19 May 2014 @ IAS, Hong Kong

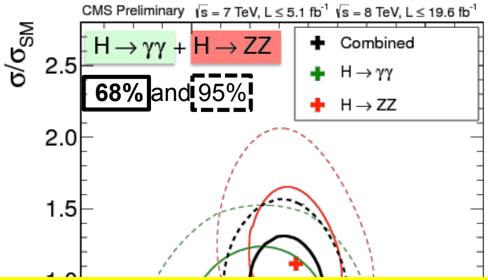
# Naturalness of
# 126 GeV Higgs and meV dark energy

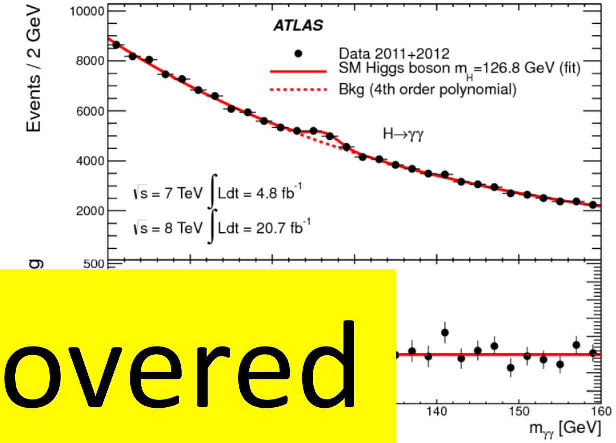
Satoshi (曉) Iso (磯) (KEK & Sokendai)

In this talk, I want to propose mechanisms (or hints) for dynamical generations of $M_{EW}$ and $\Lambda_{DE}$ from $M_{PL}$

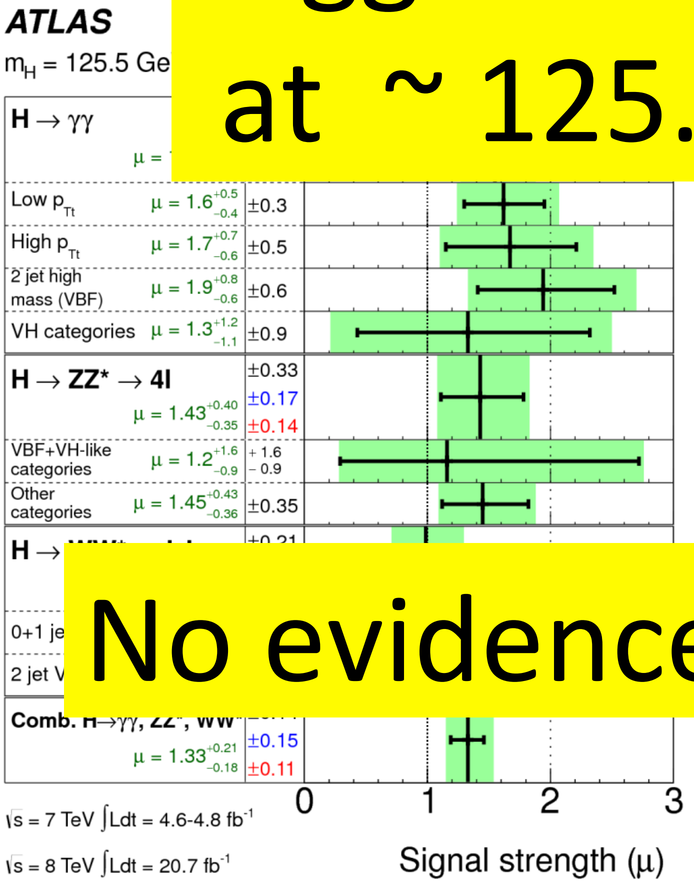
$M_{EW}$ : dimensional transmutation (Coleman Weingerg )
$\Lambda_{DE}$ : vacuum fluctuation
(the remnant of the early universe)

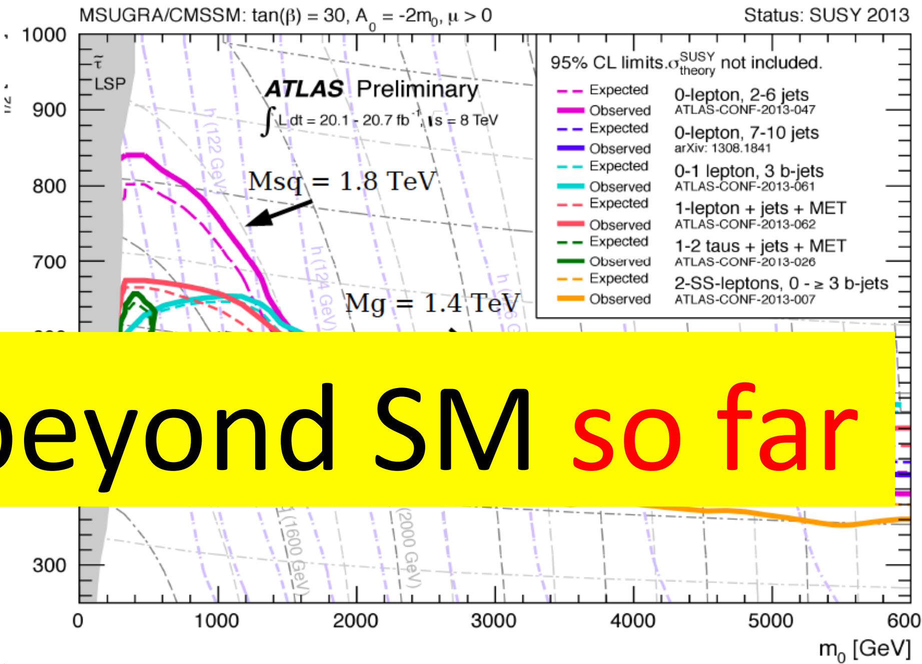
# Part 1
# Higgs at LHC





# Higgs boson is discovered at ~ 125.5 GeV





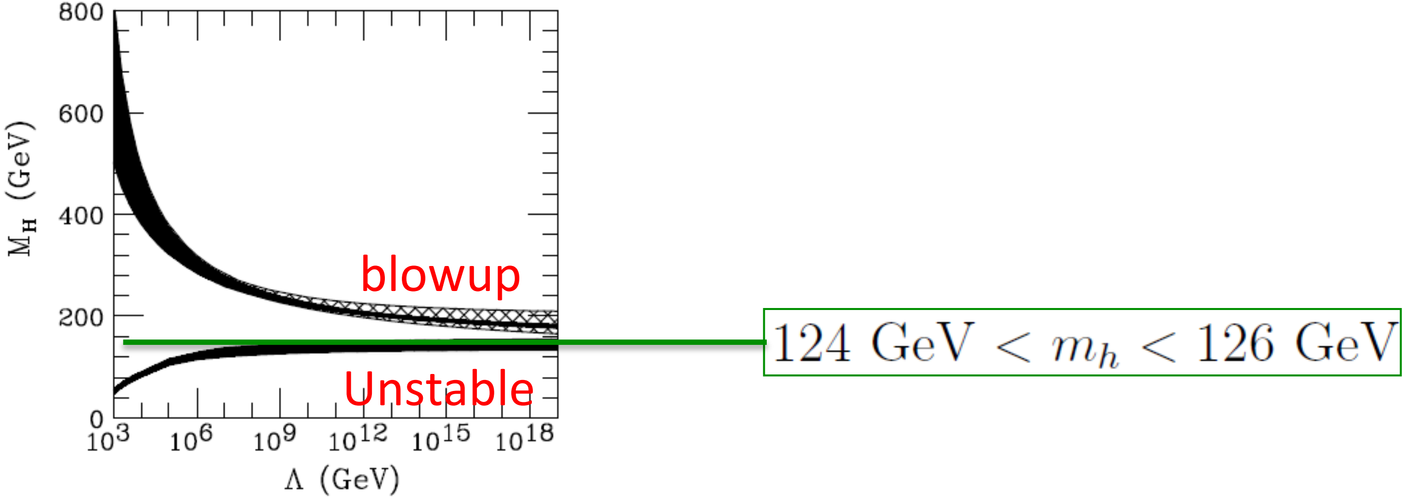
# No evidence beyond SM so far

# Two important clues for physics beyond SM from LHC

**(1) Strong constraint on SUSY** forces us to reconsider **Naturalness (Hierarchy) problem**

Why is $M_{EW}$ much smaller than $M_{PL}$ or $M_{GUT}$ ?
= quadratic divergence of Higgs mass term → TeV SUSY

**(2) 126 GeV Higgs** indicates vanishing of quartic Higgs coupling at $M_{PL}$
→ **Stability of vacuum**

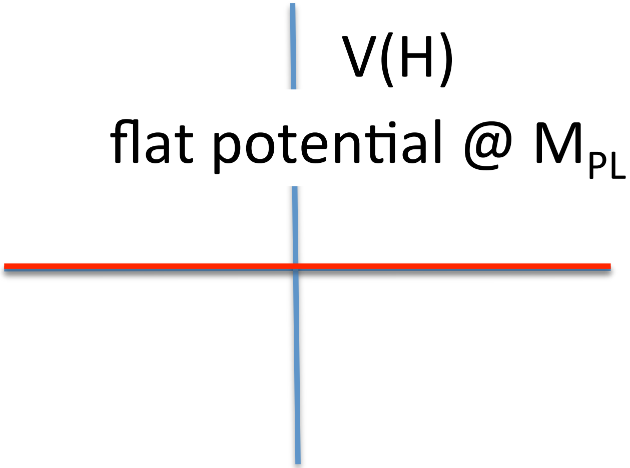


$124\ \mathrm{GeV} < m_h < 126\ \mathrm{GeV}$

# These two suggest V(H)=0 at $M_{PL}$

$$V = \xcancel{-\mu^2|H|^2} + \xcancel{\lambda(|H|^2)^2}$$

naturalness 126 GeV



V(H) may have a shift symmetry

H → H +c

(But it is broken by Yukawa and gauge couplings.)

# (1) Naturalness problem

$$V = \boxed{-\mu^2 |H|^2} + \lambda(|H|^2)^2$$

It is usually explained as the problem of quadratic divergence of $\mu^2$ .

→ cancellation of quadratic divergence by supersymmetry.

**Question: Is quadratic divergence really the issue of the hierarchy problem?**

- It can be always subtracted with no effects on physics, and different from logarithmic divergences (multiplicative renorm.)
- No quadratic divergences in dimensional regularization. (minimal subtraction)

Bardeen (1995)
Fujikawa (2011)
Aoki Iso (2012)

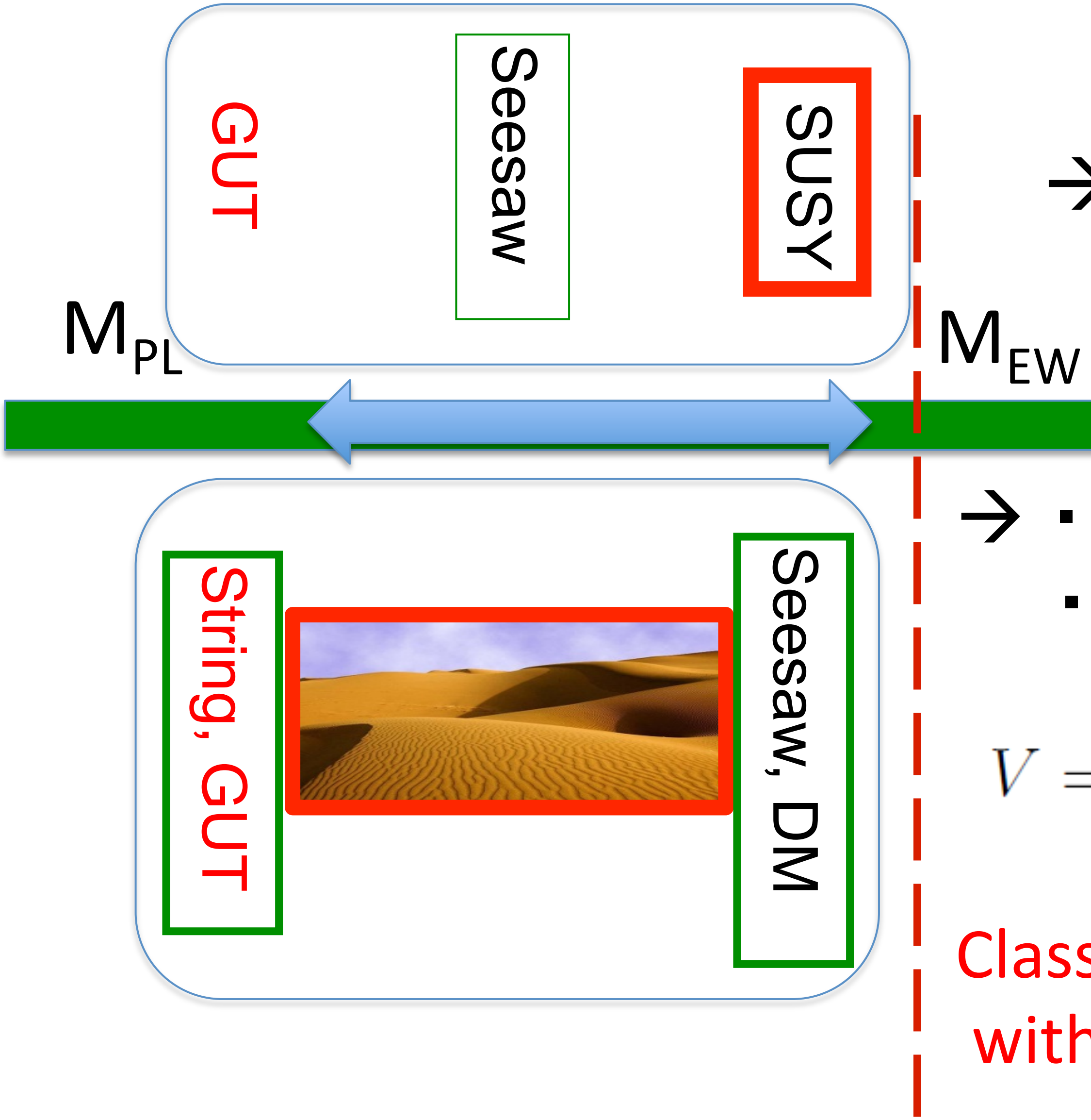
# 3 important properties of the naturalness problem

(a) SM has no dimensionful parameters if $\mu^2$-term is absent.
→ classically conformal

(b) Quadratic divergence is NOT the issue in low energy EFT.
String theory may offer a clever solution at UV.

(c) The real issue of the naturalness problem in EFT is
mixing with physical scales such as $M_{GUT}$ or $M_{seesaw}$
If $M_{GUT} < M_{PL}$ , we need something like SUSY
If $M_{GUT} = M_{String}$ , it is no longer the issue in IR.

# Two different pictures to solve the hierarchy problem



→ TeV SUSY is required

→ ・desert beyond $M_{EW}$

・$\mu^2 = 0$

$$V = -\xcancel{\mu^2|H|^2} + \lambda(|H|^2)^2$$

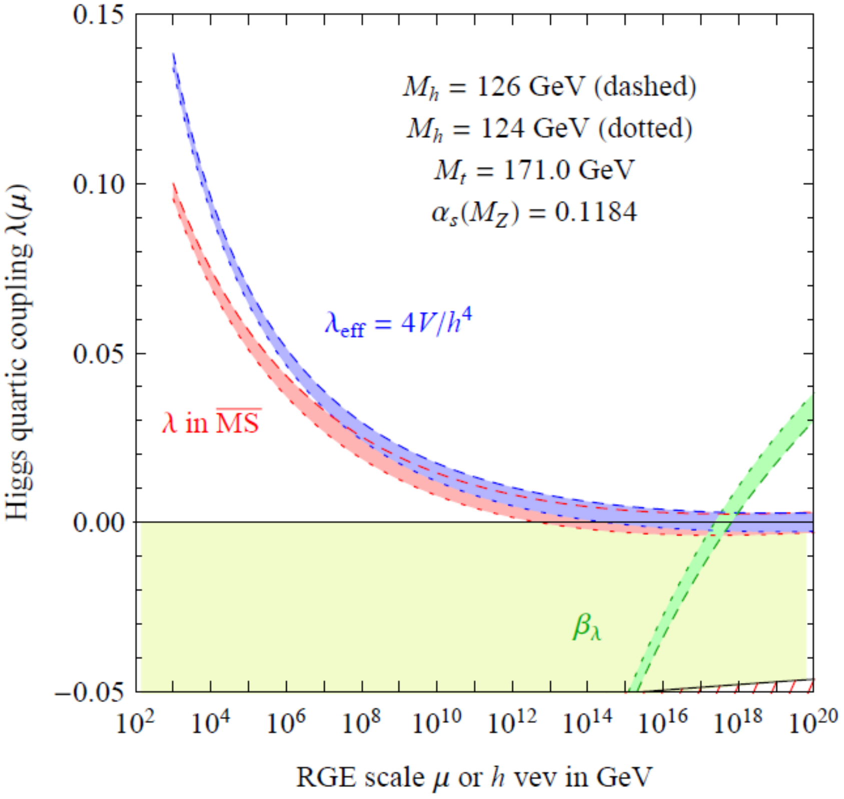
Classical conformal theory
with no intermediate scales

# (2) Stability of Vacuum

A hint for Planck scale physics from $M_H$=126 GeV

$$V = -\mu^2|H|^2 + \lambda(|H|^2)^2$$

$$m_h^2 = \boxed{2\lambda\langle h\rangle^2}$$



$$\boxed{\lambda(\Lambda_0) = \beta_\lambda(\Lambda_0) = 0}$$

$$V = -\mu^2|H|^2 + \xcancel{\lambda(|H|^2)^2}$$

Direct window to Planck scale

Froggatt Nielsen (96)
M.Shaposhnikov (07)

# Flat Higgs potential at $M_{PL}$

$$V = -\cancel{\mu^2|H|^2} + \lambda(\cancel{|H|^2})^2$$

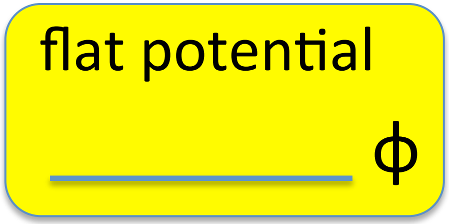
naturalness 126 GeV

Assumption

V(H) has a shift symmetry

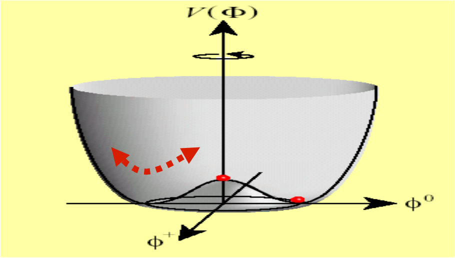
$H \rightarrow H + c$

then V(H)=0 at $M_{PL}$ .



Radiatively generate

Coleman-Weinberg mechanism



EWSB @ $M_{EW}$

# But CW does not work in SM.

the **large top Yukawa** coupling **invalidates the CW** mechanism

↓

Extension of SM is necessary !

Meissner Nicolai (07)

(B-L) extension of SM with flat Higgs potential at Planck

SM +

B-L sector

- $U(1)_{B\text{-}L}$ gauge
- SM singlet scalar $\phi$
- Right-handed $\nu$

N Okada, Y Orikasa,
M. Hashimoto & SI
0902.4050 (PLB)
0909.0128 (PRD)
1011.4769 (PRD)
1210.2848(PTEP)
1310.4304 (PRD)
1401.5944 (PRD)

**"Occam's razor" scenario**

that can explain

- 126 GeV Higgs
- Naturalness problem
- $\nu$ oscillation, baryon asymmetry

B-L symmetry is radiatively broken via CW mechanism.
How does the EWSB occur ?

Flat potential is suggested by LHC $V(H) = 0 \quad @M_{PL}$

$$\cancel{m_H^2 H^2} + \cancel{\lambda_H H^4} + \cancel{\lambda_{H\Phi} H^2}\Phi^2$$

classically
conformal

126 GeV

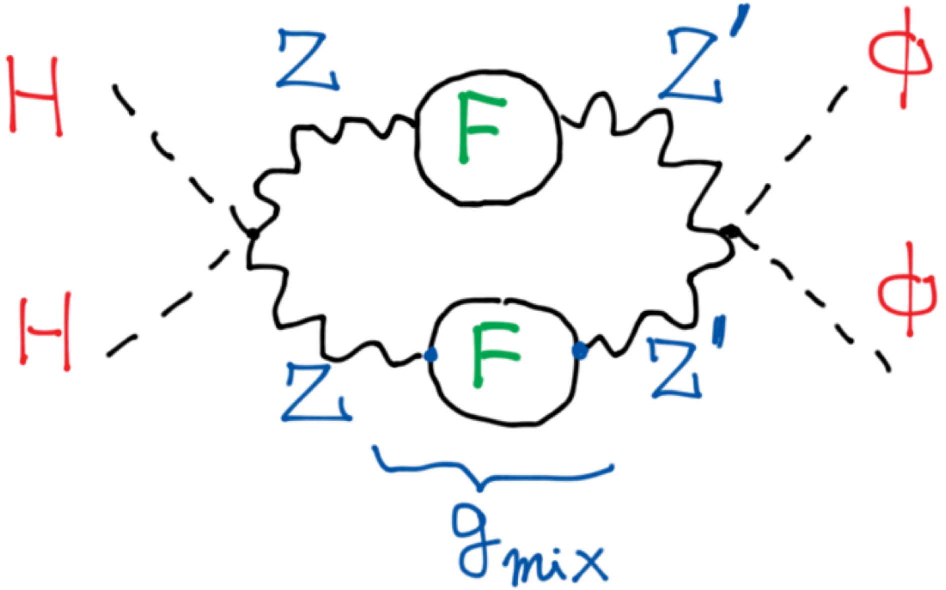
**The coefficient must be small and negative.**

$$\langle H \rangle = \sqrt{\frac{-\lambda_{H\Phi}}{\lambda_H}} M_{B-L}$$

Can the small scalar mixing be realized naturally?

→ Yes ! (Orikasa, SI 2012)

# Very small negative scalar mixing is radiatively generated



$$\lambda_{mix} \sim -\frac{1}{16\pi^2} g_Y g_{B-L} g_{mix}^2$$

$$\boxed{\langle H \rangle = \sqrt{\frac{-\lambda_{mix}}{\lambda_H}} M_{B-L}}$$

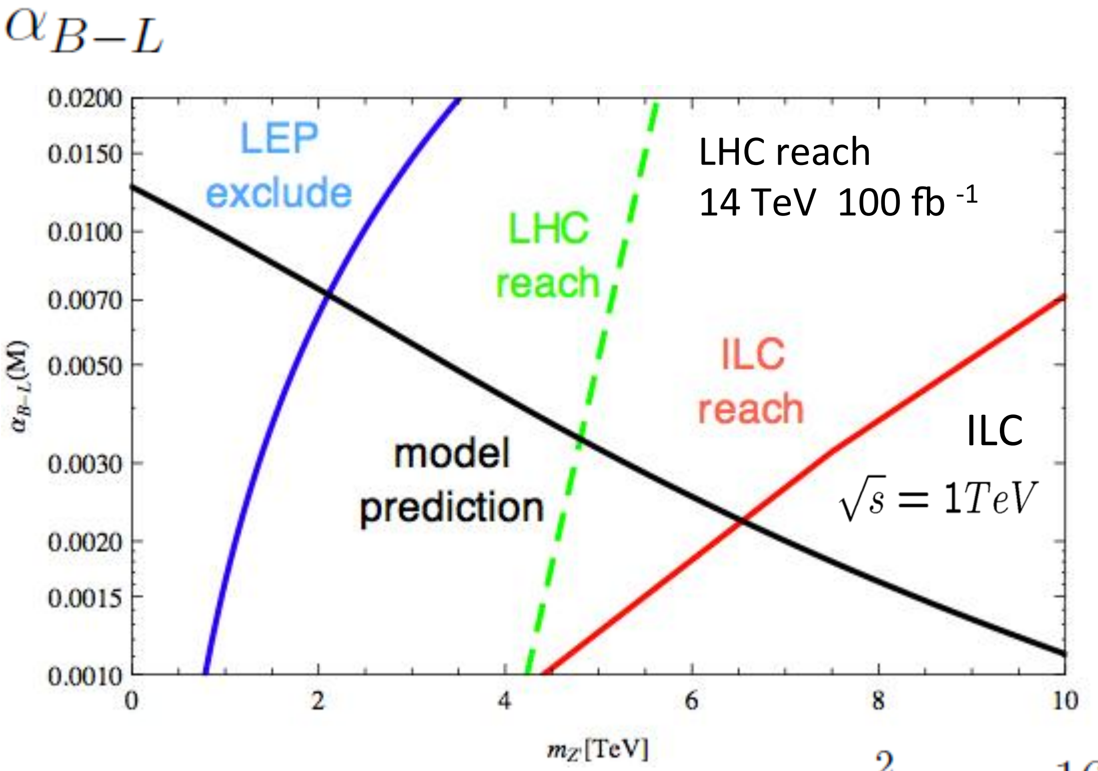
negative and very small $\lambda_{mix}$

→ B-L breaking triggers EWSB

and small hierarchy between $M_{B-L}$ & $M_{EW}$

# Prediction of the model

In order to realize EWSB at 246 GeV,
B-L scale must be around TeV (for a typical value of $\alpha_{B\text{-}L}$ ).



$$M_{B-L} \sim \frac{1}{\alpha_{B-L}} \times 35 \text{ GeV}$$

$$m_{Z'} \sim \frac{1}{\sqrt{\alpha_{B-L}}} \times 250 \text{ GeV}$$

$$\boxed{m_\phi \sim 0.1\ m_{Z'}}$$

$$m_{Z'}^2 = 16\pi\alpha_{B-L}(0)M_{B-L}^2$$

Y Orikasa, SI; 1210.2848(PTEP)

## Summary of part 1 :

- LHC No SUSY → Naturalness reconsidered

  126 GeV → Stability of vacuum

  → Flat potential at $M_{PL}$

  → "Classically conformal B-L model with flat potential at $M_{PL}$"

  prediction : TeV scale B-L

  $M_{\Phi} < M_{Z'}$ and TeV scale seesaw $M\nu_R$

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## Future problems

- Planck scale boundary condition: gauge-Higgs at Planck ??

  or String (or something beyond ordinary field theories)

  nonsupersymmetric vacuum

  GUT is broken at string scale

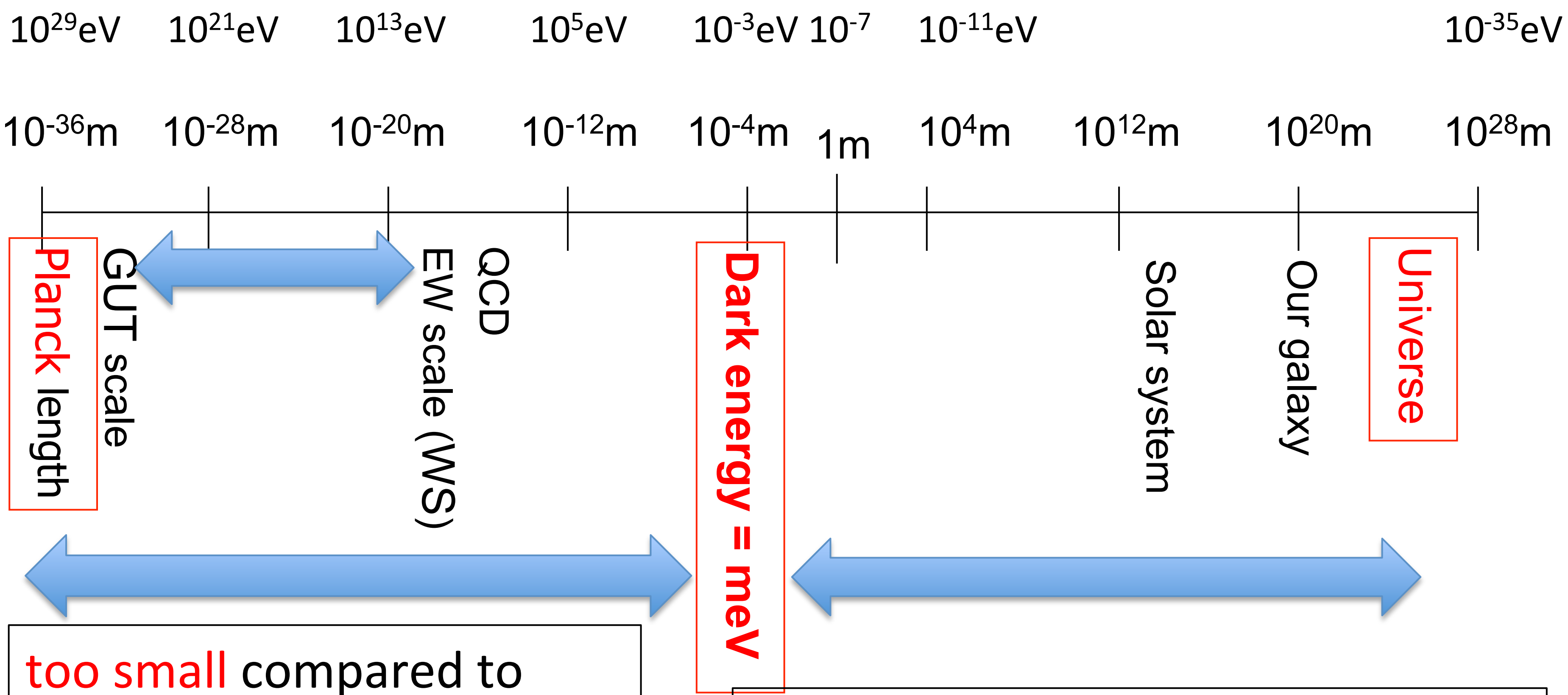
  massless scalar with flat potential
- finite temperature effect (1st order PT, supercooling problem)

# Part 2

# Vacuum energy in the universe

*H. Aoki(Saga), Y.Sekino (KEK), SI   arXiv:1402.6900  (PRD)*

# Naturalness of DE



too small compared to

$M_{pl}=10^{27}$ eV,

$\Lambda_{QCD}$ =100 MeV,

$\langle h \rangle$=246 GeV

too large compared to the Hubble parameter at present

$H_0 = 10^{-33}$ eV

$\rho_{cr}^{1/4} = 3^{1/4} (M_{pl} H_0)^{1/2}$ =2.4 meV Coincidence problem

Cosmological constant problem is usually recognized as the problem of quartic divergence in field theory.

Quartic divergence is the real issue of the cosmological constant problem ?

Quartic divergence is not the cosmological constant.

ex.) EMT of a massive field in a curved space-time

Energy=$\int d^3k\, \omega$ , pressure=$\int d^3k\, k^2/3\omega$, $\omega^2=(k^2+m^2)$

So $\Lambda^4$ term has $w=1/3$ (so it is not proportional to $g^{\mu\nu}$).

Logarithmically divergent term $m^4 \log \Lambda$

gives the cosmological constant with $w=-1$ (DE).

In the following, we assume that
"the vacuum energy is set classically zero"

$\Lambda_{\text{classical}}=0$

and ask how we can dynamically generate meV DE

$\Lambda_{\text{DE}} = \text{meV} ?$

Remnant of the vacuum fluctuation generated in the early universe

|| ?

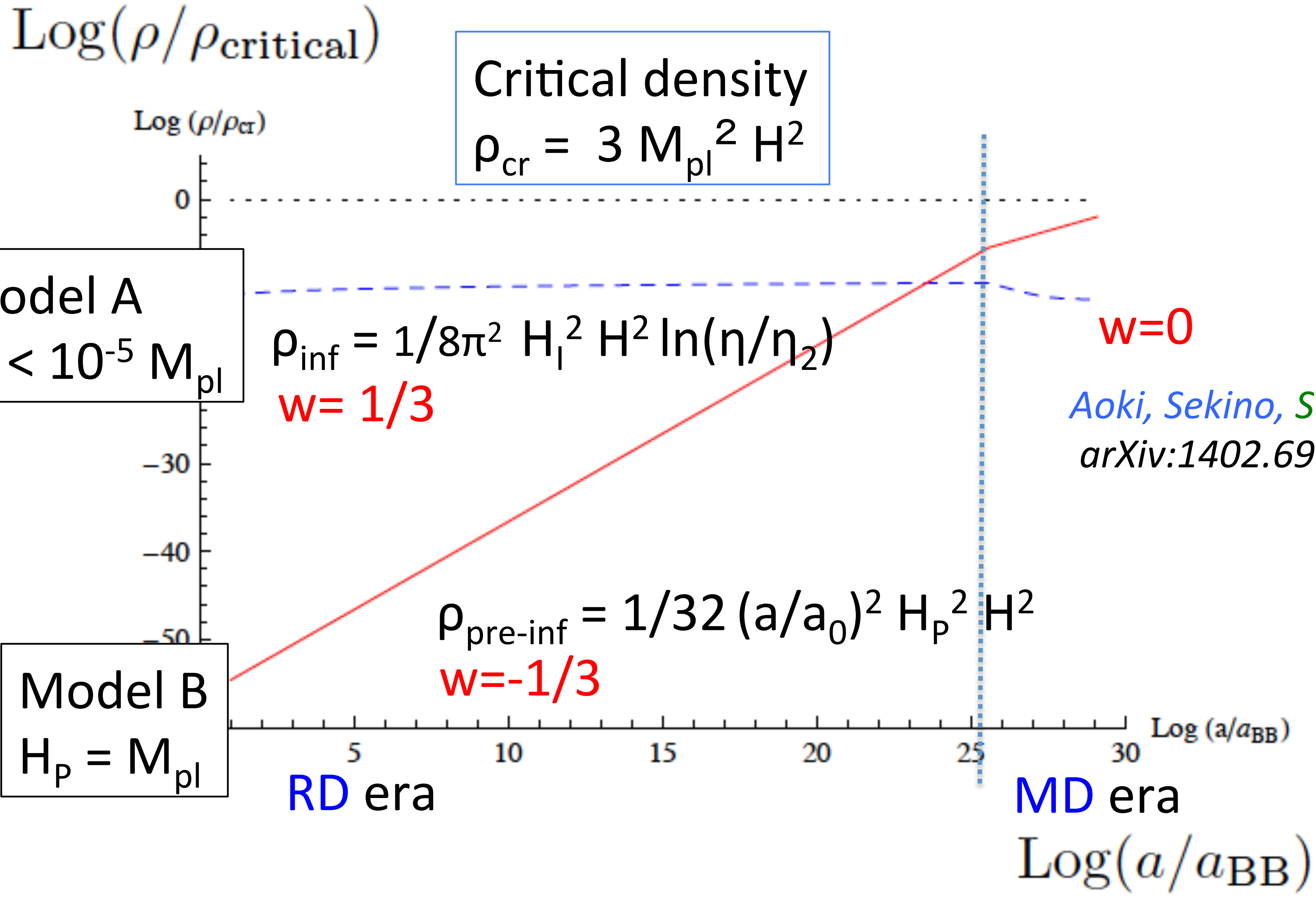
Dark energy at the present universe

We calculate ⟨EMT⟩ in the following two models.

- Model A :ordinary history of universe
  Inflation → radiation dom. → matter dom.

- Model B :pre-inflation before ordinary inflation
  pre-inflation + inflation → RD → MD
  (Hubble $H_P \sim M_{pl}$)

# Time evolution of energy density after big-bang (end of inflation)



Critical density
$\rho_{cr} = 3 M_{pl}^2 H^2$

**Model A**
$H_I < 10^{-5} M_{pl}$

$\rho_{inf} = 1/8\pi^2 H_I^2 H^2 \ln(\eta/\eta_2)$
w= 1/3

w=0

*Aoki, Sekino, SI*
*arXiv:1402.6900 (PRD)*

**Model B**
$H_P = M_{pl}$

$\rho_{pre\text{-}inf} = 1/32 (a/a_0)^2 H_P^2 H^2$
w=-1/3

RD era

MD era

# Model A (Inflation – RD – MD) $ds^2 = a(\eta)^2 \left[d\eta^2 - (dx^i)^2\right]$

(massless) minimally coupled scalar field $\phi_k = \chi_k / a(\eta)$ is described by

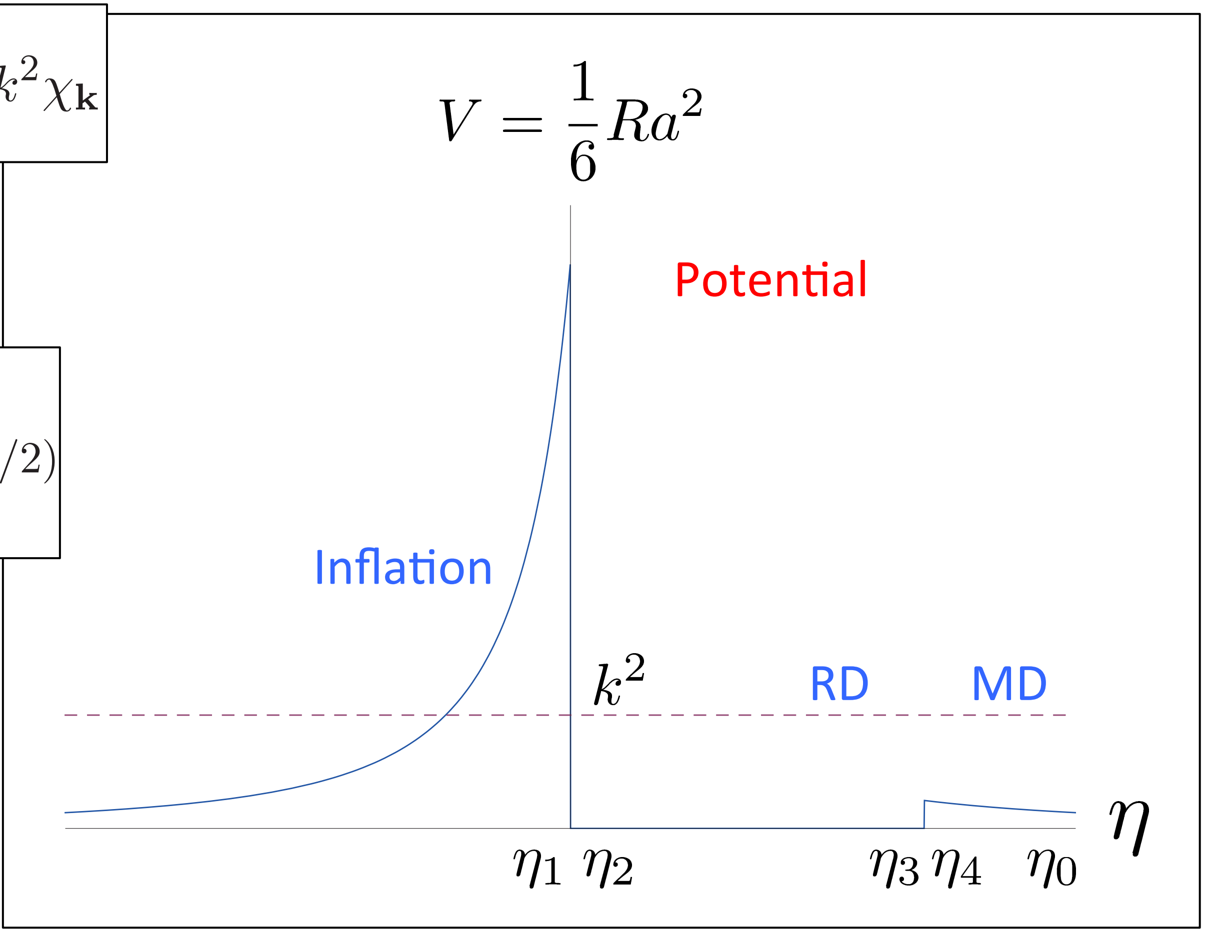
1-dim Schrodinger eq

$$\boxed{\left[-\partial_\eta^2 + \frac{1}{6}Ra^2 - m^2a^2\right]\chi_{\mathbf{k}} = k^2\chi_{\mathbf{k}}}$$

with a potential

$$\frac{1}{6}Ra^2 = \boxed{\begin{cases} 2/\eta^2 & (\eta < -|\eta_1|) \\ 0 & (|\eta_1| < \eta < \eta_4/2) \\ 2/\eta^2 & (\eta_4 < \eta < \eta_0) \end{cases}}$$

$$V = \frac{1}{6}Ra^2$$



# [Step 1] Solve the Schrodinger eq
# with Bunch-Davis initial condition in the Inflation era

$$\chi_{\mathrm{BD},\mathbf{k}} = \frac{1}{\sqrt{2k}}\left(1-\frac{i}{k\eta}\right)e^{-ik\eta}$$

Inflation period

Bogoliubov transformation (particle creation)

RD $$\chi_{\mathrm{RD},\mathbf{k}} = A(k)\ \chi_{\mathrm{PW},\mathbf{k}} + B(k)\ \chi^*_{\mathrm{PW},-\mathbf{k}}$$

MD $$\chi_{\mathrm{MD},\mathbf{k}} = C(k)\ \chi_{\mathrm{BD},\mathbf{k}} + D(k)\ \chi^*_{\mathrm{BD},-\mathbf{k}}$$

$$\chi_{\mathrm{PW},\mathbf{k}} = \frac{1}{\sqrt{2k}}e^{-ik\eta}$$

with the coefficients A, B, C, D s.t. $|A|^2-|B|^2 = |C|^2-|D|^2=1$

$$\begin{pmatrix} A(k) \\ B(k) \end{pmatrix} = \begin{pmatrix} \left(1-\frac{i}{k\eta_1}-\frac{1}{2k^2\eta_1^2}\right)e^{ik\eta_2} \\ \frac{1}{2k^2\eta_1^2}e^{-ik\eta_2} \end{pmatrix} e^{-ik\eta_1} ,$$

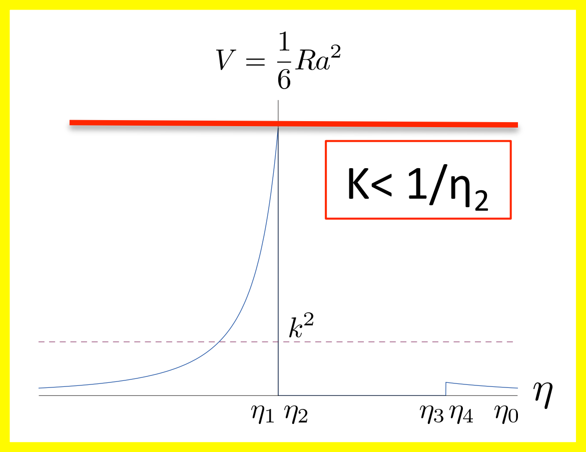
B(k) ~ $k^{-2}$

$$\begin{pmatrix} C(k) \\ D(k) \end{pmatrix} = \begin{pmatrix} \left(1+\frac{i}{k\eta_4}-\frac{1}{2k^2\eta_4^2}\right)e^{ik(\eta_4-\eta_3)} & -\frac{1}{2k^2\eta_4^2}e^{ik(\eta_4+\eta_3)} \\ -\frac{1}{2k^2\eta_4^2}e^{-ik(\eta_4+\eta_3)} & \left(1-\frac{i}{k\eta_4}-\frac{1}{2k^2\eta_4^2}\right)e^{-ik(\eta_4-\eta_3)} \end{pmatrix}\begin{pmatrix} A(k) \\ B(k) \end{pmatrix}$$

Wave functions are largely enhanced in the IR region.

# [Step 2] Calculate UV regularized EMT

B(k) should vanish
beyond potential height



## Renormalized EMT in the RD period

$$\rho^{\rm ren}_{\rm RD} = \frac{1}{8\pi^2 a^4}\int_0^{\eta_2^{-1}} dk \Bigg[ (|A|^2+|B|^2-1)\left(2k^3+\frac{k}{\eta^2}\right) + A^*B\left(-2i\frac{k^2}{\eta}+\frac{k}{\eta^2}\right)e^{2ik\eta} + AB^*\left(2i\frac{k^2}{\eta}+\frac{k}{\eta^2}\right)e^{-2ik\eta}\Bigg]$$

$$p^{\rm ren}_{\rm RD} = \frac{1}{8\pi^2 a^4}\int_0^{\eta_2^{-1}} dk \Bigg( |A|^2+|B|^2-1)\left(\frac{2}{3}k^3+\frac{k}{\eta^2}\right) + A^*B\left(-\frac{4}{3}k^3-2i\frac{k^2}{\eta}+\frac{k}{\eta^2}\right)e^{2ik\eta} + AB^*\left(-\frac{4}{3}k^3+2i\frac{k^2}{\eta}+\frac{k}{\eta^2}\right)e^{-2ik\eta}\Bigg]$$

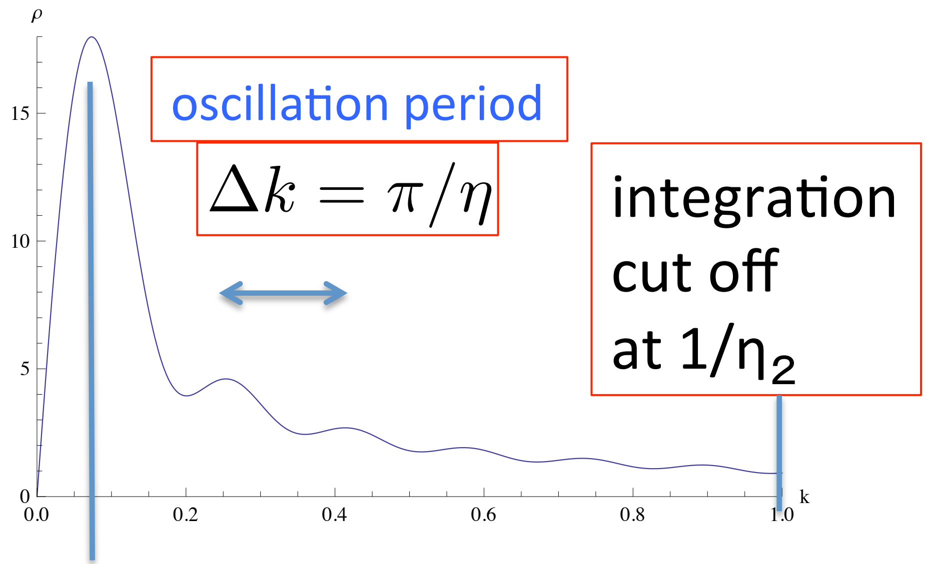
Quartic divergence

# Integrands $\rho(k)$ and $p(k)$

- $\eta_2 = -\eta_1 = 1$, and $\eta = 20$

$\rho$

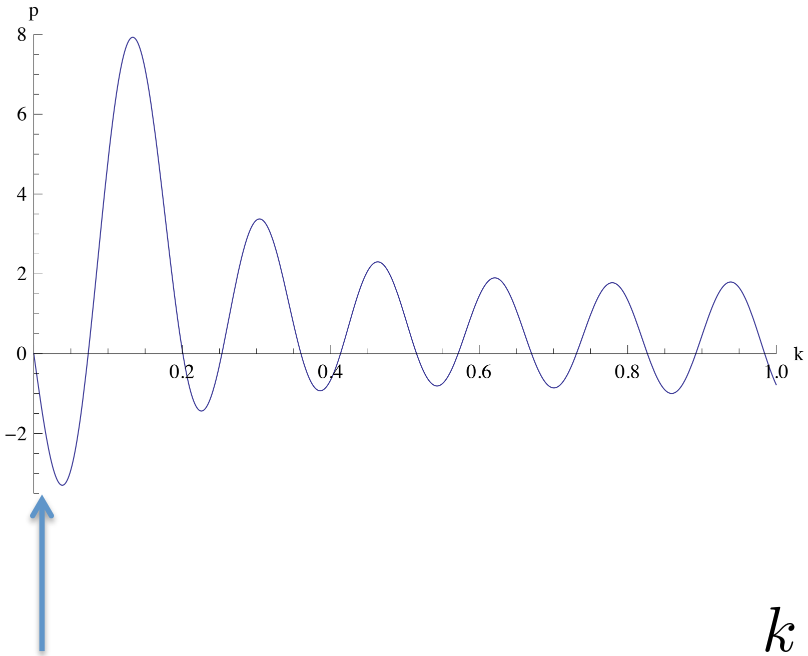
$p$



oscillation period

$\Delta k = \pi/\eta$

integration cut off at $1/\eta_2$



$\pi/2\eta$

$k$

$k$

η is conformal time at each moment.

Negative pressure at IR with w = - 1/3 .

# $\rho$ and $p$ after $k$-integration

$$s = \sin\left[2k(\eta - \eta_2)\right]$$
$$c = \cos\left[2k(\eta - \eta_2)\right]$$

$$\begin{aligned}\rho_{\mathrm{RD}} \;=\; & \frac{1}{8\pi^2 a^4 \eta_1^4}\int_0^{\eta_2^{-1}} \frac{dk}{k}\Big[1 - (k\eta)^{-1}\left(s - 2k\eta_1 c - 2(k\eta_1)^2 s\right) \\ & +(k\eta)^{-2}\left(\frac{1}{2} - \frac{1}{2}c - k\eta_1 s + (k\eta_1)^2 c\right)\Big] = \frac{1}{8\pi^2}(H_I H)^2 \ln\left(\frac{\eta}{\eta_2}\right)\end{aligned}$$

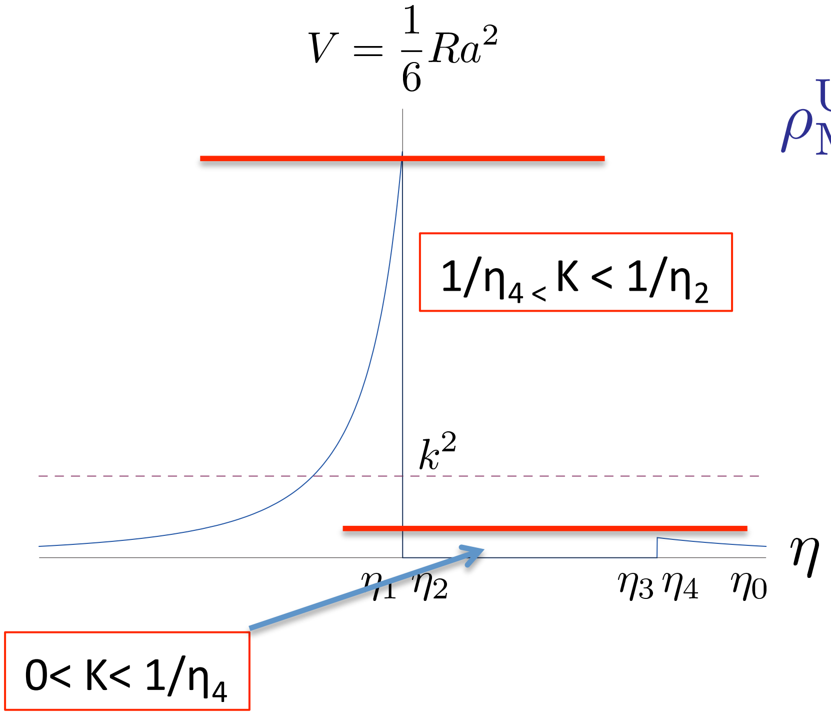
$$\boxed{\frac{1}{8\pi^2}(H_I H)^2}$$

$$\begin{aligned}p_{\mathrm{RD}} \;=\; & \frac{1}{8\pi^2 a^4 \eta_1^4}\int_0^{\eta_2^{-1}} \frac{dk}{k}\Big[\frac{1}{3} + \frac{2}{3}c + \frac{4}{3}k\eta_1 s - \frac{4}{3}(k\eta_1)^2 c \\ & -(k\eta)^{-1}\left(s - 2k\eta_1 c - 2(k\eta_1)^2 s\right) \\ & +(k\eta)^{-2}\left(\frac{1}{2} - \frac{1}{2}c - k\eta_1 s + (k\eta_1)^2 c\right)\Big] = \frac{1}{3}\cdot\frac{1}{8\pi^2}(H_I H)^2 \ln\left(\frac{\eta}{\eta_2}\right)\end{aligned}$$

w=1/3

# MD period:

We need to separate the k-integration into two regions
IR region [0< k < 1/η$_4$] and UV region [1/η$_4$ < k < 1/η$_2$ ]



1/η$_4$ < K < 1/η$_2$

0< K< 1/η$_4$

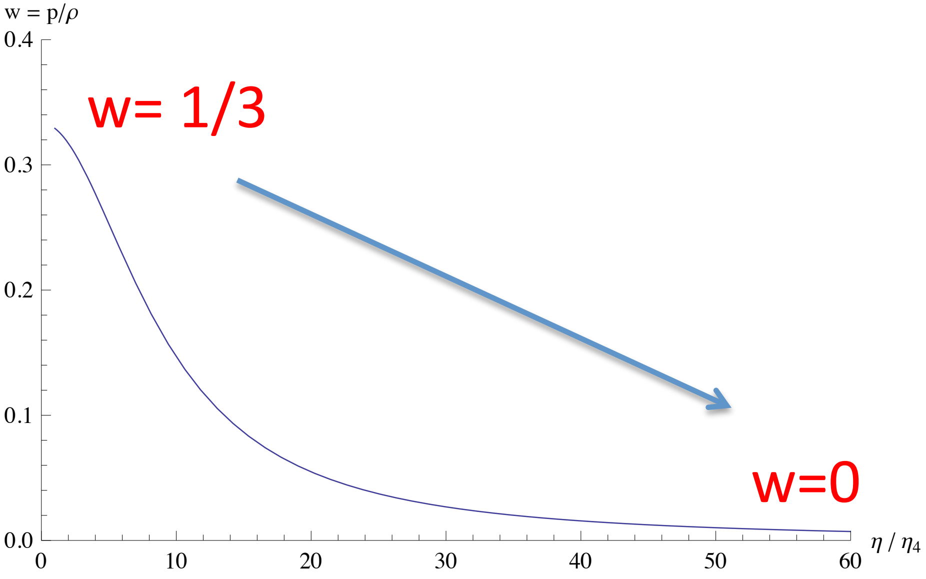
$$\rho_{\rm MD}^{\rm UV} \simeq \frac{1}{8\pi^2}(H_I H)^2 \left(\frac{a_{\rm eq}}{a}\right) \ln\left(\frac{\eta_4}{\eta_2}\right)$$

$$p_{\rm MD}^{\rm UV} \simeq 1/3\ \rho_{\rm MD}^{\rm UV}$$

$$\boxed{\begin{aligned} \rho_{\rm MD}^{\rm IR} &\simeq \frac{3}{4}\frac{1}{8\pi^2}(H_I H)^2 \\ p_{\rm MD}^{\rm IR} &\simeq 0 \end{aligned}}$$

# Time evolution of w in MD era

$$w = p/\rho$$

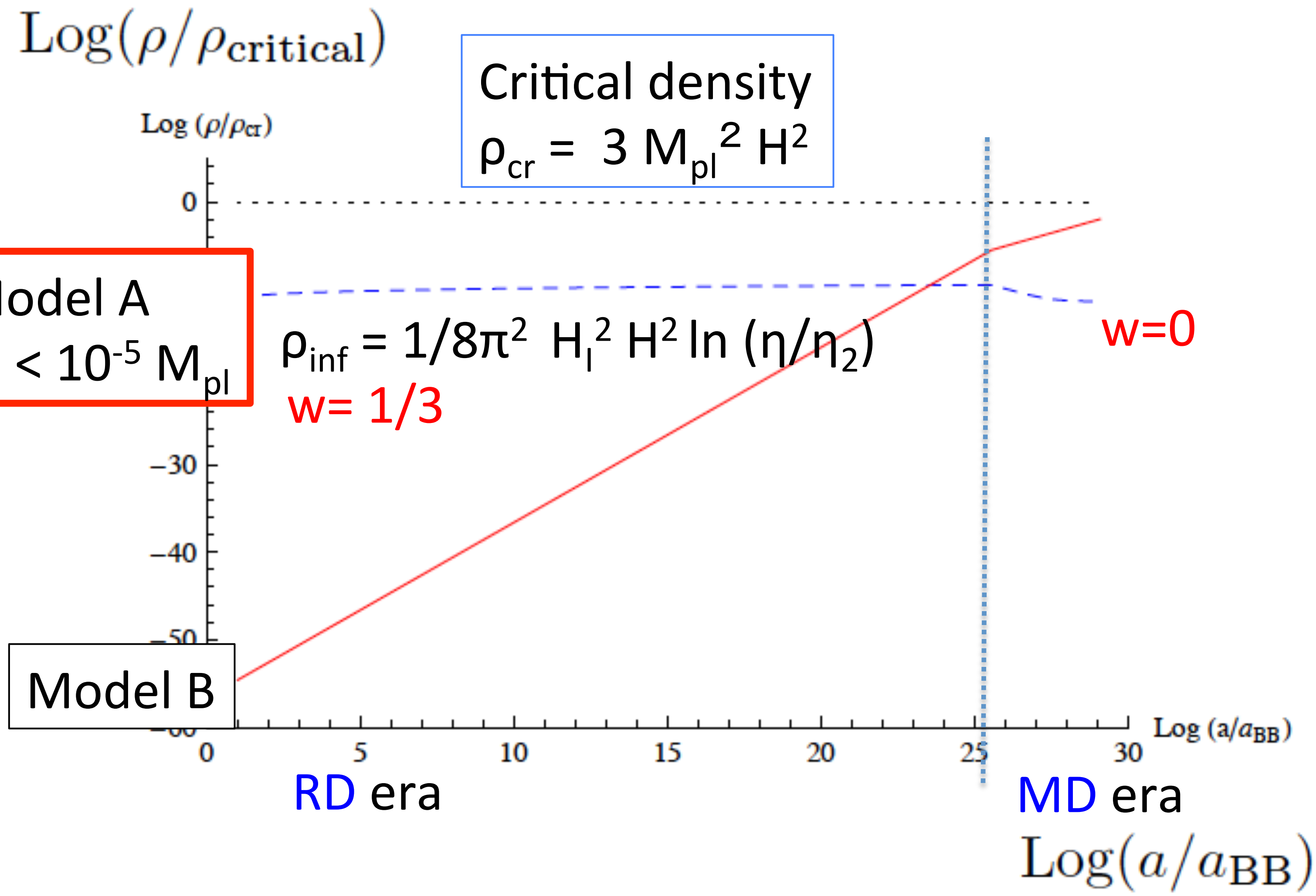


w= 1/3

w=0

$$\eta/\eta_4$$

# Time evolution of energy density after big-bang (end of inflation)



Critical density
$\rho_{cr}$ = 3 $M_{pl}^2$ $H^2$

Model A
$H_I < 10^{-5}$ $M_{pl}$

$\rho_{inf} = 1/8\pi^2 \; H_I^2 \, H^2 \ln(\eta/\eta_2)$

w= 1/3

w=0

Model B

RD era

MD era

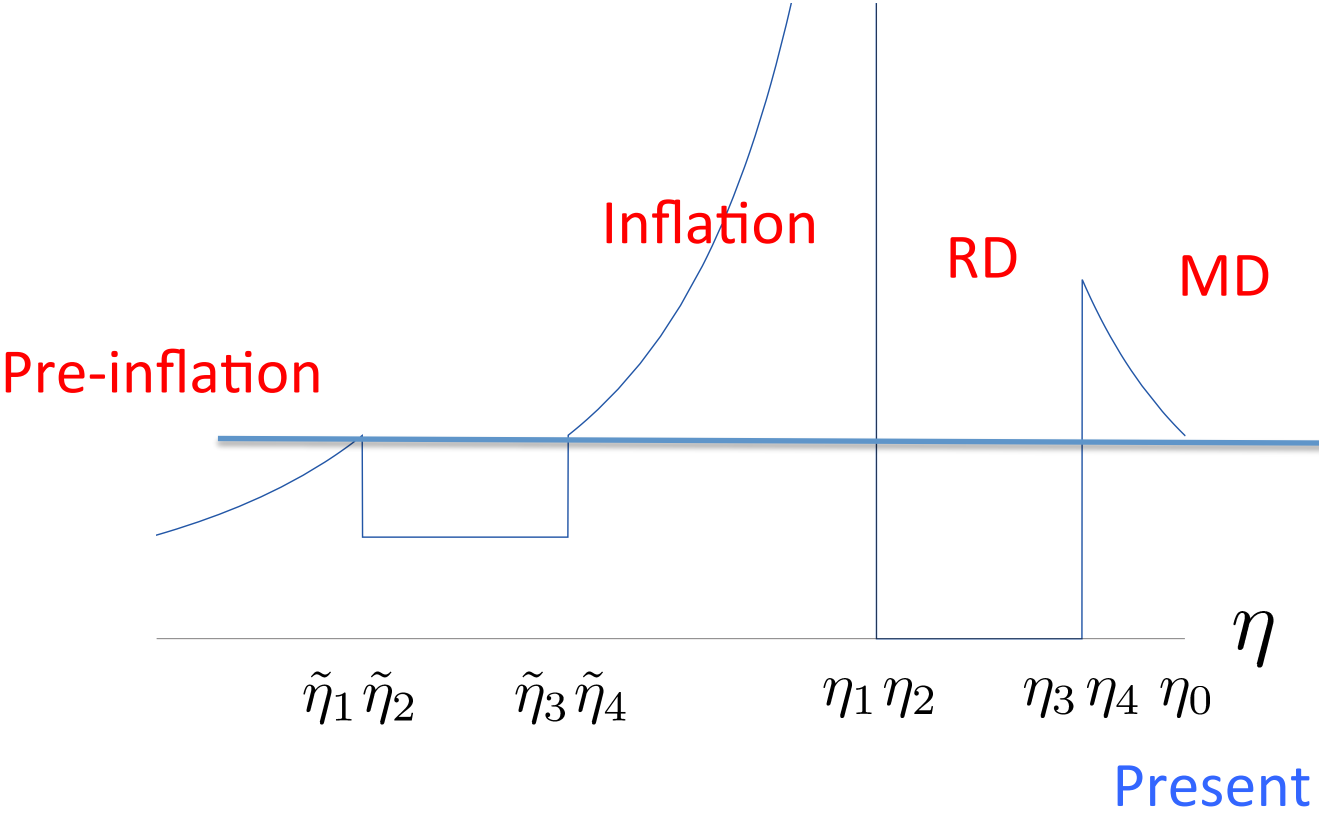
# Model B (Planckian era - Inflation – RD – MD)

Planckian era: de Sitter with larger Hubble $H_{PL} \sim M_{PL}$

(1) At $M_{PL}$, Starobinski type inflation naturally occurs.
(2) Bubble nucleation in eternal inflation
and our universe surrounded by $H_{PL}$ de Sitter

→ Further enhancement of IR modes is expected.

$$V = \frac{1}{6} R a^2$$



k< $1/\tilde{\eta}_2$

The enhanced wave functions have larger wave lengths than the current Hubble horizon size.

# $\rho$ and $p$ generated by pre-inflation

$$\gamma^{-1} = -\tilde{\eta}_1$$

$$\rho^{\mathrm{pre-inflation}} \sim \frac{H_P^2}{8\pi^2 a^2} 2 \int_0^{\gamma/2} dk\ k = \frac{H_P^2}{32\pi^2 a^2}\gamma^2$$

$$p^{\mathrm{pre-inflation}} \sim -\frac{1}{3} \cdot \frac{H_P^2}{8\pi^2 a^2} 2 \int_0^{\gamma/2} dk\ k = -\frac{1}{3} \cdot \frac{H_P^2}{32\pi^2 a^2}\gamma^2$$

EOS is given by w = - 1 /3

because only far IR mode contributes to EMT.

$$\rho(\eta)^{\mathrm{un-ren}} = \frac{1}{a^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \left[\cancel{|u_k'(\eta)|^2} + k^2 |u_k(\eta)|^2\right]$$

for far IR modes

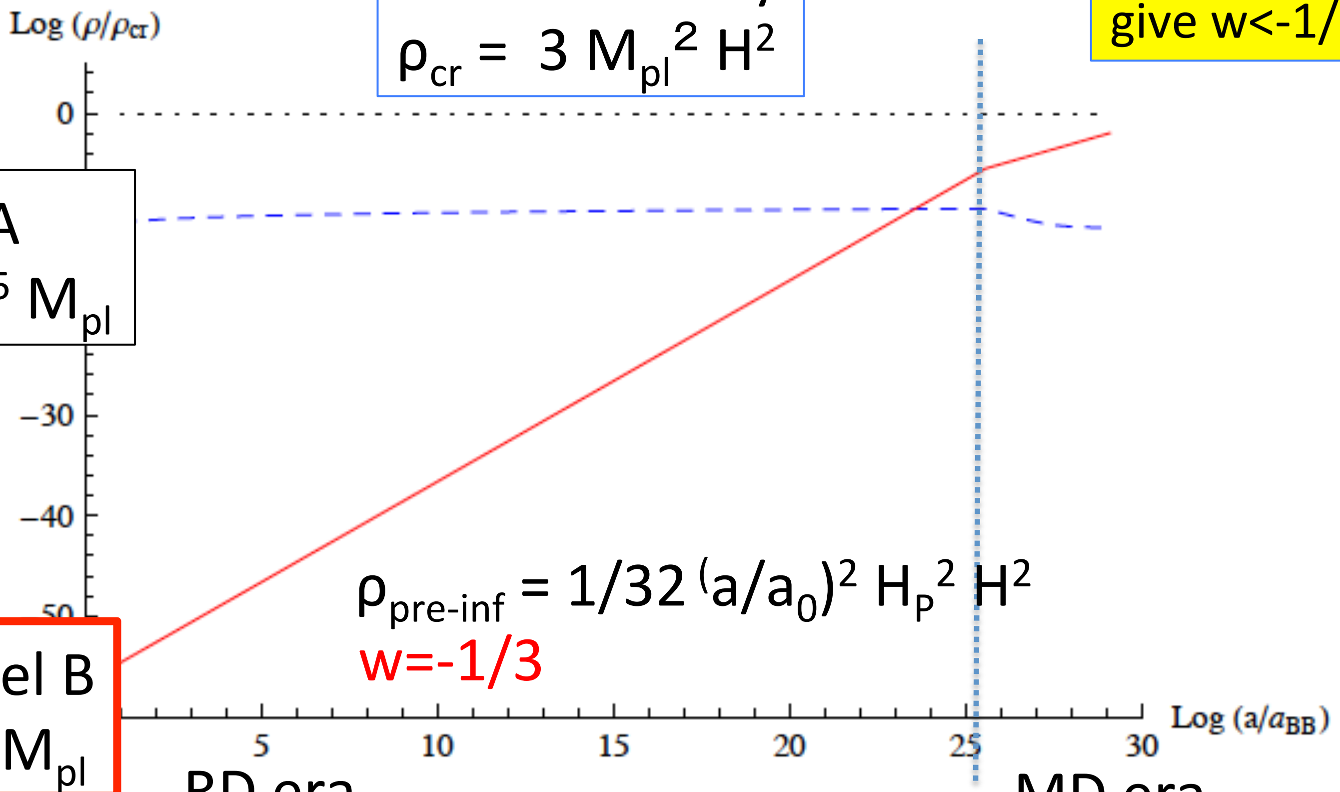
$$p(\eta)^{\mathrm{un-ren}} = \frac{1}{a^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \left[\cancel{|u_k'(\eta)|^2} - \frac{1}{3} k^2 |u_k(\eta)|^2\right]$$

# Time evolution of energy density after big-bang (end of inflation)

$\mathrm{Log}(\rho/\rho_{\mathrm{critical}})$

Critical density
$\rho_{cr} = 3 M_{pl}^2 H^2$

BEC via gravitational attraction (negative pressure) may give w<-1/3.



Model A
$H_I < 10^{-5} M_{pl}$

$\rho_{pre\text{-}inf} = 1/32 (a/a_0)^2 H_P^2 H^2$

w=-1/3

Model B
$H_P = M_{pl}$

RD era

MD era

$\mathrm{Log}(a/a_{\mathrm{BB}})$

# To summarize

Vacuum energy produced by the standard inflation **Model A**

$$\rho_{\rm inf} \simeq \begin{cases} \frac{1}{8\pi^2}(H_I H)^2 \ln\left(\frac{a}{a_{\rm BB}}\right) & \text{w =1/3} \quad \text{(RD)} \\ \frac{1}{8\pi^2}(H_I H)^2 \left[\frac{3}{4} + \ln\left(\frac{a_{\rm eq}}{a_{\rm BB}}\right) \left(\frac{a_{\rm eq}}{a}\right)\right] & \text{(MD)} \end{cases}$$

w =0

Vacuum energy generated by the pre-inflation **Model B**

$$\rho_{\rm pre-inf} \simeq \frac{1}{8} M_P^2 \frac{1}{a^2 \eta_0^2} = \begin{cases} \frac{1}{32}(M_P H)^2 \left(\frac{a_{\rm eq}}{a_0}\right) \left(\frac{a}{a_{\rm eq}}\right)^2 & \text{(RD)} \\ \frac{1}{32}(M_P H)^2 \left(\frac{a}{a_0}\right) & \text{(MD)} \end{cases}$$

w =-1/3

# Conclusions

“Naturalness of $M_{EM}$ and $\Lambda_{DE}$” are usually regarded as the problem of quadratic and quartic divergences in field theories.

But power divergences may be just calculational artifacts that should be simply subtracted.

They may be dynamically generated via

dimensional transmutation = $M_{EW}$

remnant of the early universe = $\Lambda_{DE}$

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Part 1: prediction = TeV B-L and light scalar

future issues = top down approach and finite Temp effect

Part 2: future issues = How can we make it ($w<-1/3$) ?

BEC, gravity effect ... interaction is necessary to be considered.

Thank you

多谢！