

Global constraints on the top-quark EFT at lepton colliders

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(DESY)

preliminary results
based on work with Cen Zhang (IHEP),
Martín Perelló, Marcel Vos (Valencia)

Next steps for particle physics: LHC and beyond
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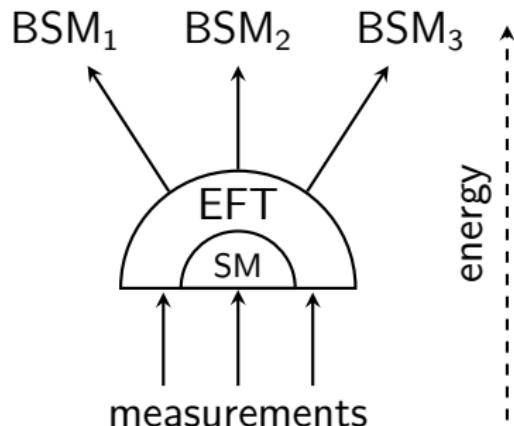


Top EFT for lepton colliders

The standard model effective field theory

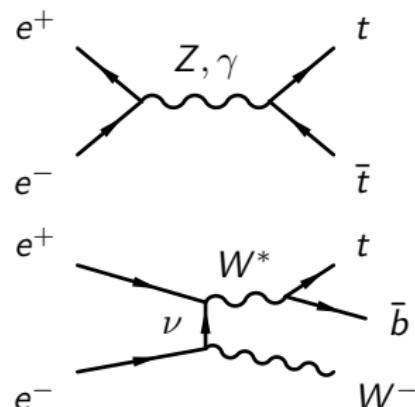
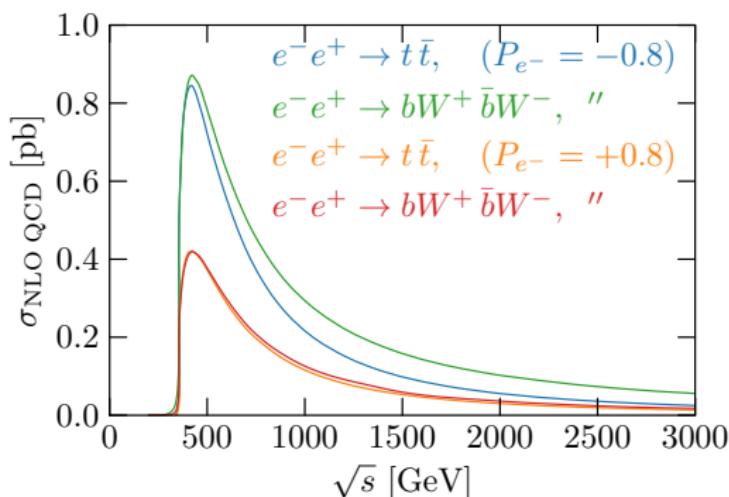
systematically parametrizes the theory space
in direct vicinity of the SM

- ▶ in a low-energy limit,
- ▶ through a proper QFT,
- ▶ when global.



Aiming at a global EFT analysis of $e^+e^- \rightarrow bW^+ \bar{b}W^-$

- including notably four-fermion operators
- exploiting statistically optimal observables
- allowing for NLO QCD corrections



Up-sector SMEFT

[Grzadkowski et al '10]

Two-quark operators:

Scalar: $O_{u\varphi} \equiv \bar{q}u \tilde{\varphi} \quad \varphi^\dagger \varphi,$

Vector: $O_{\varphi q}^1 \equiv \bar{q}\gamma^\mu q \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi$

$$\mathcal{L}_{\text{EFT}} = \sum_i \frac{C_i}{\Lambda^2} O_i$$

$$\equiv O_{\varphi q}^+ + O_{\varphi q}^V - O_{\varphi q}^A,$$

$$\equiv O_{\varphi q}^+ - O_{\varphi q}^V + O_{\varphi q}^A$$

(CC also)

$$O_{\varphi u} \equiv \bar{u}\gamma^\mu u \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi$$

$$\equiv O_{\varphi q}^V + O_{\varphi q}^A$$

(CC only, m_b int.)

Tensor: $O_{uB} \equiv \bar{q}\sigma^{\mu\nu}u \tilde{\varphi} g_Y B_{\mu\nu}, \quad \equiv O_{uA} - \tan \theta_W O_{uZ}$

$$O_{uW} \equiv \bar{q}\sigma^{\mu\nu}\tau' u \tilde{\varphi} g_W W_{\mu\nu}', \quad \equiv O_{uA} + \cotan \theta_W O_{uZ}$$

$$O_{dW} \equiv \bar{q}\sigma^{\mu\nu}\tau' d \tilde{\varphi} g_W W_{\mu\nu}',$$

$$O_{uG} \equiv \bar{q}\sigma^{\mu\nu} T^A u \tilde{\varphi} g_s G_{\mu\nu}^A.$$

(CC also)

(CC only, m_b int.)

(NLO only)

Two-quark–two-lepton operators:

Scalar: $O_{lequ}^S \equiv \bar{l}e \varepsilon \bar{q}u,$

$$O_{ledq} \equiv \bar{l}e \bar{d}q,$$

(CC also, m_e int.)

(CC only, m_e int.)

Vector: $O_{lq}^1 \equiv \bar{l}\gamma_\mu l \quad \bar{q}\gamma^\mu q$

$$\equiv O_{lq}^+ + O_{lq}^V - O_{lq}^A,$$

$$O_{lq}^3 \equiv \bar{l}\gamma_\mu\tau' l \quad \bar{q}\gamma^\mu\tau' q$$

$$\equiv O_{lq}^+ - O_{lq}^V + O_{lq}^A,$$

(CC also)

$$O_{lu} \equiv \bar{l}\gamma_\mu l \quad \bar{u}\gamma^\mu u$$

$$\equiv O_{lq}^V + O_{lq}^A,$$

$$O_{eq} \equiv \bar{e}\gamma^\mu e \quad \bar{q}\gamma_\mu q$$

$$\equiv O_{eq}^V - O_{eq}^A,$$

$$O_{eu} \equiv \bar{e}\gamma_\mu e \quad \bar{u}\gamma^\mu u$$

$$\equiv O_{eq}^V + O_{eq}^A,$$

Tensor: $O_{lequ}^T \equiv \bar{l}\sigma_{\mu\nu}e \quad \varepsilon \quad \bar{q}\sigma^{\mu\nu}u.$

(CC also, m_e int.)

Anomalous vertices

$$t\bar{t}\gamma : \quad \gamma_\mu \underbrace{(F_{1V}^\gamma + \gamma_5 F_{1A}^\gamma)}_{\sim \emptyset} + \frac{\sigma_{\mu\nu} iq^\nu}{2m_t} \underbrace{(F_{2V}^\gamma + i\gamma_5 F_{2A}^\gamma)}_{\sim \text{Re,Im}\{C_{uA}\}}$$
$$t\bar{t}Z : \quad \gamma_\mu \underbrace{(F_{1V}^Z + \gamma_5 F_{1A}^Z)}_{\sim C_{\varphi q}^V, C_{\varphi q}^A} + \frac{\sigma_{\mu\nu} iq^\nu}{2m_t} \underbrace{(F_{2V}^Z + i\gamma_5 F_{2A}^Z)}_{\sim \text{Re,Im}\{C_{uZ}\}}$$
$$t\bar{b}W : \quad \gamma_\mu \underbrace{(F_{1V}^W + \gamma_5 F_{1A}^W)}_{\sim C_{\varphi q}^+ - \frac{1}{2}(C_{\varphi q}^V - C_{\varphi q}^A) \pm C_{\varphi ud}} + \frac{\sigma_{\mu\nu} iq^\nu}{2m_t} \underbrace{(F_{2V}^W + i\gamma_5 F_{2A}^W)}_{\sim s_W^2 C_{uA} + s_W c_W C_{uZ} \pm C_{dW}^*}$$

Insufficiencies:

- miss four-fermion operators,
- conflict with gauge invariance,
do not allow for radiative corrections to be computed,
- complex couplings where the tree-level EFT prescribes real ones,
- hide correlations induced by gauge invariance,
preclude the combination of measurements in various sectors

Two ideas for global EFT analyses

Statistically optimal observables

[Atwood,Soni '92]

[Davier et al '93]

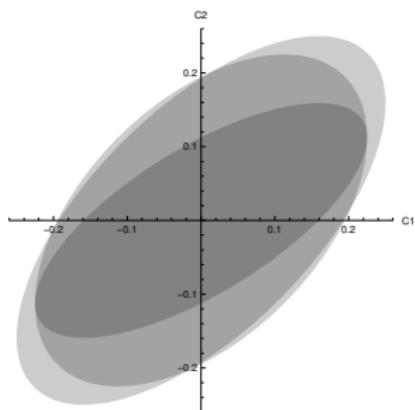
[Diehl,Nachtmann '94]

minimize the one-sigma ellipsoid in EFT parameter space

(joint efficient set of estimators,

saturating the Rao-Cramér-Fréchet bound: $V^{-1} = I$,
just like MEM)

For small C_i , with a phase-space distribution $\sigma(\Phi) = \sigma_0(\Phi) + \sum_i C_i \sigma_i(\Phi)$,
the stat. opt. obs. are the average values of $O_i(\Phi) = \sigma_i(\Phi)/\sigma_0(\Phi)$.



e.g. $\sigma(\phi) = 1 + \cos(\phi) + C_1 \sin(\phi) + C_2 \sin(2\phi)$

1. asymmetries: $O_i \sim \text{sign}\{\sin(i\phi)\}$

2. moments: $O_i \sim \sin(i\phi)$

3. statistically optimal: $O_i \sim \frac{\sin(i\phi)}{1 + \cos\phi}$

➡ area ratios $1.9 : 1.7 : 1$

Previous applications in $e^+ e^- \rightarrow t \bar{t}$, on different distributions:

[Grzadkowski, Hioki '00]

[Janot '15]

[Khiem et al '15]

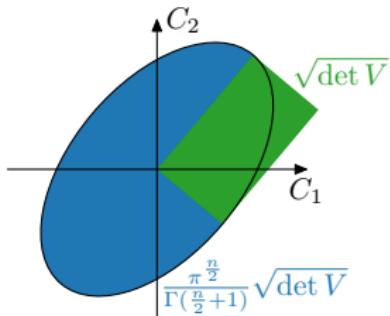
Global determinant parameter

[GD, Grojean, Gu, Wang, '17]

In a n -dimensional Gaussian fit,
with covariance matrix V ,

$$\text{GDP} \equiv \sqrt[2n]{\det V}$$

provides a geometric average
of the constraints strengths.



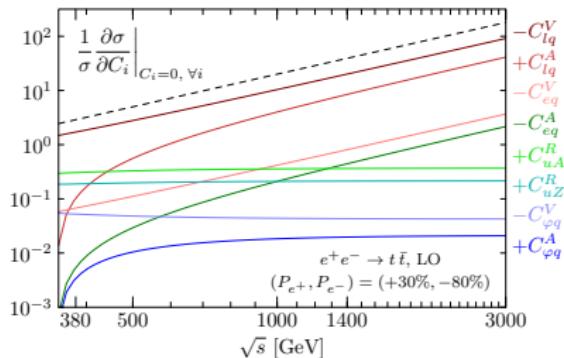
Interestingly, GDP ratios are operator-basis independent!

- as the volume scales linearly with coefficient normalization
 - as the volume is invariant under rotations
- ➡ conveniently assess constraint strengthening.

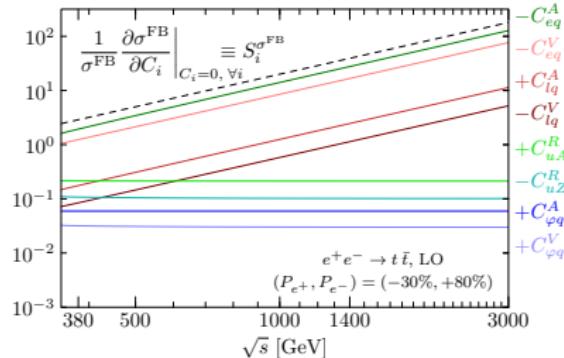
Operator sensitivities

σ and A^{FB} sensitivities

Total cross section (left pol.):



FB-integrated cross section (right pol.):



Few features:

- quadratic energy growth for four-fermion operators
- no growth for two-fermion operators (dipoles included)
- p -wave $\beta = \sqrt{1 - 4m_t^2/s}$ suppression of axial vectors at threshold
- enhanced sensitivity of axial vector operators in σ^{FB}
- sensitivity sign flip for $C_{\varphi q}^V$ and C_{uZ}^R when polarization is reversed
- etc.

Helicity amplitude decomposition in $bW^+\bar{b}W^-$

[Jacob,Wick '59]

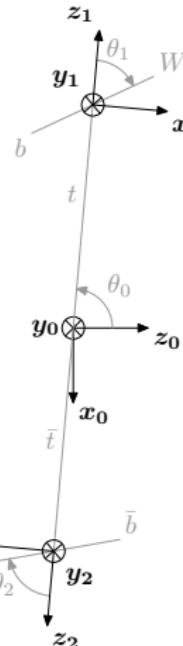
Production amplitudes:

- $++ : A_1 \sim \frac{2m_t}{\sqrt{s}} V + \sqrt{s} (D - \beta \tilde{D})$
- $-- : A_2 \sim \frac{2m_t}{\sqrt{s}} V + \sqrt{s} (D + \beta \tilde{D})$
- $+- : A_3 \sim (V + \beta A) + 2m_t D$
- $-+ : A_4 \sim (V - \beta A) + 2m_t D$

[Schmidt '95]

In terms of $\Omega = \{\theta_0, \theta_1, \phi_1, \theta_2, \phi_2\}$ helicity angles:

$+3/4$	$(A_3 ^2 + A_4 ^2)$	$ a_2 ^2 + a_4 ^2$	$ b_1 ^2 + b_3 ^2$	$(1 + \cos^2 \theta_0)$		
$+3/4$	$(A_3 ^2 - A_4 ^2)$	$ a_2 ^2 + a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$(1 + \cos^2 \theta_0)$		
$+3/4$	$(A_3 ^2 - A_4 ^2)$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2 + b_3 ^2$	$(1 + \cos^2 \theta_0)$		
$+3/4$	$(A_3 ^2 + A_4 ^2)$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$(1 + \cos^2 \theta_0)$		
$-3/2$	$(A_3 ^2 - A_4 ^2)$	$ a_2 ^2 + a_4 ^2$	$ b_1 ^2 + b_3 ^2$	$\cos \theta_0$		
$-3/2$	$(A_3 ^2 + A_4 ^2)$	$ a_2 ^2 + a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$\cos \theta_0$		
$-3/2$	$(A_3 ^2 + A_4 ^2)$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2 + b_3 ^2$	$\cos \theta_0$	$\cos \theta_1$	
$-3/2$	$(A_3 ^2 - A_4 ^2)$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$\cos \theta_0$	$\cos \theta_1$	
$+3/2$	$(A_1 ^2 + A_2 ^2)$	$ a_2 ^2 + a_4 ^2$	$ b_1 ^2 + b_3 ^2$	$\sin^2 \theta_0$		
$-3/2$	$(A_1 ^2 - A_2 ^2)$	$ a_2 ^2 + a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin^2 \theta_0$		
$+3/2$	$(A_1 ^2 - A_2 ^2)$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2 + b_3 ^2$	$\sin^2 \theta_0$	$\cos \theta_1$	
$-3/2$	$(A_1 ^2 + A_2 ^2)$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin^2 \theta_0$	$\cos \theta_1$	
$+3/2$	$\sqrt{2} \operatorname{Re}\{A_1^* A_4\}$	$ a_2 ^2 - a_4 ^2$	$ b_3 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$\sin \theta_1$	$(1 + \cos \theta_2)$
$+3/2$	$\sqrt{2} \operatorname{Re}\{A_1^* A_4\}$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$\sin \theta_1$	$(1 - \cos \theta_2)$
$+3/2$	$\sqrt{2} \operatorname{Re}\{A_2^* A_3\}$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2$	$\sin \theta_0 (1 - \cos \theta_0)$	$\sin \theta_1$	$(1 + \cos \theta_2)$
$+3/2$	$\sqrt{2} \operatorname{Re}\{A_2^* A_3\}$	$ a_2 ^2 - a_4 ^2$	$ b_3 ^2$	$\sin \theta_0 (1 - \cos \theta_0)$	$\sin \theta_1$	$(1 - \cos \theta_2)$
$-3/2$	$\sqrt{2} \operatorname{Re}\{A_2^* A_4\}$	$ a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$(1 + \cos \theta_1)$	$\sin \theta_2$
$-3/2$	$\sqrt{2} \operatorname{Re}\{A_2^* A_4\}$	$ a_2 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$(1 - \cos \theta_1)$	$\sin \theta_2$
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$-3/2$	$\sqrt{2} \operatorname{Re}\{A_1^* A_3\}$	$ a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin \theta_0 (1 - \cos \theta_0)$	$(1 - \cos \theta_1)$	$\sin \theta_2$
-3	$\operatorname{Re}\{A_1^* A_2\}$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin^2 \theta_0$	$\sin \theta_1$	$\sin \theta_2$
$-3/2$	$\operatorname{Re}\{A_3^* A_4\}$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin^2 \theta_0$	$\sin \theta_1$	$\cos(\phi_1 + \phi_2)$
$+3/2$	$\sqrt{2} \operatorname{Im}\{A_1^* A_4\}$	$ a_2 ^2 - a_4 ^2$	$ b_3 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$\sin \theta_1$	$(1 + \cos \theta_2)$
$+3/2$	$\sqrt{2} \operatorname{Im}\{A_1^* A_4\}$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$\sin \theta_1$	$(1 - \cos \theta_2)$
$-3/2$	$\sqrt{2} \operatorname{Im}\{A_2^* A_3\}$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2$	$\sin \theta_0 (1 - \cos \theta_0)$	$\sin \theta_1$	$(1 + \cos \theta_2)$
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$+3/2$	$\sqrt{2} \operatorname{Im}\{A_2^* A_4\}$	$ a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$\sin \theta_2$	$\sin \phi_2$
$+3/2$	$\sqrt{2} \operatorname{Im}\{A_2^* A_4\}$	$ a_2 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$(1 - \cos \theta_1)$	$\sin \phi_2$
$-3/2$	$\sqrt{2} \operatorname{Im}\{A_1^* A_3\}$	$ a_2 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin \theta_0 (1 - \cos \theta_0)$	$(1 + \cos \theta_1)$	$\sin \phi_2$
$+3/2$	$\sqrt{2} \operatorname{Im}\{A_1^* A_3\}$	$ a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin \theta_0 (1 - \cos \theta_0)$	$(1 - \cos \theta_1)$	$\sin \phi_2$



Helicity amplitude decomposition in $bW^+\bar{b}W^-$

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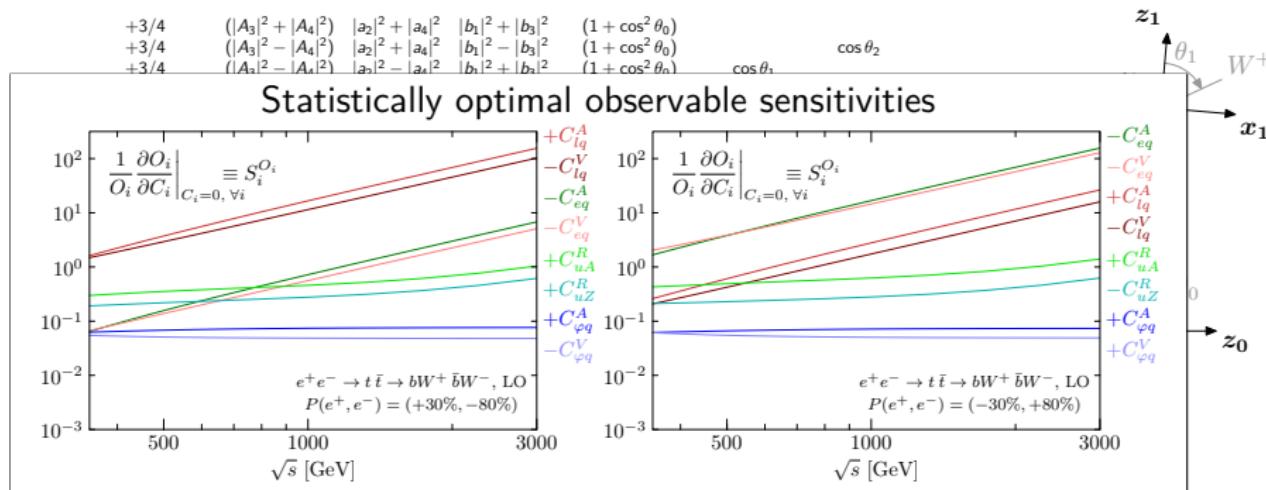
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$$\begin{array}{llllll} +3/4 & (|A_3|^2 + |A_4|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 + |b_3|^2 & (1 + \cos^2 \theta_0) \\ +3/4 & (|A_3|^2 - |A_4|^2) & |a_2|^2 + |a_4|^2 & |b_1|^2 - |b_3|^2 & (1 + \cos^2 \theta_0) \\ +3/4 & (|A_2|^2 - |A_4|^2) & |a_2|^2 - |a_4|^2 & |b_1|^2 + |b_3|^2 & (1 + \cos^2 \theta_0) \end{array} \quad \begin{array}{c} \cos \theta_1 \\ \cos \theta_2 \end{array}$$



$+3/2$	$\sqrt{2}$	$\text{Im}\{A_1^* A_4\}$	$ a_2 ^2 - a_4 ^2$	$ b_3 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$\sin \theta_1$	$(1 + \cos \theta_2)$	$\sin \phi_1$	x_2
$+3/2$	$\sqrt{2}$	$\text{Im}\{A_1^* A_4\}$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$\sin \theta_1$	$(1 - \cos \theta_2)$	$\sin \phi_1$	\bar{b}
$-3/2$	$\sqrt{2}$	$\text{Im}\{A_2^* A_3\}$	$ a_2 ^2 - a_4 ^2$	$ b_1 ^2$	$\sin \theta_0 (1 - \cos \theta_0)$	$\sin \theta_1$	$(1 + \cos \theta_2)$	$\sin \phi_1$	W^-
$-3/2$	$\sqrt{2}$	$\text{Im}\{A_2^* A_3\}$	$ a_2 ^2 - a_4 ^2$	$ b_3 ^2$	$\sin \theta_0 (1 - \cos \theta_0)$	$\sin \theta_1$	$(1 - \cos \theta_2)$	$\sin \phi_1$	y_2
$+3/2$	$\sqrt{2}$	$\text{Im}\{A_2^* A_4\}$	$ a_4 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$(1 + \cos \theta_2)$	$\sin \theta_2$	$\sin \phi_2$	x_2
$+3/2$	$\sqrt{2}$	$\text{Im}\{A_2^* A_4\}$	$ a_2 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin \theta_0 (1 + \cos \theta_0)$	$(1 - \cos \theta_1)$	$\sin \theta_2$	$\sin \phi_2$	\bar{b}
$-3/2$	$\sqrt{2}$	$\text{Im}\{A_1^* A_3\}$	$ a_2 ^2$	$ b_1 ^2 - b_3 ^2$	$\sin \theta_0 (1 - \cos \theta_0)$	$(1 + \cos \theta_1)$	$\sin \theta_2$	$\sin \phi_2$	W^-
$-3/2$	$\sqrt{2}$	$\text{Im}\{A_1^* A_3\}$	$ a_2 ^2$	$ b_1 ^2 + b_3 ^2$	$\sin \theta_0 (1 - \cos \theta_0)$	$(1 + \cos \theta_1)$	$\sin \theta_2$	$\sin \phi_2$	y_2

Benchmark analysis

resonant $e^+e^- \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$

$m_b/m_t \rightarrow 0$

analytically at LO

with perfect detector, $\text{Br} \times \epsilon = 20\%$

and statistical uncertainties only

500 fb^{-1} at $\sqrt{s} = 500 \text{ GeV}$

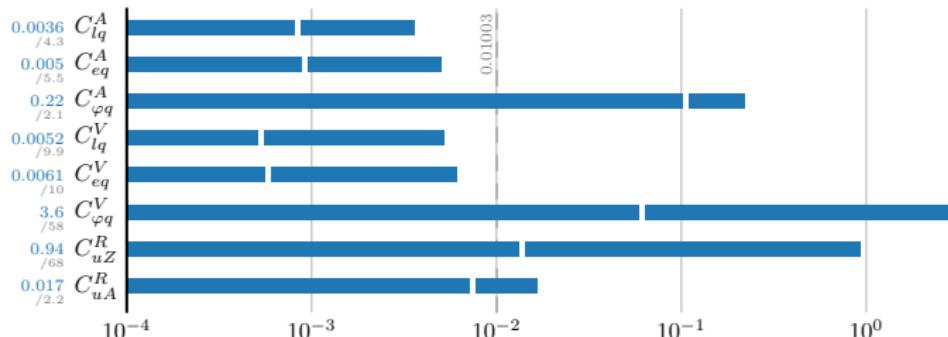
1 ab^{-1} at $\sqrt{s} = 1 \text{ TeV}$

50% with $P(e^+, e^-) = (+30\%, -80\%)$

50% with $P(e^+, e^-) = (-30\%, +80\%)$

Global constraints

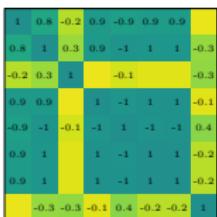
$\sigma + A^{\text{FB}}$:



- in TeV⁻²

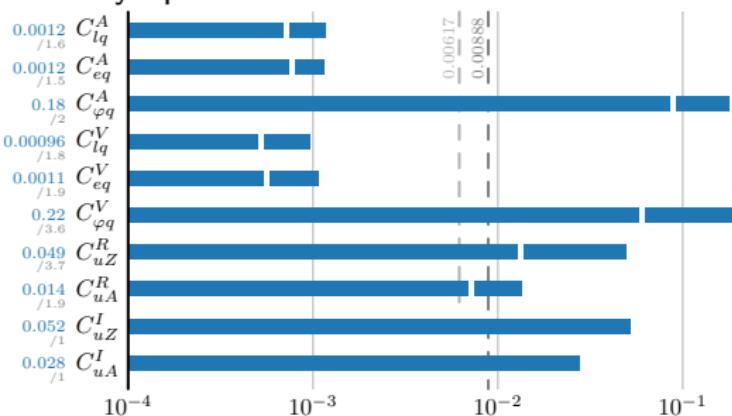
- white marks: individual constraints
- dashed vertical lines: GDPs

- gray numbers: global/individual ratios



correlation matrices

Statistically optimal observables:



factor of 1.6 improvement
of the 8-coefficient GDP

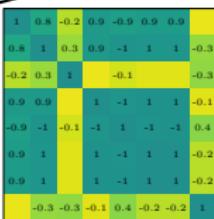
with linear coefficient
dependence only

Global constraints

$\sigma + A^{\text{FB}}$:

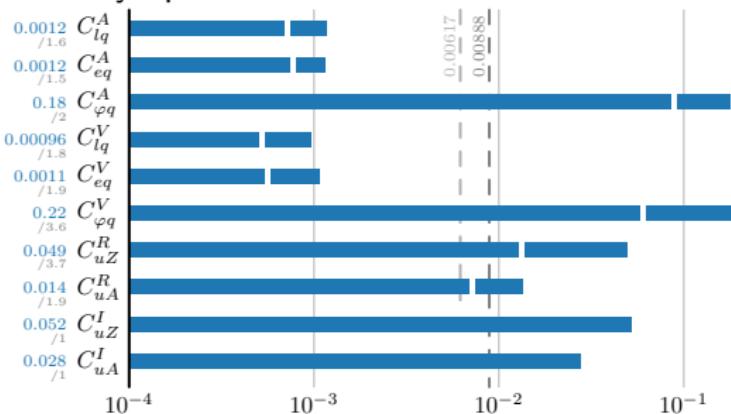


- in TeV⁻²
- white marks: individual constraints
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- gray numbers: global/individual ratios



correlation matrices

Statistically optimal observables:

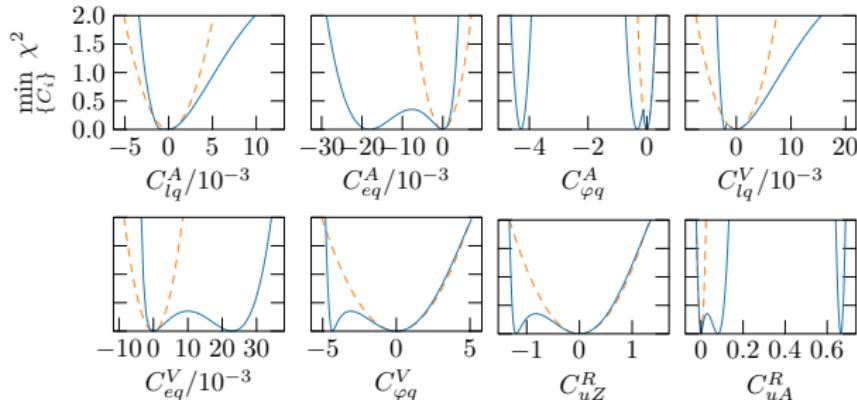


factor of 1.6 improvement
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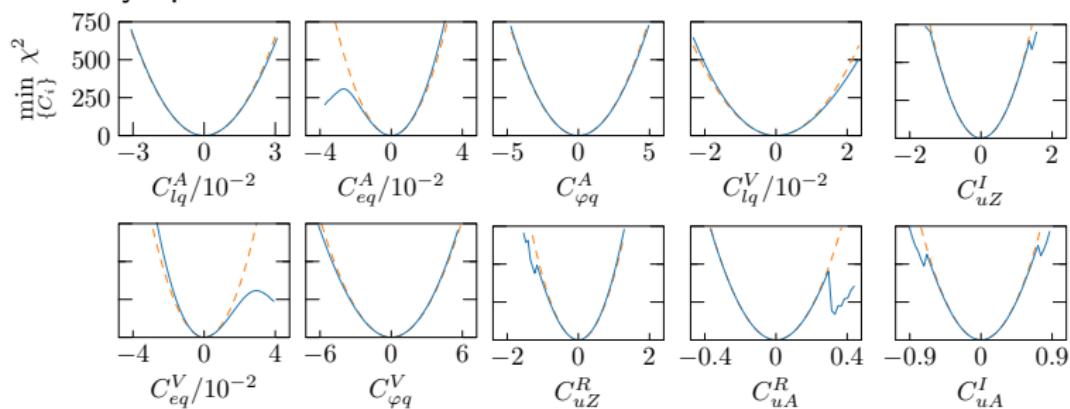
Adding quadratic coefficient dependences

$\sigma + A^{\text{FB}}$:



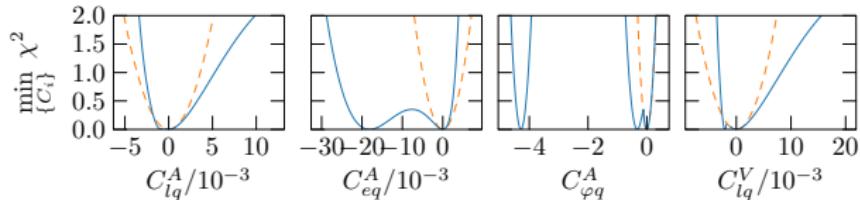
Statistically optimal observables:

Note the vertical scale!

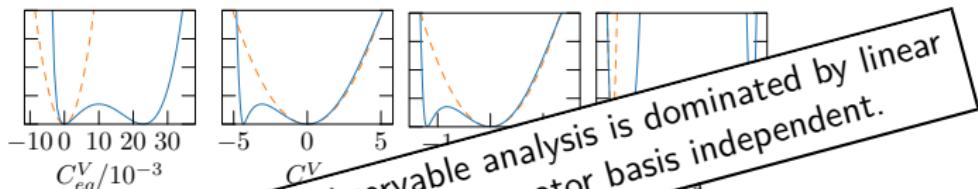


Adding quadratic coefficient dependences

$\sigma + A^{\text{FB}}$:



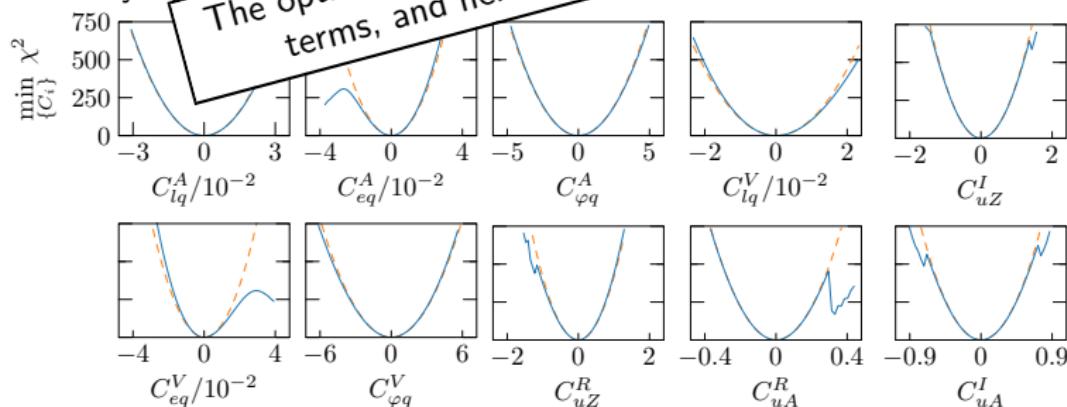
dashed: linear dependence only
plain: linear+quadratic dep.



Statistically optimal

The optimal-observable analysis is dominated by linear terms, and hence operator basis independent.

Note the vertical scale!



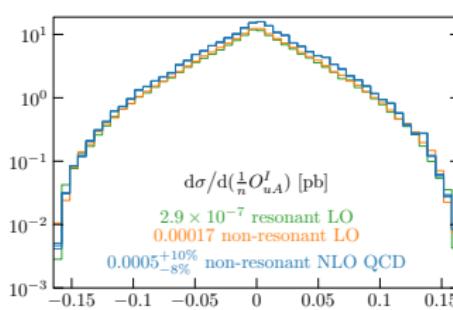
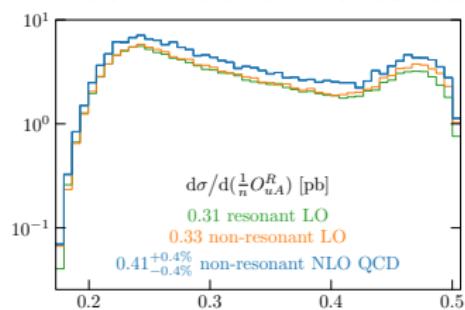
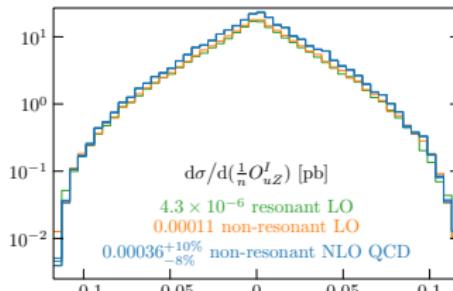
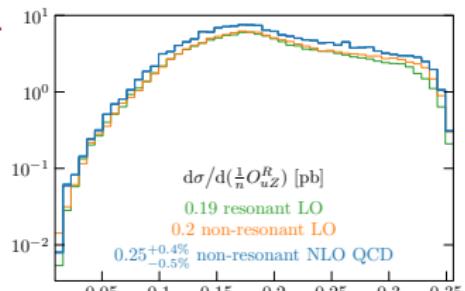
Getting more realistic

$\sqrt{s} = 500 \text{ GeV}$, $P(e^+, e^-) = (+30\%, -80\%)$,
quoted average values of distribution are $\bar{\sigma}_i / \mathcal{L}$ in pb,
QCD scale variation from $m_t/2$ to $2m_t$

Theoretical robustness

non-resonant and NLO QCD effects can be studied

e.g.



Experimental reconstruction

full detector simulations show moderate impact of reconstruction

[details will appear separately]

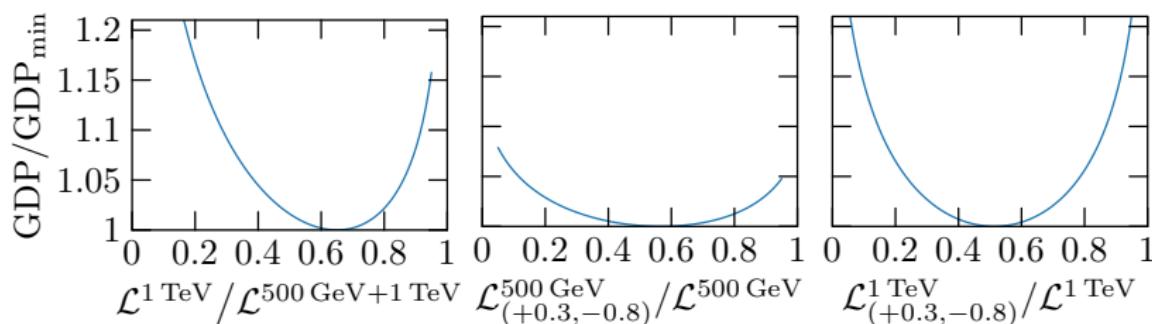
Run parameters optimization,
GDP-based

Examples of run parameters optimization

Given 1.5 ab^{-1} to share between two energies and polarizations,
the optimal repartition is:

$$\begin{array}{lll} \sqrt{s} = 500 \text{ GeV} & 570 \text{ fb}^{-1} & 61\% \text{ with } P(e^+, e^-) = (+0.3, -0.8) \\ 1 \text{ TeV} & 930 \text{ fb}^{-1} & 52\% \end{array}$$

→ GDP is 1.02 times better than the benchmark one
for the optimal-observable analysis with all 10 coefficients.



Same performances require 5.6 ab^{-1} with only $\sqrt{s} = 380 + 500 \text{ GeV}$:

$$\begin{array}{lll} \sqrt{s} = 380 \text{ GeV} & 1.7 \text{ ab}^{-1} & 59\% \text{ with } P(e^+, e^-) = (+0.3, -0.8) \\ 500 \text{ GeV} & 3.9 \text{ ab}^{-1} & 53\% \end{array}$$

Summary

The EFT parametrizes systematically the theory space in direct vicinity of the standard model.

A global analysis of future-lepton-collider constraints on the top EFT is ongoing.

Statistically optimal observables greatly help covering efficiently the multidimensional phase space.

Global determinant parameter ratios assess the strengthening of global constraints, basis independently.