

# Covariant diagrams for one-loop matching and applications in trans-TeV SUSY

Zhengkang (Kevin) Zhang (University of Michigan)

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Based on:

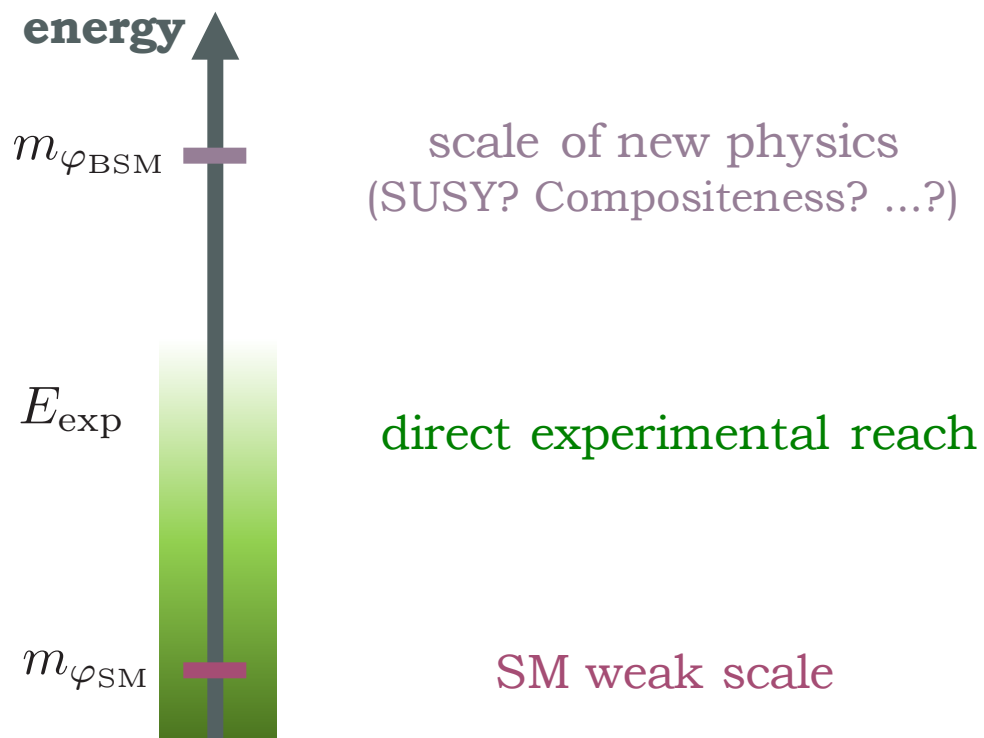
- *ZZ [1610.00710]* “Covariant diagrams for one-loop matching”
- *J. Wells, ZZ [1711.04774]* “EFT approach to trans-TeV SUSY”

See also:

- *S. Ellis, J. Quevillon, T. You, ZZ [1604.02445, 1706.07765]*

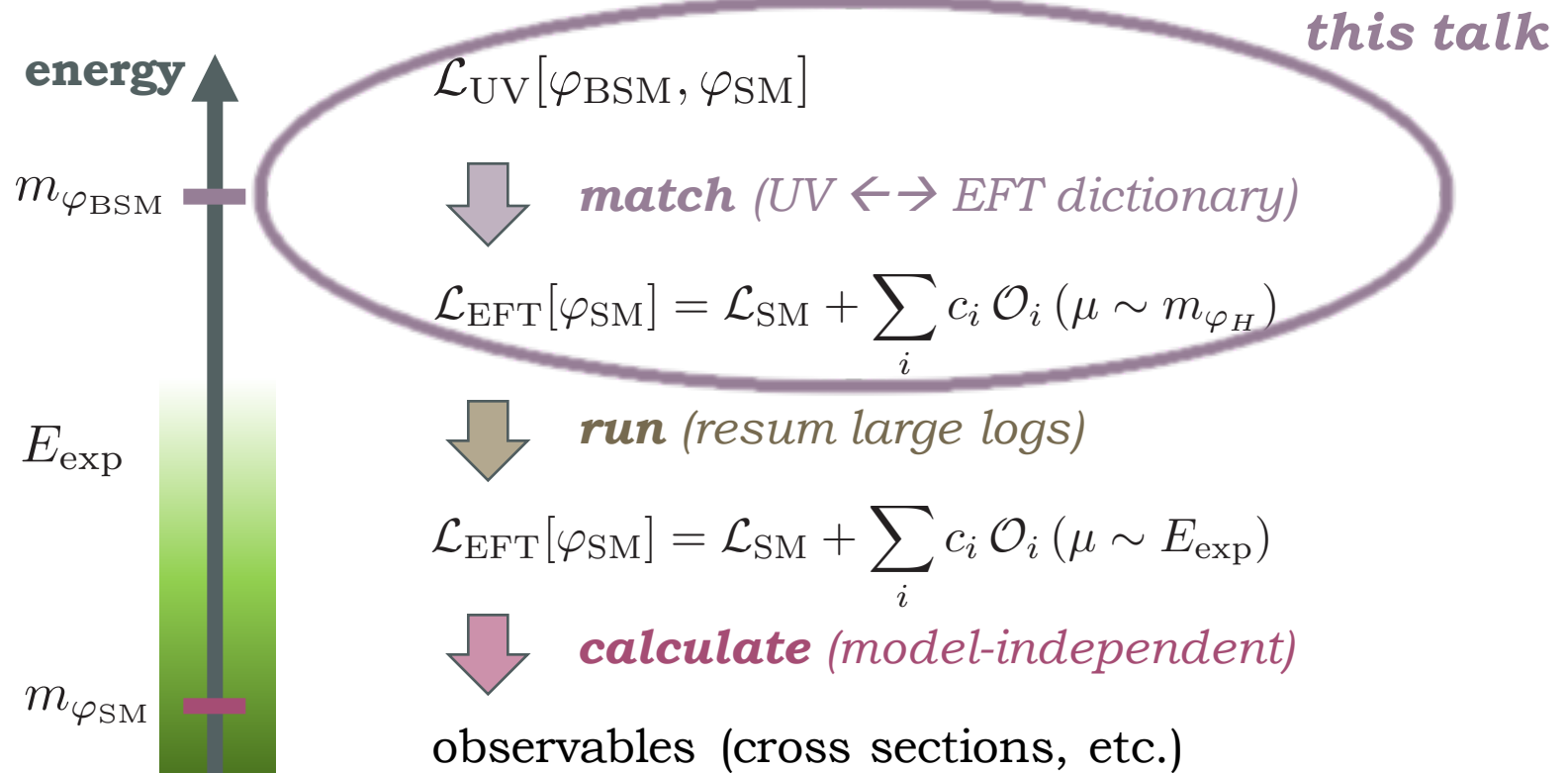
# Intro: EFT matching

- New physics may be somewhat decoupled from weak scale.



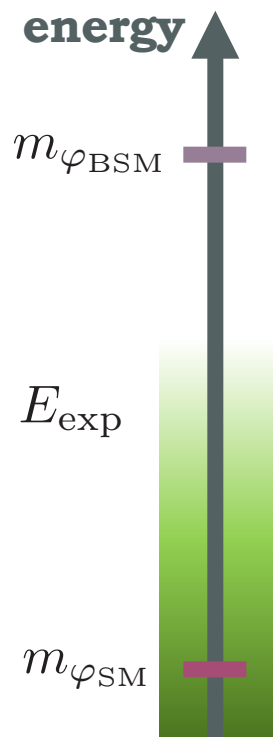
# Intro: EFT matching

- **Appropriate** and **convenient** framework: EFT.



# Intro: EFT matching

- Conventional approach to matching —



$$\mathcal{L}_{\text{UV}}[\varphi_{\text{BSM}}, \varphi_{\text{SM}}] \xrightarrow{\text{Feynman diagrams}}$$

$$\langle \varphi_L \varphi_L \dots \varphi_L \rangle$$

$$\mathcal{L}_{\text{EFT}}[\varphi_{\text{SM}}] = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i (\mu \sim m_{\varphi_H})$$

↓ **run** (resum large logs)

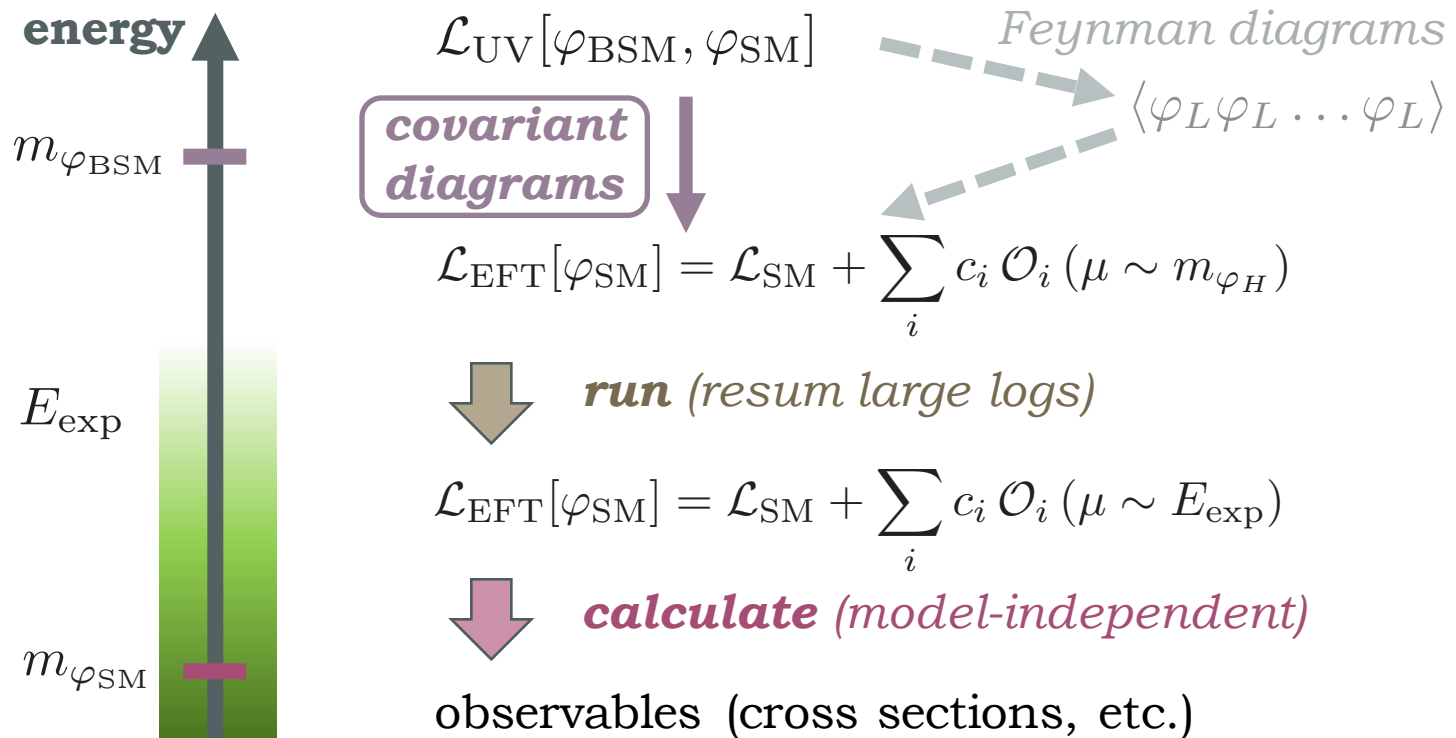
$$\mathcal{L}_{\text{EFT}}[\varphi_{\text{SM}}] = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i (\mu \sim E_{\text{exp}})$$

↓ **calculate** (model-independent)

observables (cross sections, etc.)

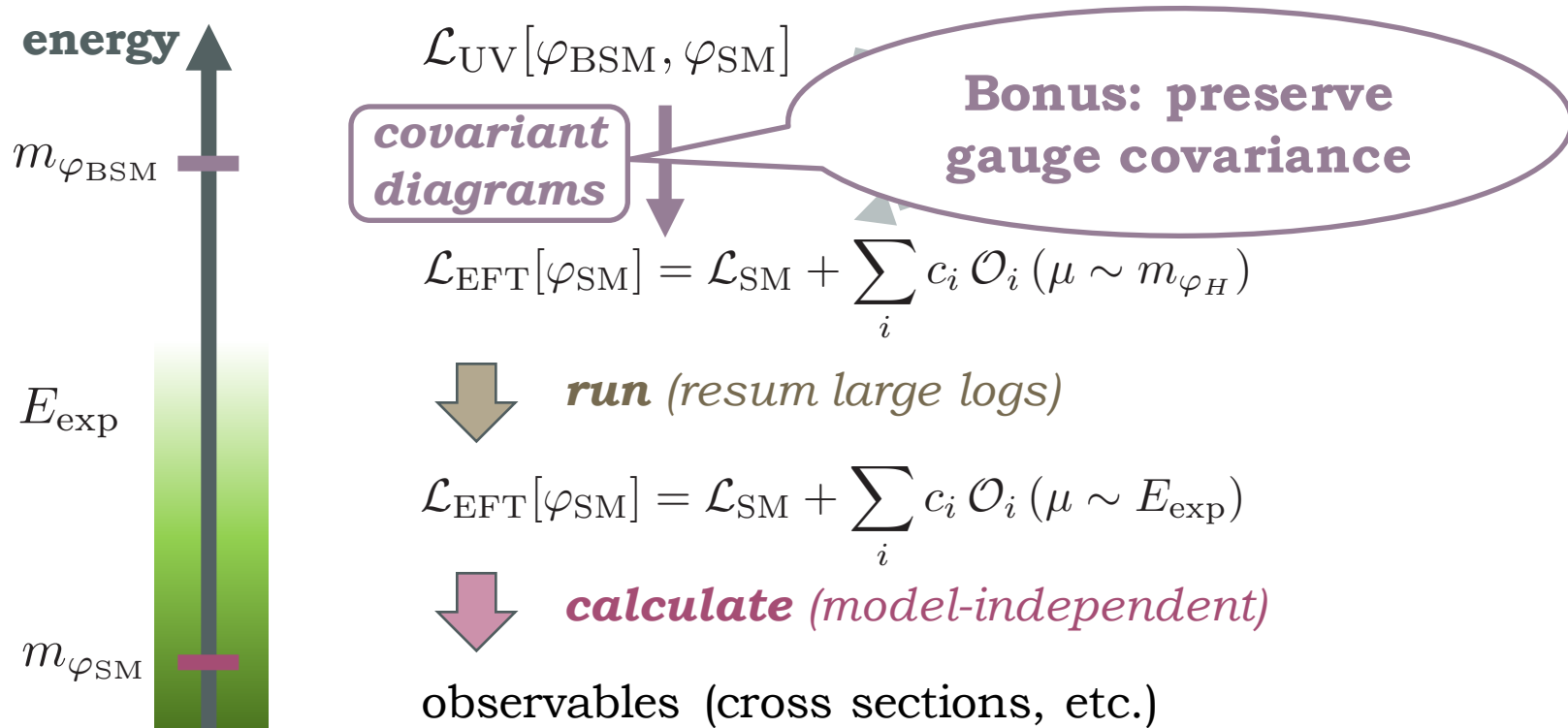
# Intro: EFT matching

- I will introduce a more **direct** and elegant approach.



# Intro: EFT matching

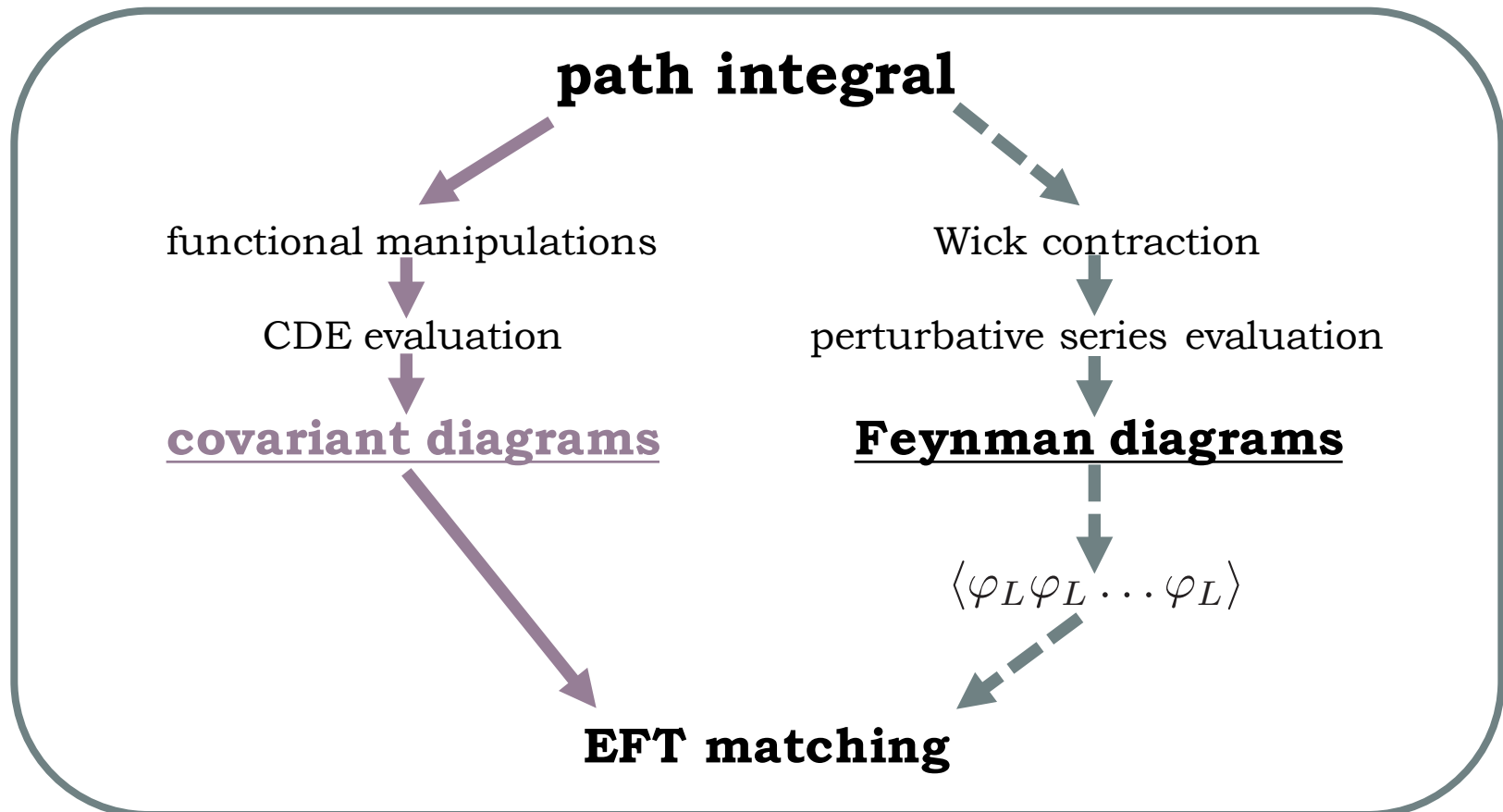
- I will introduce a more direct and **elegant** approach.



# Previous literature on correlation-function-free matching that covariant diagrams build upon

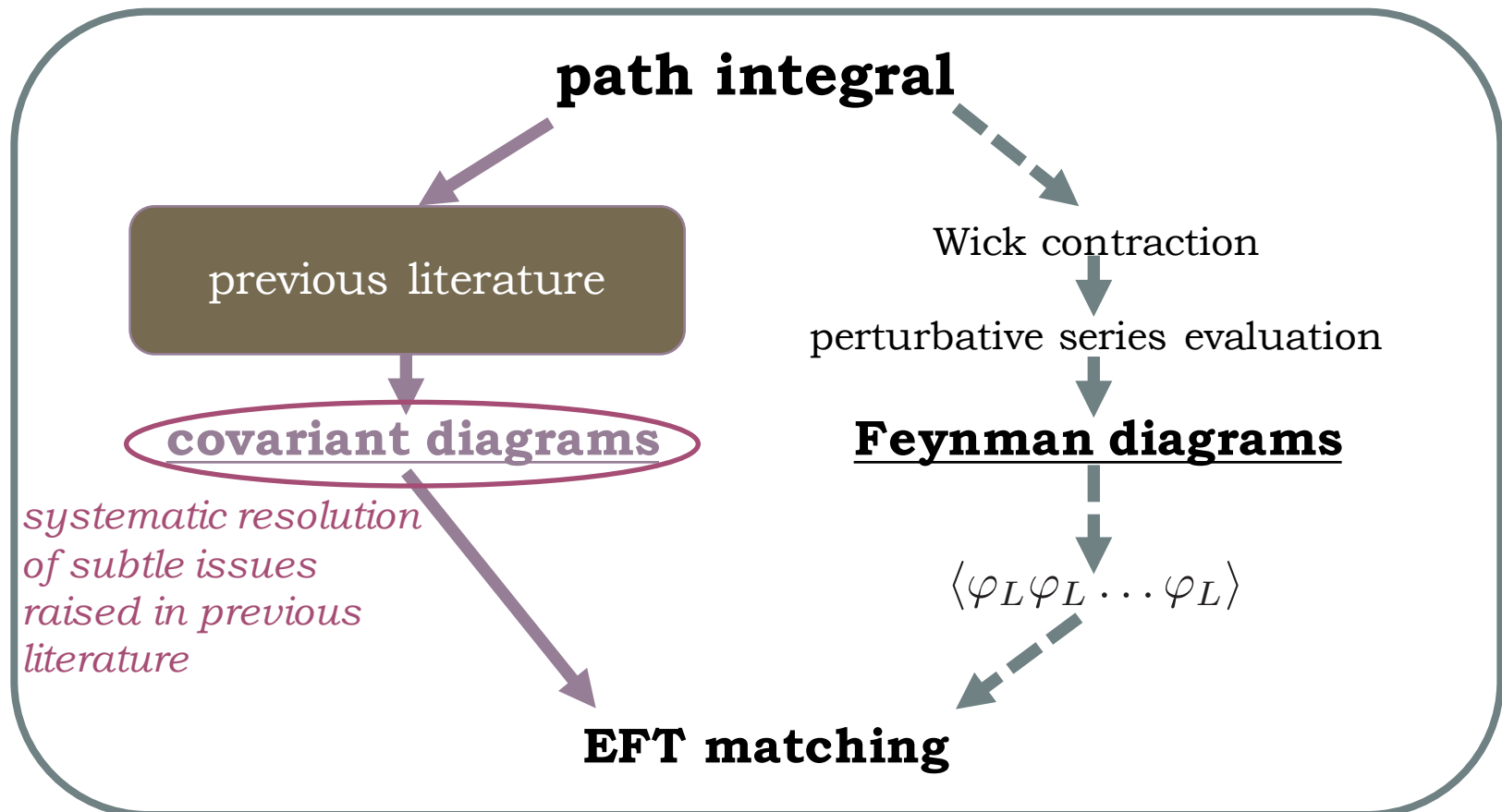
- Gaillard [Nucl.Phys.B268,669 (1986)];
  - Chan [Phys.Rev.Lett.57,1199 (1986)];
  - Cheyette [Nucl.Phys.B297,183 (1988)].
  - **Henning, Lu, Murayama [1412.1837];**
  - Chiang, Huo [1505.06334];
  - Huo [1506.00840, 1509.05942];
  - **Drozd, Ellis, Quevillon, You [1512.03003].**
  - Del Aguila, Kunszt, Santiago [1602.00126];
  - Boggia, Gomez-Ambrosio, Passarino [1603.03660];
  - **Henning, Lu, Murayama [1604.01019];**
  - **Ellis, Quevillon, You, ZZ [1604.02445];**
  - **Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142].**
- early studies
- ← revival
- application
- ← generalization
- criticism
- partial resolution
- ← simplification

# Previous literature on correlation-function-free matching that covariant diagrams build upon

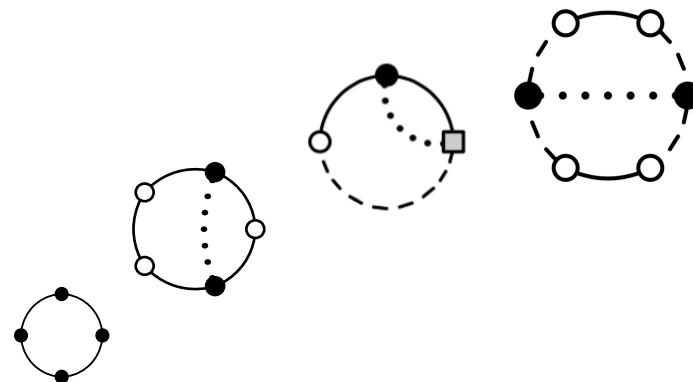




# Previous literature on correlation-function-free matching that covariant diagrams build upon



# Outline



- **Preliminaries.**
  - **Path integral** at tree and one-loop levels.
- **Core techniques.**
  - **Expansion** by regions.
  - Covariant Derivative **Expansion** (CDE).
- **Covariant diagrams** (to systematically **keep track of expansion**).
  - Basic rules with a simple example.
- **Application.**
  - Matching the MSSM onto the SMEFT.

# Preliminary: path integral at tree level

$$\int [D\varphi_H][D\varphi_L] e^{i \int d^d x \mathcal{L}_{UV}[\varphi_H, \varphi_L]} = \int [D\varphi_L] e^{i \int d^d x \mathcal{L}_{EFT}[\varphi_L]}$$

- **Tree level = stationary point approximation.**
  - Solve classical equations of motion:

$$\left. \frac{\delta \mathcal{L}_{UV}}{\delta \varphi_H} \right|_{\varphi_H = \varphi_{H,c}} = 0$$

$$\Rightarrow \mathcal{L}_{EFT}^{\text{tree}}[\varphi_L] = \mathcal{L}_{UV}[\varphi_{H,c}[\varphi_L], \varphi_L]$$

# Preliminary: path integral at one-loop level

$$\int [D\varphi_H][D\varphi_L] e^{i \int d^d x \mathcal{L}_{UV}[\varphi_H, \varphi_L]} = \int [D\varphi_L] e^{i \int d^d x \mathcal{L}_{EFT}[\varphi_L]}$$

- **One-loop level = Gaussian approximation.**

- Background field method:

$$\varphi_H = \varphi_{H,b} + \varphi'_H, \quad \varphi_L = \varphi_{L,b} + \varphi'_L$$

$$\Rightarrow \mathcal{L}_{UV}[\varphi_H, \varphi_L] + J_L \varphi_L = \mathcal{L}_{UV}[\varphi_{H,c}[\varphi_{L,b}], \varphi_{L,b}] + J_L \varphi_{L,b}$$

terms **quadratic** in  
quantum fluctuations



$$-\frac{1}{2} \begin{pmatrix} \varphi_H'^T & \varphi_L'^T \end{pmatrix} \mathcal{Q}_{UV}[\varphi_{H,c}[\varphi_{L,b}], \varphi_{L,b}] \begin{pmatrix} \varphi_H' \\ \varphi_L' \end{pmatrix} + \mathcal{O}(\varphi'^3)$$

- Path integral is *Gaussian* at this order  
=> **functional determinant** of the **quadratic operator**  $\mathcal{Q}_{UV}$ .

# 1LPI effective action vs. EFT Lagrangian

- This is what we would do if we were to compute the **1LPI effective action** (Legendre transform of the path integral):

$$\begin{aligned} \Gamma_{L,UV}^{1\text{-loop}}[\varphi_{L,b}] &= i c_s \log \det Q_{UV}[\varphi_{H,c}[\varphi_{L,b}], \varphi_{L,b}] \\ &= i c_s \text{Tr} \log Q_{UV} = i c_s \int d^d x \int \frac{d^d q}{(2\pi)^d} \text{tr} \log Q_{UV}|_{P_\mu \rightarrow P_\mu - q_\mu} \end{aligned}$$

- $c_s$  is spin factor (= +1/2 for real scalar, -1/2 for Weyl fermion).
- Notation:  $P_\mu \equiv iD_\mu$  (“kinetic momentum operator,” hermitian).
- But we are interested in a **different** quantity:

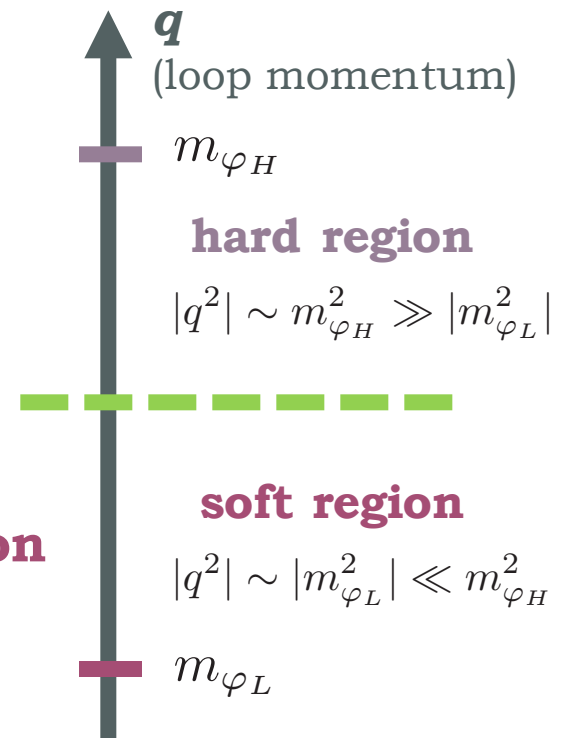
$$\int d^d x \mathcal{L}_{\text{EFT}}^{1\text{-loop}}[\varphi_L] \neq \Gamma_{L,UV}^{1\text{-loop}}[\varphi_L]$$

# Core technique #1: expansion by regions

- After careful functional manipulations, we can show (**ZZ [1610.00710]**):

$$\int d^d x \mathcal{L}_{\text{EFT}}^{\text{1-loop}}[\varphi_L] = \Gamma_{L,\text{UV}}^{\text{1-loop}}[\varphi_L] \Big|_{\text{hard}}$$

- Previously argued in *Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]*.
- Expand integrand before integrating.
- **Full integral = hard region + soft region** contributions.
  - See e.g. *Beneke, Smirnov, hep-ph/9711391*; *Jantzen, 1111.2589*.

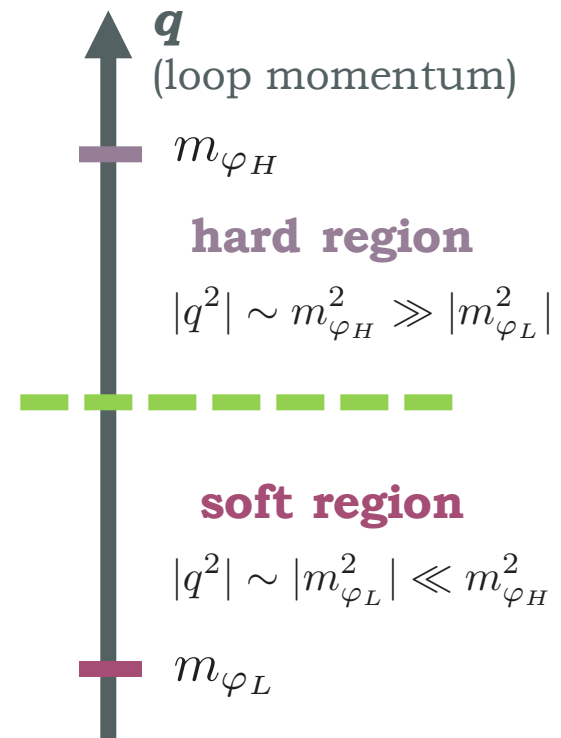


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- Previously argued in *Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]*.
- Intuition:
  - **1PI effective actions** encode quantum fluctuations at **all scales**.
  - Extract **short-distance** fluctuations => **local** operators in **EFT Lagrangian**.

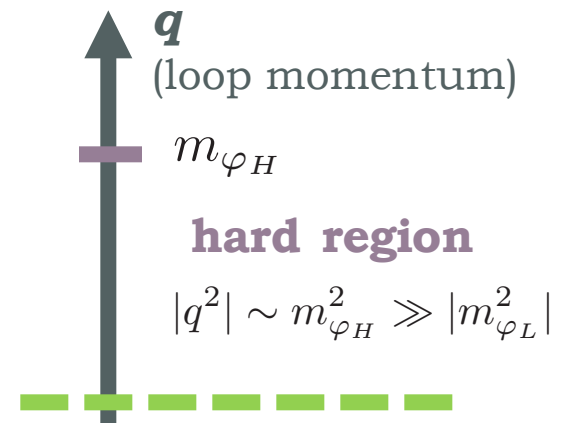


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$$\mathcal{L}_{\text{EFT}}^{\text{1-loop}}[\varphi_L] = i c_s \text{tr} \int \frac{d^d q}{(2\pi)^d} \left[ \log \mathcal{Q}_{\text{UV}}|_{P_\mu \rightarrow P_\mu - q_\mu} \right] \text{expand for } |q^2| \sim m_{\varphi_H}^2 \gg |m_{\varphi_L}^2|$$

- Covariant diagrams keep track of this series expansion.



# Core technique #2: Covariant Derivative Expansion

- CDE = expansion where derivatives are **covariant**.
  - We never separate  $D_\mu$  into  $\partial_\mu$  and  $-igA_\mu$ .

$$\mathcal{L}_{\text{EFT}}^{1\text{-loop}}[\varphi_L] = i c_s \text{tr} \int \frac{d^d q}{(2\pi)^d} \left[ \log \mathcal{Q}_{\text{UV}} \Big|_{P_\mu \rightarrow P_\mu - q_\mu} \right] \text{expand for } |q^2| \sim m_{\varphi_H}^2 \gg |m_{\varphi_L}^2|$$

- General form of quadratic operator  $\mathcal{Q}_{\text{UV}}$ :

$$\mathcal{Q}_{\text{UV}}[\varphi, P_\mu = iD_\mu; m_{\varphi_H}, m_{\varphi_L}]$$

$$= \left\{ \begin{array}{ll} -P^2 + \mathbf{M}^2 & \text{(boson)} \\ -\not{P} + \mathbf{M} & \text{(fermion)} \end{array} \right\} + \mathbf{U}[\varphi] + P_\mu \mathbf{Z}^\mu[\varphi] + \mathbf{Z}^{\dagger\mu}[\varphi] P_\mu + \dots$$

- Recall:  $\varphi_H = \varphi_{H,b} + \varphi'_H$ ,  $\varphi_L = \varphi_{L,b} + \varphi'_L$ 

$$\Rightarrow \mathcal{L}_{\text{UV}}^{\text{quadratic}} = -\frac{1}{2} \begin{pmatrix} \varphi'^T_H & \varphi'^T_L \end{pmatrix} \mathcal{Q}_{\text{UV}}[\varphi_{H,c}[\varphi_{L,b}], \varphi_{L,b}] \begin{pmatrix} \varphi'_H \\ \varphi'_L \end{pmatrix}$$

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$$\mathcal{L}_{\text{EFT}}^{1\text{-loop}}[\varphi_L] = i c_s \text{tr} \int \frac{d^d q}{(2\pi)^d} \left[ \log \right]$$

Example: real singlet scalar  
S coupling to SM Higgs H

$$\mathbf{U} = \begin{pmatrix} U_{SS} & U_{SH} & \dots \\ U_{HS} & U_{HH} & \dots \\ \dots & \dots & \dots \end{pmatrix}$$

$$H \gg |m_{\varphi_L}^2|$$

- General form of quadratic

$$\mathcal{Q}_{\text{UV}}[\varphi, P_\mu = iD_\mu; m_{\varphi_H}, m_{\varphi_L}]$$

$$= \begin{cases} -P^2 + \mathbf{M}^2 & \text{(boson)} \\ -\not{P} + \mathbf{M} & \text{(fermion)} \end{cases} + \mathbf{U}[\varphi] + P_\mu \mathbf{Z}^\mu[\varphi] + \mathbf{Z}^{\dagger\mu}[\varphi] P_\mu + \dots$$

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$$\mathcal{L}_{\text{UV}} \supset -\frac{1}{2} \lambda_{HS} |H|^2 S^2$$

$$\Rightarrow U_{SS} \supset \lambda_{HS} |H|^2$$

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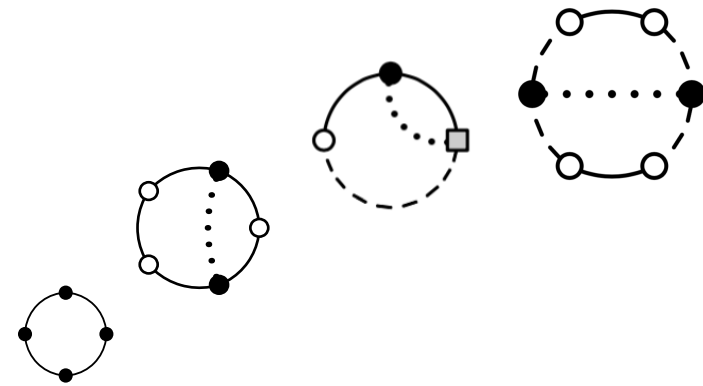
$$\mathcal{L}_{\text{EFT}}^{1\text{-loop}}[\varphi_L] = i c_s \text{tr} \int \frac{d^d q}{(2\pi)^d} \left[ \log Q_{UV} |_{P_\mu \rightarrow P_\mu - q_\mu} \right] \text{expand for } |q^2| \sim m_{\varphi_H}^2 \gg |m_{\varphi_L}^2|$$

- General form of quadratic operator  $Q_{UV}$ :

$$Q_{UV}[\varphi, P_\mu \equiv iD_\mu; m_{\varphi_H}, m_{\varphi_L}] = \left\{ \begin{array}{l} -P^2 + M^2 \quad (\text{boson}) \\ -\not{P} + M \quad (\text{fermion}) \end{array} \right\} + U[\varphi] + P_\mu Z^\mu[\varphi] + Z^{\dagger\mu}[\varphi] P_\mu + \dots$$

- Result of expansion: **operators** made of fields  $\varphi$  and **covariant** derivatives  $P_\mu \equiv iD_\mu$  (rather than correlation functions)  
=> automatically **gauge-invariant!**

# Covariant diagrams (to keep track of CDE)

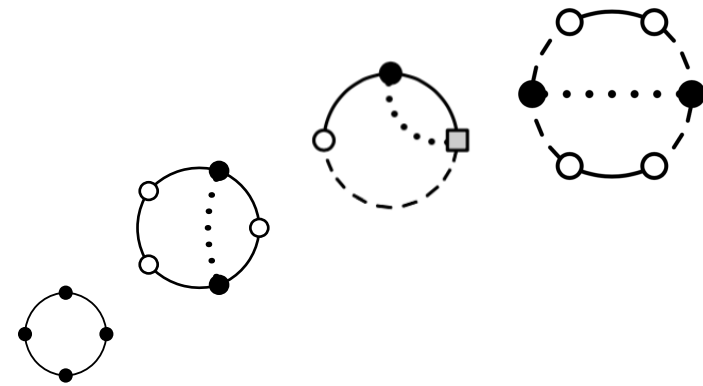


$$\mathcal{L}_{\text{EFT}}^{\text{1-loop}}[\varphi_L] = i c_s \text{tr} \int \frac{d^d q}{(2\pi)^d} \left[ \log Q_{UV} \Big|_{P_\mu \rightarrow P_\mu - q_\mu} \right] \text{expand for } |q^2| \sim m_{\varphi_H}^2 \gg |m_{\varphi_L}^2|$$

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- Collect identical terms, and encode in diagrams.
- Enumerate diagrams  $\rightarrow$  recover the full expansion.
- Rules can be derived from the general form of  $Q_{UV}$  (UV model-independent).

# Covariant diagrams (to keep track of CDE)



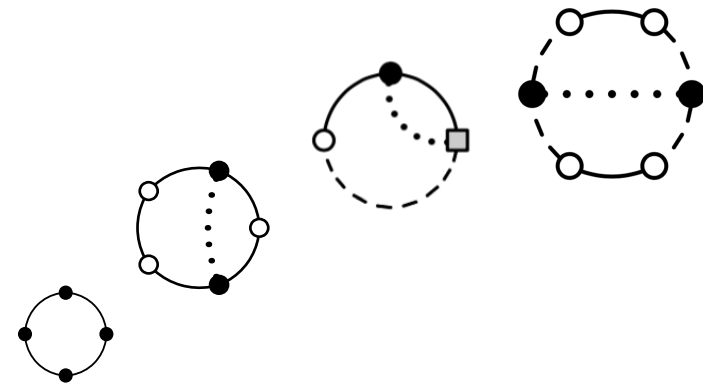
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$$\left\{ \begin{array}{ll} -P^2 + M^2 & \text{(boson)} \\ -\not{P} + M & \text{(fermion)} \end{array} \right\} + U[\varphi] + P_\mu Z^\mu[\varphi] + Z^{\dagger\mu}[\varphi] P_\mu + \dots$$

Building blocks	Bosonic	Fermionic
Propagators	$---i--- = 1$	$\frac{i}{\quad} = \begin{cases} M_i & \text{(heavy)} \\ 0 & \text{(light)} \end{cases}$ $\frac{i}{\vdots} = -\gamma_\mu$
$P$ insertions	$-\overset{i}{\bullet} \underset{\vdots}{\bullet} - = 2P_\mu$	$\overset{i}{\bullet} \underset{\vdots}{\bullet} = -\not{P}$
$U$ insertions	$-\overset{i}{\circ} \overset{j}{\circ} - , -\overset{i}{\circ} \overset{j}{\circ} \quad , \quad \overset{i}{\circ} \overset{j}{\circ} - , \overset{i}{\circ} \overset{j}{\circ} = U_{ij}[\varphi]$	
Contractions	$\overset{\mu}{\dots\dots\dots} \overset{\nu}{\dots\dots\dots} = g^{\mu\nu}$	

Analogously, there are also  $Z$  insertions

# Covariant diagrams (to keep track of CDE)



➤ Example: real singlet scalar

Recall  $\mathcal{L}_{UV} \supset -\frac{1}{2}\lambda_{HS}|H|^2 S^2$

$\Rightarrow U_{SS} \supset \lambda_{HS}|H|^2$

$\propto \text{tr}(2P^\mu \cdot U_{SS} \cdot 2P_\mu \cdot U_{SS})$

Building blocks	Bosonic	Fermionic
Propagators	$---\overset{i}{---} = 1$	$\begin{aligned} \text{---} \overset{i}{\text{---}} &= \begin{cases} M_i & \text{(heavy)} \\ 0 & \text{(light)} \end{cases} \\ \text{---} \underset{\vdots}{\overset{i}{\text{---}}} &= -\gamma_\mu \end{aligned}$
$P$ insertions	$-\overset{i}{\text{---}} \bullet \underset{\vdots}{\overset{i}{\text{---}}} = 2P_\mu$	$\text{---} \bullet \text{---} \overset{i}{\text{---}} = -\not{P}$
$U$ insertions	$-\overset{i}{\text{---}} \circ \overset{j}{\text{---}}, -\overset{i}{\text{---}} \circ \text{---} \overset{j}{\text{---}}, \text{---} \circ \overset{i}{\text{---}} \text{---}, \text{---} \circ \overset{j}{\text{---}} = U_{ij}[\varphi]$	
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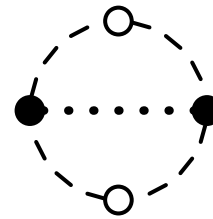
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$\propto \text{tr}(2P^\mu \cdot U_{SS} \cdot 2P_\mu \cdot U_{SS})$

▪ **Prefactor rule:**

$$-i c_s \cdot \frac{1}{S} \cdot \mathcal{I}[q^{2n_c}]_{ij\dots}^{n_i n_j \dots}$$

<b>Spin factor <math>c_s</math></b>	1/2 for each real scalar/vector, -1/2 for each Weyl fermion
<b>Symmetry factor <math>1/S</math></b>	if diagram has $Z_S$ symmetry
<b>Master integral <math>\mathcal{I}[q^{2n_c}]_{ij\dots}^{n_i n_j \dots}</math></b>	$n_i$ propagators with mass $M_i$ $n_c$ Lorentz contractions

$c_s = 1/2$   
 $S = 2$   
 $n_i = 4$   
 $n_c = 1$



# Covariant diagrams (to keep track of CDE)

➤ Example  
Recall

Defined by  $\int \frac{d^d q}{(2\pi)^d} \frac{q^{\mu_1} \dots q^{\mu_{2n_c}}}{(q^2 - M_i^2)^{n_i} (q^2 - M_j^2)^{n_j} \dots} \equiv g^{\mu_1 \dots \mu_{2n_c}} \mathcal{I}[q^{2n_c}]_{ij\dots}^{n_i n_j \dots}$

$\tilde{\mathcal{I}}[q^{2n_c}]_i^{n_i}$	$n_c = 0$	$n_c = 1$	$n_c = 2$	$n_c = 3$
$n_i = 1$	$M_i^2 (1 - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^4}{4} (\frac{3}{2} - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^6}{24} (\frac{11}{6} - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^8}{192} (\frac{25}{12} - \log \frac{M_i^2}{\mu^2})$
$n_i = 2$	$-\log \frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{2} (1 - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^4}{8} (\frac{3}{2} - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^6}{48} (\frac{11}{6} - \log \frac{M_i^2}{\mu^2})$
$n_i = 3$	$-\frac{1}{2M_i^2}$	$-\frac{1}{4} \log \frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{8} (1 - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^4}{32} (\frac{3}{2} - \log \frac{M_i^2}{\mu^2})$
$n_i = 4$	$\frac{1}{6M_i^4}$	$-\frac{1}{12M_i^2}$	$-\frac{1}{24} \log \frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{48} (1 - \log \frac{M_i^2}{\mu^2})$
$n_i = 5$	$-\frac{1}{12M_i^6}$	$\frac{1}{48M_i^4}$	$-\frac{1}{96M_i^2}$	$-\frac{1}{192} \log \frac{M_i^2}{\mu^2}$
$n_i = 6$	$\frac{1}{20M_i^8}$	$-\frac{1}{120M_i^6}$	$\frac{1}{480M_i^4}$	$-\frac{1}{960M_i^2}$

**Table 7.** Commonly-used degenerate master integrals  $\tilde{\mathcal{I}}[q^{2n_c}]_i^{n_i} \equiv \mathcal{I}[q^{2n_c}]_i^{n_i} / \frac{i}{16\pi^2}$ , with  $\frac{2}{\epsilon} = \frac{2}{\epsilon} - \gamma + \log 4\pi$  dropped. All nondegenerate (including mixed heavy-light) master integrals can be reduced to degenerate master integrals by Eq. (A.2).

▪ Prefactor

Spin factor

Symmetry

Master integral  $\mathcal{I}[q^{2n_c}]_{ij\dots}^{n_i n_j \dots}$

$n_i$  propagators with mass  $M_i$   
 $n_c$  Lorentz contractions

$n_i = 4$   
 $n_c = 1$

# Covariant diagrams (to keep track of CDE)

➤ Example  
Recall

Defined by 
$$\int \frac{d^d q}{(2\pi)^d} \frac{q^{\mu_1} \dots q^{\mu_{2n_c}}}{(q^2 - M_i^2)^{n_i} (q^2 - M_j^2)^{n_j} \dots} \equiv g^{\mu_1 \dots \mu_{2n_c}} \mathcal{I}[q^{2n_c}]_{ij\dots}^{n_i n_j \dots}$$

$\tilde{\mathcal{I}}[q^{2n_c}]_i^{n_i}$	$n_c = 0$	$n_c = 1$	$n_c = 2$	$n_c = 3$
$n_i = 1$	$M_i^2 (1 - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^4}{4} (\frac{3}{2} - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^6}{24} (\frac{11}{6} - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^8}{192} (\frac{25}{12} - \log \frac{M_i^2}{\mu^2})$
$n_i = 2$	$-\log \frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{2} (1 - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^4}{8} (\frac{3}{2} - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^6}{48} (\frac{11}{6} - \log \frac{M_i^2}{\mu^2})$
$n_i = 3$	$-\frac{1}{2M_i^2}$	$-\frac{1}{4} \log \frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{8} (1 - \log \frac{M_i^2}{\mu^2})$	$\frac{M_i^4}{32} (\frac{3}{2} - \log \frac{M_i^2}{\mu^2})$
$n_i = 4$	$\frac{1}{6M_i^4}$	$-\frac{1}{12M_i^2}$	$-\frac{1}{24} \log \frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{48} (1 - \log \frac{M_i^2}{\mu^2})$
$n_i = 5$	$-\frac{1}{12M_i^6}$	$\frac{1}{48M_i^4}$	$-\frac{1}{96M_i^2}$	$-\frac{1}{192} \log \frac{M_i^2}{\mu^2}$
$n_i = 6$	$\frac{1}{20M_i^8}$	$-\frac{1}{120M_i^6}$	$\frac{1}{480M_i^4}$	$-\frac{1}{960M_i^2}$

**Table 7.** Commonly-used degenerate master integrals  $\tilde{\mathcal{I}}[q^{2n_c}]_i^{n_i} \equiv \mathcal{I}[q^{2n_c}]_i^{n_i} / \frac{i}{16\pi^2}$ , with  $\frac{2}{\epsilon} = \frac{2}{\epsilon} - \gamma + \log 4\pi$  dropped. All nondegenerate (including mixed heavy-light) master integrals can be reduced to degenerate master integrals by Eq. (A.2).

▪ Prefactor

Spin factor

Symmetry

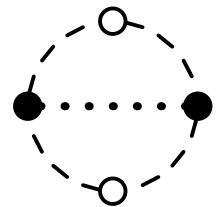
Master integral  $\mathcal{I}[q^{2n_c}]_{ij\dots}^{n_i n_j \dots}$

$n_i$  propagators with mass  $M_i$   
 $n_c$  Lorentz contractions

$n_i = 4$   
 $n_c = 1$

# Covariant diagrams (to keep track of CDE)

- Now let's put the pieces together —



$$= -\frac{i}{2} \cdot \frac{1}{2} \cdot \mathcal{I}[q^2]_i^4 \cdot \text{tr}(2P^\mu \cdot U_{SS} \cdot 2P_\mu \cdot U_{SS}) = -\frac{\lambda_{HS}^2}{192\pi^2 M_S^2} \text{tr}(P^\mu |H|^2 P_\mu |H|^2)$$

- This is part of

$$\begin{aligned} (\partial_\mu |H|^2)^2 &= [D^\mu, |H|^2][D_\mu, |H|^2] = -[P^\mu, |H|^2][P_\mu, |H|^2] \\ &= 2 \text{tr}(P^2 |H|^4) - 2 \text{tr}(P^\mu |H|^2 P_\mu |H|^2) \end{aligned}$$

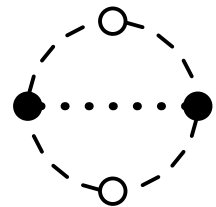
- In fact, this is the only  $\mathcal{O}(\lambda_{HS}^2)$  contribution to the 2<sup>nd</sup> term.

$$\Rightarrow \mathcal{L}_{\text{eff}} \supset \frac{\lambda_{HS}^2}{384\pi^2 M_S^2} (\partial_\mu |H|^2)^2$$

(a one-loop-generated dim-6 operator that modifies Higgs couplings)

# Covariant diagrams (to keep track of CDE)

- Now let's put the pieces together —



$$= -\frac{i}{2} \cdot \frac{1}{2} \cdot \mathcal{I}[q^2]_i^4 \cdot \text{tr}(2P^\mu \cdot U_{SS} \cdot 2P_\mu \cdot U_{SS}) = -\frac{\lambda_{HS}^2}{192\pi^2 M_S^2} \text{tr}(P^\mu |H|^2 P_\mu |H|^2)$$

- This is part of

$$\begin{aligned} (\partial_\mu |H|^2)^2 &= [D^\mu, |H|^2][D_\mu, |H|^2] = -[P^\mu, |H|^2][P_\mu, |H|^2] \\ &= 2 \text{tr}(P^2 |H|^4) - 2 \text{tr}(P^\mu |H|^2 P_\mu |H|^2) \end{aligned}$$

Aside: 1<sup>st</sup> term comes from other terms in the CDE.

- **Additional rule:** we only need to compute covariant diagrams where no Lorentz contraction is between adjacent  $P$ 's — those are sufficient to fix all independent EFT operator coefficients.

# Application: MSSM $\rightarrow$ SMEFT

- Weak-scale SUSY, a leading solution to the hierarchy problem, is falling out of favor due to:
  - Lack of superpartner discovery at the LHC
  - Measurement of  $m_h=125\text{GeV}$
- However, dismissing traditional naturalness concerns, **trans-TeV SUSY** still has attractive features:
  - Gauge coupling unification
  - Yukawa coupling unification
  - Dark matter candidate
- EFT treatment is warranted.

# Application: MSSM $\rightarrow$ SMEFT

## $d \leq 4$ operators & threshold corrections

- SM couplings (gauge, Yukawa, ...) are numerically different above and below SUSY threshold.

$$\int [D\varphi_{\text{BSM}}][D\varphi_{\text{SM}}] e^{i \int d^d x \mathcal{L}[\varphi_{\text{BSM}}, \varphi_{\text{SM}}]} = \int [D\varphi_{\text{SM}}] e^{i \int d^d x \mathcal{L}_{\text{SMEFT}}[\varphi_{\text{SM}}]}$$

$$\begin{aligned} \mathcal{L}_{\text{SMEFT}} = & \mathcal{L}_{\text{SM}} + \delta Z_\phi |D_\mu \phi|^2 + \sum_{f=q,u,d,l,e} \bar{f} \delta Z_f i \not{D} f - \frac{1}{4} \delta Z_G G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4} \delta Z_W W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4} \delta Z_B B_{\mu\nu} B^{\mu\nu} \\ & + \delta m^2 |\phi|^2 + \delta \lambda |\phi|^4 + (\bar{u} \delta y_u q \cdot \epsilon \cdot \phi + \bar{d} \delta y_d q \cdot \phi^* + \bar{e} \delta y_e l \cdot \phi^* + \text{h.c.}) + \text{dimension 6 ...} \end{aligned}$$

$$\begin{aligned} g_3 - g_3^{\text{eff}} &= \frac{1}{2} g_3 \delta Z_G, & g_2 - g_2^{\text{eff}} &= \frac{1}{2} g_2 \delta Z_W, & g_1 - g_1^{\text{eff}} &= \frac{1}{2} g_1 \delta Z_B, \\ m^2 - m_{\text{eff}}^2 &= \delta m^2 + m^2 \delta Z_\phi, & \lambda - \lambda_{\text{eff}} &= \delta \lambda + 2 \lambda \delta Z_\phi, \\ \mathbf{y}_u - \mathbf{y}_u^{\text{eff}} &= \delta \mathbf{y}_u + \frac{1}{2} (\mathbf{y}_u \delta Z_q + \delta Z_u \mathbf{y}_u + \mathbf{y}_u \delta Z_\phi), \\ \mathbf{y}_d - \mathbf{y}_d^{\text{eff}} &= \delta \mathbf{y}_d + \frac{1}{2} (\mathbf{y}_d \delta Z_q + \delta Z_d \mathbf{y}_d + \mathbf{y}_d \delta Z_\phi), \\ \mathbf{y}_e - \mathbf{y}_e^{\text{eff}} &= \delta \mathbf{y}_e + \frac{1}{2} (\mathbf{y}_e \delta Z_l + \delta Z_e \mathbf{y}_e + \mathbf{y}_e \delta Z_\phi). \end{aligned}$$

# Application: MSSM $\rightarrow$ SMEFT

## $d \leq 4$ operators & threshold corrections

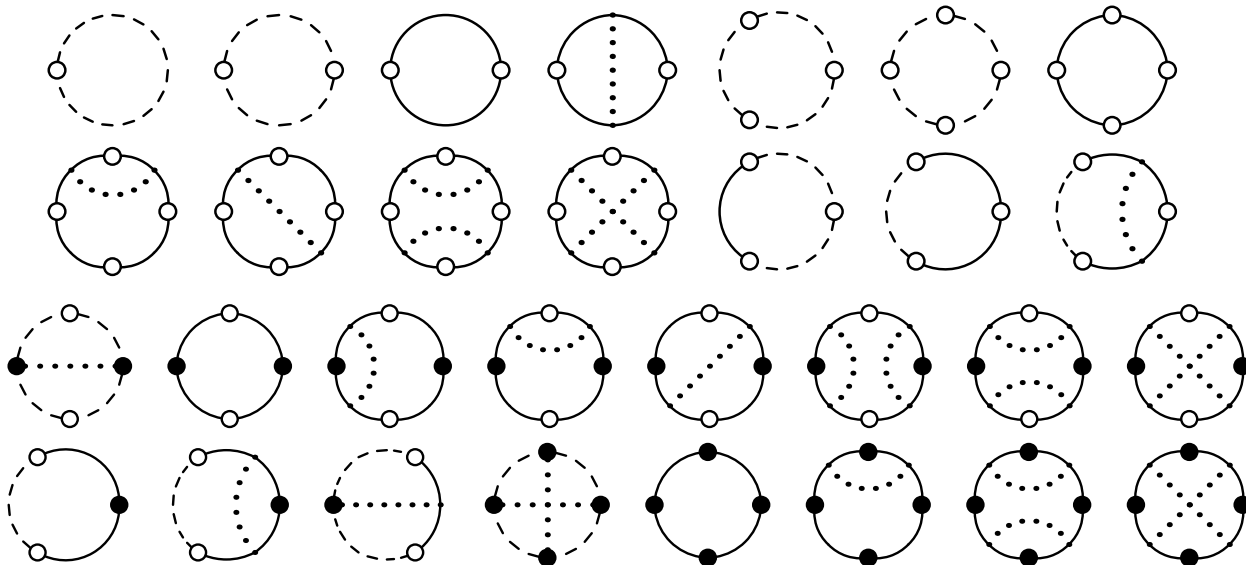
- SM couplings (gauge, Yukawa, ...) are numerically different above and below SUSY threshold.
- SUSY threshold corrections are important for —
  - Consistent matching of the Higgs quartic
  - Bottom-tau Yukawa unification

$$\begin{aligned}g_3 - g_3^{\text{eff}} &= \frac{1}{2} g_3 \delta Z_G, & g_2 - g_2^{\text{eff}} &= \frac{1}{2} g_2 \delta Z_W, & g_1 - g_1^{\text{eff}} &= \frac{1}{2} g_1 \delta Z_B, \\m^2 - m_{\text{eff}}^2 &= \delta m^2 + m^2 \delta Z_\phi, & \lambda - \lambda_{\text{eff}} &= \delta \lambda + 2 \lambda \delta Z_\phi, \\y_u - y_u^{\text{eff}} &= \delta y_u + \frac{1}{2} (y_u \delta Z_q + \delta Z_u y_u + y_u \delta Z_\phi), \\y_d - y_d^{\text{eff}} &= \delta y_d + \frac{1}{2} (y_d \delta Z_q + \delta Z_d y_d + y_d \delta Z_\phi), \\y_e - y_e^{\text{eff}} &= \delta y_e + \frac{1}{2} (y_e \delta Z_l + \delta Z_e y_e + y_e \delta Z_\phi).\end{aligned}$$

# Application: MSSM $\rightarrow$ SMEFT

## $d \leq 4$ operators & threshold corrections

- **Just 30** covariant diagrams  $\Rightarrow$  full 1-loop SUSY threshold corrections in the MSSM.
  - Full agreement with (much more involved) Feynman diagram calculation in *Bagger, Matchev, Pierce, Zhang [hep-ph/9606211]*.





# Application: MSSM $\rightarrow$ SMEFT

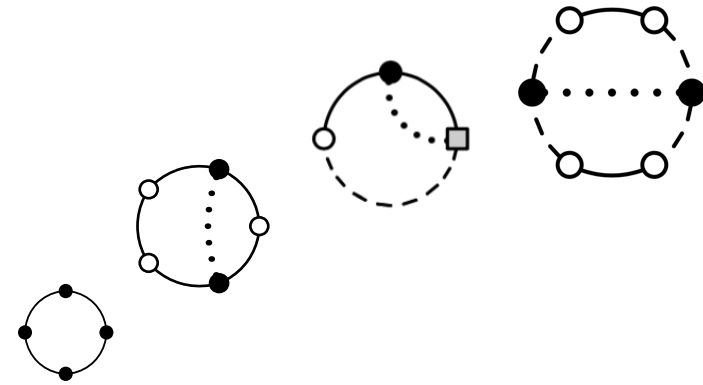
## d=6 operators & Higgs couplings

- The matching calculation can be extended to dim-6.
- d>4 operators lead to observable deviations from the SM.
  - Generically small for one-loop generated operators, but there can be parametric enhancement.
  - Example:  $hbb$  coupling shift due to  $\mathcal{L}_{(d=6)} \supset |\phi|^2 (\bar{\psi}_q \mathbf{C}_{d\phi} \psi_d) \cdot \phi$

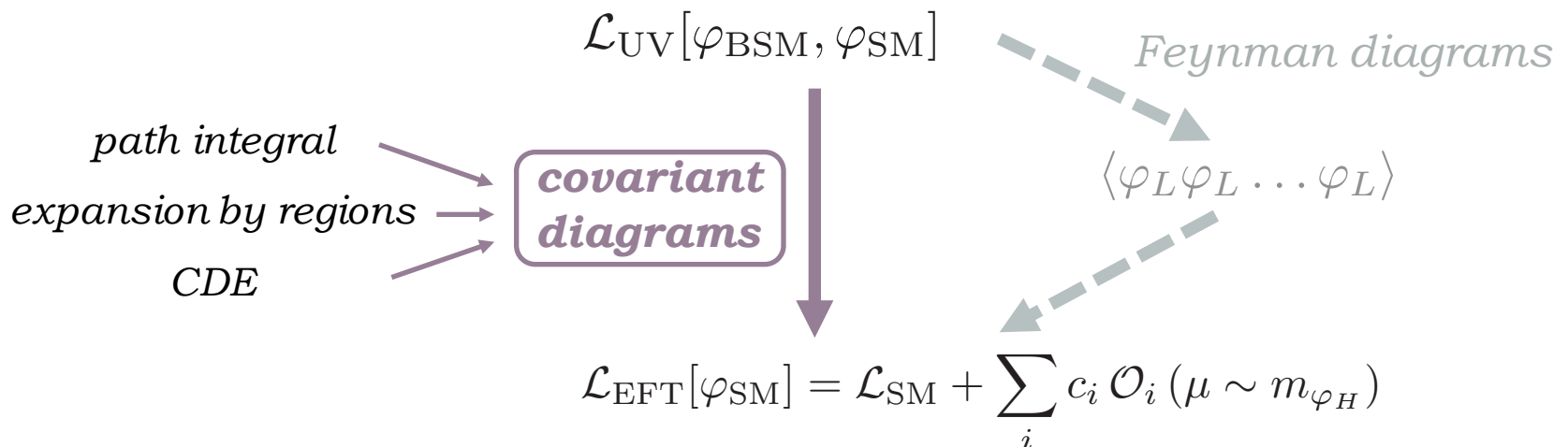
$$C_{b\phi} \simeq -\frac{y_b}{2M_{\Phi}^2} \left[ (g^2 + g'^2) - \frac{\tan\beta}{16\pi^2} y_t^4 \left( \frac{\mu}{M_s} \right) x_t (x_t^2 - 6) \right]$$

- One-loop matching contribution can be enhanced for large  $\tan\beta$ , and can dominate over tree-level contribution.
- Heavy sfermions and gauginos out of reach of direct search can lead to sizable  $hbb$  coupling deviation.

# Summary

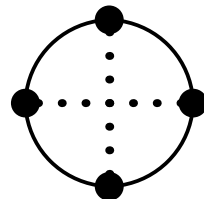


- Covariant diagrams: a systematic diagrammatic representation of functional approaches to one-loop matching, which
  - avoids the detour of computing correlation functions;
  - preserves gauge covariance;
  - can make EFT matching calculations easier.



# Backup

- Another example —



$$\begin{aligned}
 &= -i \cdot \frac{1}{4} \cdot \mathcal{I}[q^4]_i^4 \cdot \text{tr}(2P^\mu \cdot 2P^\nu \cdot 2P_\mu \cdot 2P_\nu) \\
 &= -\frac{1}{96\pi^2} \log \frac{M_i^2}{\mu^2} \text{tr}(P^\mu P^\nu P_\mu P_\nu)
 \end{aligned}$$

- This is part of

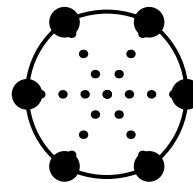
$$\begin{aligned}
 g^2 \text{tr}(G^{\mu\nu} G_{\mu\nu}) &= -\text{tr}([D^\mu, D^\nu][D_\mu, D_\nu]) = -\text{tr}([P^\mu, P^\nu][P_\mu, P_\nu]) \\
 &= 2 \text{tr}(P^2 P^2) - 2 \text{tr}(P^\mu P^\nu P_\mu P_\nu)
 \end{aligned}$$

- In fact, this is the only possible contribution to the 2<sup>nd</sup> term.

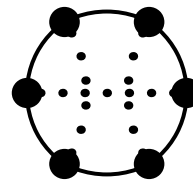
$$\Rightarrow \boxed{\mathcal{L}_{\text{eff}} \supset -\frac{g^2}{48\pi^2} \log \frac{M_i^2}{\mu^2} \left[ -\frac{1}{4} \text{tr}(G^{\mu\nu} G_{\mu\nu}) \right]} \Rightarrow \frac{g_{\text{eff}}^2(\mu)}{g^2(\mu)} = 1 + \frac{g^2}{48\pi^2} \overset{\text{Dynkin index}}{\uparrow} T(R_i) \log \frac{M_i^2}{\mu^2}$$

# Backup

- Similarly, we can compute **dim-6 pure-gauge operators**.
  - Need covariant diagrams with **6 P insertions**.
  - 2 ways of Lorentz contraction => **2 independent operators**.



$$= -i \frac{1}{6} \mathcal{I}[q^6]_i^6 \cdot 2^6 \text{tr}(P^\mu P^\nu P^\rho P_\mu P_\nu P_\rho)$$



$$= -i \frac{1}{2} \mathcal{I}[q^6]_i^6 \cdot 2^6 \text{tr}(P^\mu P^\nu P^\rho P_\nu P_\mu P_\rho)$$

$$\Rightarrow \mathcal{L}_{\text{eff}} \supset \frac{g^2}{480\pi^2} \frac{T(R_i)}{M_i^2} \left[ -\frac{1}{2} (D^\mu G_{\mu\nu}^a)^2 + \frac{g}{3!} f^{abc} G_\mu^{a\nu} G_\nu^{b\rho} G_\rho^{c\mu} \right]$$

# Backup

- The MSSM  $\mathbf{U}$  matrix (schematic, assuming  $R$ -parity):

*heavy fields*

	$\Phi$	$\tilde{q}$	$\tilde{u}$	$\tilde{d}$	$\tilde{l}$	$\tilde{e}$	$\tilde{\chi}$	$\tilde{g}$	$\tilde{W}$	$\tilde{B}$	$\phi$	$q$	$u$	$d$	$l$	$e$	$G$	$W$	$B$	
$\Phi$	$\varphi^2$										$v^2, \varphi^2$	$u, d$	$q$	$q$	$e$	$l$		$D\Phi$	$D\Phi$	
$\tilde{q}$		$\varphi^2$	$\varphi$	$\varphi$			$u, d$	$q$	$q$	$q$										
$\tilde{u}$		$\varphi$	$\varphi^2$	$\Phi\phi$			$q$	$u$		$u$										
$\tilde{d}$		$\varphi$	$\Phi\phi$	$\varphi^2$			$q$	$d$		$d$										
$\tilde{l}$					$\varphi^2$	$\varphi$	$e$		$l$	$l$										
$\tilde{e}$					$\varphi$	$\varphi^2$	$l$			$e$										
$\tilde{\chi}$		$u, d$	$q$	$q$	$e$	$l$			$\varphi$	$\varphi$										
$\tilde{g}$		$q$	$u$	$d$																
$\tilde{W}$		$q$			$l$		$\varphi$													
$\tilde{B}$		$q$	$u$	$d$	$l$	$e$	$\varphi$													
$\phi$	$v^2, \varphi^2$										$\varphi^2$	$u, d$	$q$	$q$	$e$	$l$		$D\phi$	$D\phi$	
$q$	$u, d$										$u, d$		$\varphi$	$\varphi$			$q$	$q$	$q$	
$u$	$q$										$q$	$\varphi$					$u$		$u$	
$d$	$q$										$q$	$\varphi$					$d$		$d$	
$l$	$e$										$e$				$\varphi$			$l$	$l$	
$e$	$l$										$l$				$\varphi$				$e$	
$G$												$q$	$u$	$d$			$G_{\mu\nu}$			
$W$	$D\Phi$										$D\phi$	$q$			$l$		$W_{\mu\nu}, \phi^2, \Phi^2$			
$B$	$D\Phi$										$D\phi$	$q$	$u$	$d$	$l$	$e$			$B_{\mu\nu}, \phi^2, \Phi^2$	

*light fields*

# Backup

- The MSSM **U** matrix (schematic, assuming *R*-parity):

	$\Phi$	$\tilde{q}$	$\tilde{u}$	$\tilde{d}$	$\tilde{l}$	$\tilde{e}$	$\tilde{\chi}$	$\tilde{g}$	$W$	$B$	$\phi$	$q$	$u$	$d$	$l$	$e$	$G$	$W$	$B$
<i>heavy fields</i>	$\Phi$	$\varphi^2$									$v^2, \varphi^2$	$u, d$	$q$	$q$	$e$	$l$		$D\Phi$	$D\Phi$
	$\tilde{q}$	$\varphi^2$	$\varphi$	$\varphi$			$u, d$	$q$	$q$	$q$									
	$\tilde{u}$		$\varphi$	$\varphi^2$	$\Phi\phi$		$q$	$u$	$u$	$u$									
	$\tilde{d}$		$\varphi$	$\Phi\phi$	$\varphi^2$		$q$	$d$	$d$	$d$									
	$\tilde{l}$					$\varphi^2$	$e$	$l$	$l$	$l$									
	$\tilde{e}$					$\varphi$	$\varphi^2$	$l$	$l$	$l$									
	$\tilde{\chi}$		$u, d$	$q$	$q$	$e$	$l$		$\varphi$										
	$\tilde{g}$		$q$	$u$	$d$														
	$\tilde{W}$		$q$			$l$	$\varphi$												
	$\tilde{B}$		$q$	$u$	$d$	$l$	$e$	$\varphi$											
<i>light fields</i>	$\phi$	$v^2, \varphi^2$									$\varphi^2$	$u, d$	$q$	$q$	$e$	$l$		$D\phi$	$D\phi$
	$q$	$u, d$									$u, d$		$\varphi$	$\varphi$			$q$	$q$	$q$
	$u$	$q$									$q$	$\varphi$					$u$		$u$
	$d$	$q$									$q$	$\varphi$					$d$		$d$
	$l$	$e$									$e$				$\varphi$			$l$	$l$
	$e$	$l$									$l$								$l$
	$G$											$q$	$u$	$d$			$G_{\mu\nu}$		
	$W$	$D\Phi$									$D\phi$	$q$			$l$			$W_{\mu\nu}, \phi^2, \Phi^2$	
	$B$	$D\Phi$									$D\phi$	$q$	$u$	$d$	$l$	$e$			$B_{\mu\nu}, \phi^2, \Phi^2$

squark-gluino interaction with background quark field

$$U_{\tilde{q}\tilde{g}} = \sqrt{2} g_3 \begin{pmatrix} (T^B \bar{\psi}_q^c)_{i\alpha} \\ (\bar{\psi}_q T^B)_{i\alpha} \end{pmatrix}$$

# Backup

- The MSSM  $\mathbf{U}$  matrix (schematic, assuming  $R$ -parity):

heavy fields

light fields

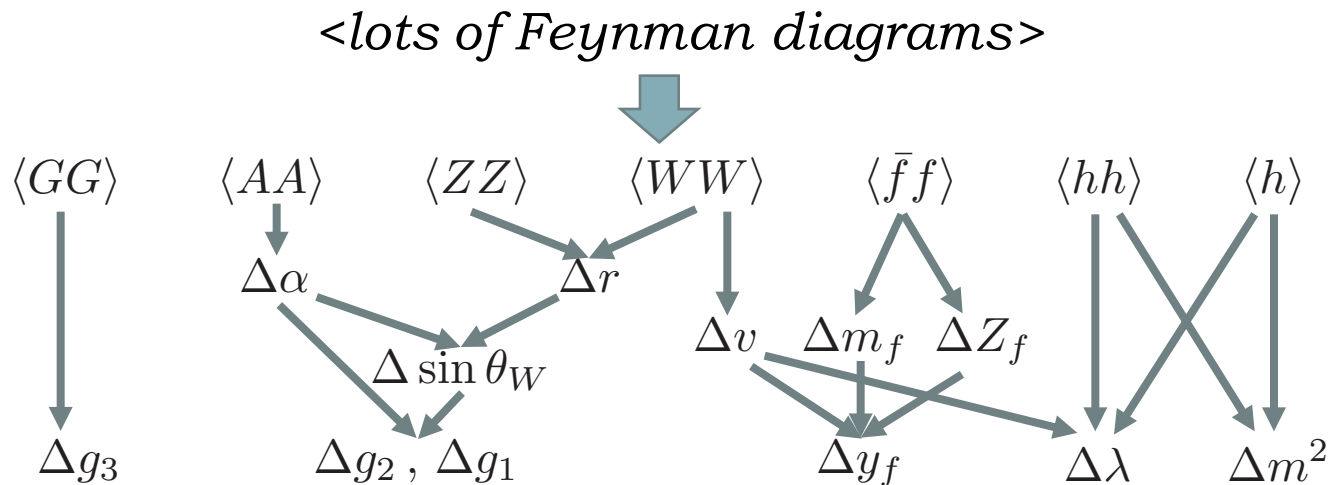
	$\Phi$	$\tilde{q}$	$\tilde{u}$	$\tilde{d}$	$\tilde{l}$	$\tilde{e}$	$\tilde{\chi}$	$\tilde{g}$	$W$	$B$	$\phi$	$q$	$u$	$d$	$l$	$e$	$G$	$W$	$B$
$\Phi$	$\varphi^2$										$v^2, \varphi^2$	$u, d$	$q$	$q$	$e$	$l$		$D\Phi$	$D\Phi$
$\tilde{q}$		$\varphi^2$	$\varphi$	$\varphi$			$u, d$	$q$	$q$	$q$									
$\tilde{u}$																			
$\tilde{d}$																			
$\tilde{l}$																			
$\tilde{e}$																			
$\tilde{\chi}$																			
$\tilde{g}$																			
$\tilde{W}$																			
$\tilde{B}$																			
$\phi$	$v^2, \varphi^2$										$\varphi^2$	$u, d$	$q$	$q$	$e$	$l$		$D\phi$	$D\phi$
$q$	$u, d$										$u, d$		$\varphi$	$\varphi$			$q$	$q$	$q$
$u$	$q$										$q$		$\varphi$				$u$		$u$
$d$	$q$										$q$		$\varphi$				$d$		$d$
$l$	$e$										$e$				$\varphi$			$l$	$l$
$e$	$l$										$l$				$\varphi$				$e$
$G$												$q$	$u$	$d$			$G_{\mu\nu}$		
$W$	$D\Phi$										$D\phi$	$q$			$l$		$W_{\mu\nu}, \phi^2, \Phi^2$		
$B$	$D\Phi$										$D\phi$	$q$	$u$	$d$	$l$	$e$			$B_{\mu\nu}, \phi^2, \Phi^2$

heavy Higgs-light quark interaction with background light quark fields

$$U_{\Phi q} = \begin{pmatrix} -\delta_{\alpha}^{\beta} \bar{\psi}_d^j \mathbf{y}_d \tan \beta & \epsilon_{\beta\alpha} \bar{\psi}_{uj}^c \mathbf{y}_u^* \cot \beta \\ \epsilon^{\beta\alpha} \bar{\psi}_u^j \mathbf{y}_u \cot \beta & -\delta_{\beta}^{\alpha} \bar{\psi}_{dj}^c \mathbf{y}_d^* \tan \beta \end{pmatrix}$$

# Backup

- Old way of doing this calculation:
  - See classic paper *Bagger, Matchev, Pierce, Zhang [hep-ph/9606211]*.



- In the decoupling limit, we can use covariant diagrams to easily reproduce all their results.



# Backup

- The matching calculation can be extended to dim-6.
- $d > 4$  operators lead to observable deviations from the SM.
  - Example: Higgs coupling shifts

$$\mathcal{L}_{(d=6)} \supset |\phi|^2 (\bar{\psi}_q \mathbf{C}_{d\phi} \psi_d) \cdot \phi + |\phi|^2 (\bar{\psi}_l \mathbf{C}_{e\phi} \psi_e) \cdot \phi + \text{h.c.}$$

$$\delta\kappa_b = -\frac{C_{b\phi} v^2}{y_b^{\text{eff}}}, \quad \delta\kappa_\tau = -\frac{C_{\tau\phi} v^2}{y_\tau^{\text{eff}}}$$

- One-loop matching contributions to these operators can be parametrically enhanced for large  $\tan\beta$ , and can dominate over tree-level contributions.

$$C_{b\phi} \simeq -\frac{y_b}{2M_\Phi^2} \left[ (g^2 + g'^2) - \frac{\tan\beta}{16\pi^2} y_t^4 \left( \frac{\mu}{M_s} \right) x_t (x_t^2 - 6) \right]$$

# Backup

- In our numerical analysis, we require:
  - Consistent matching onto SM parameters extracted at weak scale
  - Bottom-tau Yukawa unification

$$C_{b\phi} \simeq -\frac{y_b}{2M_\Phi^2} \left[ (g^2 + g'^2) - \frac{\tan\beta}{16\pi^2} y_t^4 \left( \frac{\mu}{M_s} \right) x_t (x_t^2 - 6) \right]$$

- Higgs coupling measurements can probe regions of SUSY parameter space inaccessible to direct searches!

