim (m_{SUSY} M_{Planck}^n)^{1/(n+1)} \quad (n \geq 0)$$

* More careful analysis of the dynamics of the model implies that the PQ symmetry is spontaneously broken during inflation for generic model parameters, and the QCD axions might be here.

$$f_a(t_0) \sim (m_{SUSY} M_{Planck}^n)^{1/(n+1)} \quad (n \geq 0)$$

➔ Axion scale SUSY (n=0):

$$m_{SUSY} \sim f_a \sim 10^9 - 10^{11}\ \mathrm{GeV}$$

or Low scale SUSY (n=1):

$$m_{SUSY} \sim \frac{f_a^2}{M_{Planck}} \sim 10^3 - 10^4\ \mathrm{GeV}$$