Self-consistent simulation of coupling impedance and beam-beam interaction in future e+/e- collider

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Outline

• Introduction of beam-beam simulation
• New effects in future e+/e- collider
• Simulation result
• Summary
Classical Beam-Beam Simulation

• Macro-particles, Particles-in-cell

• Poisson Solver

\[ \nabla^2 \phi = -\frac{\rho}{\varepsilon_0} \]

\[ \mathcal{E}_x = -\frac{\partial \phi(x, y)}{\partial x} = \frac{Q}{2\varepsilon_0 \sqrt{2\pi(\sigma_x^2 - \sigma_y^2)}} \cdot \text{Im}(\Gamma) \]

\[ \mathcal{E}_y = -\frac{\partial \phi(x, y)}{\partial y} = \frac{Q}{2\varepsilon_0 \sqrt{2\pi(\sigma_y^2 - \sigma_x^2)}} \cdot \text{Re}(\Gamma) \]

\[ \Gamma = w\left(\frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}}\right) - \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)w\left(\frac{x\sigma_y + iy\sigma_x}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}}\right) \]
Classical Beam-Beam Simulation

• Bunch Length => Hourglass effect => Slices & Interpolation

• Horizontal Crossinalg Angle

• With approximation of Gaussian distribution, Symplectic beam-beam map is given by Synchro-beam mapping method (Hirata)

• With Large Piwinski Angle, strong-strong PIC is very time-consuming, quasi-strong-strong is a feasible method
Crosstalk between beam-beam and Lattice nonlinearity

- Element-by-element tracking in lattice + beam-beam map at IP.

D. Zhou, ICFA Mini-Workshop on Commissioning of SuperKEKB and e+e- Colliders, 2013

Y. Ohnishi, 18th KEKB Review, 2014

Transverse aperture is reduced significantly.
Beamstrahlung Effect is New!

• Increase of Energy spread
• Bunch lengthening
• Beam Lifetime

\[ \tau_{BS} \approx \frac{1}{n_{LP} f_{rev}} \frac{4\sqrt{\pi}}{3} \sqrt{\frac{\delta_{acc}}{\alpha r_e}} \exp \left( \frac{2 \delta_{acc} \alpha}{3 r_e \gamma^2} \frac{\gamma \sigma_x \sigma_z}{\sqrt{2} r_e N_b} \right) \frac{\sqrt{2}}{\sqrt{\pi} \sigma_z \gamma^2} \left( \frac{\gamma \sigma_x \sigma_z}{\sqrt{2} r_e N_b} \right)^{3/2} \]

• The longitudinal oscillation is strongly distorted by transverse dynamics.
New found X-Z coherent instability

Horizontal oscillations may also be distorted by longitudinal direction even with a Large Piwinski angle.

- It is believed the crab-waist scheme help to suppress the transverse and synchro-beatron resonance in collision
- Correlated head-tail instability in beam-beam collisions with a large Piwinski angle
- Blowup of horizontal beam size
- Resonance occur near $\nu_x = 0.5 + n\nu_s$
- Some benchmarking experiments has been done at SKEKB

K. Ohmi and etal., DOI:10.1103/PhysRevLett.119.134801

K. Ohmi, eeFACT18 @ HKUST, Hongkong
Combined effect of Beam-Beam Interaction & Coupling Impedance

Conventional Method:

• Calculate bunch length with longitudinal impedance: $\sigma_z(I)$

• $\sigma_z(I)$ is an input parameter as the equilibrium bunch length w/o collision in the beam-beam simulation, since the longitudinal dynamics is nearly not affected by beam-beam interaction. (Gaussian Approximation in longitudinal direction)

\[ I(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \tilde{I}(\omega) \]

Self-consistent Method in future lepton collider:

• The effect of longitudinal wake field is considered directly

\[ \tilde{V}(\omega) = -\tilde{I}(\omega)Z_{\parallel}(\omega) \]

• (The beamstrahlung effect during collision is also considered)
Longitudinal Impedance is Considered in Beam-Beam Code. Crosscheck of longitudinal distribution between IBB and Elegant.

Thanks M. Migliorati for the FCC-ee Resistive Wall Impedance data

• Microwave instability and bunch lengthening
  • The microwave instability will rarely induce beam losses, but may reduce the luminosity due to the deformed beam distribution and increasing of the beam energy spread.
  • With the impedance model developed, the microwave instability and bunch lengthening are simulated with the code Elegant => the design bunch intensity is just above the threshold, and turbulent distributions are observed above the threshold.
  • To mitigate the effect:
    • Impedance reduction
    • Beam parameter optimization

Keil-Schnell criterion:  
\[
I_{th} = \frac{\sqrt{2\pi} \alpha_p \frac{E}{e} \sigma_e^2 \sigma_t}{R \left| \frac{Z_{\parallel}}{n} \right|_{\text{eff}}}
\]
Recheck of CEPC-Z (cdr)

- Only one working point is stable without impedance: where the input parameter of $\sigma_z(I) = \sigma_z(0)$ in simulation.
- The beam-beam interaction gets more unstable with longitudinal impedance due to the X-Z coupling.
- Bootstrapping method with impedance is also tried, stable threshold of bunch population is about 3e10.
Crosscheck of the simulation with longitudinal impedance

The e- and e+ bunches with a tilt angle move head on to each other in the Lorentz boosted frame[1]. The transverse momentum kick induced by the cross wake force is represented as follows[2], where we have ignore a term that induces a tune shift.

\[ \Delta p_{\pm}(z) = -\frac{N_{ze}}{\gamma} \int_{-\infty}^{\infty} W_{z}(z-z') \rho_{z}(z') dz' \]

This cross wake force is caused by the beam-beam interaction and is a linear part of beam-beam force. We use this localized force for particle tracking. The tracking results become worse when we consider the longitudinal impedance. The following show the tracking result.

• smaller $\beta_x^*$ is preferred for X-Z coupling #*.  
  • Larger stable region is obtained by reducing the $\beta_x^*$ from 0.2m to 0.15m  
• By including the longitudinal impedance  
  • The stable region is reduced and shifted  
  • With only RW, a smaller tune shift and larger stable region are observed  
• Beam parameters and impedance should be further optimized.

* D. Shatilov, FCC-ee parameter optimization, ICFA Beam Dynamics Newsletter 72, 30 (2017).
Nonlinear Kick in Local RF Cavity

- With cavity’s nonlinear kick (Sin wave): only one working point is stable considering impedance
  \[ \text{\(3\sigma_z = 22\text{mm} \sim 3\text{ deg}\), \(\phi_s = 158.9\text{ deg}\)}\]
  \(<z>\) shift 3.6mm

Linear Oscillation in Longitudinal Direction

- \([x/\sigma_x \cdot Q_x]\) half ring
  - \(\beta_x = 0.15\), w/o impedance
  - \(\beta_x = 0.15\), w/ impedance
  - \(0.5 + 4\nu_s\)
  - \(0.5 + 5\nu_s\)

Nonlinear Kick in RF Cavity

- cav, w/o impedance
- cav, w/ impedance
  - \(0.5 + 4\nu_s\)
  - \(0.5 + 5\nu_s\)
• Transverse mode coupling instability
  • The threshold for the TMCI is estimated using both analytical formula and Eigen mode analysis.

\[ I_{0}^{th} = \frac{2Q_{s}\omega_{0}E/e}{\sum \beta_{\perp,j}k_{y,j}} \]

• When considering bunch lengthening due to the impedance or beamstrahlung, the transverse effective impedance decreases due to its dependence on the bunch distribution => TMCI threshold increases.

Threshold is comparable with the design current without considering bunch lengthening.

Larger safety margin obtained when considering bunch lengthening effects.
• No significant influence of the transverse wake field on collision.
  • $\beta_x^* = 0.15\text{m}$
  • With longitudinal and horizontal impedance
General Chromaticity

- Here only include a linear horizontal tune chromaticity to check if the chromaticity could affect the X–Z instablitiy.

\[
\bar{x} = M_4(\delta)x,
\bar{z} = z + g(x, p_x, y, p_y, z, \delta),
\delta = \bar{\delta},
g = -x^T M_4^T(\delta)S_4(\partial_\delta M_4(\delta))x/2
\]


\[\sigma_p \sim 1.1e-3,\]
FCCee-Z

Only a tune shift (~0.004) is found
Stable width of working point space: 0.004

Only resistive wall impedance of FCCee is considered.

Figure 2.27: RMS bunch length (left) and RMS energy spread (right) as a function of the bunch intensity obtained from numerical simulations, without beamstrahlung (BS), considering only the RW impedance produced by a NEG film with 100 nm thickness.
Parameter Optimization

• $\xi_x \ll Q_s$, to suppress X-Z coherent instability

• Since $\xi_x \propto \frac{\beta_x N}{\beta_x \epsilon_x + \sigma_z^2 \theta^2}$ and $\phi = \frac{\sigma_z \theta}{\sigma_x} \gg 1$, smaller $\beta_x$ is more preferred.

• Longer bunch length $\sigma_z$ is preferred for small $\xi_x$. Increasing $\alpha_p$ help increase bunch length (reduce $\xi_x$), and $Q_s$. To keep $\xi_y$ unchanged, bunch population increase. $L \propto \frac{N \xi_y}{\beta_y}$ keep unchanged. Microwave instability threshold $N_{th} \propto \frac{\alpha_p \sigma_x^2 \sigma_z}{\text{Re}[Z/n]}$ also increase.

• Without beamstrahlung and wakefield, decrease Vrf, do not change $\xi_x/Q_s$, but increase the synchro-betatron resonance order.

• D. Shatilov, FCC-ee parameter optimization, ICFA Beam Dynamics Newsletter 72, 30 (2017).
Summary

• Self-consistent simulation considering beam-beam interaction and single-bunch collective effect has been done.

• The longitudinal wakefield has a serious effect on the stability of collision for CEPC-Z

• The impedance budget needs to be further squeezed

• The machine parameters needs to be further optimized:
  • Squeeze $\beta_x$
  • Increasing $\alpha_p$
• Backup
FCC IMPEDANCE + CEPC Z PARAMETER

\[ \frac{\sigma_y}{\sigma_{x,0}} \]

\[ Q_x \text{ [half ring]} \]

\[ 0.5 + 4v_s \quad 0.5 + 5v_s \]

\[ qx = 0.554, 0.556, 0.558, 0.560, 0.562, 0.564, 0.566, 0.568, 0.570, 0.572, 0.576 \]

\[ \text{turns} \]

\[ 0 \quad 1000 \quad 2000 \quad 3000 \quad 4000 \quad 5000 \]
FCC IMPEDANCE, BX=0.15, CEPC
## CEPC VS FCC (CDR)

<table>
<thead>
<tr>
<th></th>
<th>CEPC</th>
<th>FCC</th>
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<tbody>
<tr>
<td>Bunch Popualtion</td>
<td>8e10</td>
<td>1.7e11</td>
</tr>
<tr>
<td>Horizontal Emittance</td>
<td>0.18 nm</td>
<td>0.27 nm</td>
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<tr>
<td>Vertical Emittance</td>
<td>1.6 pm</td>
<td>1.0 pm</td>
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<tr>
<td>Arc Cell Phase Advance</td>
<td>90/90</td>
<td>60/60 [deg]</td>
</tr>
<tr>
<td>Momentum Compaction</td>
<td>11.1e-6</td>
<td>14.8e-6</td>
</tr>
<tr>
<td>Horizontal $\beta_x^*$</td>
<td>0.2 m</td>
<td>0.15 m</td>
</tr>
<tr>
<td>Vertical $\beta_y^*$</td>
<td>1 mm</td>
<td>0.8 mm</td>
</tr>
<tr>
<td>Energy Spread [SR]</td>
<td>3.8e-4</td>
<td>3.8e-4</td>
</tr>
<tr>
<td>Bunch Length [SR]</td>
<td>2.4 mm</td>
<td>3.5 mm</td>
</tr>
<tr>
<td>Piwinski Angle [SR]</td>
<td>6.6</td>
<td>8.2</td>
</tr>
<tr>
<td>Synchrotron Tune $Q_s$</td>
<td>0.028</td>
<td>0.025</td>
</tr>
<tr>
<td>Beam-Beam Parameter $\xi_{x,0}/\xi_{y,0}$</td>
<td>0.004/0.079</td>
<td>0.004/0.133</td>
</tr>
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