



Precise measurement of m_W and Γ_W using threshold scan method

Peixun Shen, Paolo Azzurri, Zhijun Liang, Gang Li, Chunxu Yu
NKU, INFN Pisa, IHEP

IAS Program on High Energy Physics 2020

2020/1/18, HKUST

Outline

- Motivation
- Methodology
- Statistical and systematic uncertainties
- Data taking schemes
- Summary

Motivation

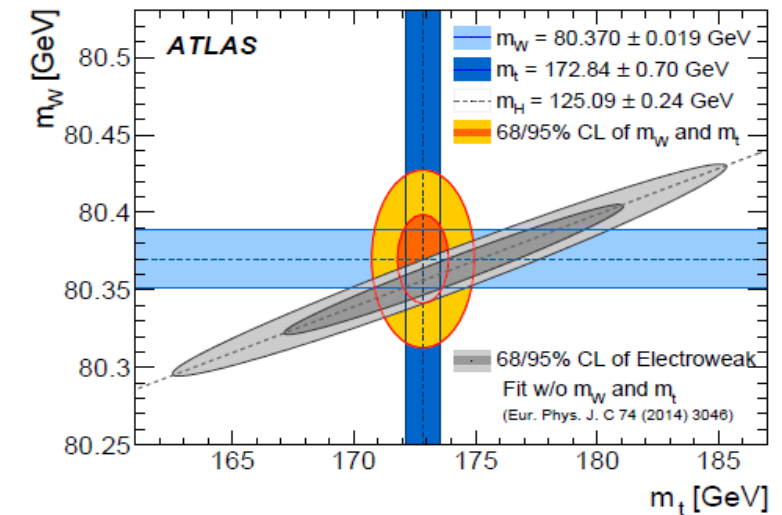
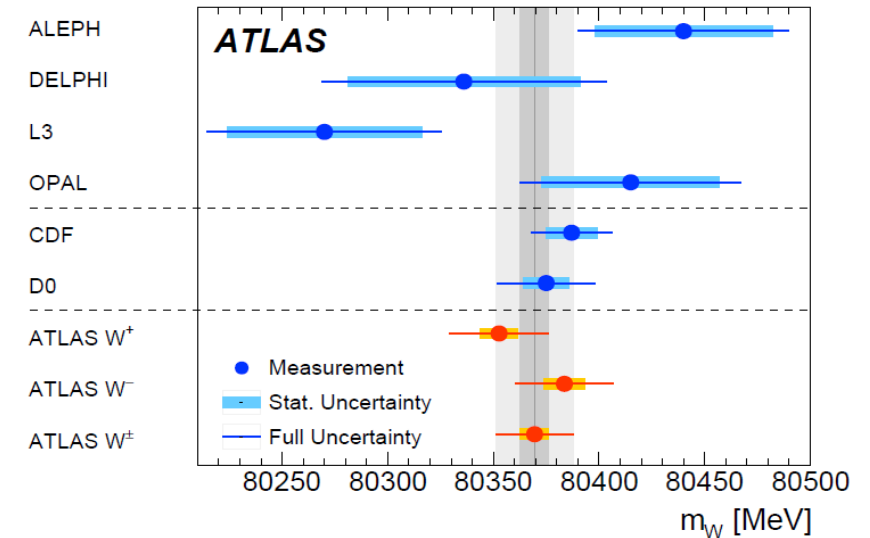
- The m_W plays a central role in precision EW measurements and in constraint on the SM model through global fit.

$$G_F = \frac{\pi\alpha}{\sqrt{2}m_W^2 \sin^2 \theta_W} \frac{1}{(1+\Delta r)}$$

Δr is the correction, whose leading-order contributions depend on the m_t and m_H

- Several ways to measure m_W :
 - The direct method, with kinematically-constrained or mass reconstructions
 - Using the lepton end-point energy
 - W^+W^- threshold scan method (used in this study)

W.J. Stirling, arXiv:hep-ph/9503320v1 14 Mar 1995
ILC, arXiv:1603.06016v1 [hep-ex] 18 Mar 2016
FCC-ee, arXiv:1703.01626v1 [hep-ph] 5 Mar 2017



Methodology

➤ Why?

$$\sigma_{WW}(m_W, \Gamma_W, \sqrt{s}) = \frac{N_{obs}}{L\epsilon P} \quad \left(P = \frac{N_{WW}}{N_{WW} + N_{bkg}}\right)$$

so m_W, Γ_W can be obtained by fitting the N_{obs} , with the theoretical formula σ_{WW}

➤ How?

$\Delta m_W, \Delta \Gamma_W$						
N_{obs}	L	ϵ	N_{bkg}	E	σ_E

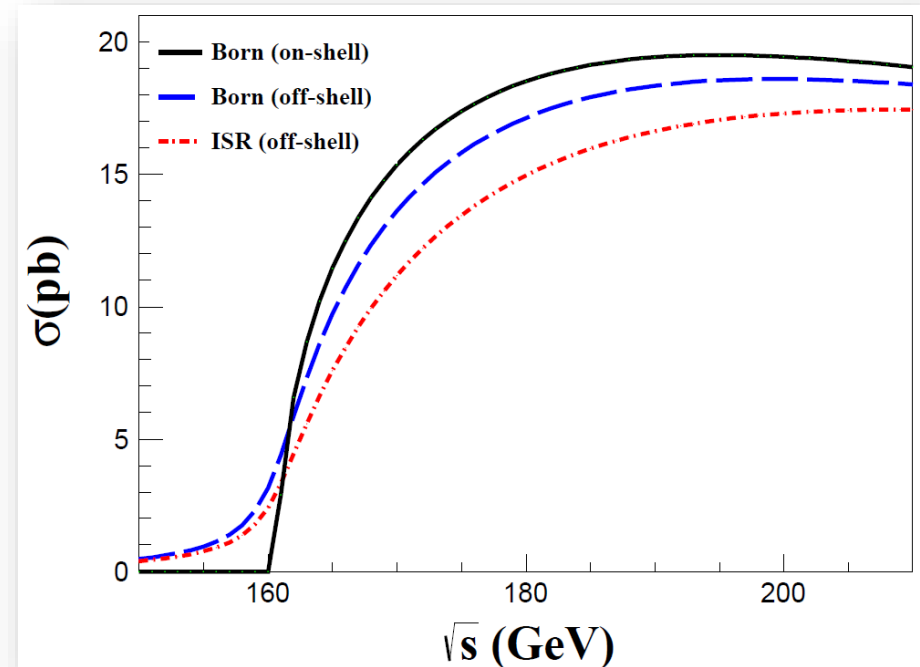
In general, these uncertainties are dependent on \sqrt{s} , so it is a optimization problem when considering the data taking.

➤ If ..., then?

With the configurations of $L, \Delta L, \Delta E$..., we can obtain: $m_W \sim ? \Gamma_W \sim ?$

Theoretical Tool

- The σ_{WW} is a function of \sqrt{s} , m_W and Γ_W , which is calculated with the GENTLE package in this work (CC03)
- The ISR correction is also calculated by convoluting the Born cross sections with QED structure function, with the radiator up to NL $O(\alpha^2)$ and $O(\beta^3)$



Statistical and systematic uncertainties

Statistical uncertainty

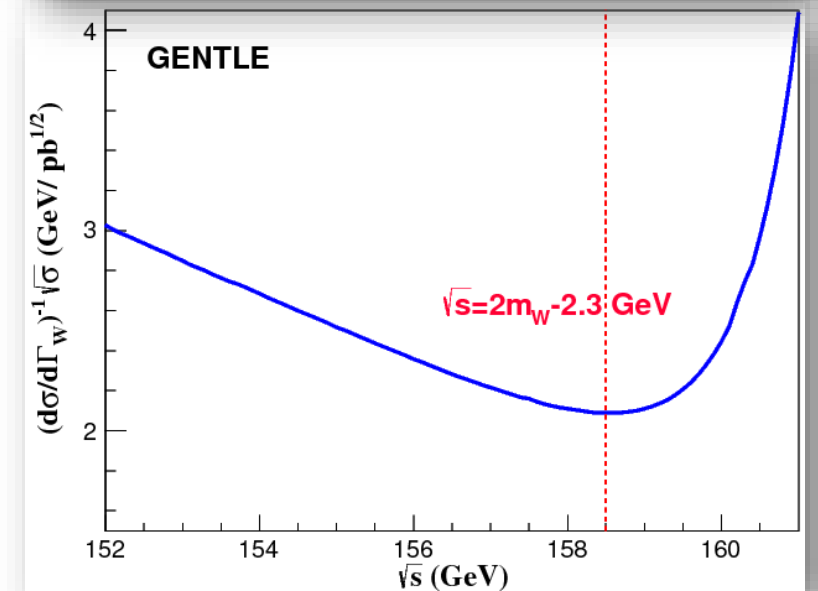
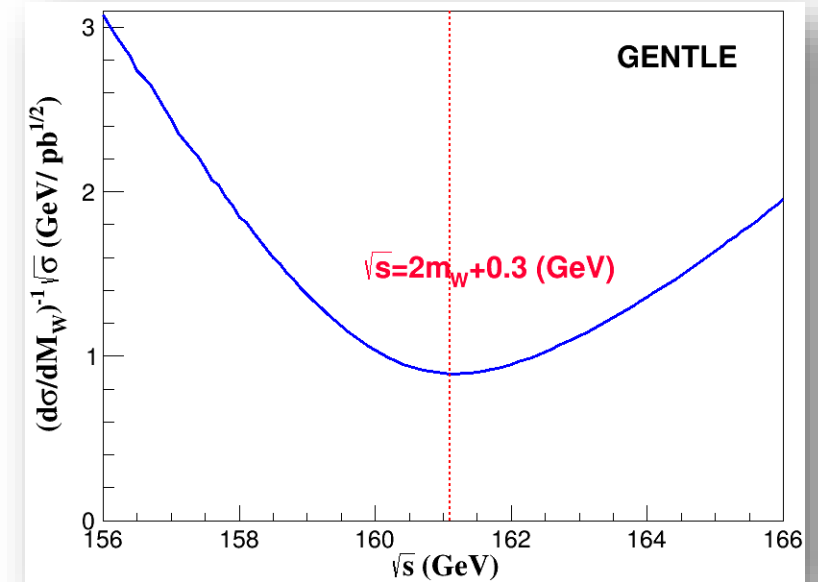
$$\begin{aligned}\triangleright \Delta\sigma_{WW} &= \sigma_{WW} \times \frac{\Delta N_{WW}}{N_{WW}} = \sigma_{WW} \times \frac{\sqrt{N_{WW}+N_{bkg}}}{N_{WW}} \\ &= \sqrt{\frac{\sigma_{WW}}{L\epsilon P}} \quad \left(P = \frac{N_{WW}}{N_{WW}+N_{bkg}}\right)\end{aligned}$$

$$\triangleright \Delta m_W = \left(\frac{\partial\sigma_{WW}}{\partial m_W}\right)^{-1} \times \Delta\sigma_{WW} = \left(\frac{\partial\sigma_{WW}}{\partial m_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$$

$$\triangleright \Delta\Gamma_W = \left(\frac{\partial\sigma_{WW}}{\partial\Gamma_W}\right)^{-1} \times \Delta\sigma_{WW} = \left(\frac{\partial\sigma_{WW}}{\partial\Gamma_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$$

With $L=3.2ab^{-1}$, $\epsilon=0.8$, $P=0.9$:

$\Delta m_W=0.6$ MeV, $\Delta\Gamma_W=1.4$ MeV (individually)



Statistical uncertainty

- When there are more than one data point, we can measure both m_W and Γ_W .
- With the chisquare defined as:

$$\chi^2 = \sum_i \frac{(N_{\text{fit}}^i - N_{\text{obs}}^i)^2}{N_{\text{obs}}^i} = \frac{(\mathcal{L}EP)^i (\sigma_{\text{fit}}^i - \sigma_{\text{obs}}^i)^2}{\sigma_{\text{obs}}^i}$$

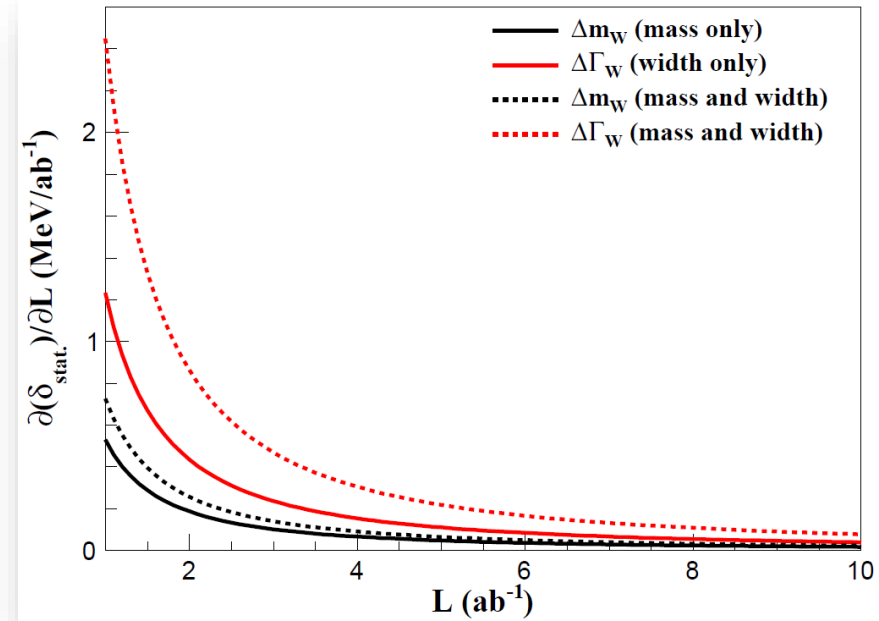
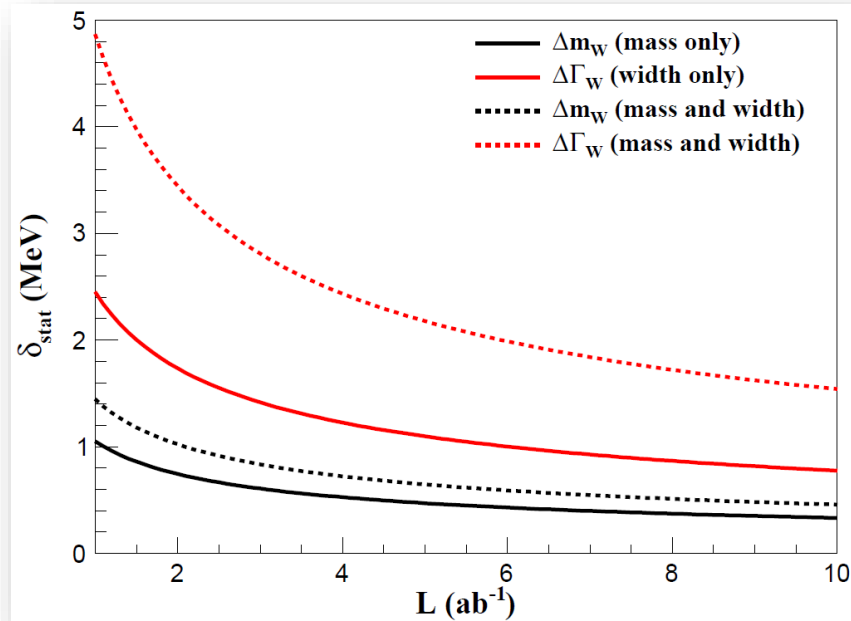
the error matrix is in the form:

$$V = \frac{1}{2} \times \begin{pmatrix} \frac{\partial^2 \chi^2}{\partial m_W^2} & \frac{\partial^2 \chi^2}{\partial m_W \partial \Gamma_W} \\ \frac{\partial^2 \chi^2}{\partial m_W \partial \Gamma_W} & \frac{\partial^2 \chi^2}{\partial \Gamma_W^2} \end{pmatrix}^{-1} = \sum_i \begin{pmatrix} \frac{(\mathcal{L}EP)^i}{\sigma_{\text{obs}}^i} \left(\frac{\partial \sigma}{\partial m_W}\right)^2 & \frac{(\mathcal{L}EP)^i}{\sigma_{\text{obs}}^i} \frac{\partial \sigma}{\partial m_W} \frac{\partial \sigma}{\partial \Gamma_W} \\ \frac{(\mathcal{L}EP)^i}{\sigma_{\text{obs}}^i} \frac{\partial \sigma}{\partial m_W} \frac{\partial \sigma}{\partial \Gamma_W} & \frac{(\mathcal{L}EP)^i}{\sigma_{\text{obs}}^i} \left(\frac{\partial \sigma}{\partial \Gamma_W}\right)^2 \end{pmatrix}^{-1}$$

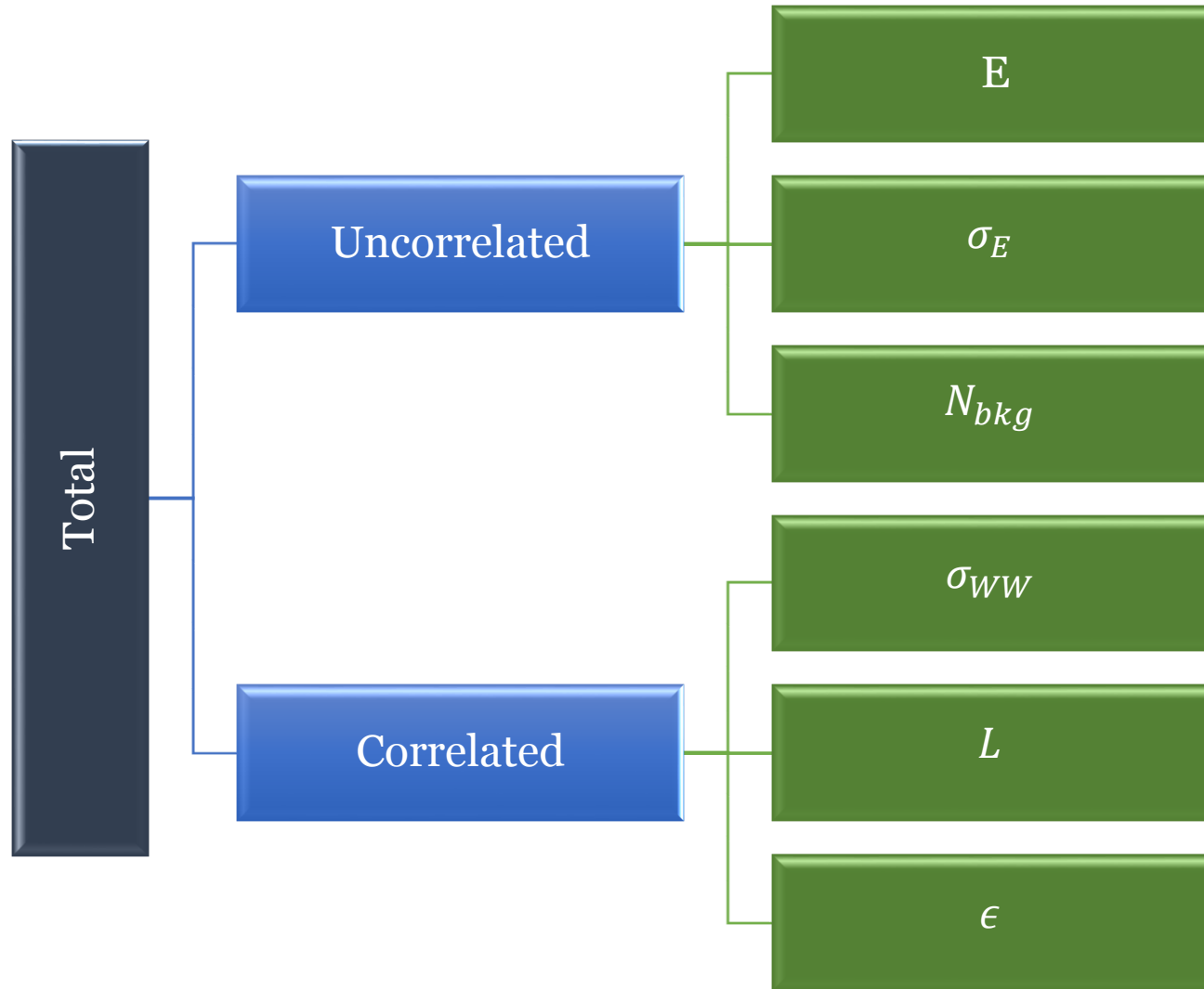
- When the number of fit parameter reduce to 1:

$$\Delta m_W = \left(\frac{\partial \sigma_{WW}}{\partial m_W}\right)^{-1} \times \Delta \sigma_{WW} = \left(\frac{\partial \sigma_{WW}}{\partial m_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{\mathcal{L}EP}}$$


Statistical uncertainty



Systematic uncertainty



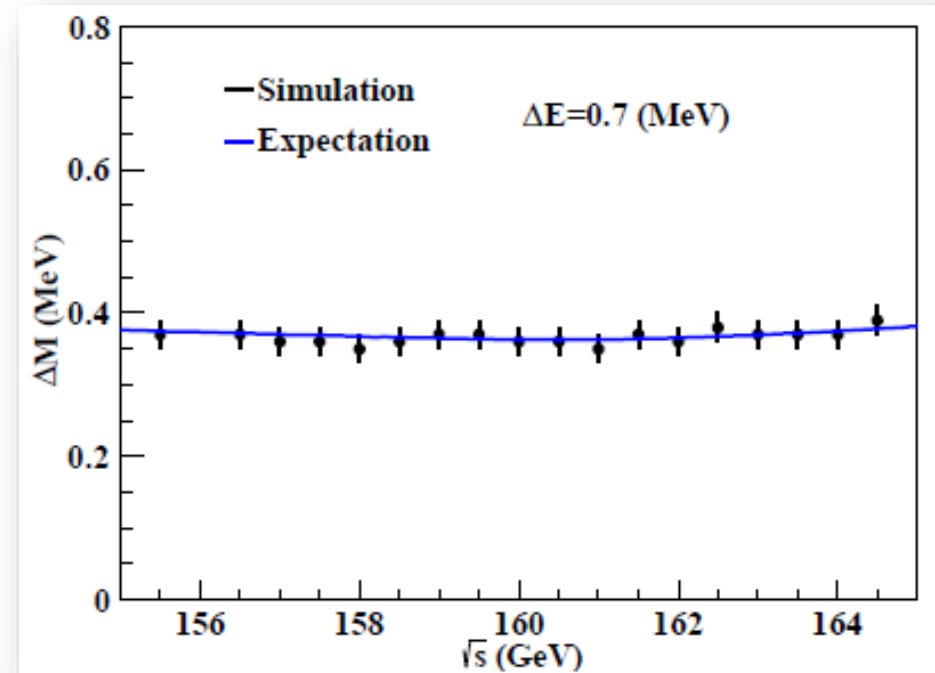
Energy calibration uncertainty

- With ΔE , the total energy becomes:

$$E = G(E_p, \Delta E) + G(E_m, \Delta E)$$

- $$\Delta m_W = \frac{\partial m_W}{\partial \sigma_{WW}} \frac{\partial \sigma_{WW}}{\partial E} \Delta E$$

- The Δm_W will be large when ΔE increase, and **almost independent with \sqrt{s} .**



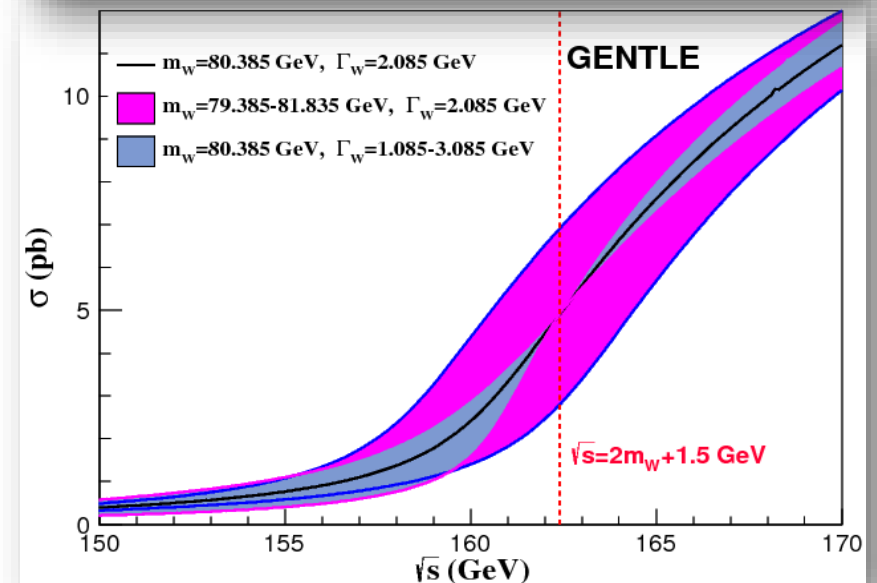
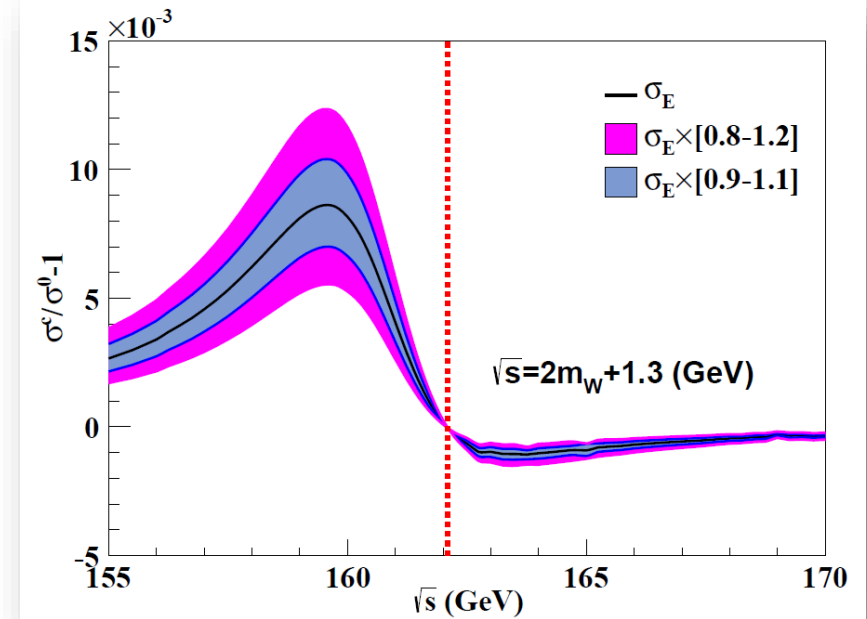
Energy spread uncertainty

➤ With E_{BS} , the σ_{WW} becomes:

$$\begin{aligned}\sigma_{WW}(E) &= \int_0^\infty \sigma_{WW}(E') \times G(E, E') dE' \\ &= \int \sigma(E') \times \frac{1}{\sqrt{2\pi}\delta_E} e^{-\frac{(E-E')^2}{2\sigma_E^2}} dE'\end{aligned}$$

➤ $\sigma_E + \Delta\sigma_E$ is used in the simulation, and σ_E is for the fit formula.

➤ The m_W insensitive to δ_E when taking data around 162.3 GeV



Background uncertainty

The effect of background are in two different ways

1. Stat. part:
$$\Delta m_W(N_B) = \frac{\partial m_W}{\partial \sigma_{WW}} \cdot \frac{\sqrt{L\epsilon_B\sigma_B}}{L\epsilon}$$

2. Sys. part:
$$\Delta m_W(\sigma_B) = \frac{\partial m_W}{\partial \sigma_{WW}} \cdot \frac{L\epsilon_B\sigma_B}{L\epsilon} \cdot \Delta\sigma_B$$

With $L=3.2\text{ab}^{-1}$, $\epsilon_B\sigma_B = 0.3\text{pb}$, $\Delta\sigma_B = 10^{-3}$:

$\Delta m_W(N_B) \sim 0.2 \text{ MeV}$, which has been embodied in the product of $\epsilon \cdot P$,
and $\Delta m_W(\sigma_B)$ is considerable with the former.

Correlated sys. uncertainty

- The correlated sys. uncertainty includes: ΔL , $\Delta\epsilon$, $\Delta\sigma_{WW}$...
- Since $N_{obs} = L \cdot \sigma \cdot \epsilon$, these uncertainties affect σ_{WW} in same way.
- We use the total correlated sys. uncertainty in data taking optimization:

$$\delta_c = \sqrt{\Delta L^2 + \Delta\epsilon^2}$$

$$\Delta m_W = \frac{\partial m_W}{\partial \sigma_{WW}} \sigma_{WW} \cdot \delta_c, \Delta \Gamma_W = \frac{\partial \Gamma_W}{\partial \sigma_{WW}} \sigma_{WW} \cdot \delta_c$$

Correlated sys. uncertainty

$$\Delta m_W = \frac{\partial m_W}{\partial \sigma_{WW}} \sigma_{WW} \cdot \delta_c$$

Two ways to consider to effect:

➤ Gaussian distribution

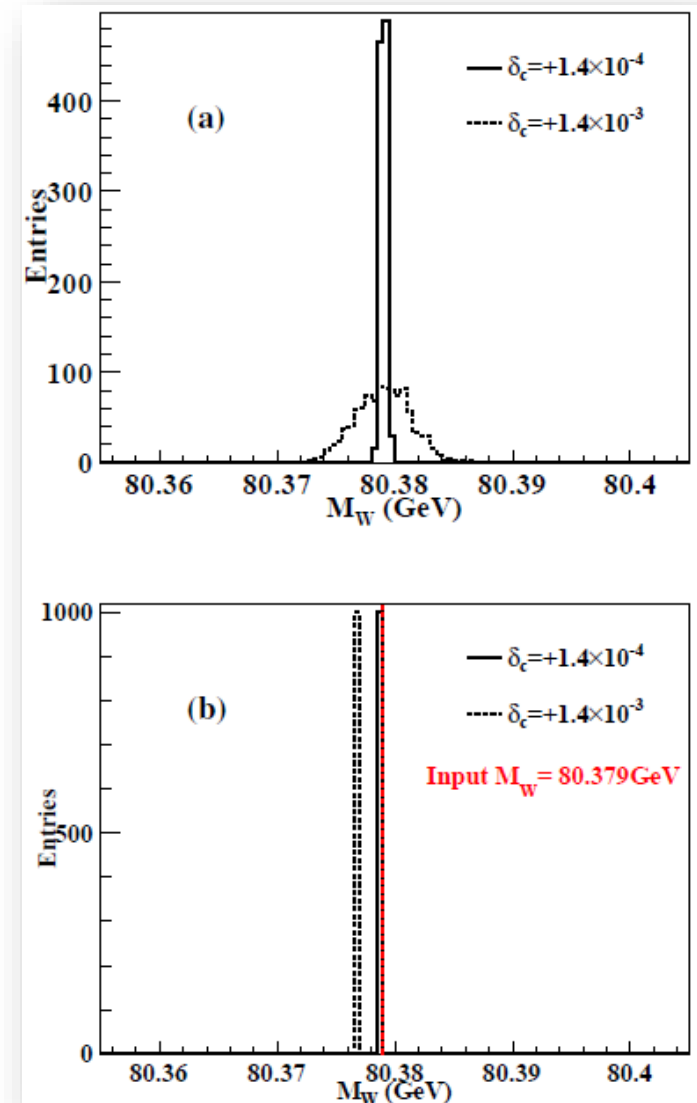
$$\sigma_{WW} = G(\sigma_{WW}^0, \delta_c \cdot \sigma_{WW}^0)$$

➤ Non-Gaussian (will cause shift)

$$\sigma_{WW} = \sigma_{WW}^0 \times (1 + \delta_c)$$

With $\delta_c = +1.4 \cdot 10^{-4} (10^{-3})$ at 161.2GeV

$$\Delta m_W \sim 0.24 \text{MeV} (3 \text{MeV})$$



Correlated sys. uncertainty

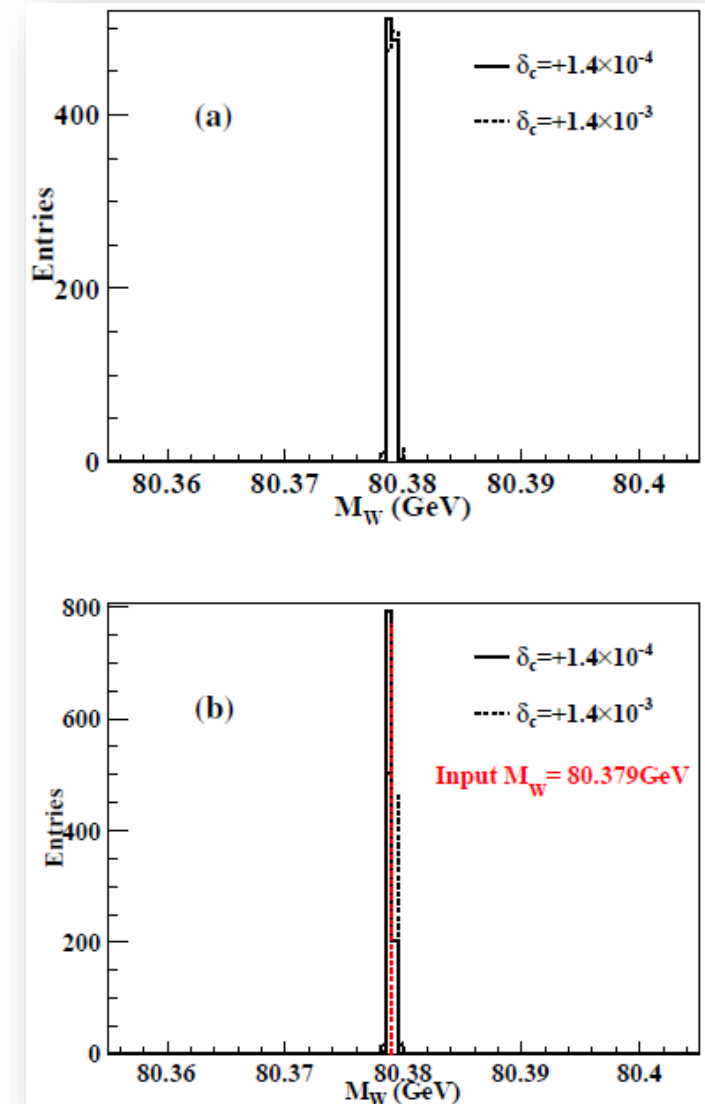
To consider the correlation, the scale factor method is used,

$$\chi^2 = \sum_i^n \frac{(y_i - h \cdot x_i)^2}{\delta_i^2} + \frac{(h-1)^2}{\delta_c^2},$$

where y_i , x_i are the true and fit results, h is a free parameter, δ_i and δ_c are the independent and correlated uncertainties.

For the Gaussian consideration, the scale factor can reduce the effect.

For the non-Gaussian case, the shift of the m_W is controlled well



Data taking scheme

Data taking schemes

One point

- Smallest $\Delta m_W, \Delta \Gamma_W$ (stat.)
- Large sys. uncertainties
- Only for m_W or Γ_W , without correlation

Two points

- Measure m_W and Γ_W simultaneously
- Without the correlation

Three points or more

- Measure m_W and Γ_W simultaneously, with the correlation

Taking data at one point (just for m_W)

There are two special energy points :

- The one which most statistical sensitivity to m_W :

$$\Delta m_W(\text{stat.}) \sim 0.59 \text{ MeV at } E=161.2 \text{ GeV}$$

(with $\Delta\Gamma_W$ and ΔE_{BS} effect)

- The one $\Delta m_W(\text{stat.}) \sim 0.65 \text{ MeV}$ at $E \approx 162.3 \text{ GeV}$

(with negative $\Delta\Gamma_W, \Delta E_{BS}$ effects)

With $\Delta L (\Delta\epsilon) < 10^{-4}, \Delta\sigma_B < 10^{-3}, \Delta E=0.7\text{MeV},$
 $\Delta\sigma_E=0.1, \Delta\Gamma_W=42\text{MeV}$



$\sqrt{s}(\text{GeV})$	161.2	162.3
E	0.36	0.37
σ_E	0.20	-
σ_B	0.17	0.17
δ_c	0.24	0.34
Γ_W	7.49	-
Stat.	0.59	0.65
$\Delta m_W(\text{MeV})$	7.53	0.84

Taking data at two energy points

➤ To measure Δm_W and $\Delta \Gamma_W$, we scan the energies and the luminosity fraction of the two data points:

1. $E_1, E_2 \in [155, 165] \text{ GeV}, \Delta E = 0.1 \text{ GeV}$

2. $F \equiv \left(\frac{L_1}{L_2}\right) \in (0, 1), \Delta F = 0.05$

➤ We define the object function: $T = m_W + 0.1\Gamma_W$ to optimize the scan parameters (assuming m_W is more important than Γ_W).

Taking data at two energy points

➤ The 3D scan is performed, and 2D plots are used to illustrate the optimization results;

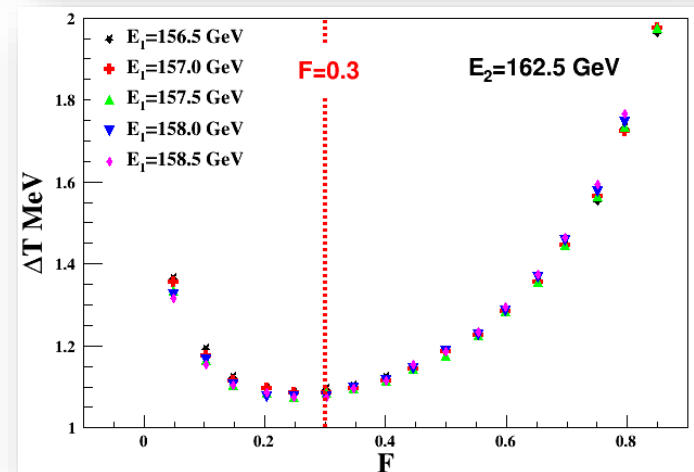
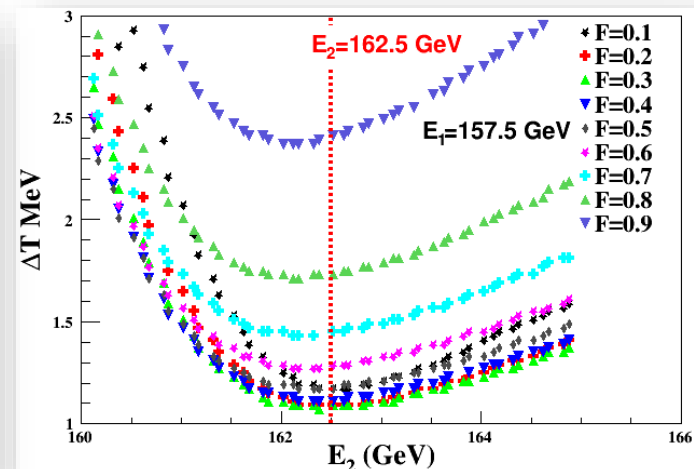
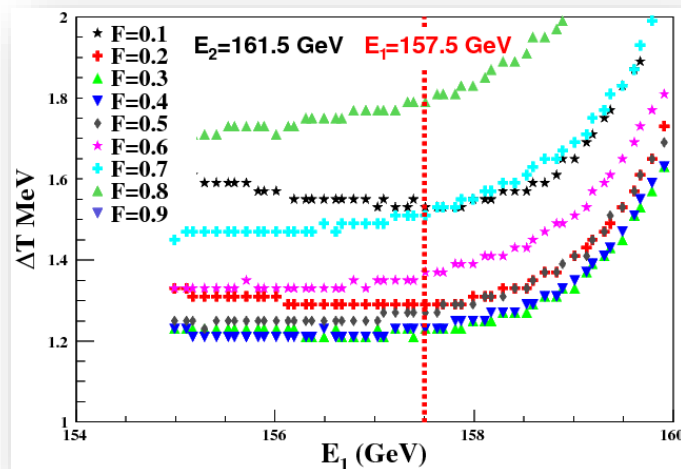
➤ When draw the ΔT change with one parameter, another is fixed with scanning of the third one;

➤ $E_1=157.5$ GeV, $E_2=162.5$ GeV (around $\frac{\partial \sigma_{WW}}{\partial \Gamma_W}=0$, $\frac{\partial \sigma_{WW}}{\partial E_{BS}}=0$) and $F=0.3$ are taken as the result.

$$\Delta L(\Delta\epsilon) < 10^{-4}, \Delta\sigma_B < 10^{-3}$$

$$\sigma_E = 1 \times 10^{-3}, \Delta E = 0.7 \text{ MeV}$$

$$\Delta\sigma_E = 0.1\%$$



(MeV)	E	σ_E	σ_B	δ_c	Stat.	Total
Δm_W	0.38	-	0.21	0.33	0.80	0.97
$\Delta \Gamma_W$	0.54	0.56	1.38	0.20	2.92	3.32

Taking data at three energy points

The procedure of three points optimization is similar to two points

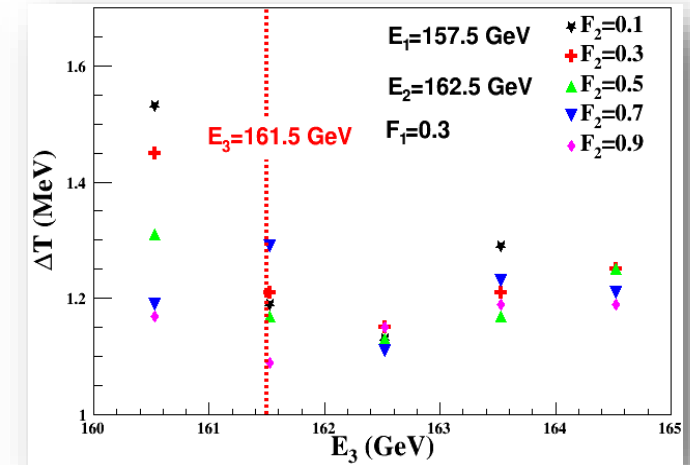
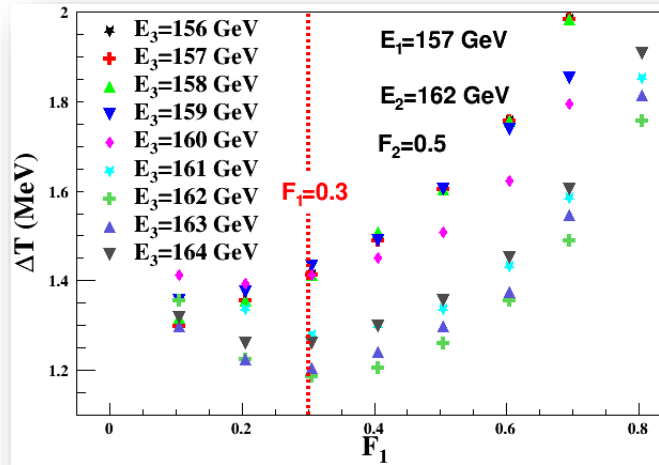
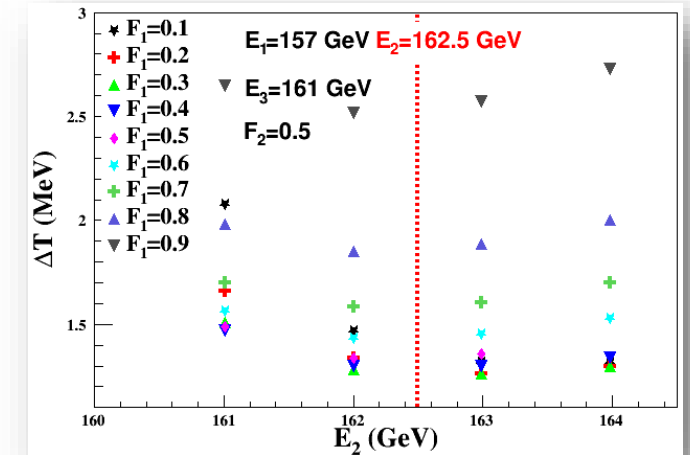
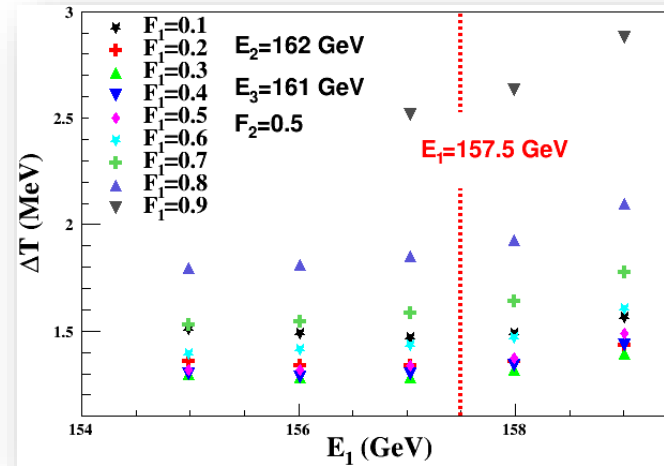
E_1	157.5 GeV
E_2	162.5 GeV
E_3	161.5 GeV
F_1	0.3
F_2	0.9



$\Delta m_W \sim 0.98 \text{ MeV}$
 $\Delta \Gamma_W \sim 3.37 \text{ MeV}$



$\Delta L(\Delta \epsilon) < 10^{-4}, \Delta \sigma_B < 10^{-3}$
 $\sigma_E = 1 \times 10^{-3}, \Delta E = 0.7 \text{ MeV}$
 $\Delta \sigma_E = 0.1$



Summary

- The precise measurement of m_W (Γ_W) is studied (threshold scan method)
- Different data taking schemes are investigated, based on the stat. and sys. uncertainties analysis.
- With the configurations :

$$\Delta L(\Delta\epsilon) < 10^{-4}, \Delta\sigma_B < 10^{-3}$$

$$\sigma_E = 1 \times 10^{-3}, \Delta E = 0.7 \text{ MeV}$$

$$\Delta\Gamma_W = 42 \text{ MeV}, \Delta\sigma_E = 0.1$$



Data-taking scheme	mass or width	δ_{stat} (MeV)	δ_{sys} (MeV)				Total (MeV)
			ΔE	$\Delta\sigma_E$	δ_B	δ_c	
One point	Δm_W	0.65	0.37	-	0.17	0.34	0.84
Two points	Δm_W	0.80	0.38	-	0.21	0.33	0.97
	$\Delta\Gamma_W$	2.92	0.54	0.56	1.38	0.20	3.32
Three points	Δm_W	0.81	0.30	-	0.23	0.29	0.98
	$\Delta\Gamma_W$	2.93	0.52	0.55	1.38	0.20	3.37



Thank you !

Backup Slides

Covariance matrix method

➤
$$y_i = \frac{n_i}{\epsilon}, \quad v_{ii} = \sigma_i^2 + y_i^2 \sigma_f^2$$

where σ_i is the stat. error of n_i , σ_f is the relative error of ϵ

➤ The correlation between data points i, j contributes to the off-diagonal matrix element v_{ij} :

➤ Then we minimize: $\chi_1^2 = \eta^T V^{-1} \eta$

For this method, The biasness is uncontrollable

(MO Xiao-Hu HEPNP 30 (2006) 140-146)

H. J. Behrend et al. (CELLO Collaboration)

Phys. Lett. B 183 (1987) 400

D'Agostini G. Nucl. Instrum. Meth. A346 (1994)

$$V = \begin{pmatrix} \sigma_1^2 + y_1^2 \sigma_f^2 & y_1 y_2 \sigma_f^2 & \cdots & y_1 y_n \sigma_f^2 \\ y_2 y_1 \sigma_f^2 & \sigma_2^2 + y_2^2 \sigma_f^2 & \cdots & y_2 y_n \sigma_f^2 \\ \vdots & \vdots & \ddots & \vdots \\ y_n y_1 \sigma_f^2 & y_n y_2 \sigma_f^2 & \cdots & \sigma_n^2 + y_n^2 \sigma_f^2 \end{pmatrix}$$

$$\eta = \begin{pmatrix} y_1 - k_1 \\ y_2 - k_2 \\ \vdots \\ y_n - k_n \end{pmatrix}$$

Scale factor method

- This method is used by introducing a free fit parameter to the χ^2 :

$$\chi_2^2 = \sum_i \frac{(fy_i - k_i)^2}{\sigma_i^2} + \frac{(f-1)^2}{\sigma_f^2}$$

σ_i includes stat. and uncorrelated sys errors, σ_f are the correlated errors.

The equivalence of this form and the one from matrix method is

proved in : MO Xiao-Hu HEPNP 30 (2006) 140-146 .

- Both the matrix and the factor approach have bias, which may be considerably striking when the data points are quite many or the scale factor is rather large.

- According to ref: MO Xiao-Hu HEPNP 31 (2007) 745-749, the unbiased χ^2 is constructed as:

$$\chi_3^2 = \sum_i \frac{(y_i - gk_i)^2}{\sigma_i^2} + \frac{(g-1)^2}{\sigma_f^2} \quad (\text{used in our previous results})$$

The central value from χ_2^2 can be re-scaled, the relative error is still larger than those from χ_3^2 estimation.

Brandelik R et al(TASSO Collab.). Phys. Lett., 1982, B113: 499—508; Brandelik R et al(TASSO Collab.). Z. Phys., 1980, C4: 87—93
Bartel W et al(JADE Collab.). Phys. Lett., 1983, B129: 145—152
D'Agostini G. Nucl. Instrum. Methods, 1994, A346: 306—311