Stability and predictability of the S-dS black-hole in dRGT

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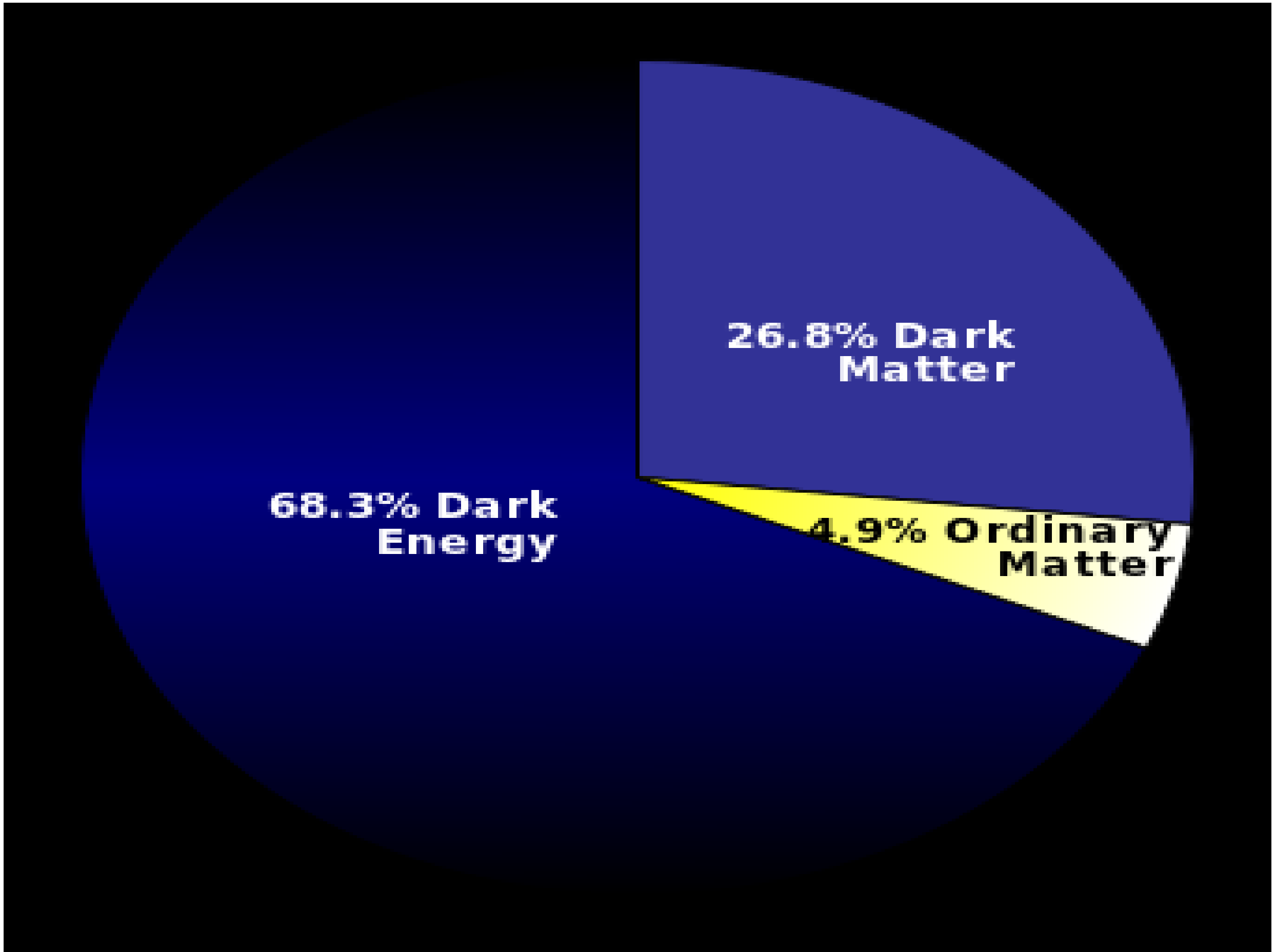
Motivation

- Composition of the universe in agreement with the standard model of Cosmology.



Dark Energy

The easiest way for explaining the acceleraded expansion of the Universe, is introducing a cosmological constant

$$S_{EH} = \int d^4x\sqrt{-g}(R - 2\Lambda) \quad \Longrightarrow \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \Lambda g_{\mu\nu}$$

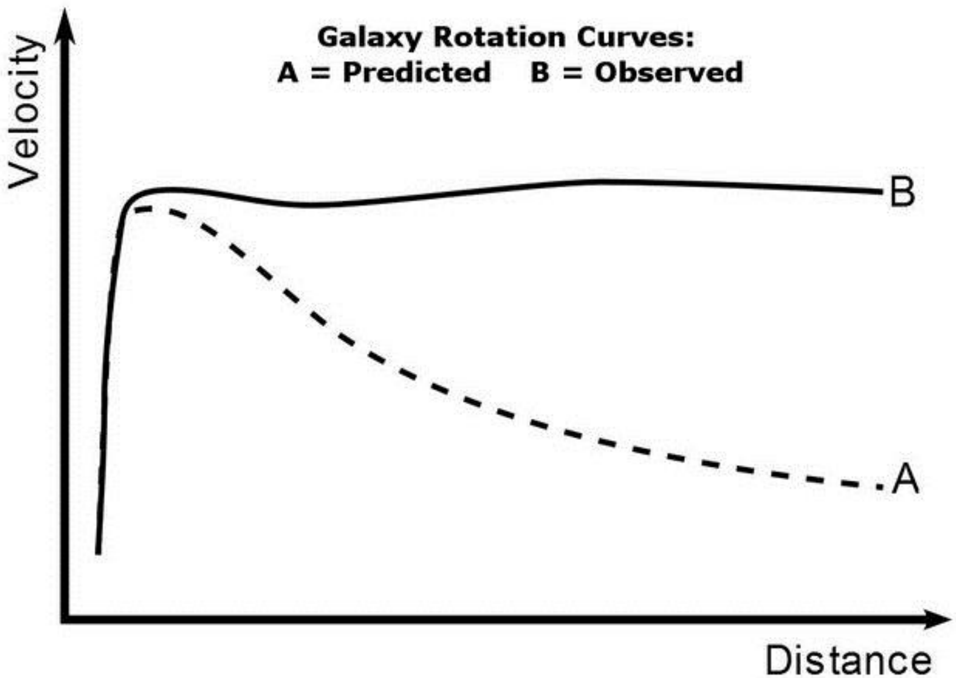
The theory is invariant under the transformation

$$\delta g_{\mu\nu} = -2\nabla_{(\mu}\zeta_{\nu)}$$

But then what's the problem with introducing the cosmological constant? We should be happy with that. What's going on?

The calculations from zero point quantum fluctuations provide a huge value in comparison with observations.

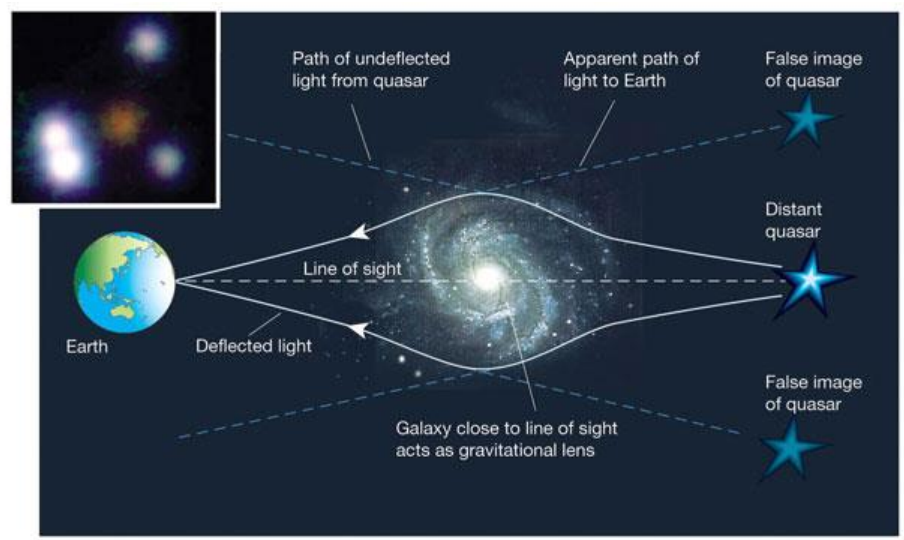
Dark Matter



We do not have a satisfactory explanation for the observed galactic dynamics.

We cannot explain the extra effect in gravitational lenses. What about CMB?

The easiest explanation is Cold Dark Matter.



Einstein-Hilbert action and Massive gravity

The Potential for General Relativity:

$\sqrt{g}R \to$ Kinetic term$= \partial h \partial h$

$h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$:

$\sqrt{g} =$ simplest potential $= 1 + \frac{h}{2} + \frac{h^2}{8} - \frac{h^2_{\mu\nu}}{4} \cdots$

Coefficients in different orders are fixed to sum up into the root g

This potential does not change the number of degrees of freedom, it's not a mass term

Pathologies of the Fierz-Pauli theory.

Mass term for gravity: Generically 6 degrees of freedom

Quadratic mass term$= b_1 h_{\mu\nu}^2 + b_2 h^2$

Fierz and Pauli 1939: $b_1 = -b_2$ to preserve unitarity

Fierz-Pauli mass term$= M_{\rm pl}^2 m^2 (h_{\mu\nu}^2 - h^2)$

The field equations are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{1}{2} m^2 \left(h_{\mu\nu} - h\eta_{\mu\nu} \right) + M_P^{-2} T_{\mu\nu},$$

What's then the problem with the Fierz-Pauli theory?

The vDVZ discontinuity: van Dam, Veltman; Zakharov 70

Non-lineartities can restore continuity Vainshtein 72

This discontinuity can be seen from the calculations of the propagators given as follows.

Massive graviton.

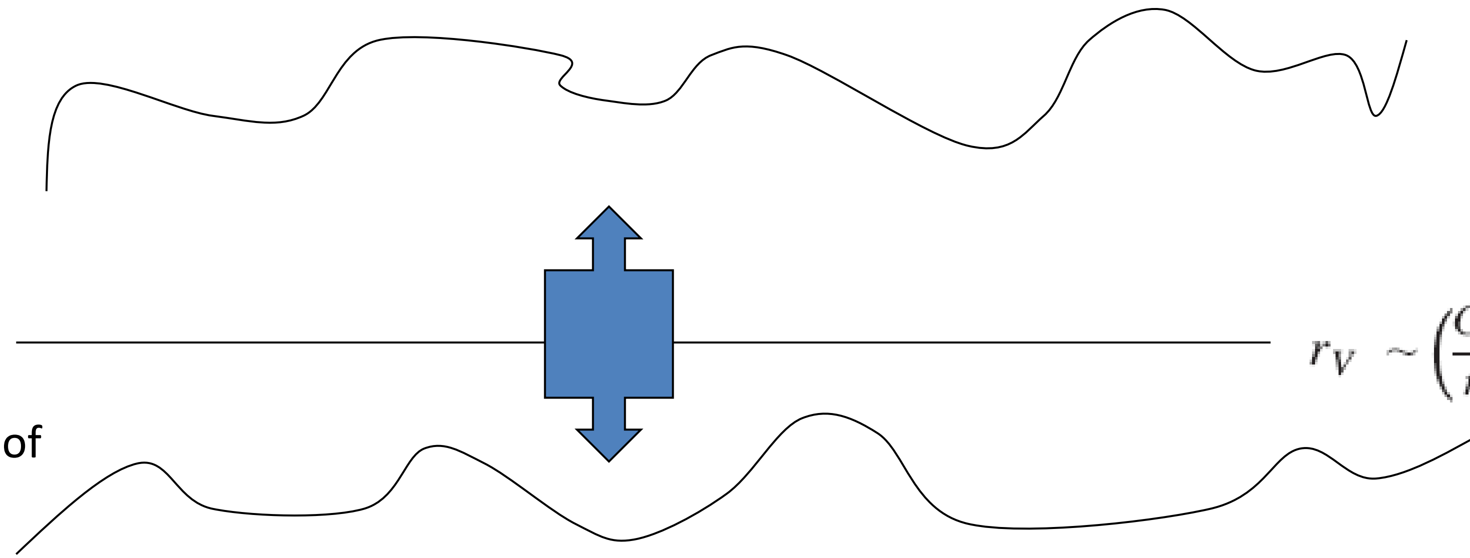
$$D^{(m\neq 0)}_{\mu\nu\rho\sigma} = \frac{1}{k^2+m^2}\left(\eta_{\rho\mu}\eta_{\sigma\nu} + \eta_{\rho\nu}\eta_{\sigma\mu} - \frac{2}{3}\eta_{\rho\sigma}\eta_{\mu\nu} - \frac{2}{3}\eta_{\rho\sigma}\frac{k_\mu k_\nu}{m^2}\right).$$

This term vanishes for the final expression of the amplitude.

Massless graviton.

$$\bar{D}^{(m=0)}_{\mu\nu\rho\sigma} \sim \frac{1}{k^2}\left(\eta_{\rho\mu}\eta_{\sigma\nu} + \eta_{\rho\nu}\eta_{\sigma\mu} - \eta_{\rho\sigma}\eta_{\mu\nu}\right),$$

How to solve this problem?



Non-linearities should restore the continuity at the massless limit (Vainshtein, 1972).

dRGT non-linear massive gravity.

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}(R + m^2 U(g, \phi)),$$

It is a ghost-free theory. The field equations are:

$$G_{\mu\nu} = -m^2 X_{\mu\nu},$$

$$X_{\mu\nu} = \frac{\delta U}{\delta g^{\mu\nu}} - \frac{1}{2} U g_{\mu\nu}.$$

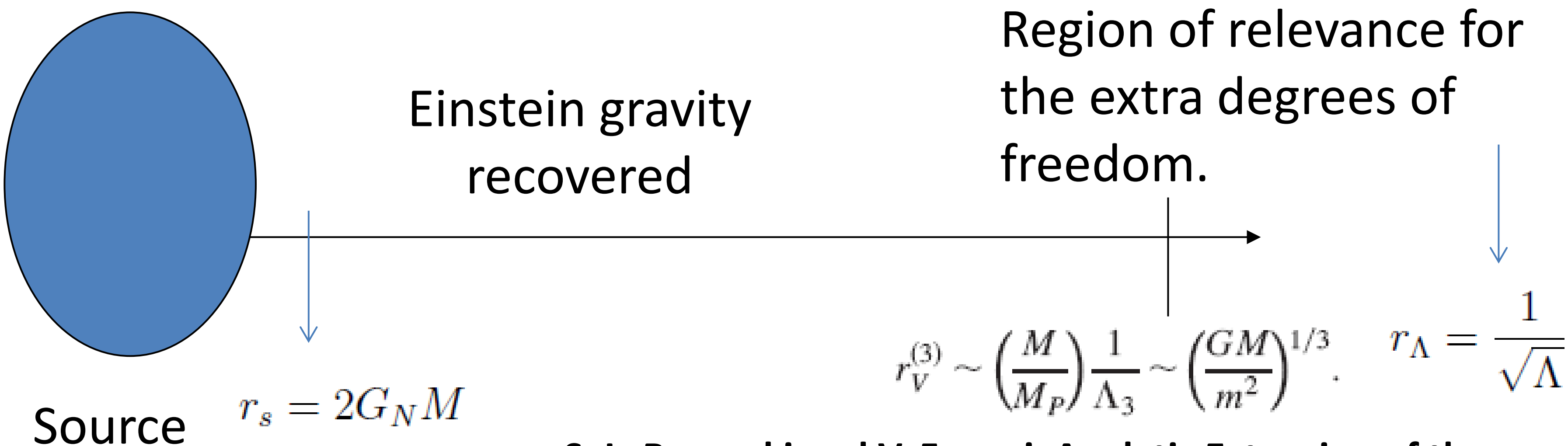
$$\nabla_\mu \left(\frac{\partial U}{\partial(\partial_\mu \phi^a)} \right) = 0.$$

Corresponding to the Bianchi identity in unitary gauge.

Hideo Kodama and Ivan Arraut, Prog. Theor. Exp. Phys. (2014)023E02.

The Vainshtein conditions

- Every theory trying to reproduce the accelerated expansion of the universe reproduces at least three scales when the local physics is analyzed:



$$r_s = 2G_N M$$

$$r_V^{(3)} \sim \left(\frac{M}{M_P}\right)\frac{1}{\Lambda_3} \sim \left(\frac{GM}{m^2}\right)^{1/3}.$$

$$r_\Lambda = \frac{1}{\sqrt{\Lambda}}$$

Check for example: **S. L. Bazanski and V. Ferrari, Analytic Extension of the Schwarzschild-de Sitter Metric, Il Nuovo Cimento Vol. 91 B, N. 1, 11 Gennaio (1986).**

The Vainshtein conditions

In order to stimate the order of magnitude of the Vainhstein scale, it is enough with evaluating the extremal conditions for the dynamical metric in unitary gauge.

$$dg_{\mu\nu} = \left(\frac{\partial g_{\mu\nu}}{\partial r}\right)_t dr + \left(\frac{\partial g_{\mu\nu}}{\partial t}\right)_r dt = 0,$$

These set of conditions, when applied to the dynamical metric, help us to evaluate the Vainshtein radius.

S-dS black-Hole solution in dRGT.

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{rt}(drdt + dtdr) + r^2 d\Omega_2^2,$$

$$g_{tt} = -f(r)(\partial_t T_0(r,t))^2, \qquad g_{rr} = -f(r)(\partial_r T_0(r,t))^2 + \frac{1}{f(r)},$$

$$g_{tr} = -f(r)\partial_t T_0(r,t)\partial_r T_0(r,t),$$

$$f(r) = 1 - \frac{2GM}{r} - \frac{1}{3}\Lambda r^2.$$

The fiducial metric just takes the form:

$$f_{\mu\nu}dx^\mu dx^\nu = -dt^2 + \frac{dr^2}{S_0^2} + \frac{r^2}{S_0^2}(d\theta^2 + r^2 sin^2\theta),$$

1). The stability analysis revealed that the solution with one free-parameter is stable with a functional degeneracy appearing through the time components of the Stückelberg field for scalar perturbations in generic modes.
2). This suggests that it is necessary to review the concept of energy inside this theory. In fact, this can be seen from the gauge transformations in dRGT for the S-dS metric defined as:

$$dt \to \partial_r T_0(r,t)dr + \partial_t T_0(r,t)dt$$

Hideo Kodama and Ivan Arraut, Prog. Theor. Exp. Phys. (2014)023E02.

Test particles moving around a source.

For the previous solution, we can define the following set of invariants:

$$g_{\mu\nu}U^{\mu}U^{\nu} = C, \quad \Rightarrow \quad g_{tt}\left(\frac{dt}{d\tau}\right)^2 + g_{rr}\left(\frac{dr}{d\tau}\right)^2 + 2g_{tr}\left(\frac{dr}{d\tau}\right)\left(\frac{dt}{d\tau}\right) + g_{\phi\phi}\left(\frac{d\phi}{d\tau}\right)^2 = C,$$

$$g_{tt}\left(\frac{dt}{d\tau}\right) + g_{rt}\left(\frac{dr}{d\tau}\right) = E_{dRGT},$$

$$\frac{d}{d\tau}\left(r^2\left(\frac{d\phi}{d\tau}\right)\right) = 0.$$

The first of these (E_{dRGT}) ⇓ Our usual notion of energy

The second ⇓ Angular momentum

In terms of the usual notion of energy, the first invariant becomes:

$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2-\left(\frac{\partial_t T_0(r,t)\partial_r T_0(r,t)}{(\partial_r T_0(r,t))^2-\frac{1}{f(r)^2}}\right)\left(\frac{dr}{d\tau}\right)\frac{E}{g_{tt}}+\frac{L^2}{2r^2g_{rr}}=-\frac{1}{2g_{rr}}\left(\frac{E^2}{g_{tt}}+1\right),$$

For the S-dS metric sol. In dRGT.

If: $$\partial_r T_0(r,t)=0,$$

Then we have the following:

$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2+U_{eff}(r)=\frac{1}{2}\left(E^2+\frac{L^2}{3r_\Lambda^2}-1\right)=C,$$

Ivan Arraut , arXiv:1311.0732 [gr-qc] (Contribution to the Karl-Schwarzschild meeting); Ivan Arraut, arXiv:1305.0475 [gr-qc].

Then T_0 recovers the role of standard time and we recover the GR results. In such a case, the standard notion of energy is valid and no degeneracy would appear.

$$g_{tt}\left(\frac{dt}{d\tau}\right)+g_{rt}\left(\frac{dr}{d\tau}\right)=E_{dRGT},$$

This term would disappear

ADM and Komar mass functions in dRGT.

In GR we know that the following current:

$$J_T^{\mu} = K_{\nu} T^{\mu\nu}.$$

Although conserved, it is not a good one since for vacuum solutions the energy-momentum tensor vanishes. Instead, we define:

$$J_R = K_{\nu} R^{\mu\nu},$$

As the conserved current related to the energy conservation. Then we can define the energy as:

$$E_R = \frac{1}{4\pi G}\int_{\Sigma} d^3x\sqrt{\gamma}n_\mu J^\mu_R.$$

Taking into account that:

$$\nabla_\mu \nabla_\nu K^\mu = K^\mu R_{\mu\nu}$$

Then:

$$J^\mu_R = \nabla_\nu(\nabla^\mu K^\nu).$$

$$E_R = \frac{1}{4\pi G}\int_{\partial\Sigma} d^2x\sqrt{\gamma}n_\mu\sigma_\nu\nabla^\mu K^\nu.$$

How to calculate the Killing vector?

$$K^\mu = \gamma_t \frac{\partial}{\partial t},$$

Normalized in agreement with:

$$\gamma_t = (-g_{00})^{-1/2}_{r=r_V}.$$

PRD 54, 10

Evaluated at the Vainshtein radius. In addition, we have to know:

$$n_0 = -\sqrt{f(r)}\partial_t T_0(r,t), \qquad \sigma_r = \sqrt{-f(r)(\partial_r T_0(r,t))^2 + \frac{1}{f(r)}}.$$

By using the normalizations:

$$g_{\mu\nu} n^{\mu} n^{\nu} = -1 \qquad \text{And:} \qquad g_{\mu\nu} \sigma^{\mu} \sigma^{\nu} = 1.$$

Then:

$$n_{\mu}\sigma_{\nu}\nabla^{\mu}K^{\nu} = \frac{1}{\partial_t T_0(r,t) f(r)} \left(-f(r)^2(\partial_r T_0(r,t))^2 + 1\right)^{3/2} \Gamma^{r}_{00}\gamma_t + f(r)\partial_r T_0(r,t)\left(\sqrt{-f(r)^2(\partial_r T_0(r,t))^2 + 1}\right)\Gamma^{r}_{r0}\gamma_t,$$

By assuming the components of the metric to be time-independent, implies:

$$T_0(r,t) \backsim t + A(r),$$

$$n_{\mu}\sigma_{\nu}\nabla^{\mu}K^{\nu} = \frac{\gamma_t}{2}\partial_t T_0(r,t) f'(r)\sqrt{-f(r)^2(\partial_r T_0(r,t))^2 + 1}.$$

The determinant of the induced metric is:

$$\sqrt{\gamma^{(2)}} = r^2 sin\theta.$$

Then, finally we get the following result:

$$E_R = \frac{\gamma_t}{2G}\partial_t T_0(r,t) r^2 f'(r)\sqrt{-f(r)^2(\partial_r T_0(r,t))^2 + 1} = M_{dRGT}.$$

This expression clearly depends on the dynamics of Stückelberg fields. It reduces to the standard Komar function if we ignore the extra degrees of freedom of dRGT.

The previous expression is well defined as:

$$-\int \frac{dr}{f(r)} + C_1(t) < T_0(r,t) < \int \frac{dr}{f(r)} + C_2(t),$$

Note that this condition is satisfied by one of the family of solutions obtained in:

Hideo Kodama and Ivan Arraut, Prog. Theor. Exp. Phys. (2014)023E02.

For the other family, the condition has to be taken into account.

Conclusions

- 1). I have derived some “conserved” expressions related to the energy definition in dRGT.
- 2). The energy in its standard definition is not conserved. It is a velocity-dependent quantity.
- 3). The Komar mass function is modified by the dynamic of the Stückelberg fields.
- 4). Further analysis are in process.
- 5). The conservation of energy in dRGT is a matter of debate, see for example PRD 88, 064008, (2013).