Inflation, Primordial Black Holes, Dark Matter, and Gravitational Waves

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Outline

- Gravitational waves (tensor modes)
- Standard slow-roll inflation and CMB B-mode polarization
- Beyond slow-roll ?!
- Anisotropic stress τ_{ij} during inflation

  Classical source – particle production

  $<\tau_{ij}>=0$, vacuum fluctuations of stress tensor
- Primordial black holes (a candidate for dark matter)
- Gravitational waves due to second-order effects

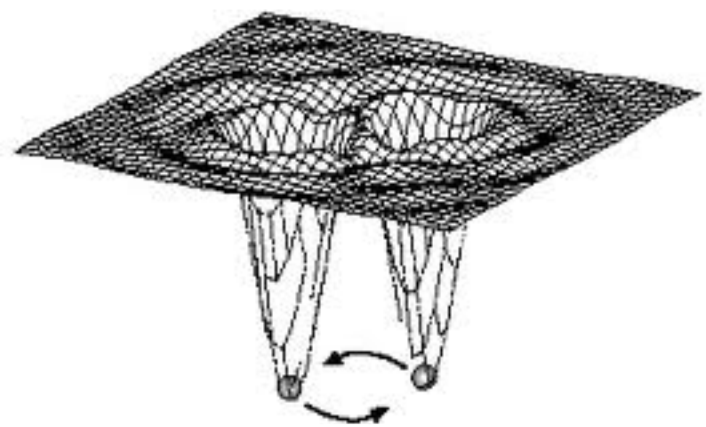
Gravitational Waves in Expanding Universe

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$$
$$= a^2(\eta)\left(d\eta^2 - d\mathbf{x}^2\right)$$

conformal time

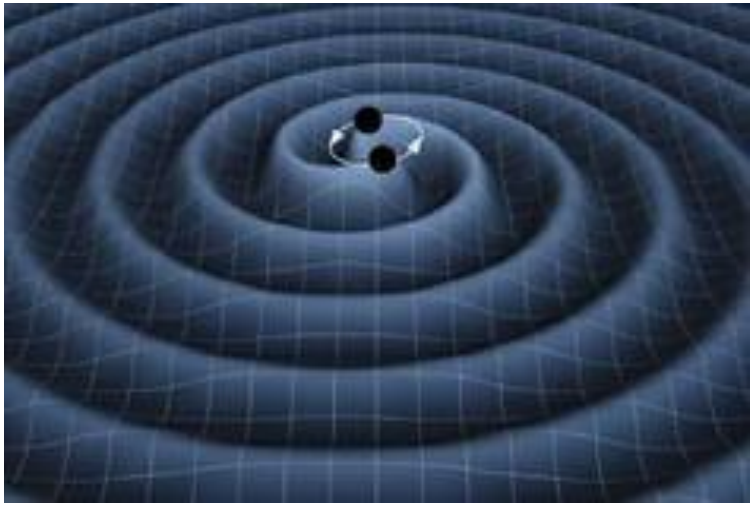
$$d\eta = dt/a(t)$$

$$g_{\mu\nu} = a^2(\eta)(\eta_{\mu\nu} + h_{\mu\nu}), \quad h_{\mu\nu} \ll 1$$

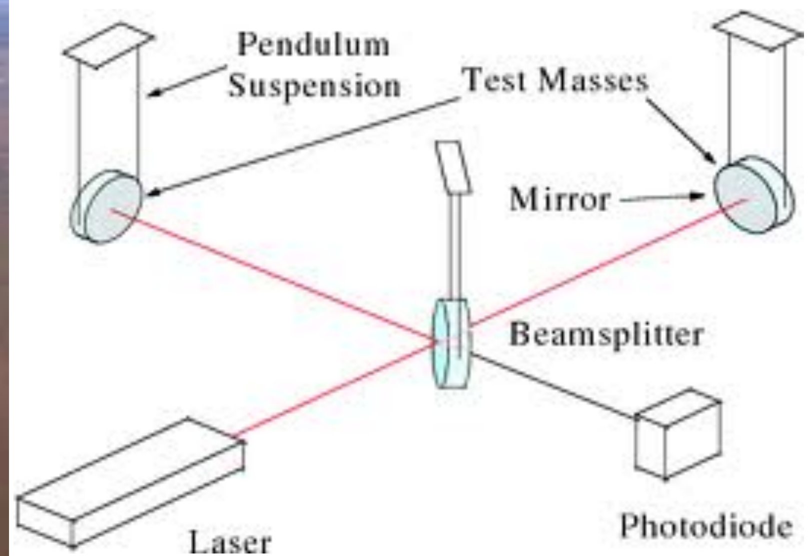


binary black holes

gravitational waves are ripples of space-time

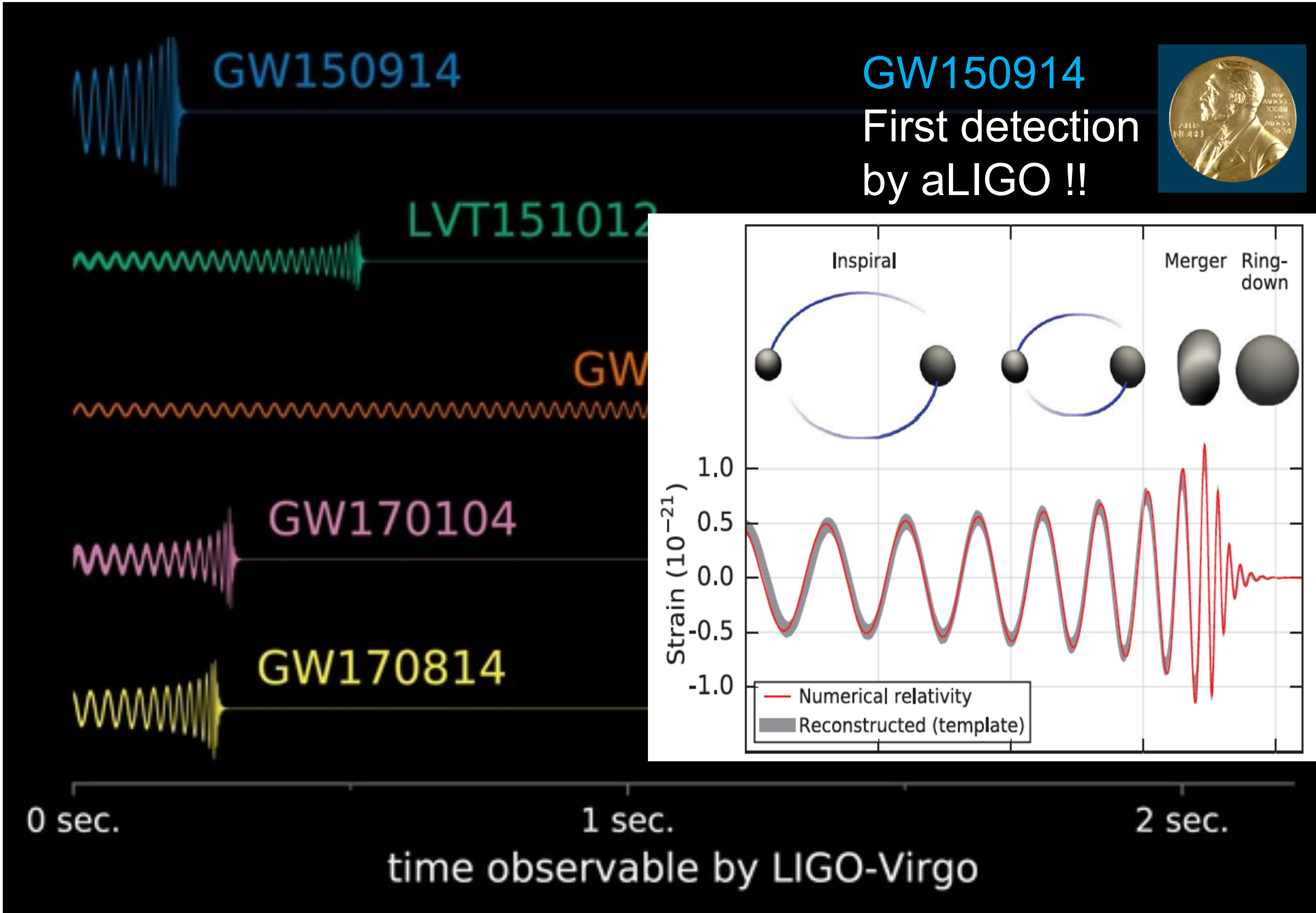


GWs Observation



LIGO interferometry experiment

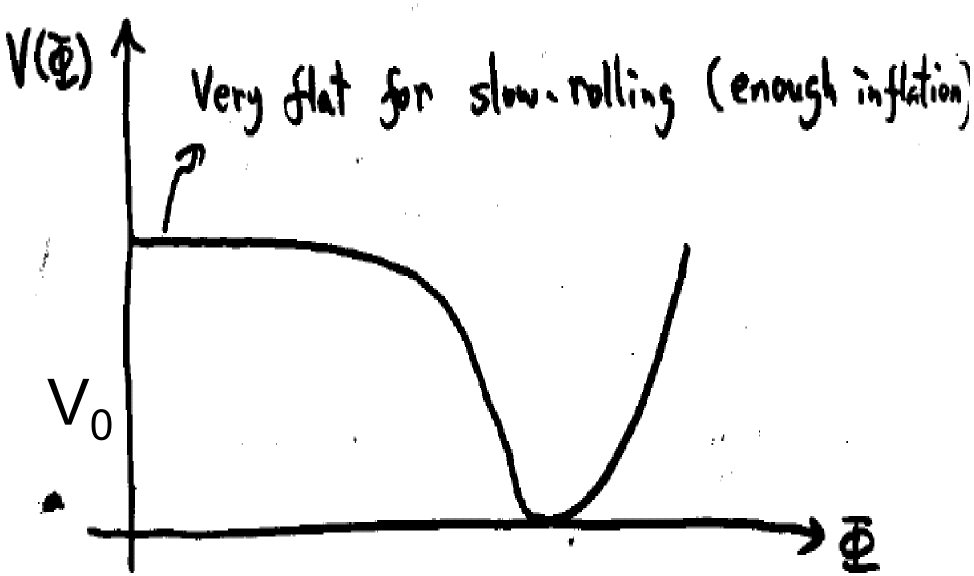
Detection of Gravitational Waves in Binary Black Hole Merger



Spectrum of the primordial curvature perturbation

Scalar mode

A generic $V(\Phi)$ for inflation



scalar field equation of motion

$$\ddot{\phi} + 3\frac{\dot{R}}{R}\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \qquad \frac{\dot{R}}{R} = H$$

Φ Inflaton

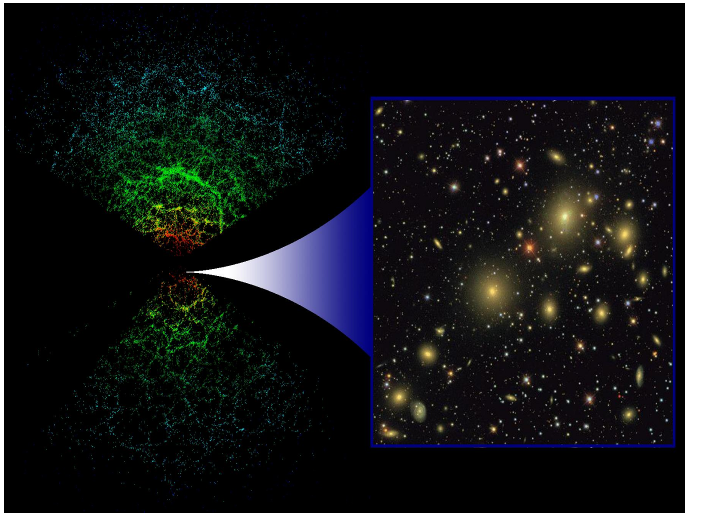
Slow-roll inflation

$$H^2 \simeq \frac{V(\phi)}{3M_{\rm Pl}^2},$$

$$3H\dot{\phi} \simeq -V'(\phi),$$

$$\mathcal{P}_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2$$

$$P_R(k) = (\delta\rho/\rho)^2$$



large scale structure of the Universe

Tensor mode

$$h_{ij}(x) = h(x; \mathbf{k}, \lambda)\, \epsilon_{ij}(\mathbf{k}; \lambda)\,,$$

polarization tensor and $\lambda = +, \times$.

$$\epsilon_{ij}(\mathbf{k}; \lambda)\epsilon^{ij}(\mathbf{k}; \lambda') = 2\delta_{\lambda\lambda'}$$

$$I_{\rm graviton} = \frac{1}{16\pi G}\int d^4x\, a^2(\eta)\, \frac{1}{2}\left[(\partial_\mu h(x; \mathbf{k}, +))^2 + (\partial_\mu h(x; \mathbf{k}, \times))^2\right]$$

$$h(x; \mathbf{k}, \lambda) = (2\pi)^{-\frac{3}{2}} h_\lambda(\eta; \mathbf{k})\, e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.}$$

equation of motion for the wave amplitude

$$\ddot{h}_\lambda + 2\frac{\dot{a}}{a}\dot{h}_\lambda + k^2 h_\lambda = 0$$

Inflation $\longrightarrow$ Radiation-dominated $\longrightarrow$ matter-dominated

choose
Bunch-Davis vacuum
or positive-energy solution

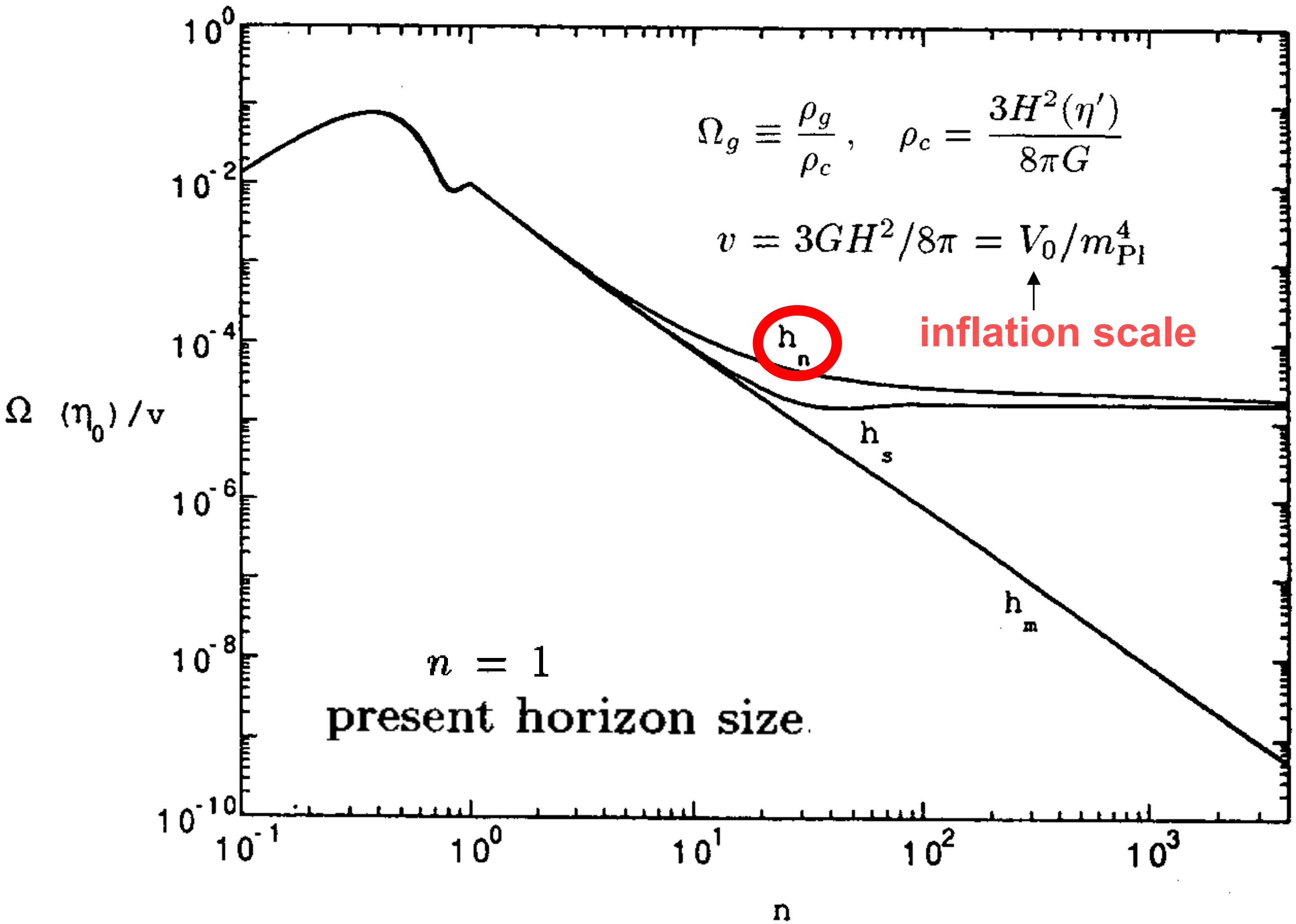
$$|h_\lambda(\eta; k)|^2 k^3 \simeq 8\pi G H^2 \left[\frac{3j_1(k\eta)}{k\eta}\right]^2$$

Almost scale-invariant power spectrum $P_h(k)$

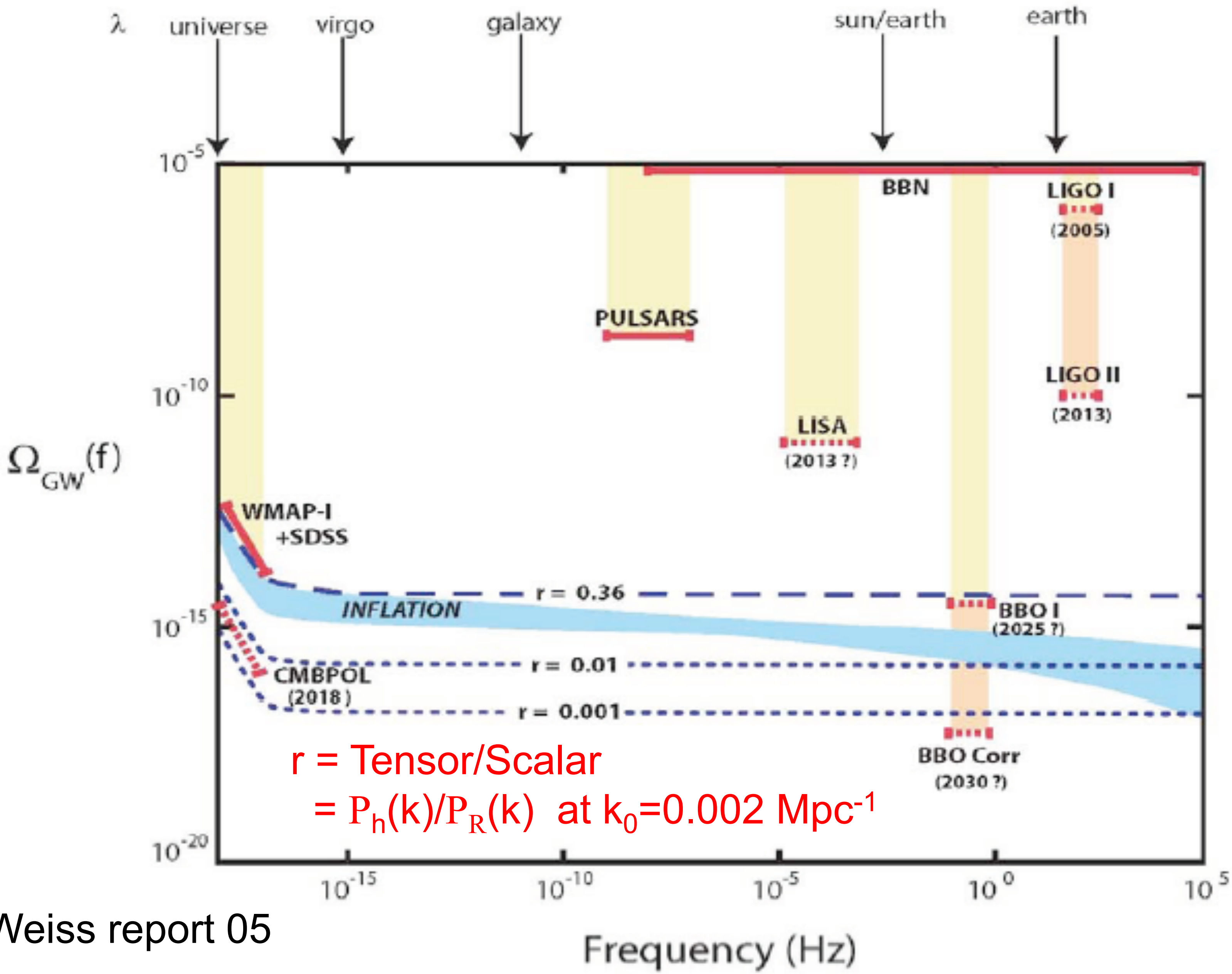
$$\frac{3j_1(k\eta)}{k\eta} \simeq 1 \quad \text{as} \quad k\eta \ll 1\,.$$

spectral energy density of gravitational waves

$$\rho_g \equiv \sum_{\lambda=+,\times} k' \frac{d\rho_\lambda}{dk'} = \sum_{\lambda=+,\times} \frac{1}{32\pi G a^2(\eta')} \left(\frac{k'}{2\pi}\right)^3 \left[k'^2 |h_\lambda|^2 + \left|\frac{dh_\lambda}{d\eta'}\right|^2\right]$$



Primordial gravitational waves



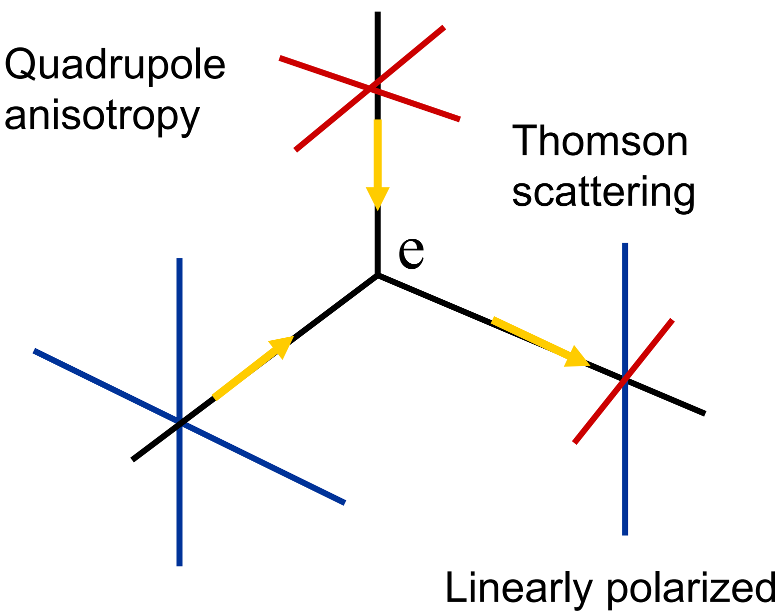
r = Tensor/Scalar

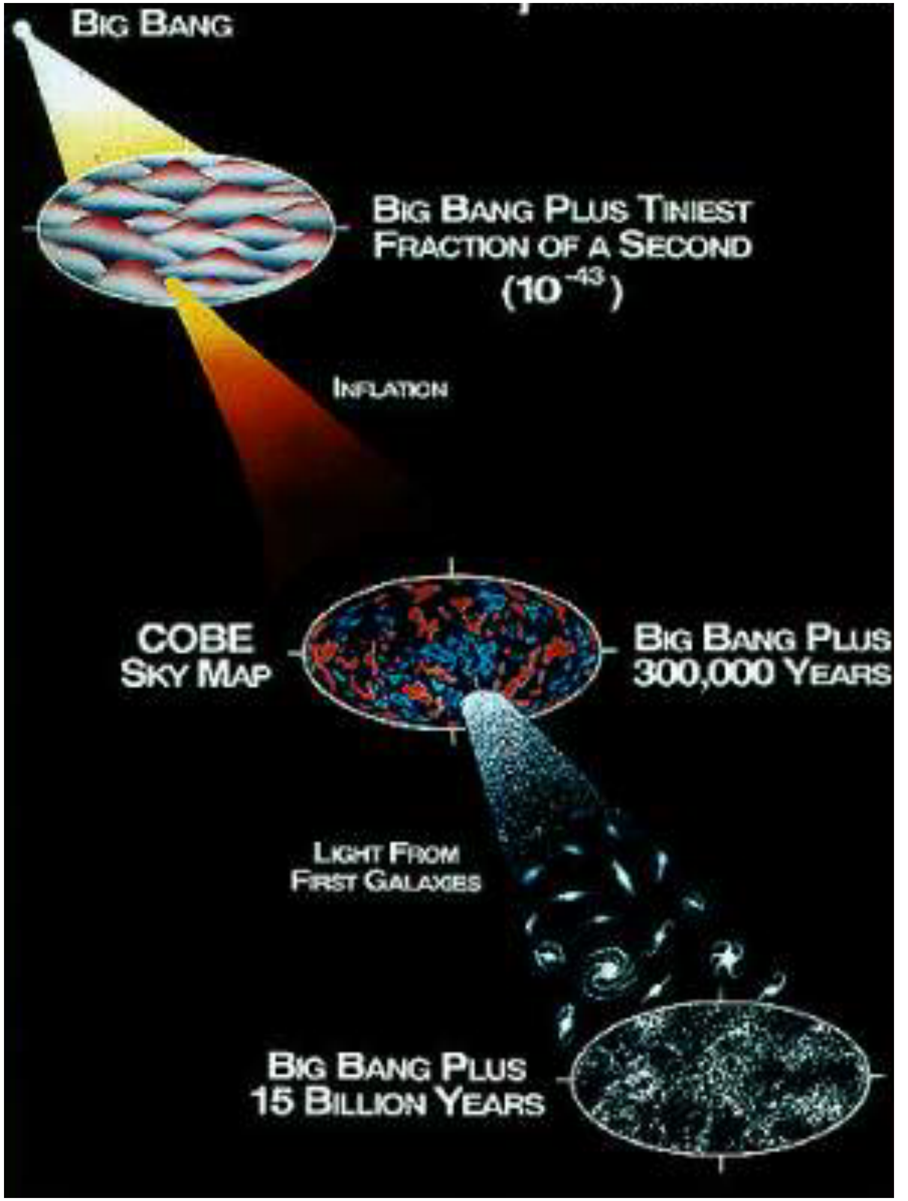
$= P_h(k)/P_R(k)$ at k_0=0.002 Mpc^{-1}

Weiss report 05

CMB Anisotropy and Polarization

- On large angular scales, matter imhomogeneities or primordial gravitational waves generate gravitational redshifts
- On small angular scales, acoustic oscillations in plasma on last scattering surface generate Doppler shifts
- Thomson scatterings with electrons generate polarization



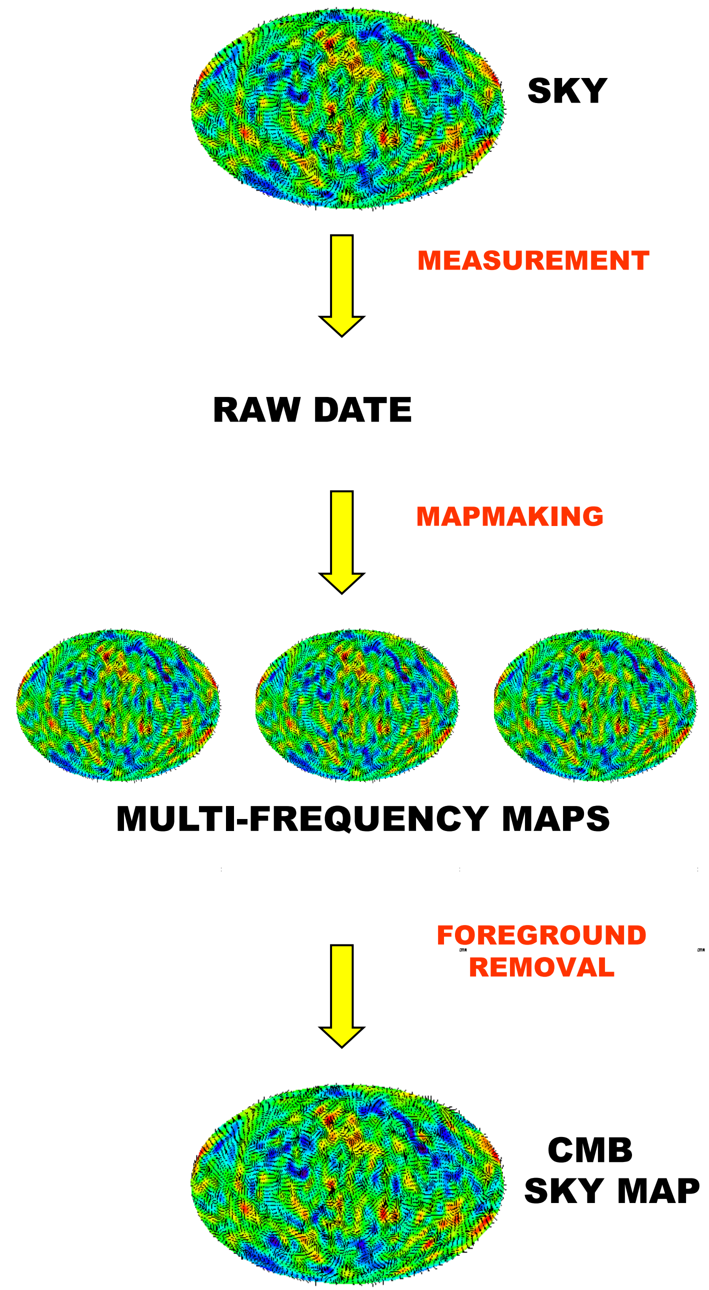


CMB Measurements

- Point the telescope to the sky
- Measure CMB Stokes parameters:

  $T = T_{CMB} - T_{mean}$,

  $Q = T_{EW} - T_{NS}$, $U = T_{SE\text{-}NW} - T_{SW\text{-}NE}$
- Scan the sky and make a sky map
- Sky map contains CMB signal, system noise, and foreground contamination including polarized galactic and extra-galactic emissions
- Remove foreground contamination by multi-frequency subtraction scheme
- Obtain the CMB sky map



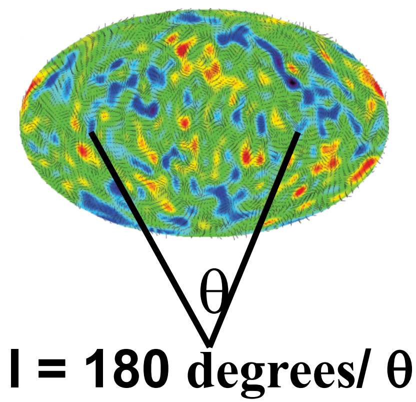
CMB Anisotropy and Polarization Angular Power Spectra

Decompose the CMB sky into a sum of spherical harmonics:

$T(\theta,\varphi) = \Sigma_{lm}\ a_{lm}\, Y_{lm}\,(\theta,\varphi)$

$(Q - iU)\,(\theta,\varphi) = \Sigma_{lm}\ a_{2,lm}\ {}_{2}Y_{lm}\,(\theta,\varphi)$

$(Q + iU)\,(\theta,\varphi) = \Sigma_{lm}\ a_{-2,lm}\ {}_{-2}Y_{lm}\,(\theta,\varphi)$

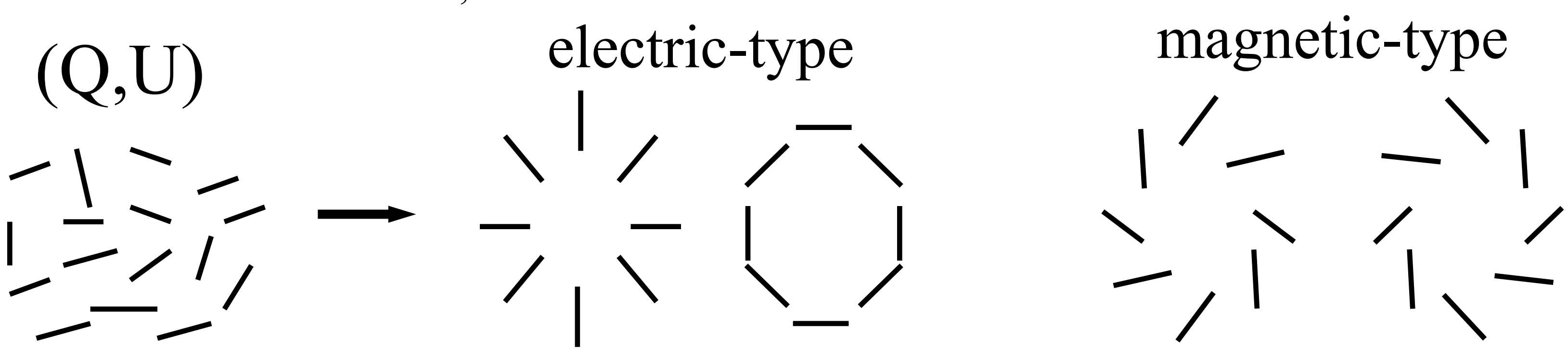


$C^{T}_{l} = \Sigma_m\,(a^*_{lm}\, a_{lm})$ anisotropy power spectrum

$C^{E}_{l} = \Sigma_m\,(a^*_{2,lm}\, a_{2,lm} + a^*_{2,lm}\, a_{-2,lm})$ E-polarization power spectrum

$C^{B}_{l} = \Sigma_m\,(a^*_{2,lm}\, a_{2,lm} - a^*_{2,lm}\, a_{-2,lm})$ B-polarization power spectrum

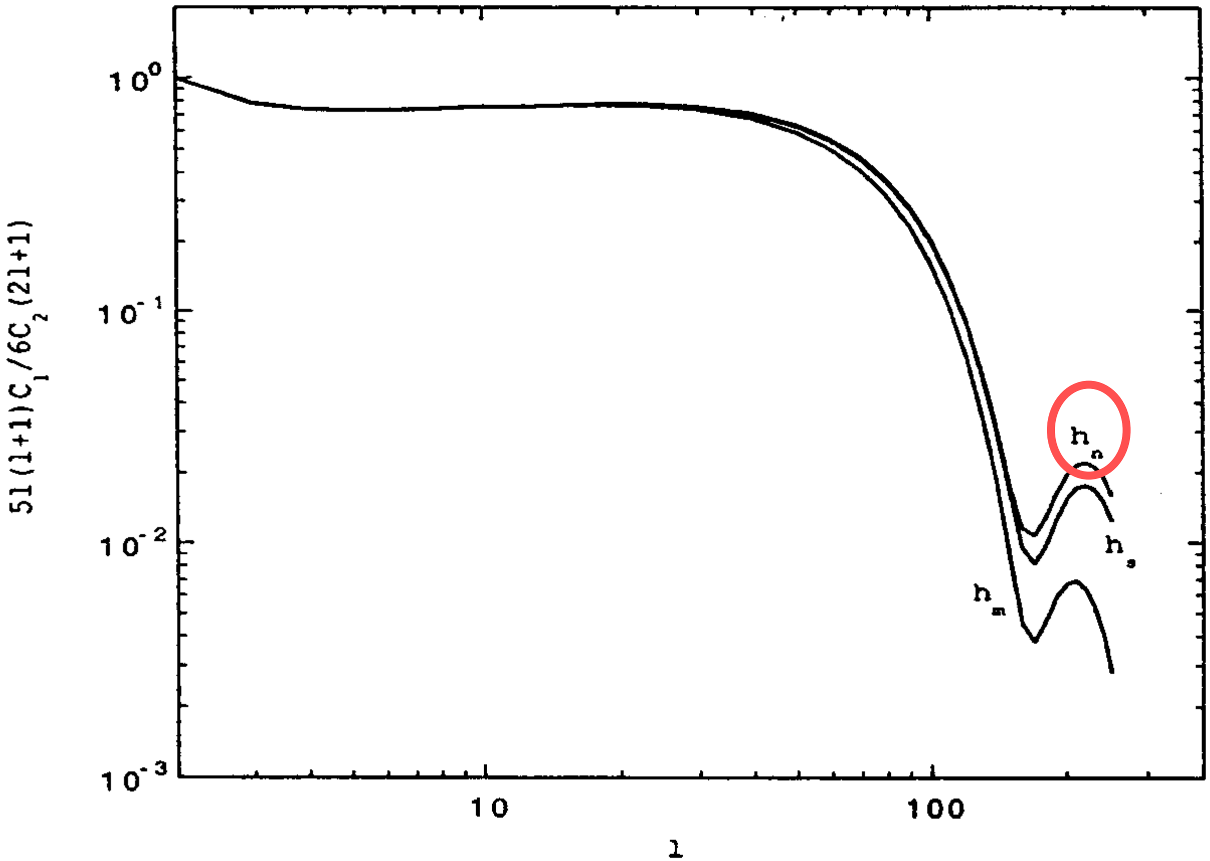
$C^{TE}_{l} = -\,\Sigma_m\,(a^*_{lm}\, a_{2,lm})$ TE correlation power spectrum



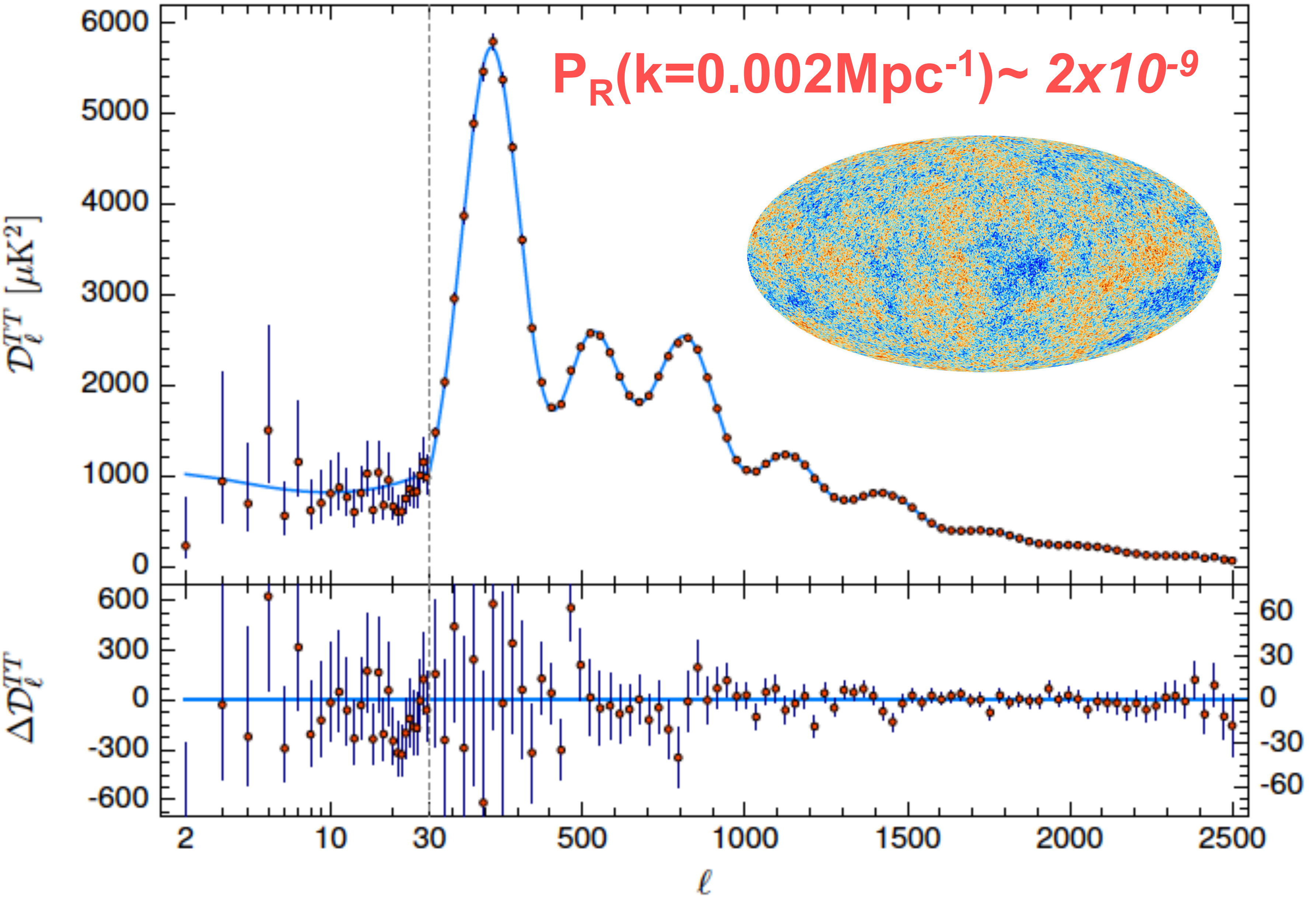
PGWs and CMB temperature anisotropy

Sachs–Wolfe effect: $$\frac{\delta T}{T}(\mathbf{e}) = -\frac{1}{2}\int_e^r d\Lambda e^i e^j \frac{\partial}{\partial \eta'} h_{ij}(\eta', \mathbf{x})$$

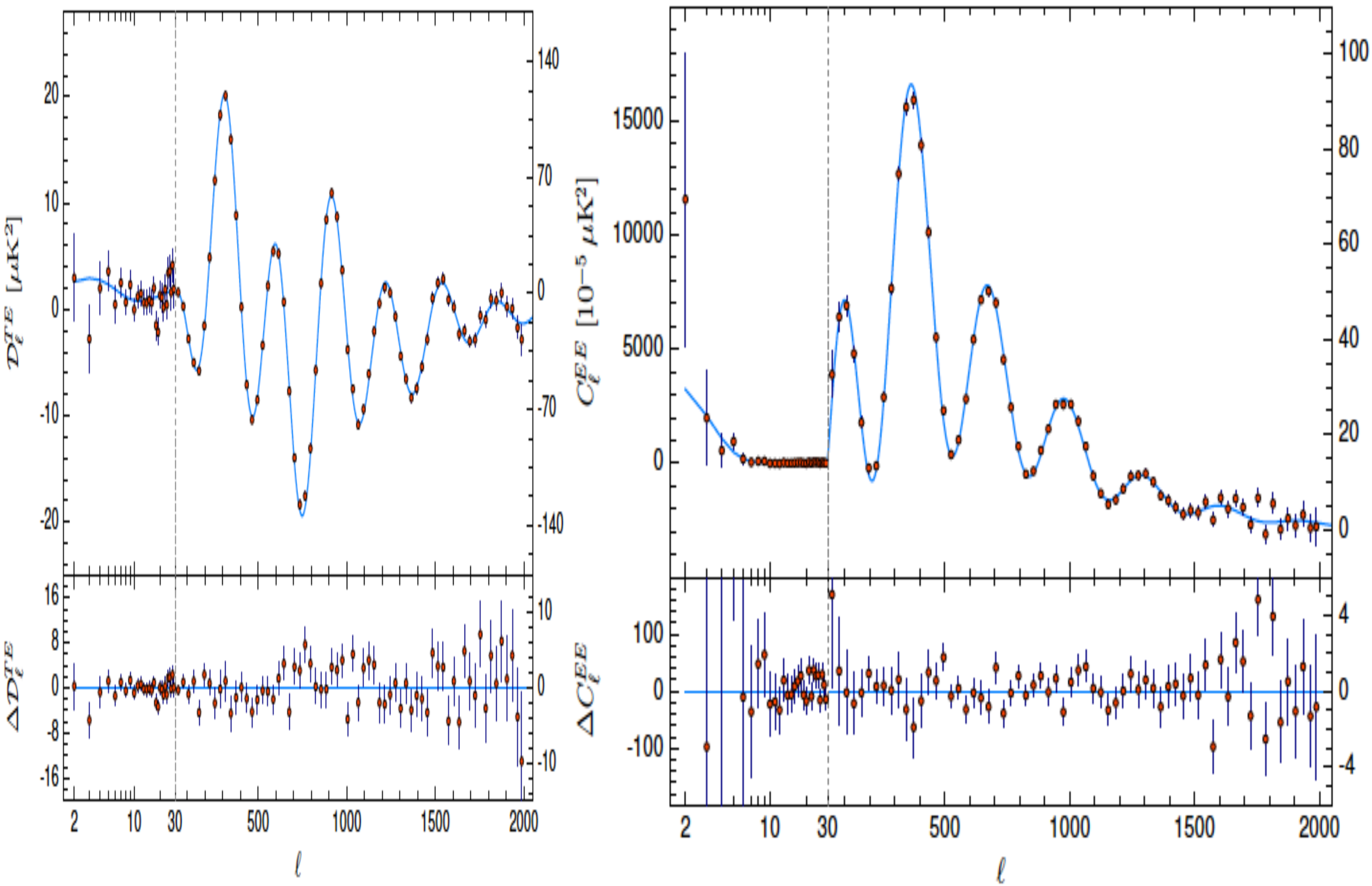
$$C_l = \frac{32\pi v}{3}(2l+1)(l+2)(l+1)l(l-1)\int_0^\infty \frac{dk'}{k'}\left[\int_{\eta'_{\rm dec}}^{\eta'_0} d\eta' \frac{dh(\eta')}{d\eta'} \frac{j_l[k'(\eta'_0-\eta')]}{k'^2(\eta'_0-\eta')^2}\right]^2$$



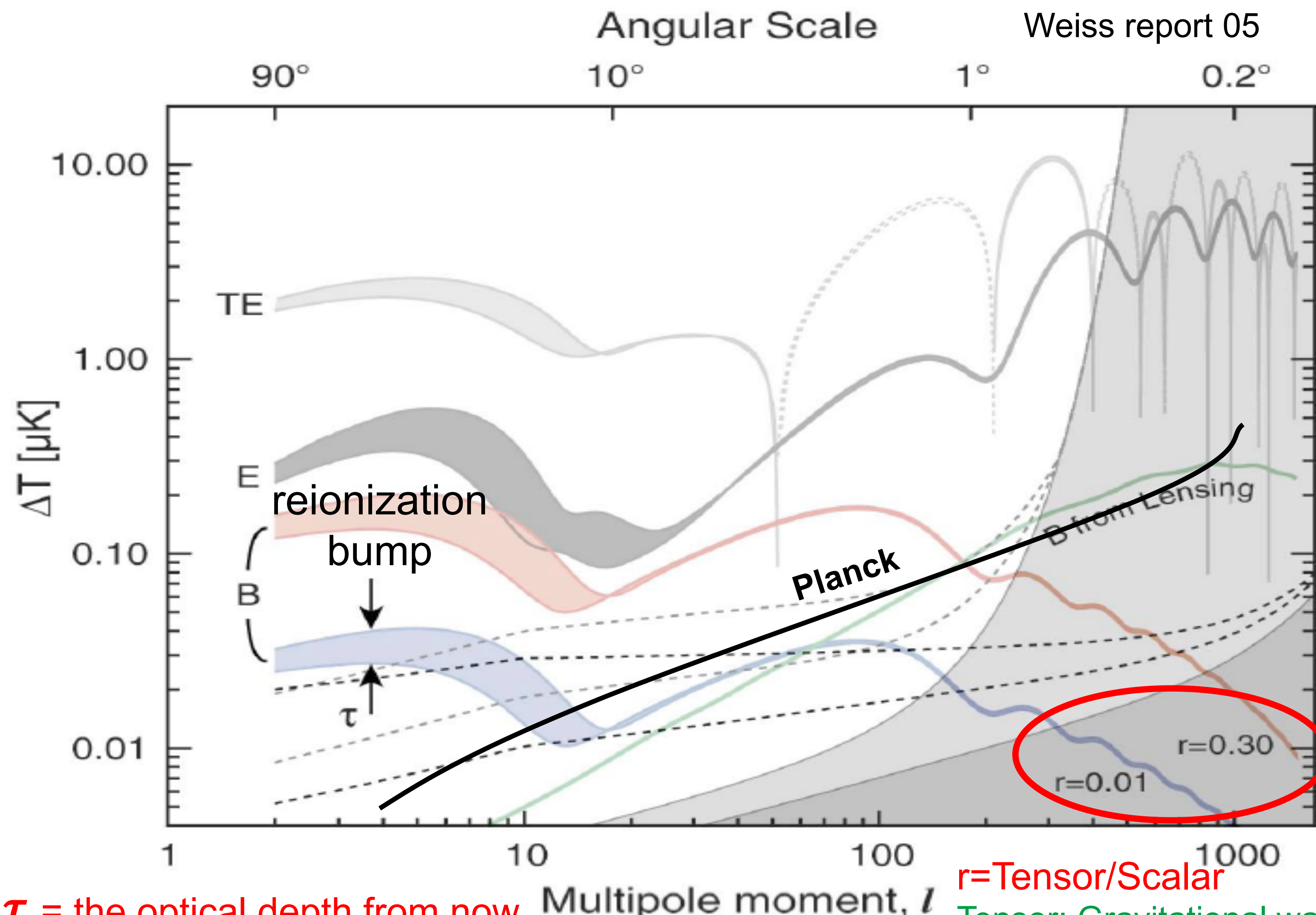
Planck CMB Anisotropy $D^{TT}_l = l(l+1)\, C^T_l$ 2018



Planck CMB Polarization Power Spectra 2018



Gravity-wave induced B-mode



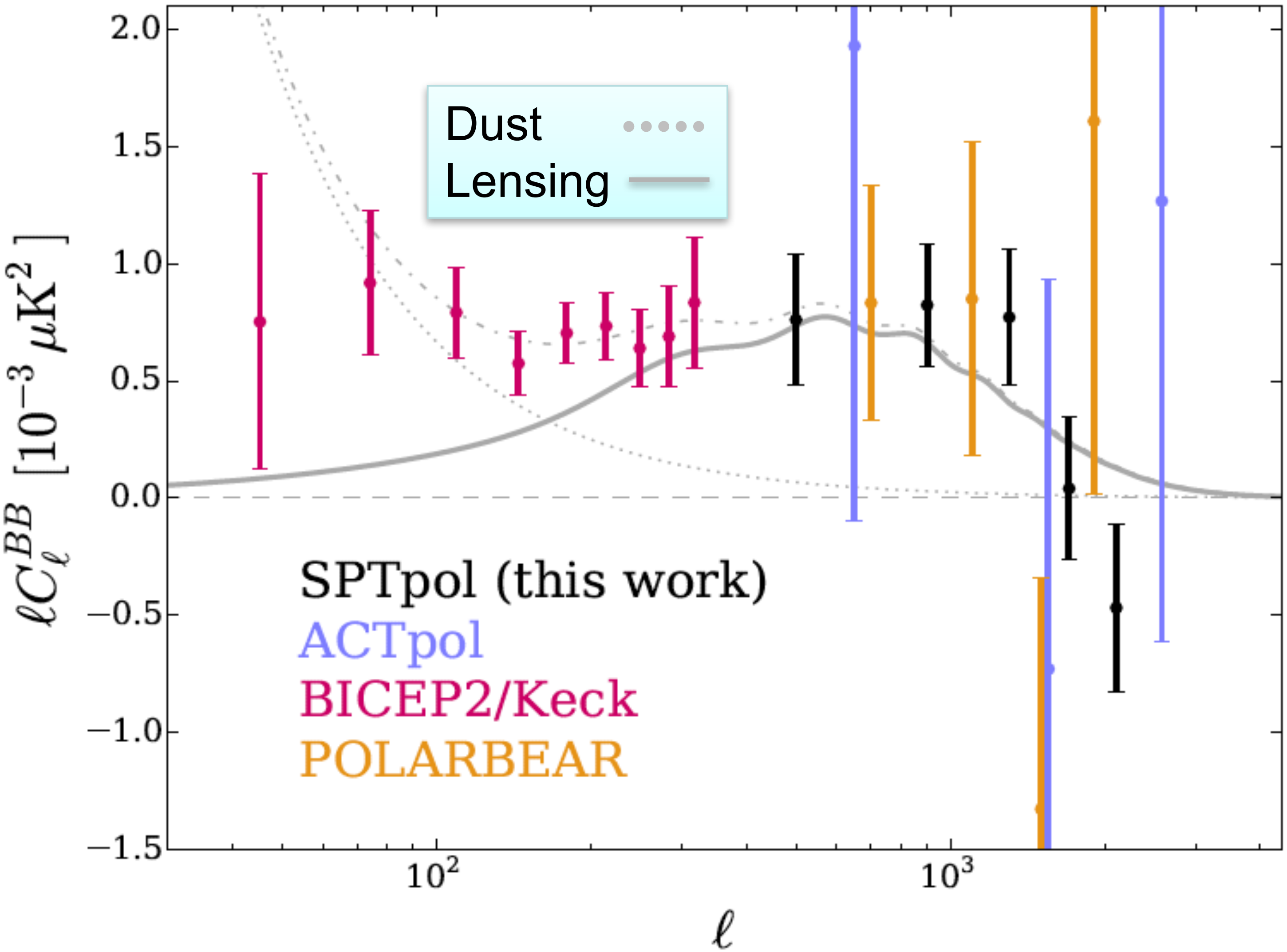
τ = the optical depth from now to the last scattering surface

r=Tensor/Scalar

Tensor: Gravitational waves

Scalar: Density fluctuations

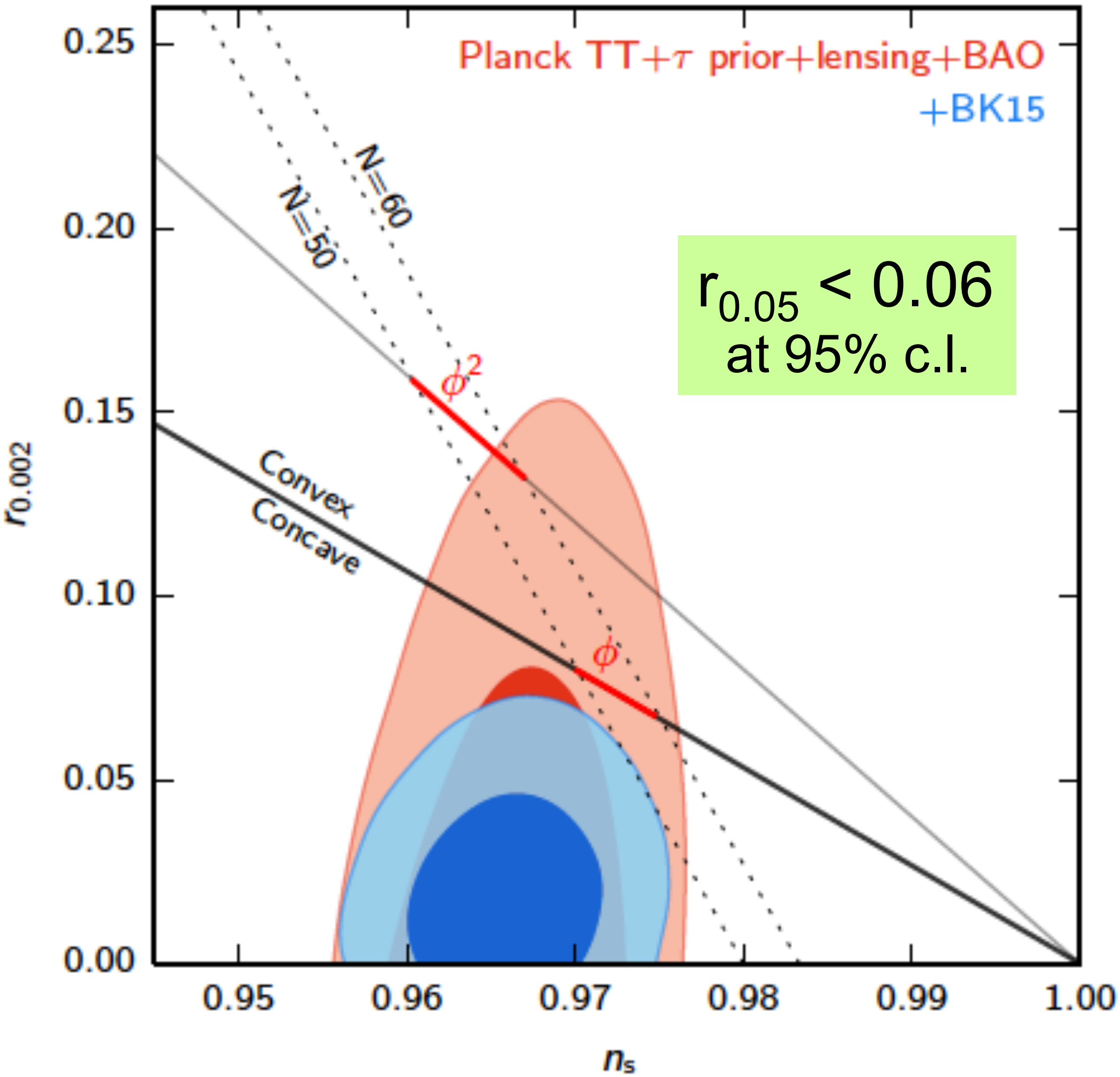
Current B-mode measurements



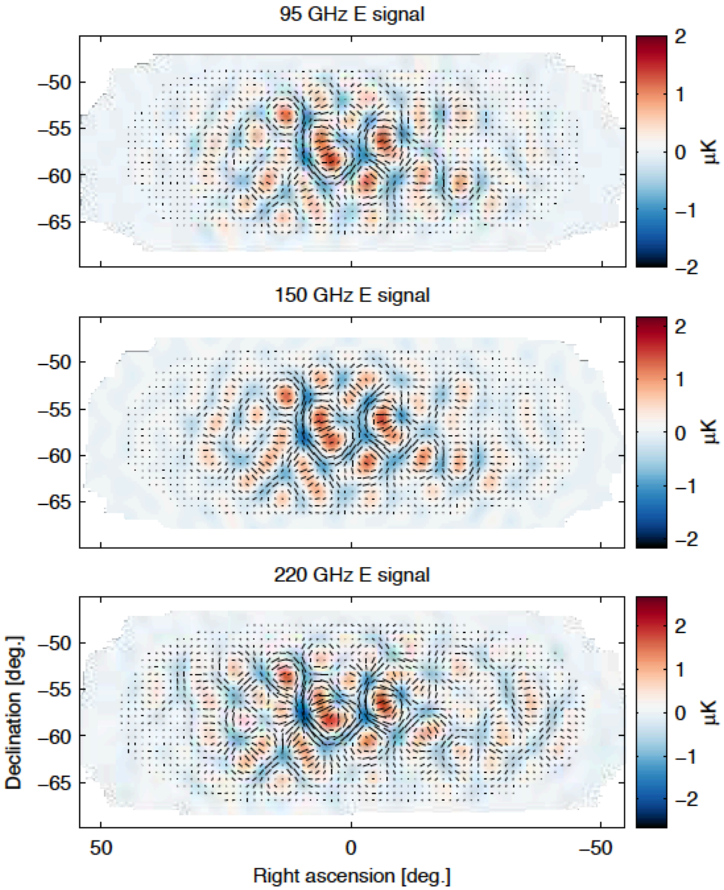
Galactic polarized dust emission
B-mode power spectrum at 150 GHz

$$\frac{l(l+1)}{2\pi} C_{l\,\text{dust}}^{BB} = 0.0118 \left(\frac{l}{80}\right)^{-0.42} \mu K^2.$$

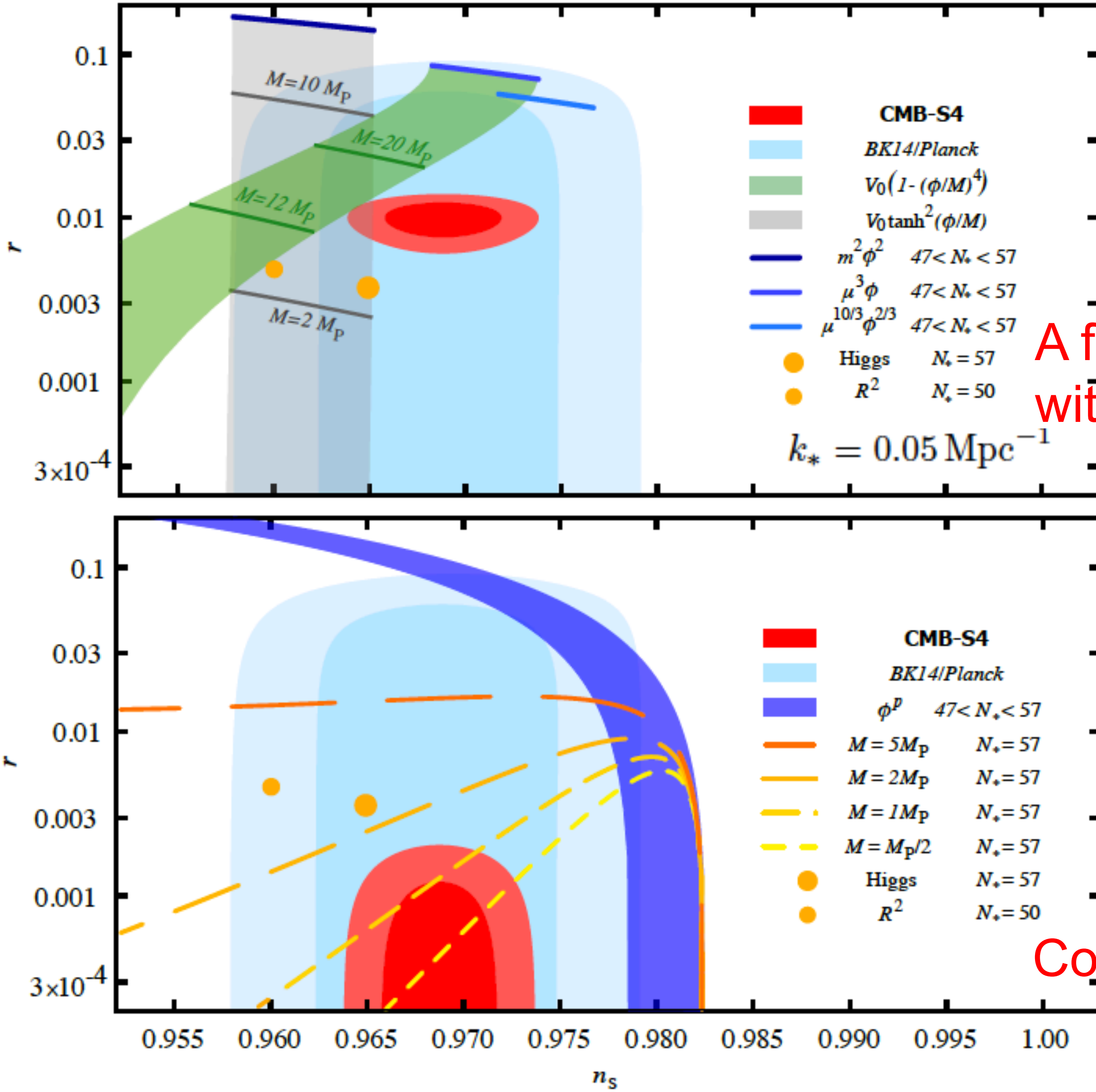
Joint Planck+BICEP2/Keck Array constraint on r by removal of dust contamination (2018)



Keck Array 2015
E-polarization data



CMB-S4 forecasts on inflation models



A fiducial model with r=0.01

Constraints on r

Beyond Standard Slow-roll ?!

- Slow-roll inflation for the first 7 e-foldings (61>N>54)

  Planck CMB + LSS + Sne + ...

- Flat inflaton potential? Interacting inflaton ?

  trapped inflation in which back reaction effectively make inflaton slow-rolling down a steep potential

- Binary black holes detected by aLIGO/VIRGO are primordial black holes produced during inflation (N ~ 40) ?

  PBHs may be dark matter and aLIGO has seen dark matter!

Primordial Black Holes

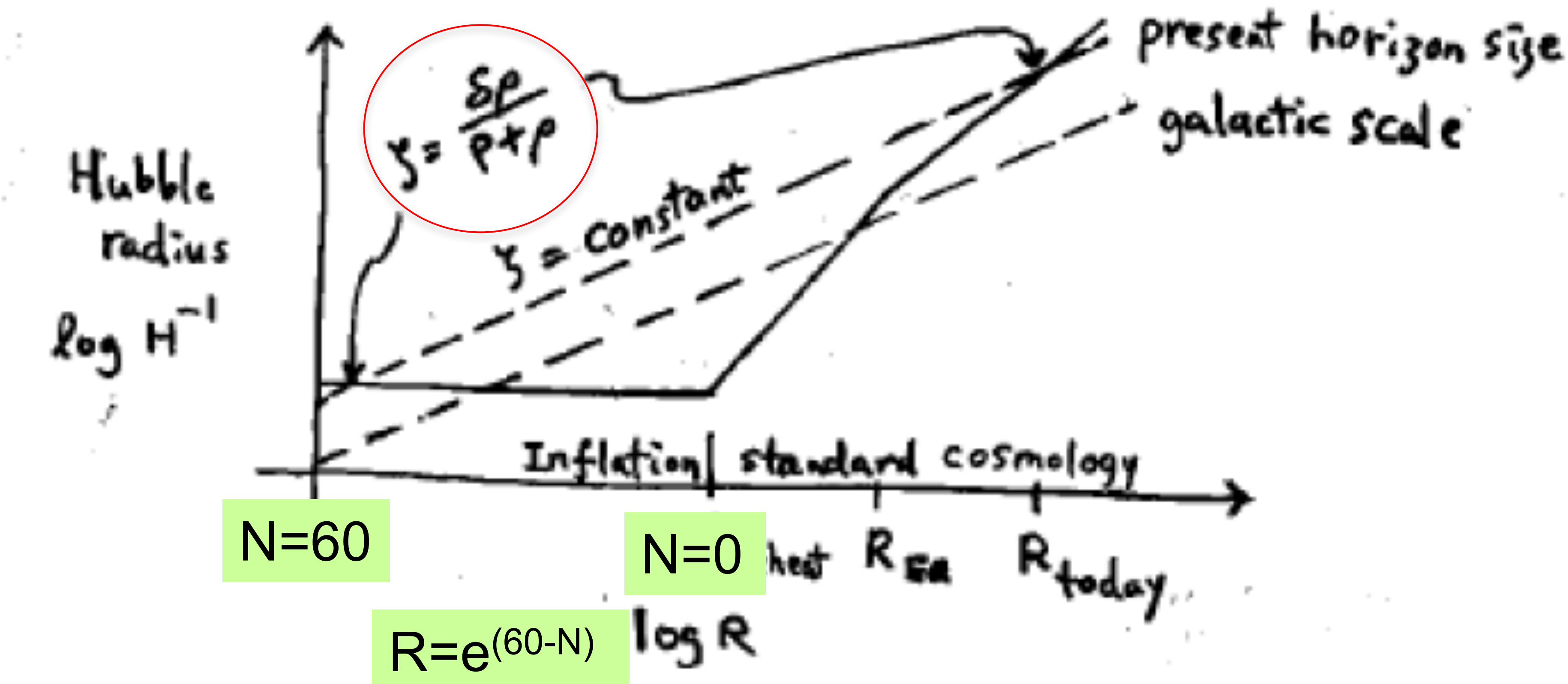
- Formed at high-density contrasts (*$\delta\rho/\rho \sim 0.5$*) over a wide range of scales or masses in the radiation-dominated Universe
- There have been stringent astrophysical and cosmological constraints on M_{PBH}
- $10 M_\odot$ PBHs could be the binary BHs observed by aLIGO gravity-wave detectors

  Bird et al. 16., Clesse et al. 16, Sasaki et al. 16
- PBHs behave like cold dark matter

  García-Bellido, Linde, Wands 96
- They, although being of baryonic origin, do not participate in big-bang nucleosynthesis

Production of PBHs in inflation

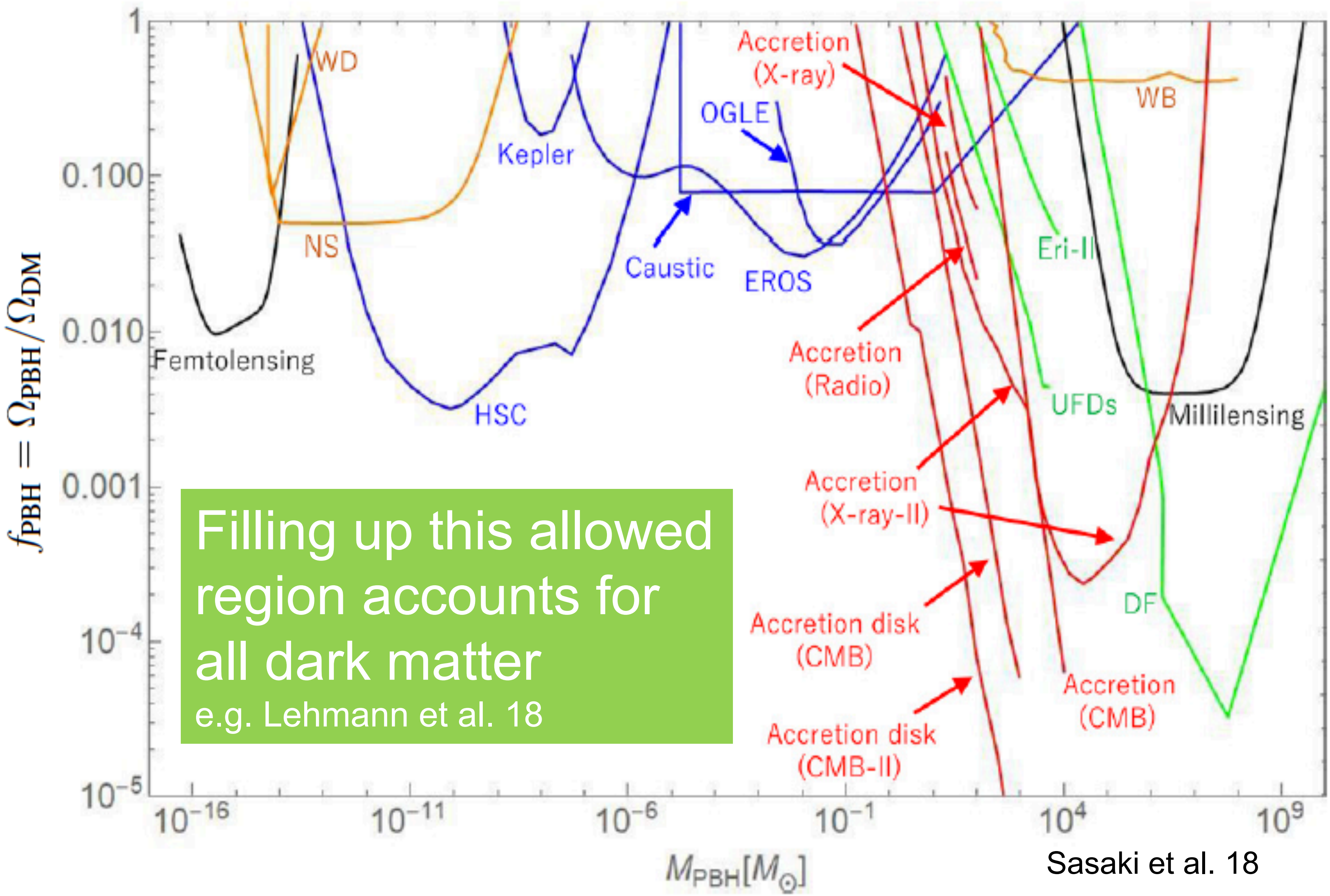
Evolution of cosmological density perturbation



$$\boxed{M_{\mathrm{BH}} = \frac{4\pi M_p^2}{H} e^{2N} = 2.74 \times 10^{-38} e^{2N} \left(\frac{M_p}{H}\right) M_{\odot}}$$

N is e-foldings before the end of inflation

Astrophysical and Cosmological Constraints on PBHs



Filling up this allowed region accounts for all dark matter

e.g. Lehmann et al. 18

Sasaki et al. 18

PBH Production in Inflation

- Single-field slow-roll inflation models, matter density perturbation ($\delta\rho/\rho \sim 10^{-5}$) too small
- Modified inflation potential to achieve blue-tilted matter power spectra or running spectral indices, leading to large density perturbation at the end of inflation, but mostly $M_{PBH} << M_{\odot}$
  García-Bellido, Linde, Wands 96
- To boost M_{PBH} , hybrid inflation, double inflation, curvaton models by inflating small-scale density perturbation to the size of a stellar-mass to supermassive PBH Kawasaki, Kohri, Yokoyama, Yanagida….
- Inflation with an inflecton point Garcia-Bellido, Morales,…
- Trapped inflation Peloso, Unal, Ng+,…

Trapped axion Inflation

We consider a version of the trapped inflation driven by a pseudoscalar φ that couples to a $U(1)$ gauge field A_μ:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\alpha}{4f}\varphi\tilde{F}^{\mu\nu}F_{\mu\nu}\right], \quad (3)$$

$$\varphi = \phi(\eta) + \delta\varphi(\eta, \vec{x})$$

$d\eta$=dt/a

Under the temporal gauge, $A_\mu = (0, \vec{A})$, we decompose $\vec{A}(\eta, \vec{x})$ into its right and left circularly polarized Fourier modes, $A_\pm(\eta, \vec{k})$, whose equation of motion is then given by

$k/(aH) < 2|\xi|$

Spinoidal instability

$$\left[\frac{d^2}{d\eta^2} + k^2 \mp 2aHk\xi\right]A_\pm(\eta, k) = 0, \quad \xi \equiv \frac{\alpha}{2fH}\frac{d\phi}{dt}. \quad (5)$$

Background

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} + \frac{dV}{d\phi} = \frac{\alpha}{f}\langle \vec{E}\cdot\vec{B}\rangle,$$

$$3H^2 = \frac{1}{M_p^2}\left[\frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + V(\phi) + \frac{1}{2}\langle \vec{E}^2 + \vec{B}^2\rangle\right]$$

$$\langle \vec{E}\cdot\vec{B}\rangle \simeq -2.4\cdot 10^{-4}\frac{H^4}{\xi^4}\,\mathrm{e}^{2\pi\xi},$$
$$\left\langle\frac{\vec{E}^2+\vec{B}^2}{2}\right\rangle \simeq 1.4\cdot 10^{-4}\frac{H^4}{\xi^3}\,\mathrm{e}^{2\pi\xi}.$$

$$\frac{1}{2}\langle \vec{E}^2 + \vec{B}^2\rangle = \int \frac{dk\,k^2}{4\pi^2 a^4}\sum_{\lambda=\pm}\left(\left|\frac{dA_\lambda}{d\eta}\right|^2 + k^2|A_\lambda|^2\right),$$

$$\langle \vec{E}\cdot\vec{B}\rangle = -\int \frac{dk\,k^3}{4\pi^2 a^4}\frac{d}{d\eta}\left(|A_+|^2 - |A_-|^2\right).$$

Perturbation

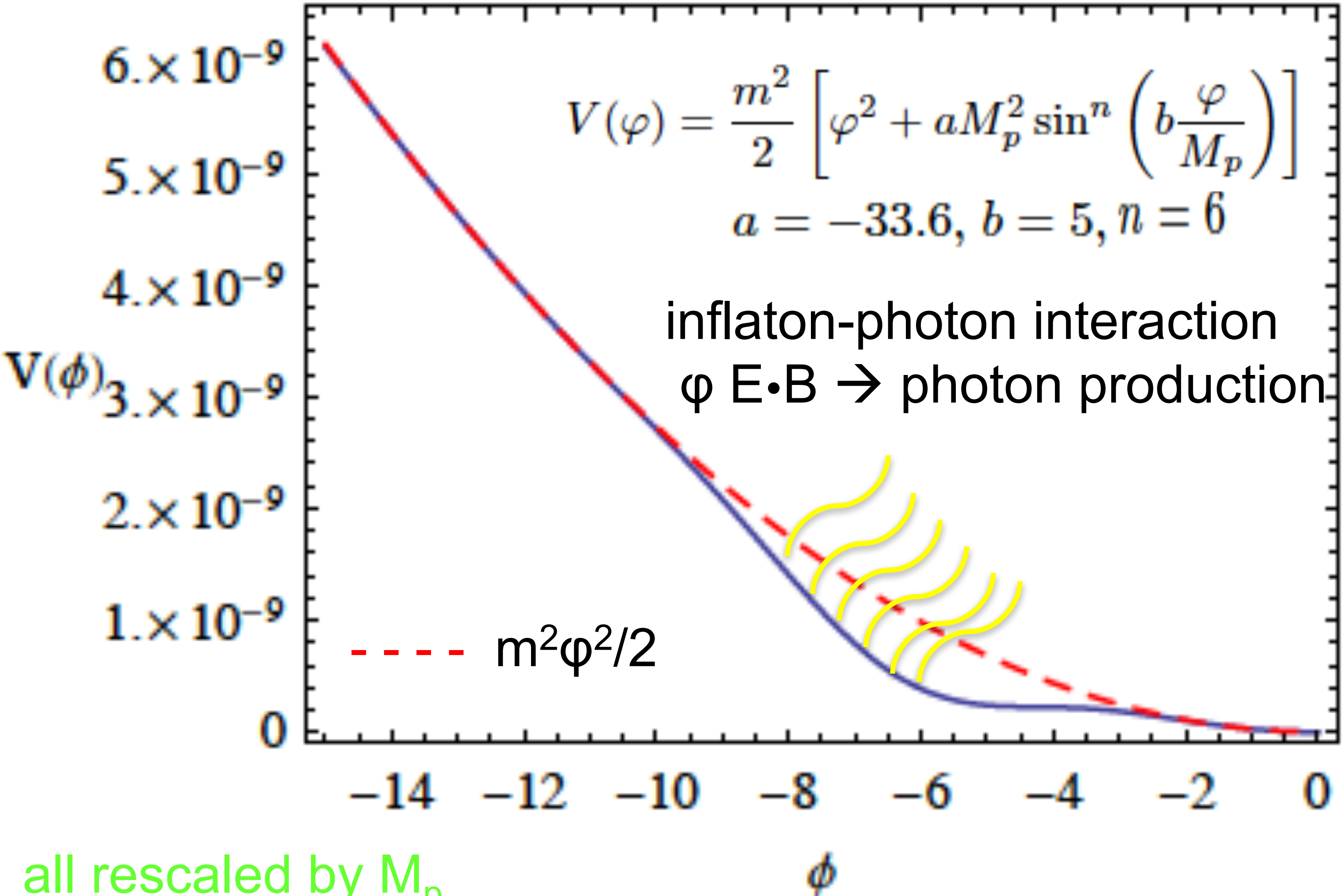
$$\left[\frac{\partial^2}{\partial t^2} + 3\beta H\frac{\partial}{\partial t} - \frac{\vec{\nabla}^2}{a^2} + \frac{d^2V}{d\phi^2}\right]\delta\varphi(t,\vec{x}) = \frac{\alpha}{f}\left(\vec{E}\cdot\vec{B} - \langle \vec{E}\cdot\vec{B}\rangle\right)$$

$$\delta\varphi = \frac{\alpha}{3\beta f H^2}\left(\vec{E}\cdot\vec{B} - \langle \vec{E}\cdot\vec{B}\rangle\right)$$

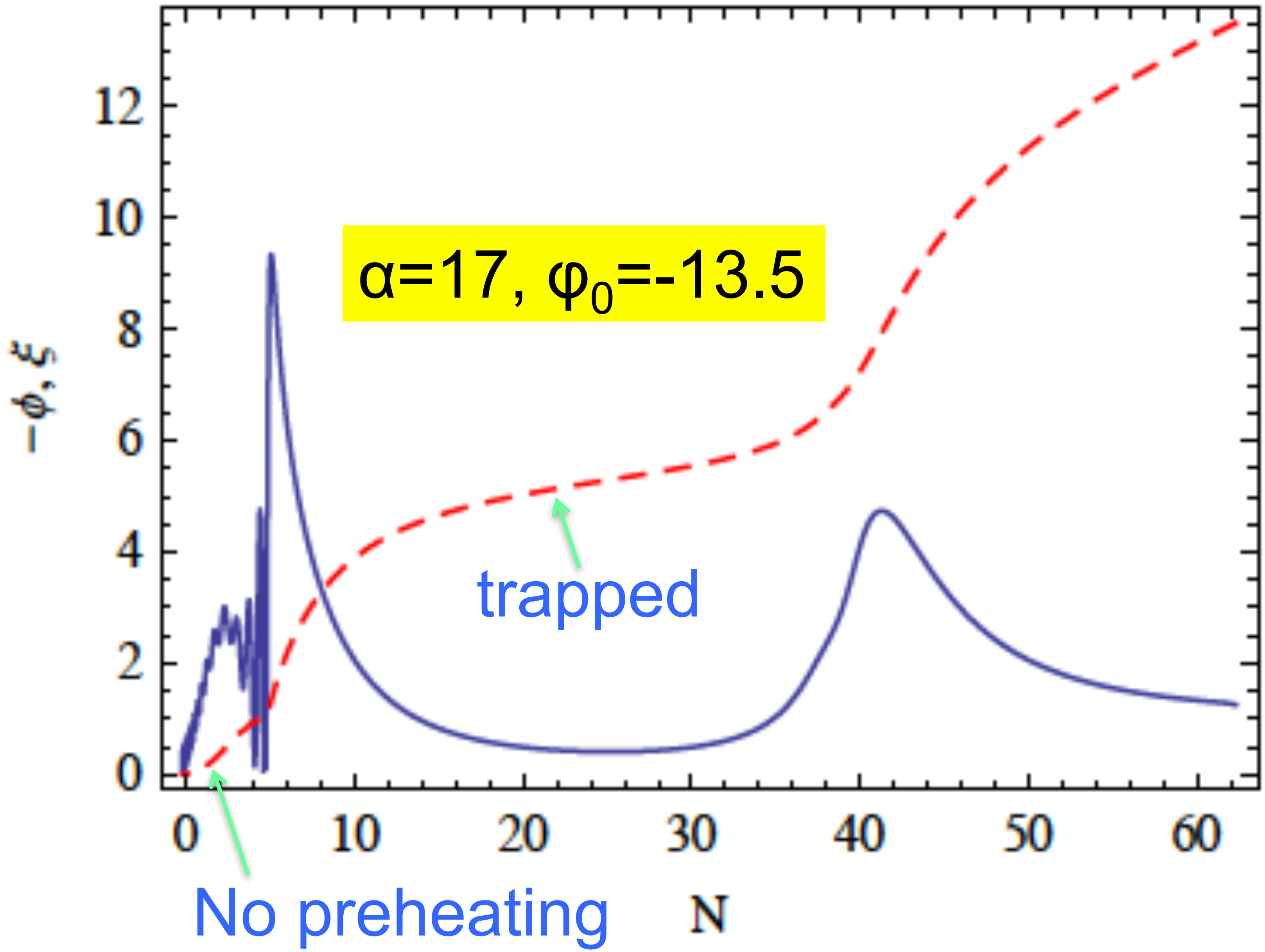
$$\beta \equiv 1 - 2\pi\xi\frac{\alpha}{f}\frac{\langle \vec{E}\cdot\vec{B}\rangle}{3H(d\phi/dt)}$$

$$\Delta_\zeta^2(k) = \langle \zeta(x)^2\rangle = \frac{H^2\langle\delta\varphi^2\rangle}{(d\phi/dt)^2} = \left[\frac{\alpha\langle \vec{E}\cdot\vec{B}\rangle}{3\beta f H(d\phi/dt)}\right]^2$$

e.g. Trapped axion inflation with a steep potential Cheng, Lee, Ng 16

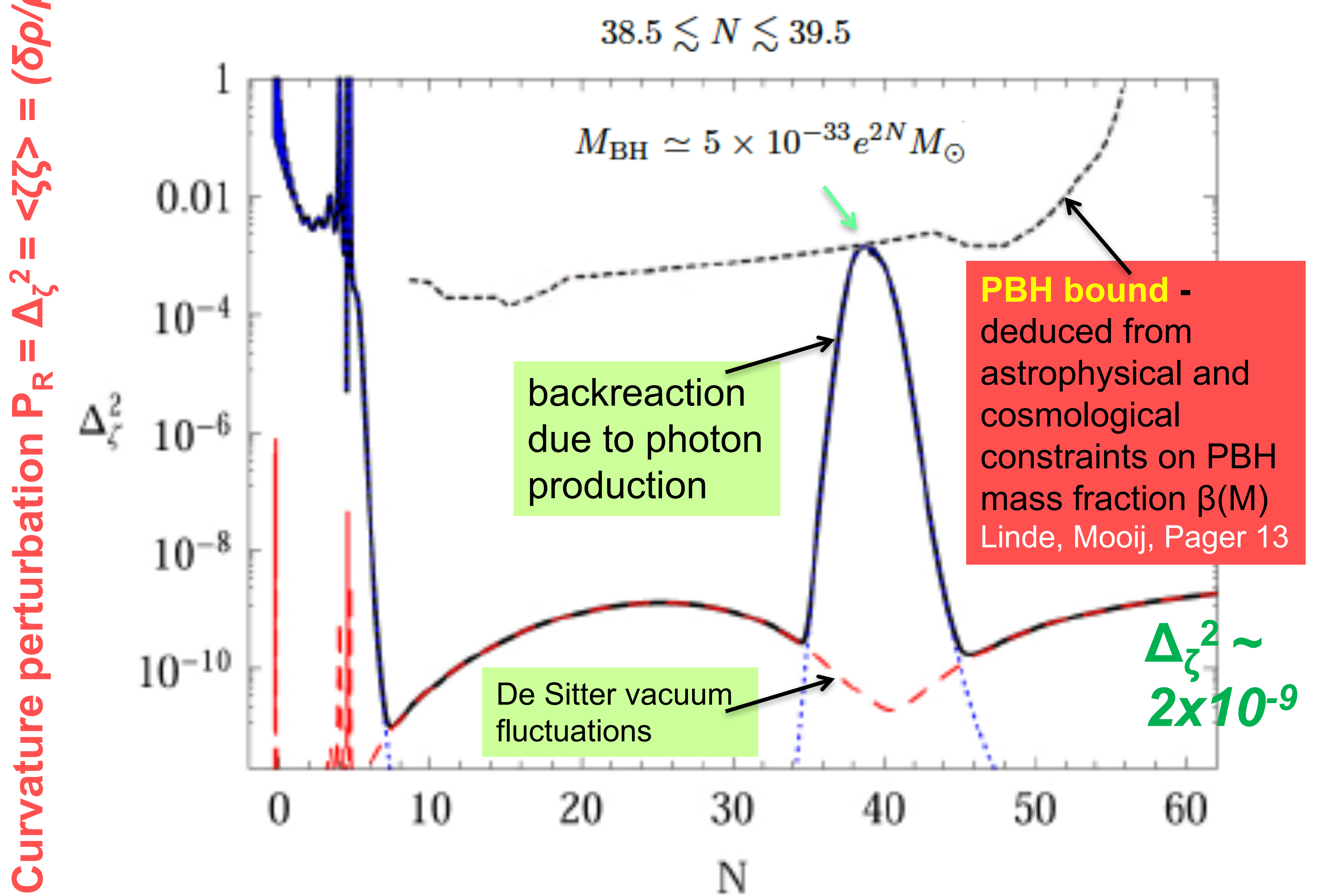


all rescaled by M_p



Note that inflation starts at N~60

Production of 10-100$M_\odot$ PBHs



Anisotropy Stress

energy-momentum tensor $\delta T_{ij} = \bar{p} h_{ij} + a^2 \delta_{ij} \delta p + a^2 \pi_{ij}$

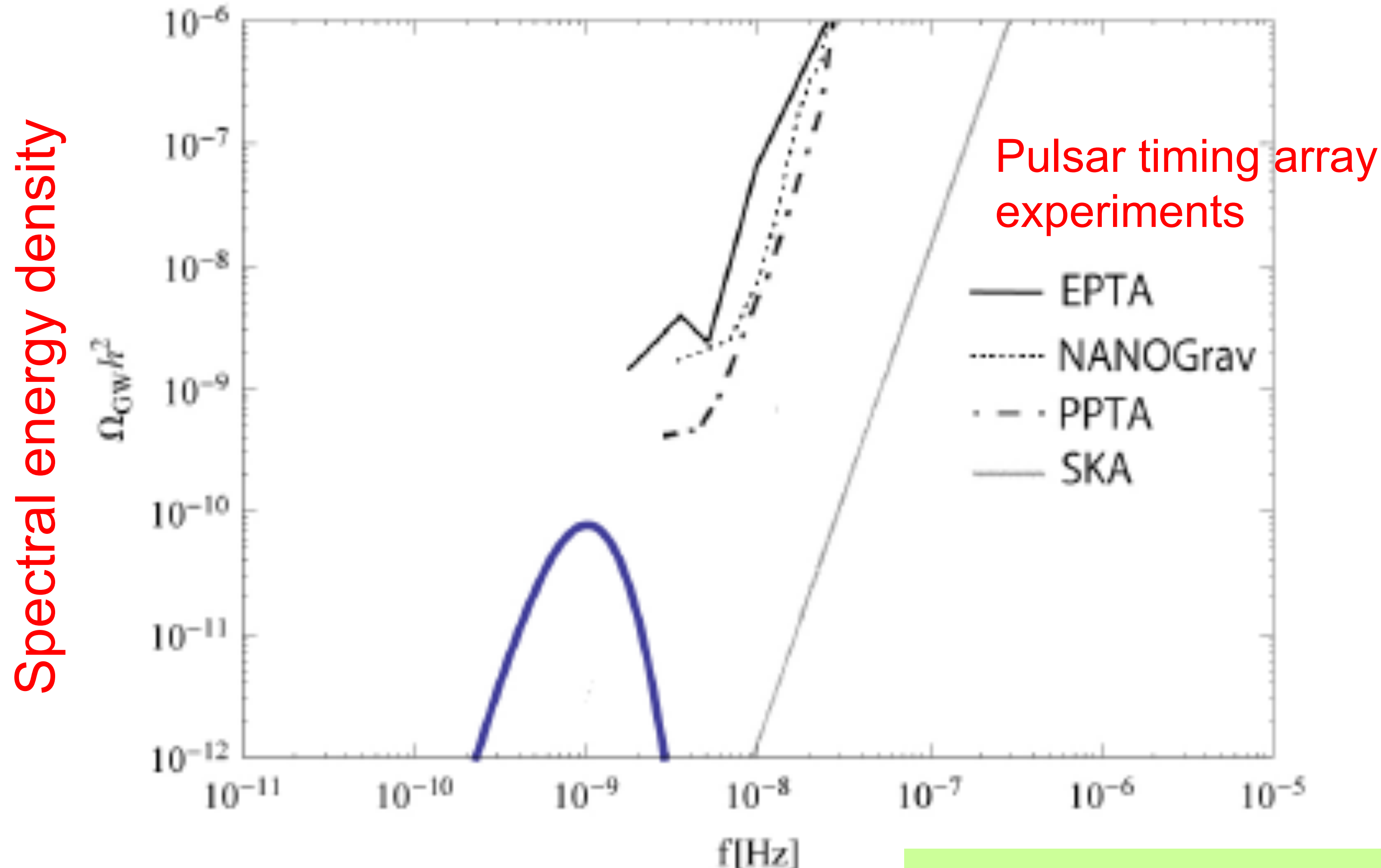
perfect fluid

anisotropic stress tensor

$$\ddot{h}_{ij} + 2\frac{\dot{a}}{a}\dot{h}_{ij} + k^2 h_{ij} = 16\pi G a^2 \pi_{ij}.$$

traceless transverse anisotropic stress tensor $\pi_{ii} = 0, \quad \partial_i \pi_{ij} = 0$

Associated Gravitational Waves in Trapped Inflation



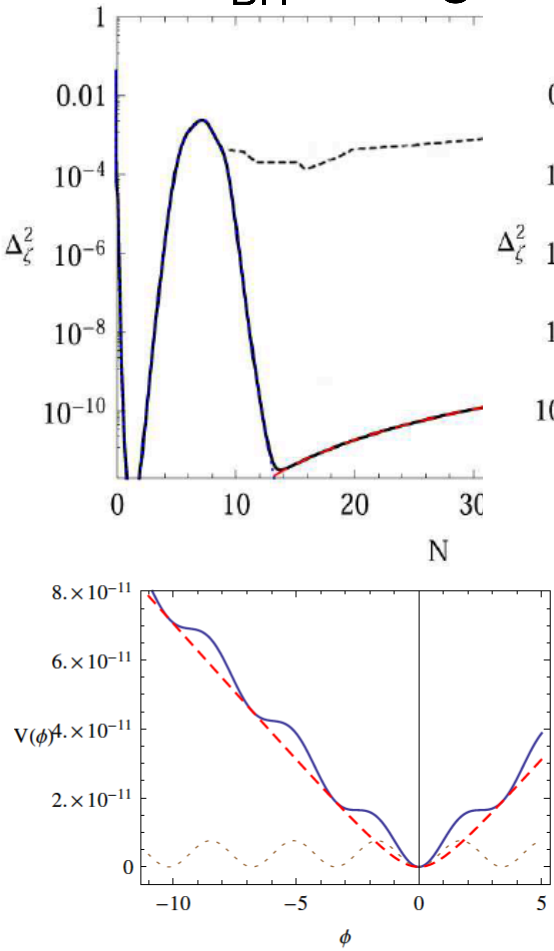
$$\left[\frac{\partial^2}{\partial\eta^2}+\frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta}-\vec{\nabla}^2\right]h_{ij}=\frac{2a^2}{M_p^2}\left(-E_iE_j-B_iB_j\right)^{TT}$$

Transverse traceless part of photon energy-momentum

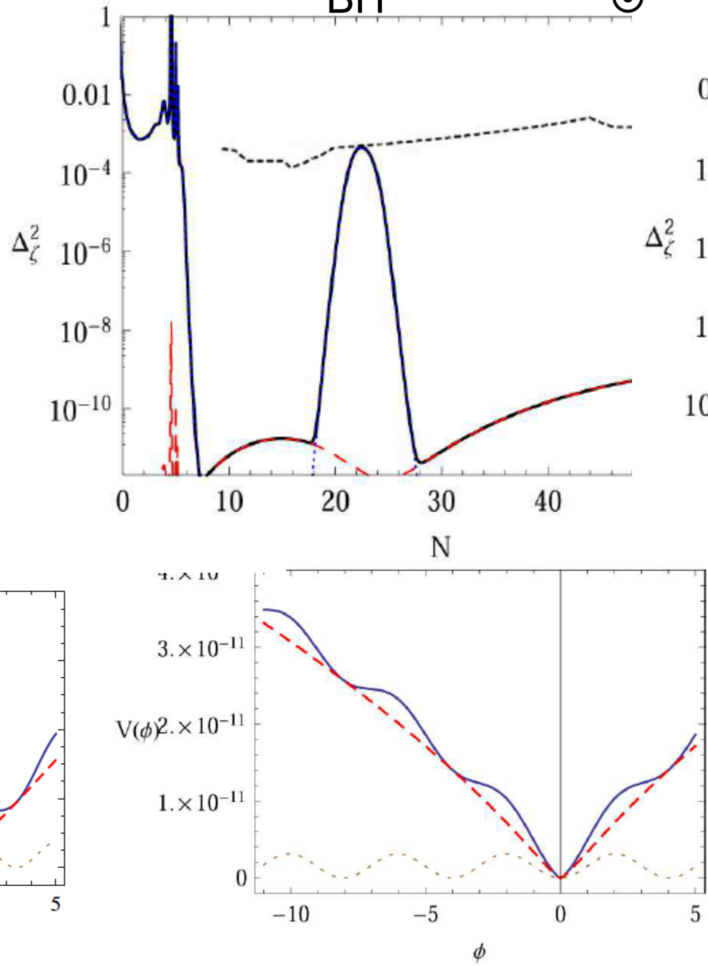
<E·E> ~ <E·B> ~ ζ

Production of PBHs realized in axion monodromy inflation with sinusoidal modulations

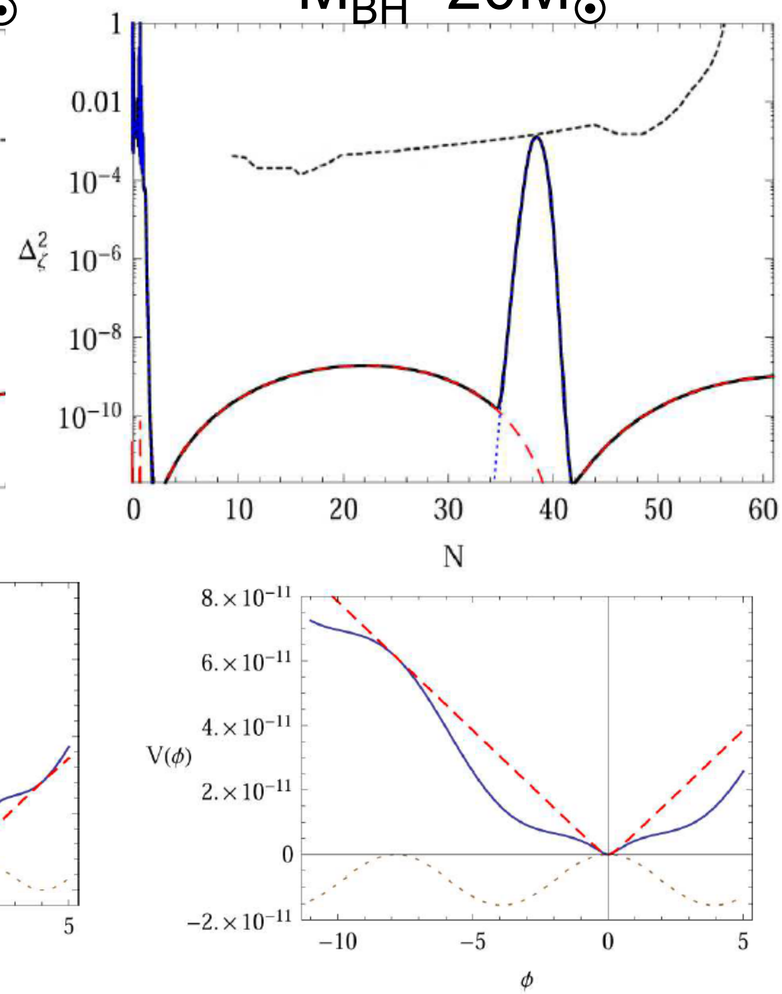
$M_{BH} \sim 10^8$ g



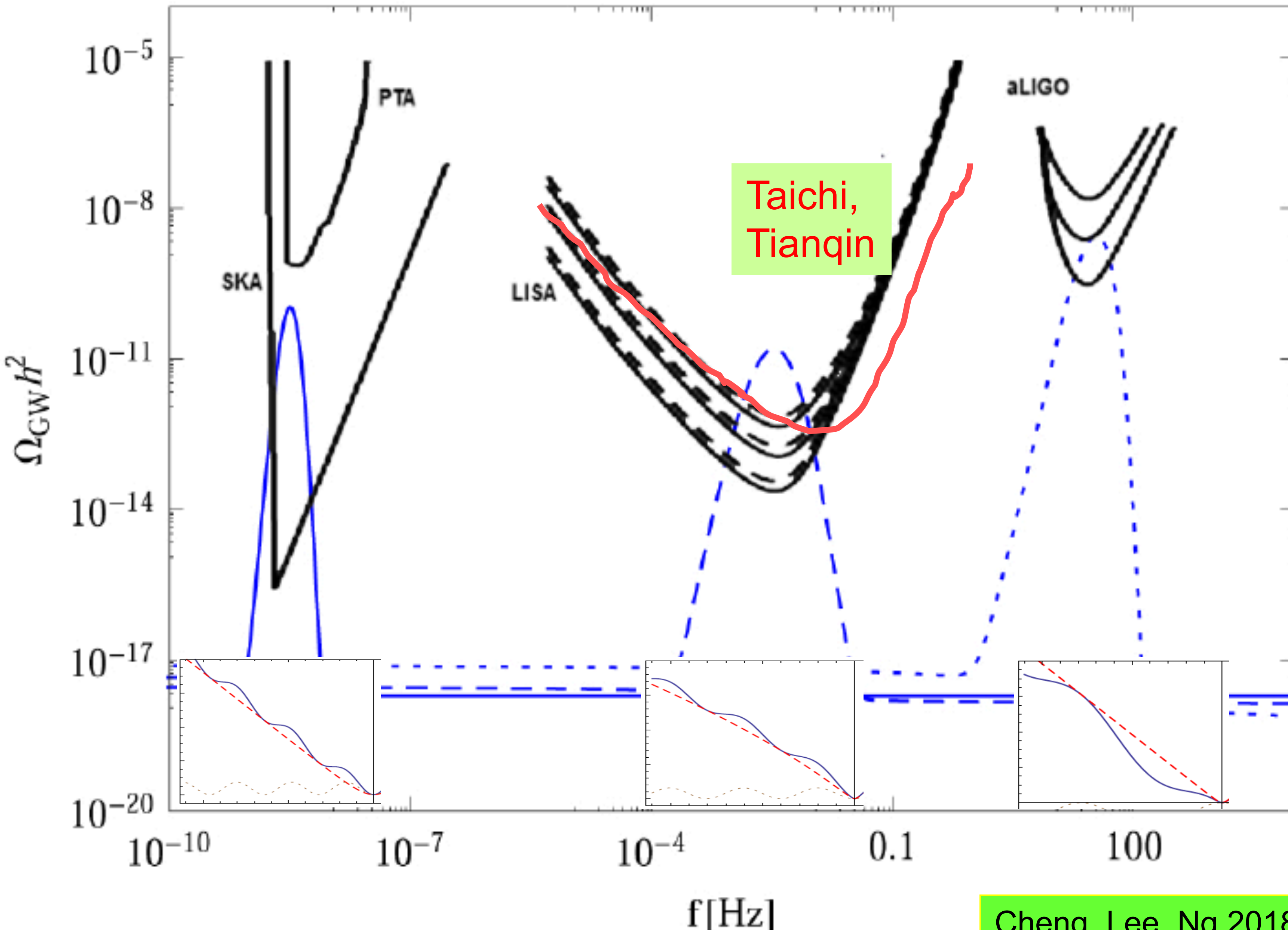
$M_{BH} \sim 10^{-13} M_\odot$



$M_{BH} \sim 20 M_\odot$



GWs associated with PBHs in modulated axion inflation



Cheng, Lee, Ng 2018

PBH seeds or Large Curvature Perturbation Associated Gravitational Waves (continue)

$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta} - \vec{\nabla}^2\right] h_{ij} = 0$$

Free gravitational wave equation

De Sitter vacuum fluctuations during inflation lead to almost scale-invariant primordial gravitational waves

$P_h = 8\pi GH^2$ and $\Delta_\zeta^2 = \langle\zeta\zeta\rangle = (\delta\rho/\rho)^2 \sim 2\text{x}10^{-9}$ on CMB scales

$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{\partial}{\partial\eta} - \vec{\nabla}^2\right] h_{ij} = 16\pi G T_{ij}$$

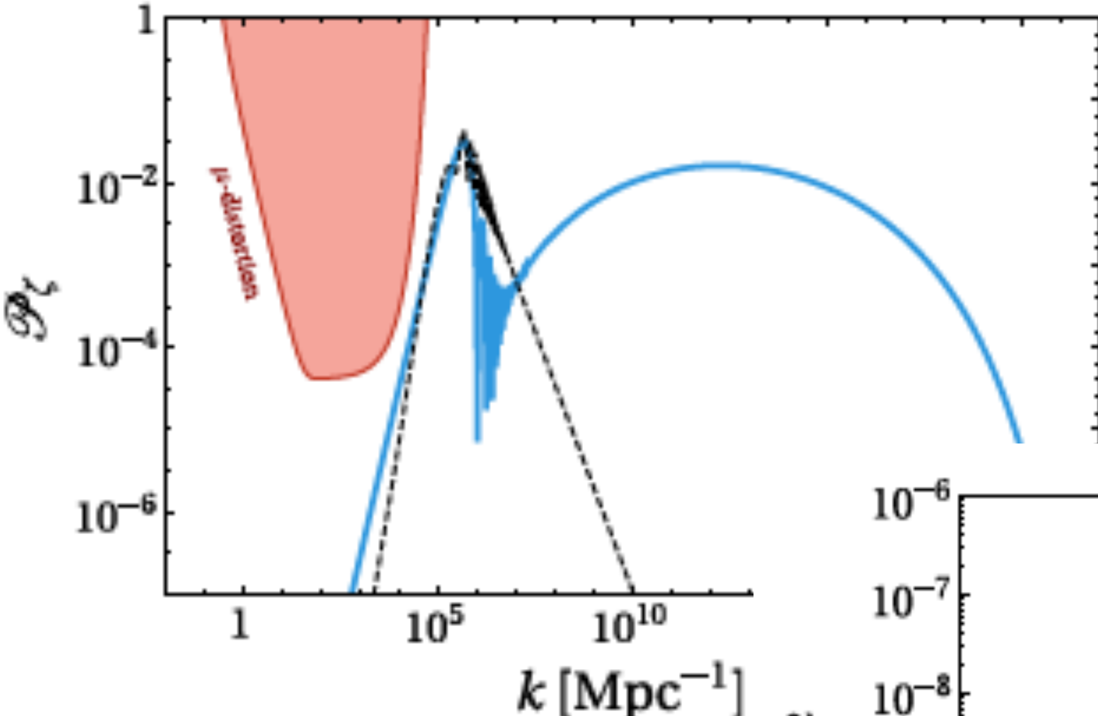
Stress due to transverse traceless part of 2nd order curvature perturbation $T_{ij}(\zeta^2)$

$\Delta_\zeta^2 = \langle\zeta\zeta\rangle = (\delta\rho/\rho)^2 \sim 10^{-3}$

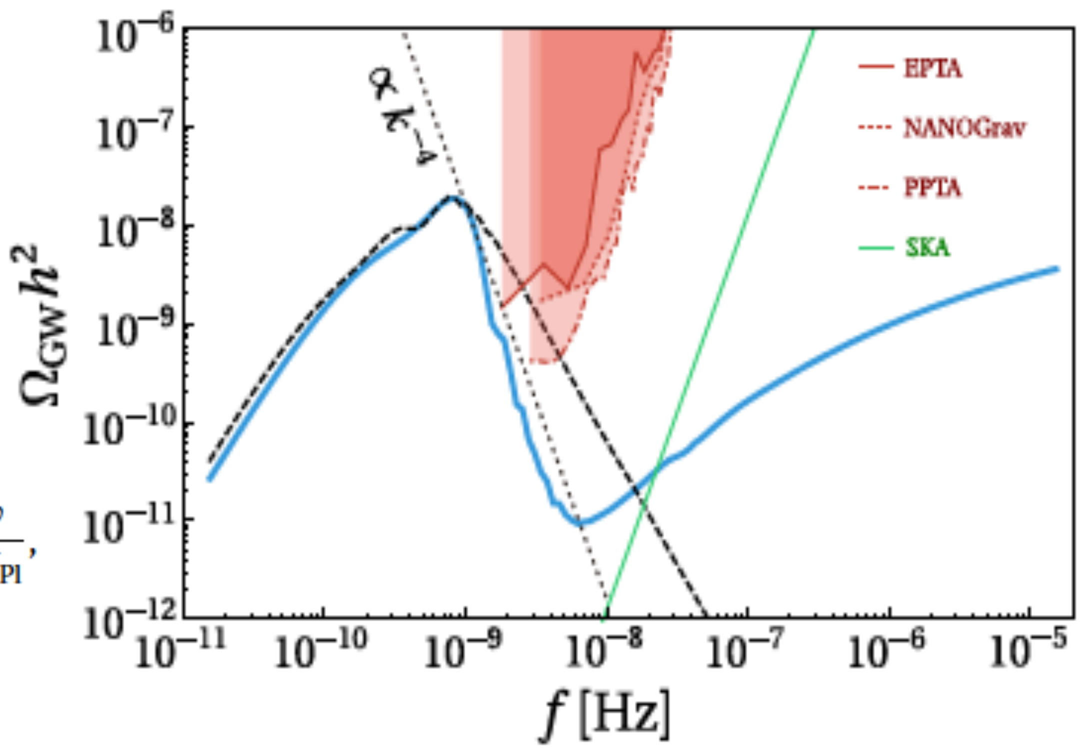
When large curvature perturbation re-enter the horizon during the radiation-dominated era and collapse to form PBHs, they induce gravitational waves at short wavelengths

Ananda, Clarkson,Wands 2007, Baumann, Steinhardt,Takahashi,Ichiki 2007

Broader spectrum of PBHs and GWs in other inflation models



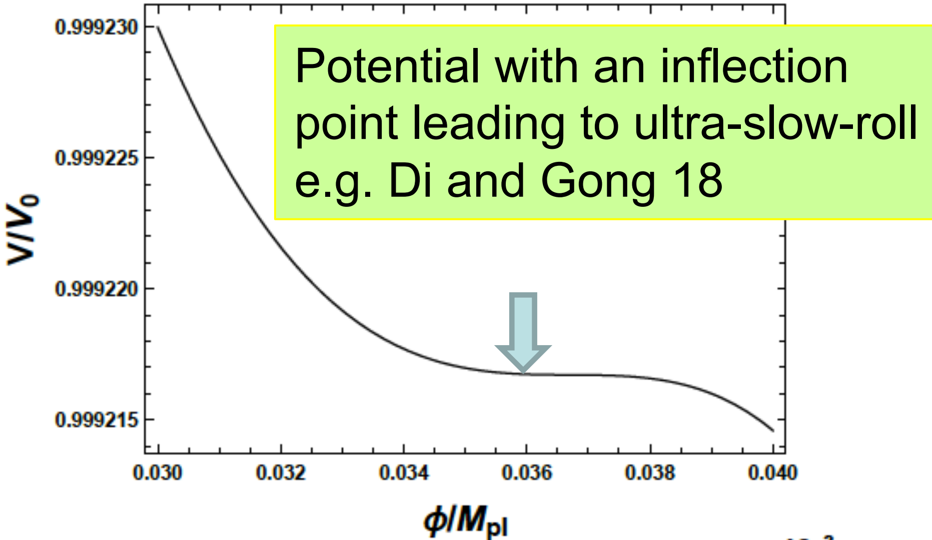
Double inflation
e.g. Inomata et al. 16

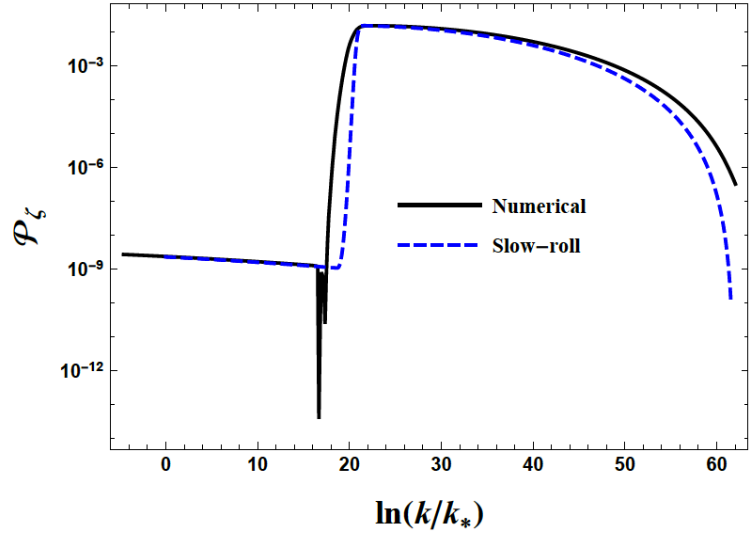


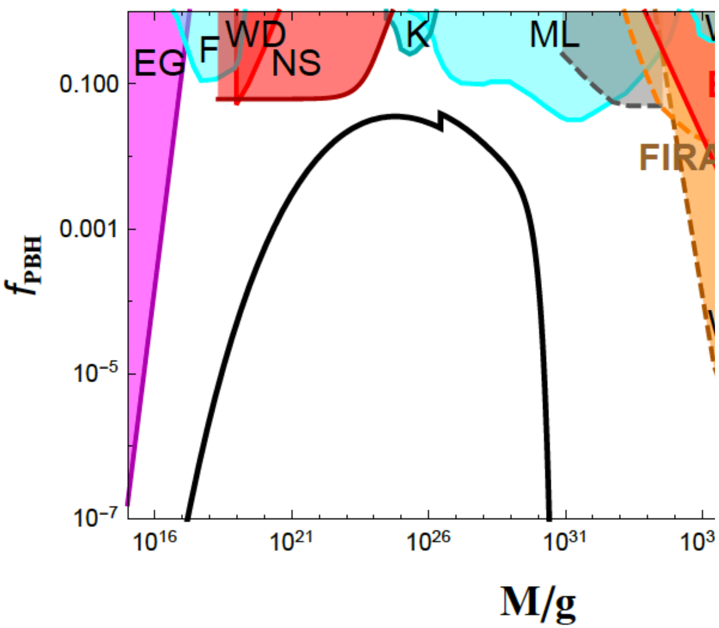
$$V(\phi,\varphi)=V_{\rm pre}(\phi)+V_{\rm stb}(\phi,\varphi)+V_{\rm new}(\varphi),$$

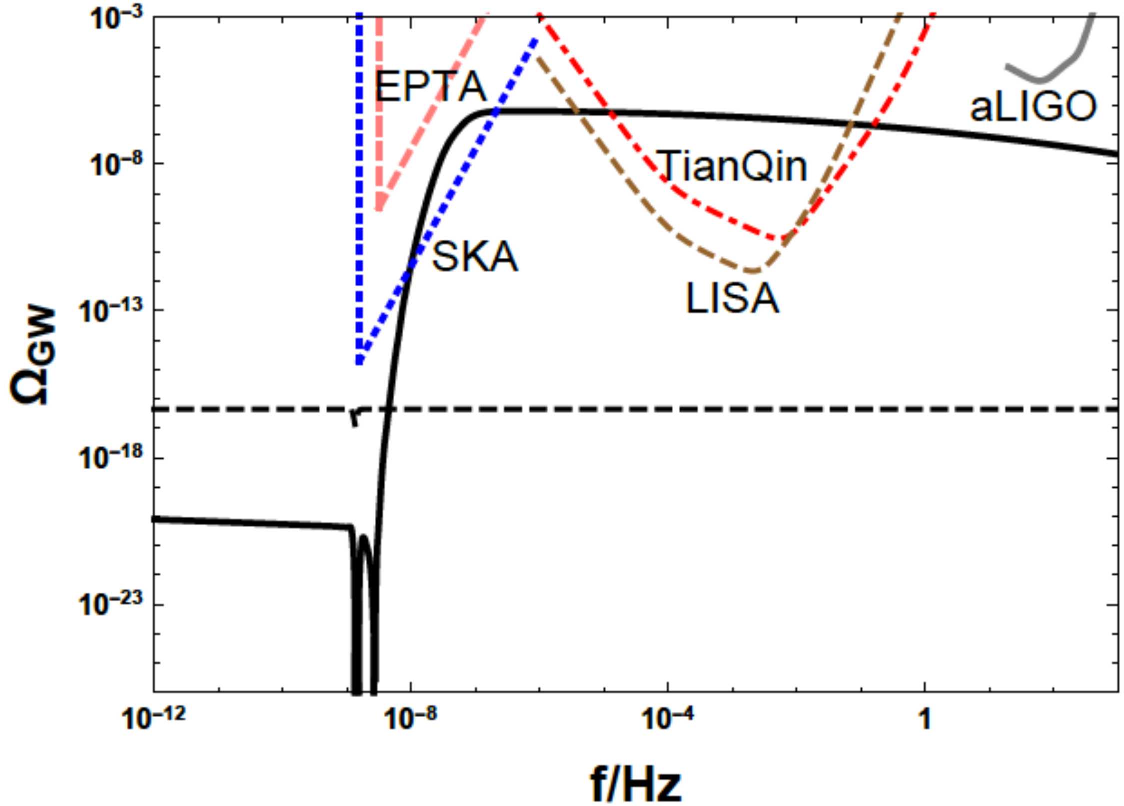
$$V_{\rm new}(\varphi)=\left(v^2-g\frac{\varphi^n}{M_{\rm Pl}^{n-2}}\right)^2-\kappa\frac{\varphi^2}{2M_{\rm Pl}^2}-\varepsilon v^4\frac{\varphi}{M_{\rm Pl}},$$

$$V_{\rm stb}(\phi,\varphi)=c_{\rm pot}\frac{V_{\rm pre}(\phi)}{2M_{\rm Pl}^2}\varphi^2,$$


Potential with an inflection point leading to ultra-slow-roll e.g. Di and Gong 18
V/V0
0.999230
0.999225
0.999220
0.999215
0.030
0.032
0.034
0.036
0.038
0.040
ϕ/Mpl

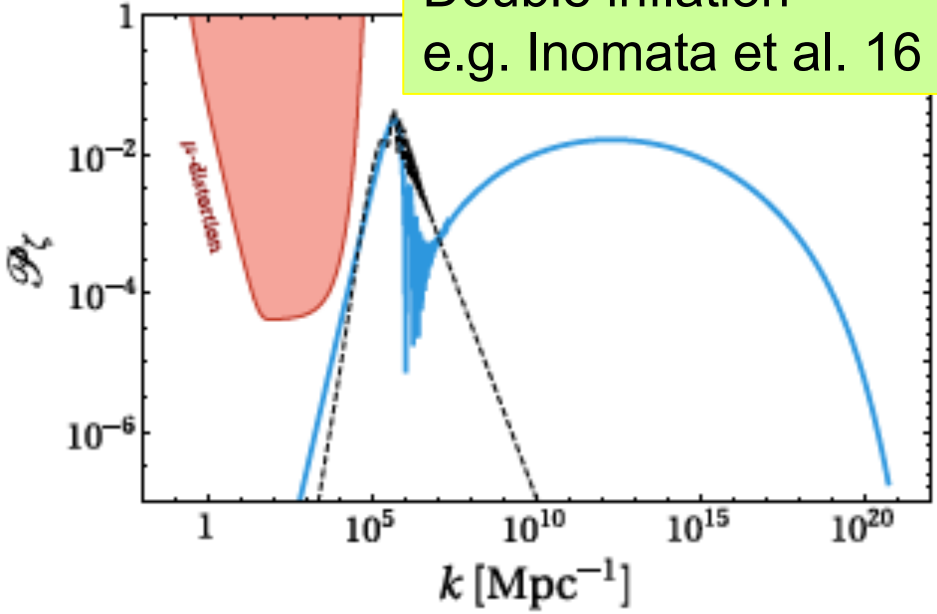

Numerical
Slow−roll
$\mathcal{P}_\zeta$
10^{-3}
10^{-6}
10^{-9}
10^{-12}
0
10
20
30
40
50
60
$\ln(k/k_*)$

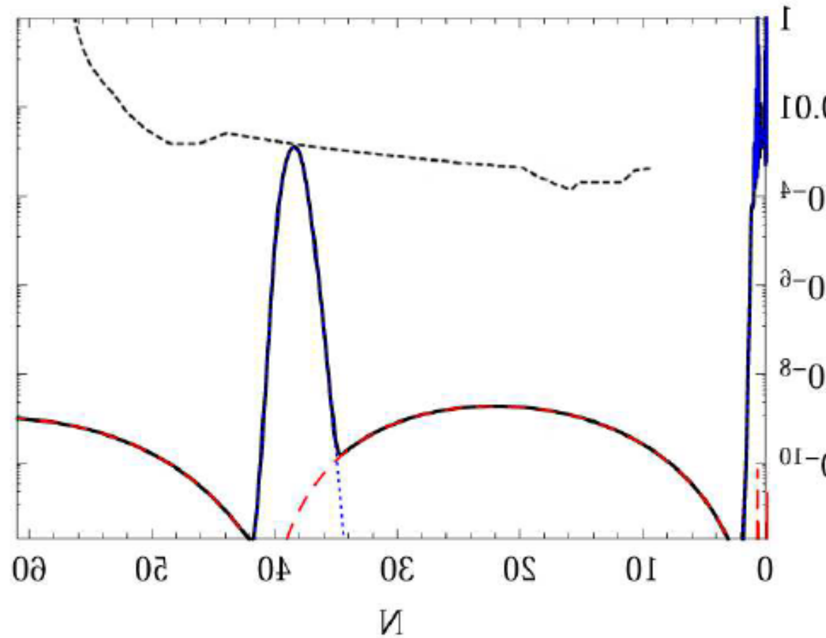

EG
F
WD
NS
K
ML
FIRA
$f_{\rm PBH}$
0.100
0.001
10^{-5}
10^{-7}
10^{16}
10^{21}
10^{26}
10^{31}
M/g


EPTA
SKA
TianQin
LISA
aLIGO
$\Omega_{\rm GW}$
10^{-3}
10^{-8}
10^{-13}
10^{-18}
10^{-23}
10^{-12}
10^{-8}
10^{-4}
1
f/Hz

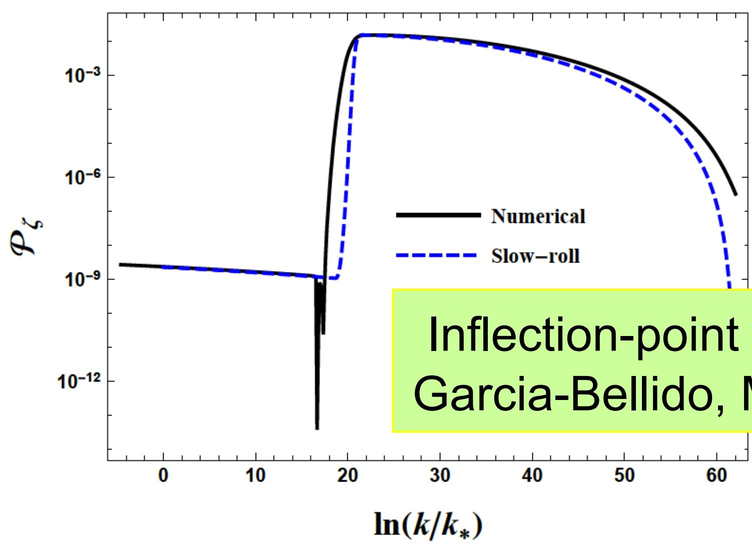
Broad vs narrow spectrum

Double inflation
e.g. Inomata et al. 16



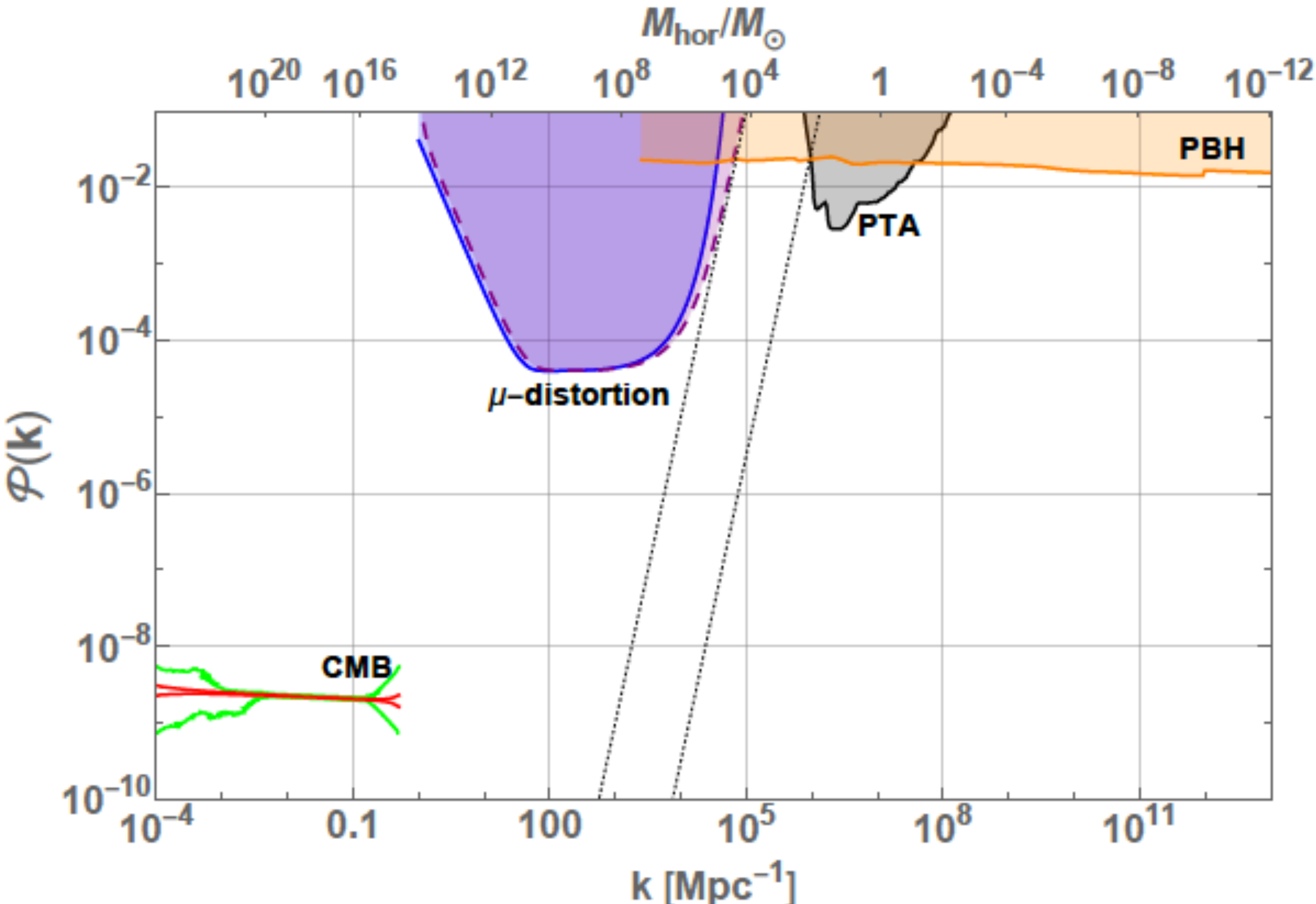


Trapped inflation
e.g. Cheng et al.. 16



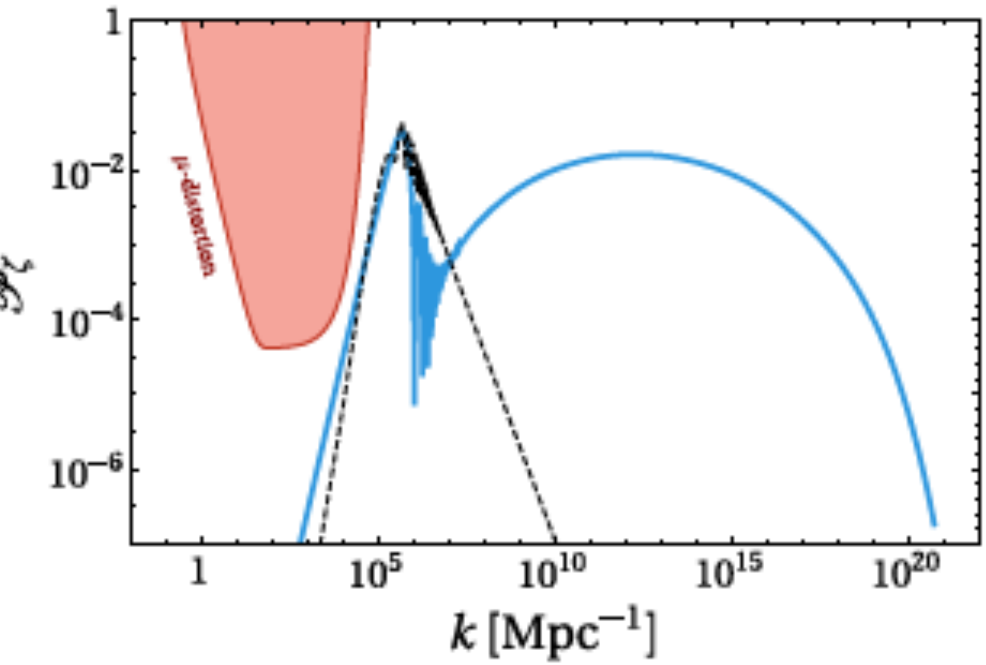
Inflection-point inflation
Garcia-Bellido, Morales 16

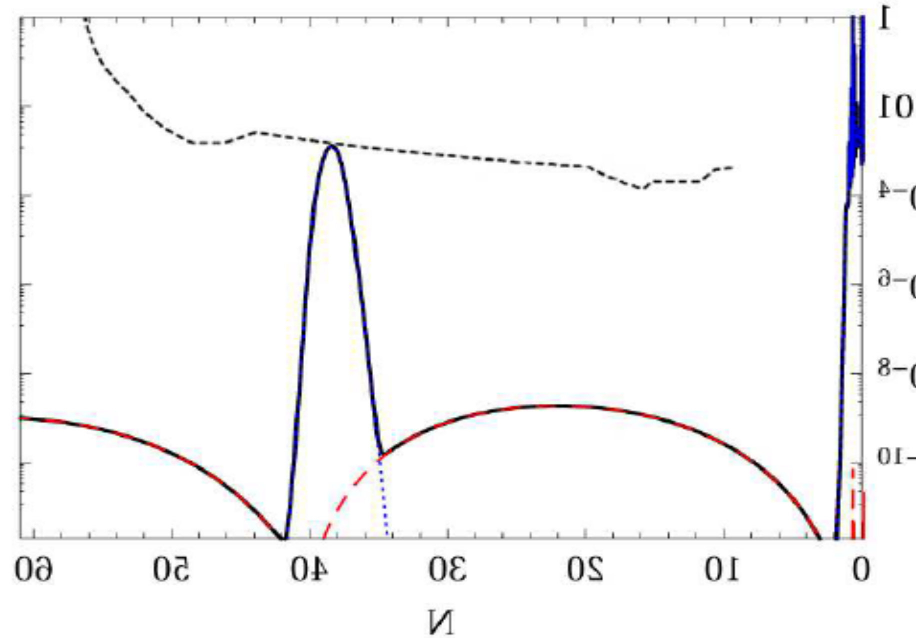
How many and how big PBHs from Inflation



Byrnes+ 18

Biggest PBH with $M_{BH} \sim$ 10-100 $M_\odot$





Double inflation
e.g. Inomata et al. 16

Trapped inflation
e.g. Cheng et al.. 16

Vacuum Anisotropic Stress

Generation of gravity waves by a fluctuating source:

Integrate $$\Box_S h_\mu{}^\nu = -16\pi\, G\, S_\mu{}^\nu$$

using a retarded Green's function

Hsiang,Ford,Ng,Wu 2011,2017

$$\Box_S G_R(x,x') = -\frac{\delta(x-x')}{\sqrt{-\gamma}}$$

$$g_{\mu\nu} = \gamma_{\mu\nu} + h_{\mu\nu}$$

and form the metric correlation function

$$K_\mu{}^\nu{}_\rho{}^\sigma(x,x') = \langle h_\mu{}^\nu(x)\, h_\rho{}^\sigma(x')\rangle - \langle h_\mu{}^\nu(x)\rangle\langle h_\rho{}^\sigma(x')\rangle$$

in terms of a stress tensor correlation function

$$C_\mu{}^\nu{}_\rho{}^\sigma(x,x') = \langle S_\mu{}^\nu(x)\, S_\rho{}^\sigma(x')\rangle - \langle S_\mu{}^\nu(x)\rangle\langle S_\rho{}^\sigma(x')\rangle$$

Conformally invariant fields:

$$C^{RW}_{\mu\nu\alpha\beta}(x,x') = a^{-4}(\eta)\, a^{-4}(\eta')\, C^{flat}_{\mu\nu\alpha\beta}(x,x')$$

Take spatial Fourier transforms:

$$\hat{A}(\eta,\mathbf{k}) \equiv \frac{1}{(2\pi)^3}\int d^3x\, e^{i\mathbf{k}\cdot\mathbf{x}} A(\eta,\mathbf{x})$$

and take $\mathbf{k}$ to be in the z-direction.

Need only x & y components of $\hat{C}_{\mu}{}^{\nu}{}_{\rho}{}^{\sigma}(\eta_1,\eta_2,k)$

Note: "power spectrum" in cosmology usually refers to

$$\mathcal{P}(k) = 4\pi k^3\, P(k)$$

Here $P(k)$ is a spatial component of $\hat{K}_{\mu}{}^{\nu}{}_{\rho}{}^{\sigma}(\eta,\eta,k)$ ***equal time correlation***

First model: assume that the gravity wave fluctuations vanish at some initial time $\eta = \eta_0$ (the beginning of inflation) and then integrate forward in time to the end of inflation at $\eta = \eta_r$

Power spectrum:

$$P_s(k) = 64(2\pi)^8 \int_{\eta_0}^{\eta_r} d\eta_1 \int_{\eta_0}^{\eta_r} d\eta_2 \, \hat{G}(\eta, \eta_1, k) \, \hat{G}(\eta, \eta_2, k) \, \hat{C}_{flat}(\eta_1 - \eta_2, k)$$

Result:

$$P_s(k) = -\frac{H^2 \, S^2}{3\pi^2 \, k}(1 + k^2 \, H^{-2})$$

S = expansion factor during inflation

Conclusion

- Primordial gravitational waves probe inflationary paradigm
- Long-wavelength GWs induced by metric vacuum fluctuations

  CMB B-mode polarization
- Primordial black hole seeds and short-wavelength GWs induced by particle production during inflation
- Short-wavelength GWs induced by 2nd order curvature perturbation during radiation era
- The binary BHs observed by aLIGO/VIRGO gravity-wave detectors may be primordial BHs

  PBHs could be dark matter!

  How to distinguish PBHs from astrophysical BHs?
- Associated GWs in the frequency ranges of

  PTAs, LISA, aLIGO/VIRGO, KAGRA

  polarized (+ mode / - mode) GWs in axion inflation