``HKIAS Program on High Energy Physics'', Jan. 2018, HKIAS

TESTING NATURALNESS

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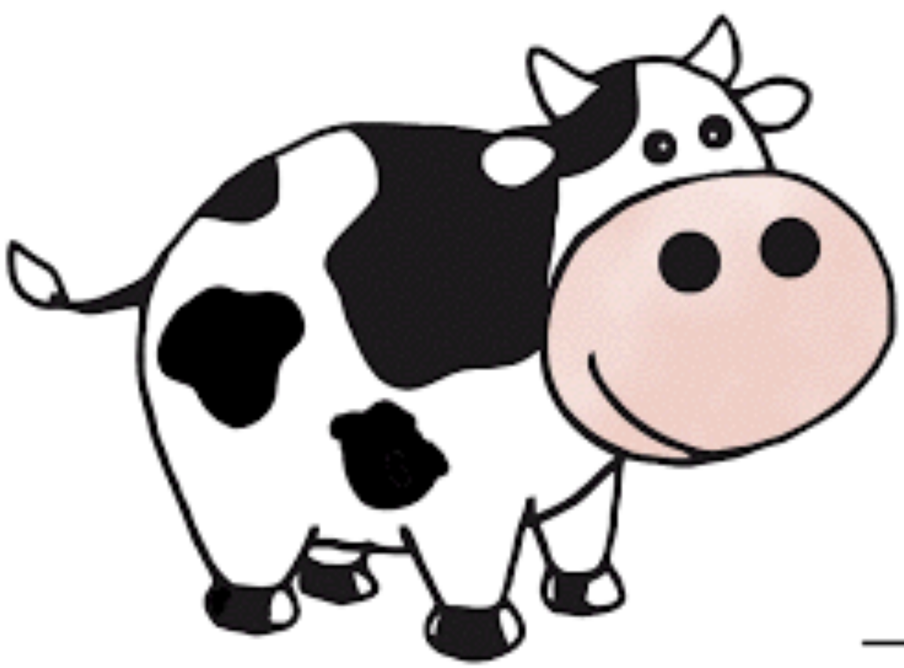
Based on

[C. Chen, J. Hajer, TL, I. Low and H. Zhang, arXiv: 1705.07743 (JHEP 2017)]

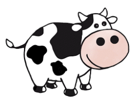
and in-progress work with J. Bernon, J. Hajer, Y. Jiang, I. Low, et. al.

Naturalness Problem in Particle Physics

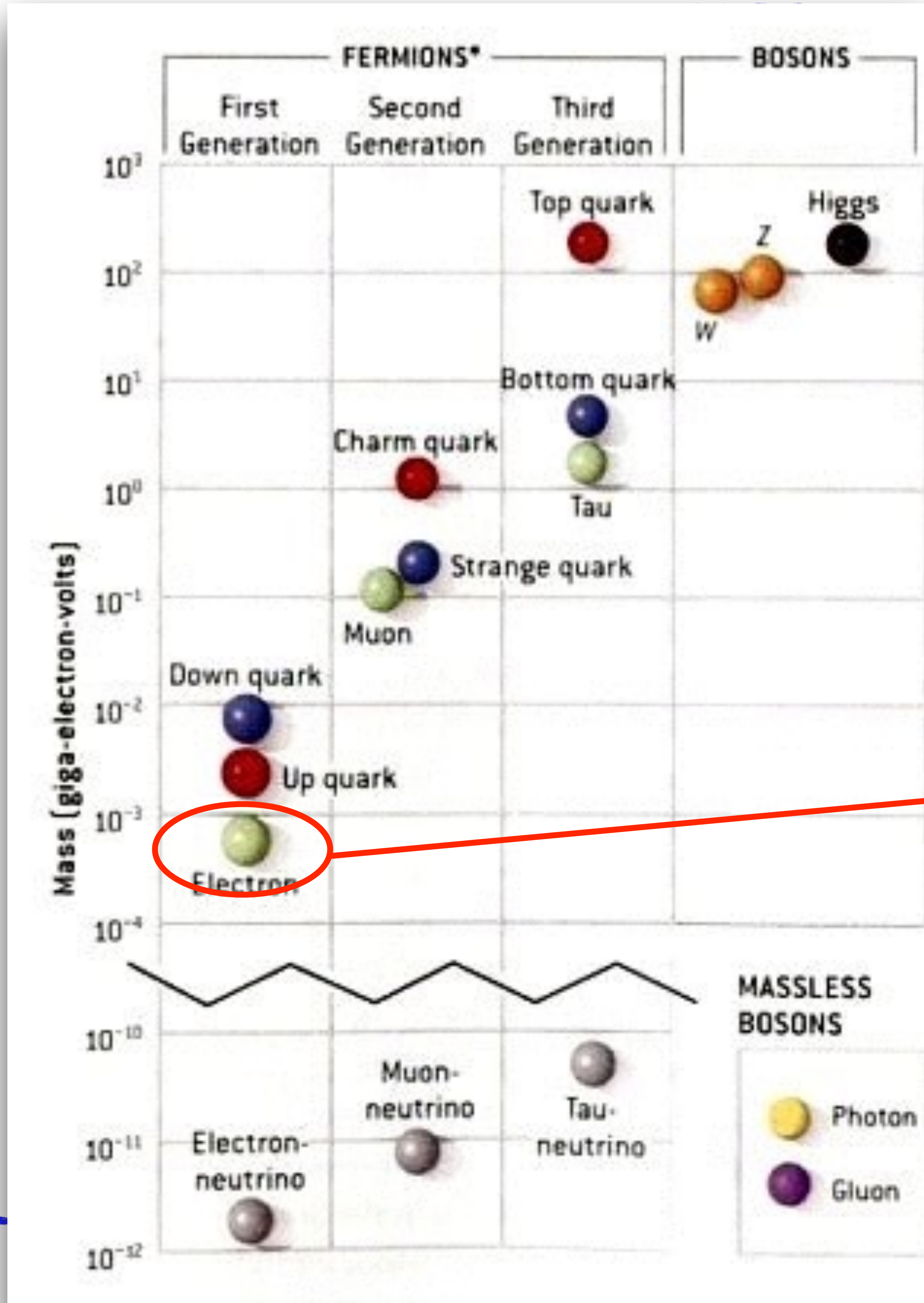
A large discrepancy between two energy scales strongly correlated



Unnatural!



Naturalness Problem in Particle Physics



EW scale ~ 100 GeV

The electron mass is orders smaller than the EW scale — Unnatural!

Naturalness Problem in Particle Physics

Observations: (1) zero mass limit => chiral symmetry; (2) chiral symmetry breaking => a logarithmically divergent contribution from the cutoff at quantum level

$$m_e \sim m^0_e \,[1 + 3\alpha/4\pi \; 1n(\Lambda/m_e)]$$

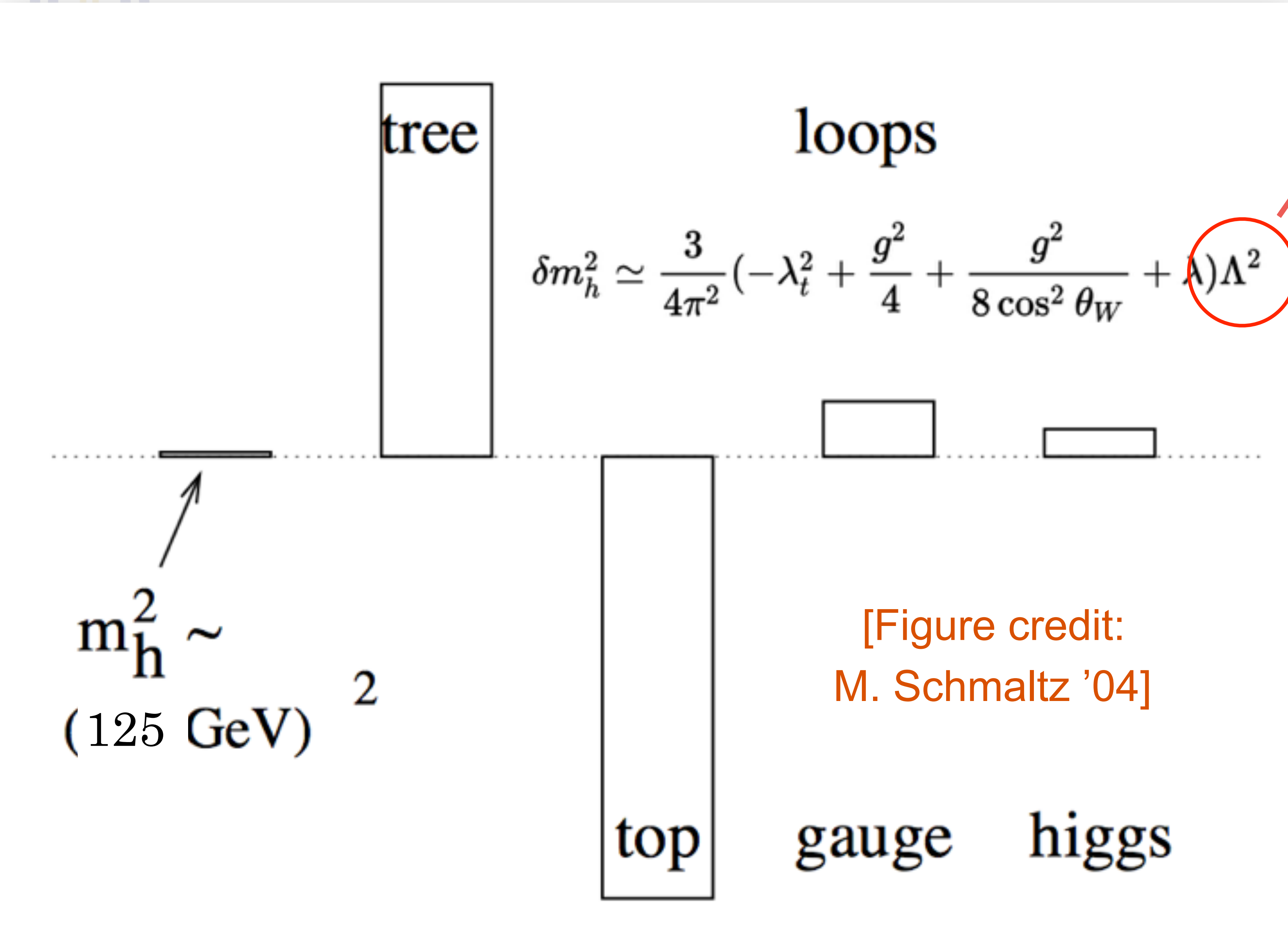
t'Hooft statement for "technical naturalness"

If the turning off of an ``unnatural" parameter results in an enhanced symmetry which can be (approximately) softly broken, this parameter is ``technically" natural.

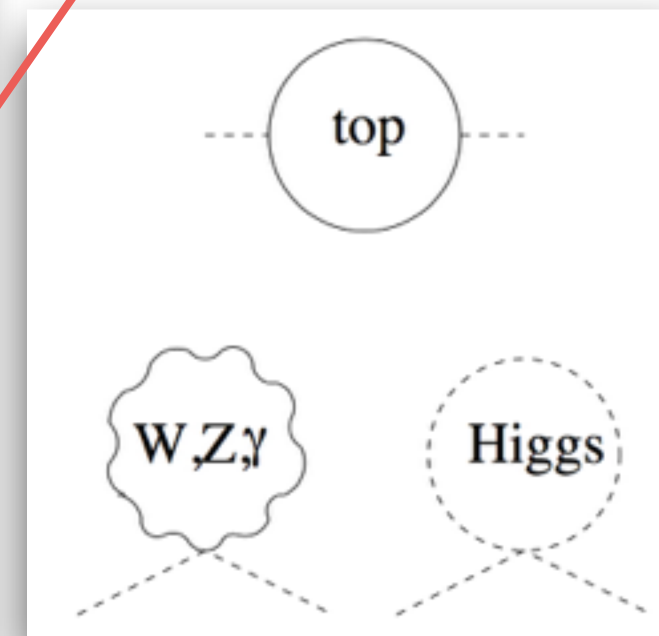
=> The smallness of m_e: not natural, but technically natural !

However, not all particles have a masse technically natural in the SM

Naturalness Problem in Particle Physics



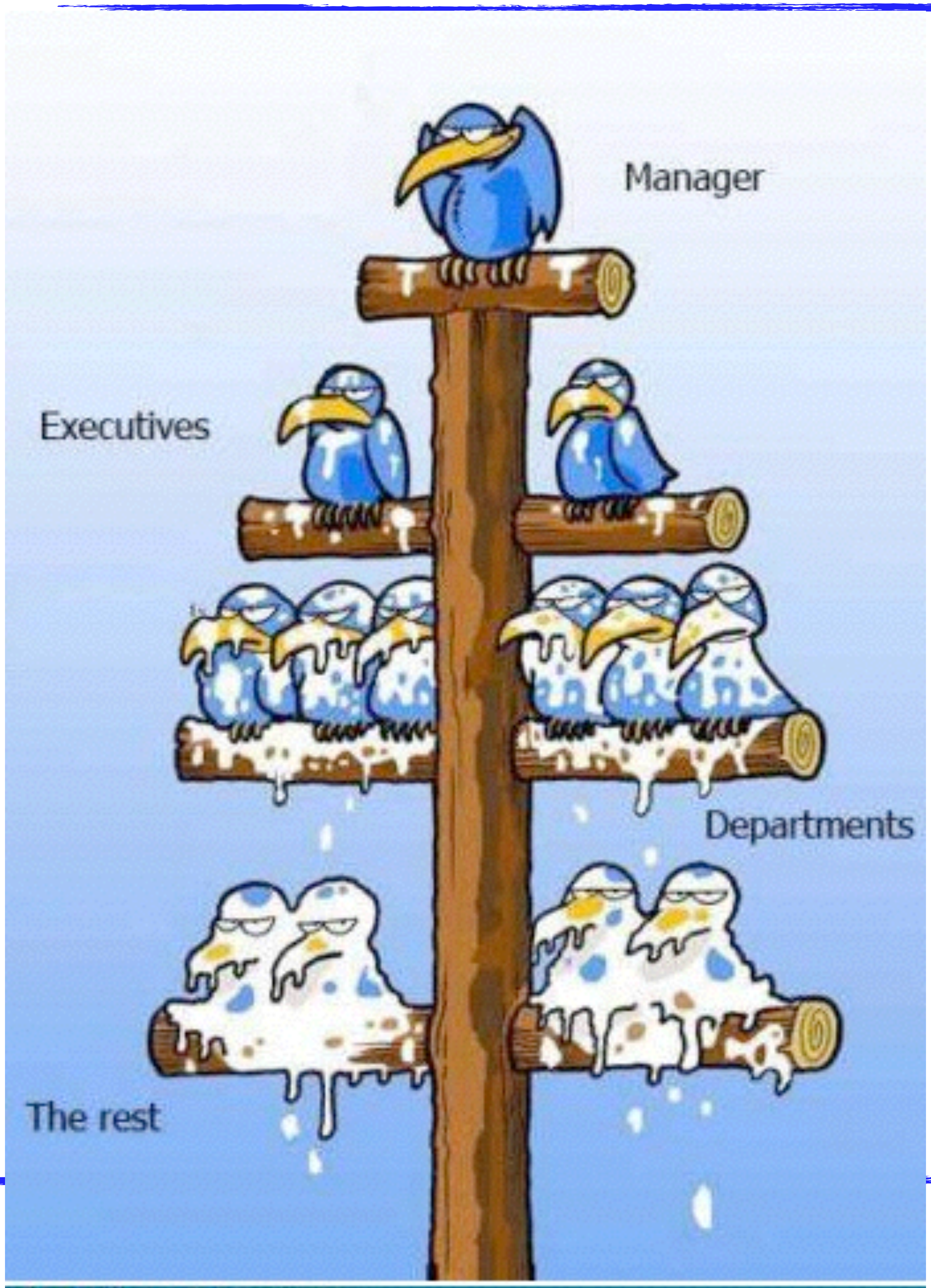
$\sim M_{Planck}$^2



A hierarchy of 30 orders!

- Unnatural!

``Hierarchy'' Problem



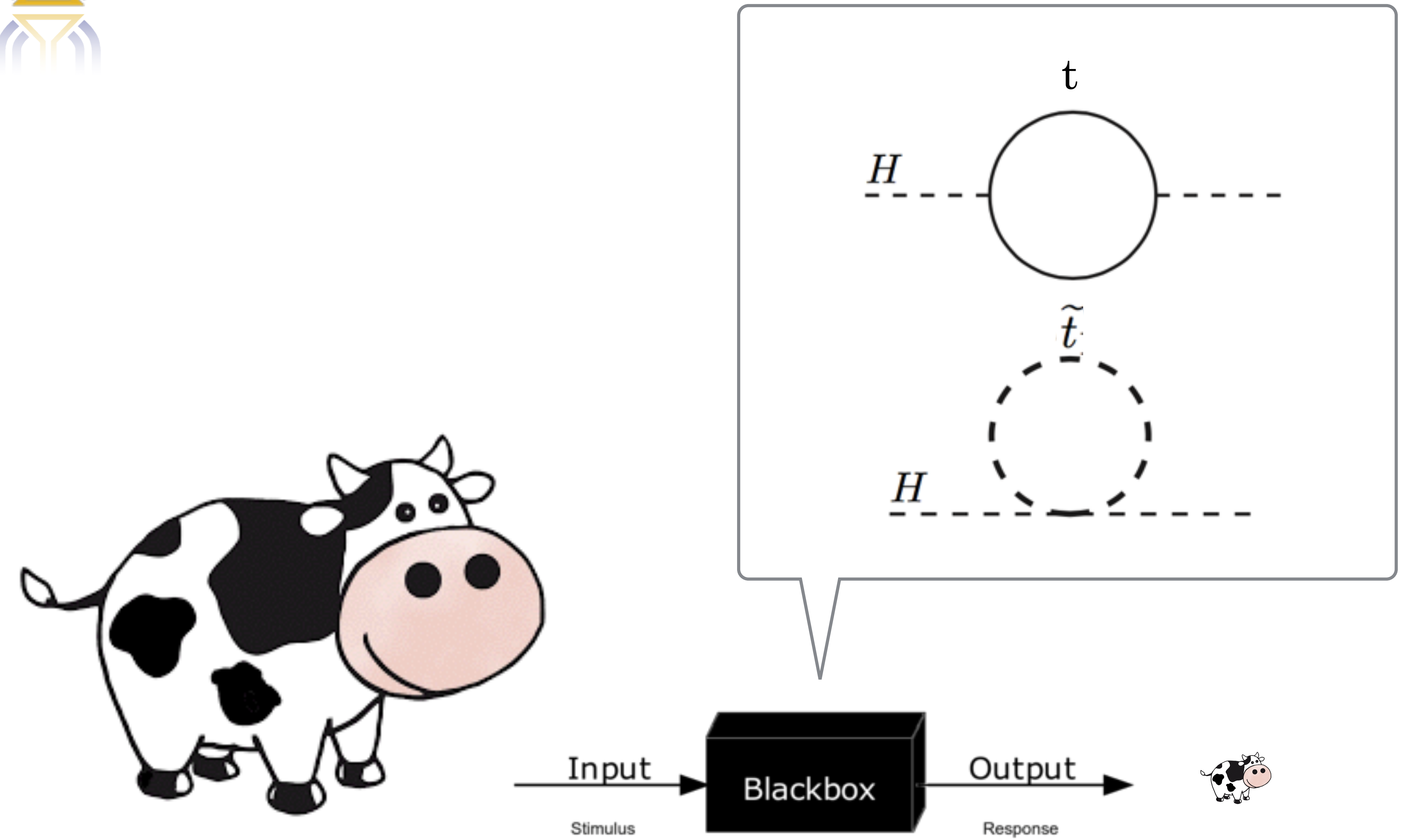
Picture credit: www

→ Planck scale

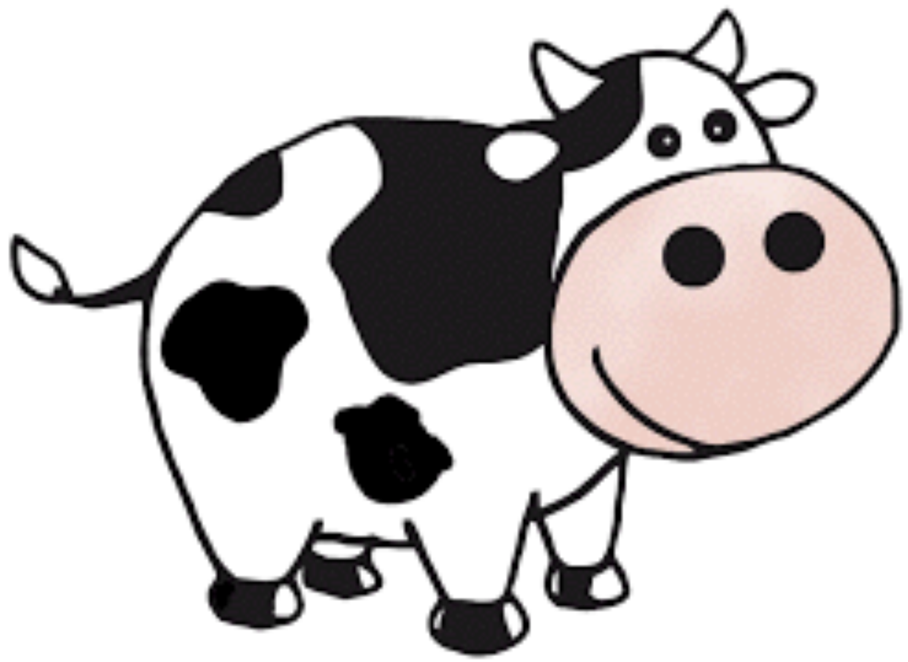
Is there a ``technically natural'' solution?

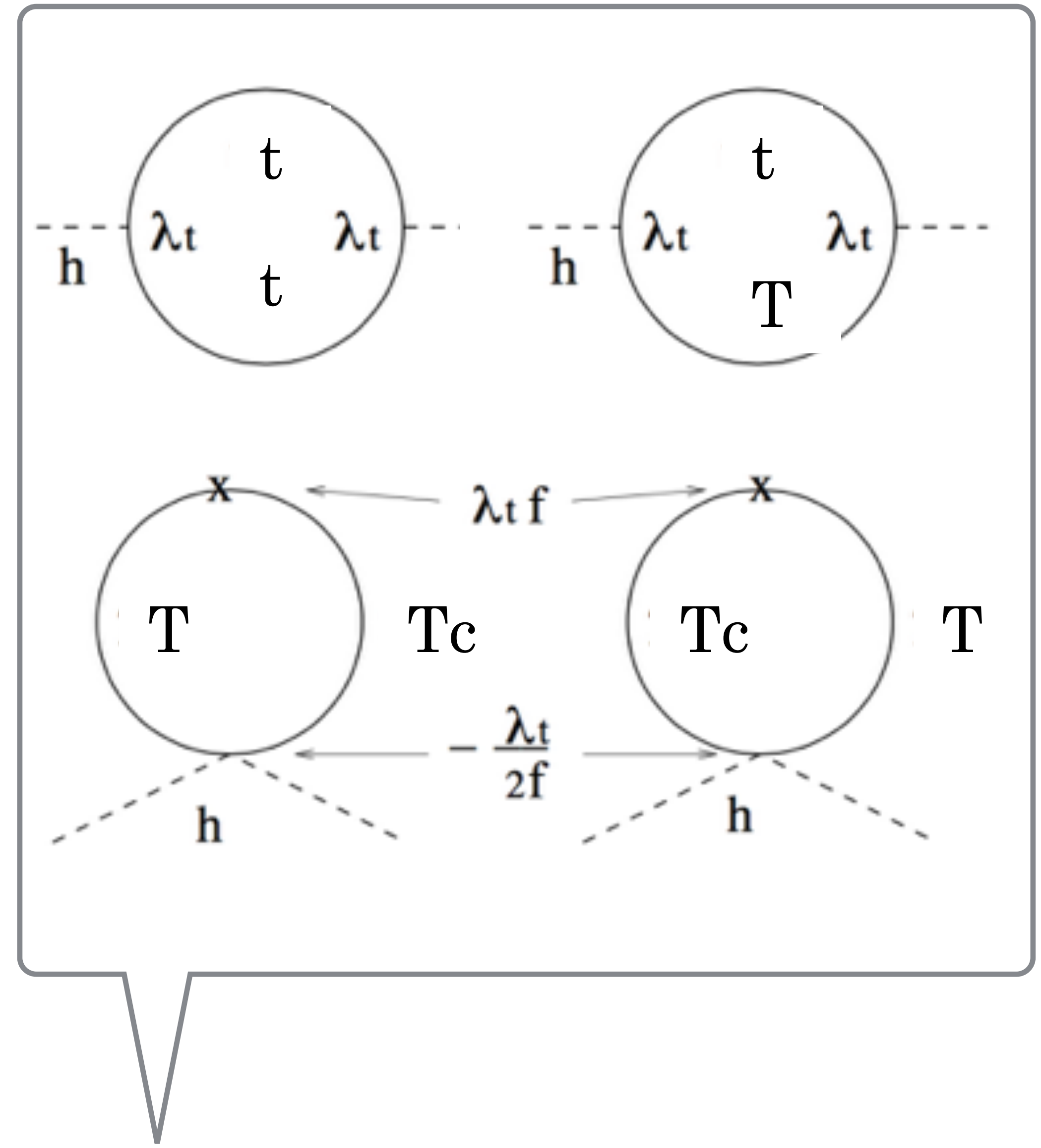
→ EW scale

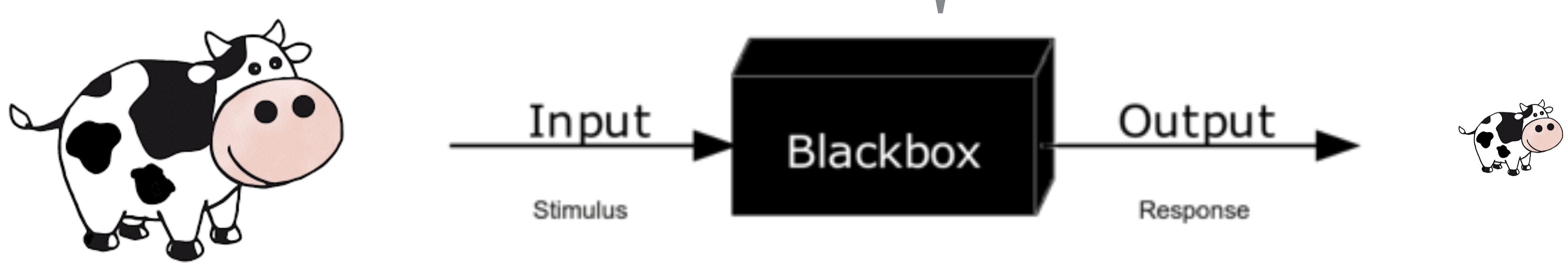
Solution I - Fermionic Symmetry (Supersymmetry)



Solution II - Bosonic Symmetry (Little/Twin Higgs)







Some Wisdoms

The underlying symmetry =>

(1) orders a spectrum of ``partner" particles

(2) predicts a sum rule for canceling quadratic divergence in mh^2, either completely or at a leading quantum level

Motivated an amount of searches for ``partner" particles at, e.g., LEP, Tevatron, LHC, for decades

A must-be-done task post the discovery of any partner-like particle:

Measuring the sum rule

One might ask:

why worry about step 2) when we have seen no empirical sign of a top partner??

The answer:

We live in a unique juncture in history!

As part of the planning for a new generation of particle accelerators, we would like to know the ability of a new hadron machine to unambiguously establish the Naturalness principle, should a top partner-like particle be discovered.

Ian Low, 2017 CERN-KCK workshop

Simplified Model

SM + one pair of vector-like (weak isospin singlet) top partners

$$\mathcal{L}_U = u_3^c \left(c_0 f U + c_1 H q_3 + \frac{c_2}{f} H^2 U + \ldots \right) + U^c \left(\hat{c}_0 f U + \hat{c}_1 H q_3 + \frac{\hat{c}_2}{f} H^2 U + \ldots \right) + \text{h.c.} \, .$$

Simplified Model

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$$\mathcal{L}_U = u_3^c \left(c_0 f U + c_1 H q_3 + \frac{c_2}{f} H^2 U + \dots \right) + U^c \left(\hat{c}_0 f U + \hat{c}_1 H q_3 + \frac{\hat{c}_2}{f} H^2 U + \dots \right) + \text{h.c.}\ .$$

Model	Coset		SU(2)	c_0	c_1	c_2	$\hat{c}_0$	$\hat{c}_1$	$\hat{c}_2$
Toy model	$\frac{SU(3)}{SU(2)}$	[22]	**1**	λ_1	$-\lambda_1$	$-\lambda_1$	λ_2	0	0
Simplest	$\left(\frac{SU(3)}{SU(2)}\right)^2$	[23]	**1**	λ	$-\lambda$	$-\lambda$	λ	λ	$-\lambda$
Littlest Higgs	$\frac{SU(5)}{SO(5)}$	[14]	**1**	λ_1	$-\sqrt{2}i\lambda_1$	$-2\lambda_1$	λ_2	0	0
Custodial	$\frac{SO(9)}{SO(5)\,SO(4)}$	[20]	**2**	y_1	$\frac{i}{\sqrt{2}}y_1$	$-\frac{1}{2}y_1$	y_2	0	0
T-parity invariant	$\frac{SU(3)}{SU(2)}$	[19]	**1**	λ	$-\lambda$	$-\lambda$	$-\lambda$	$-\lambda$	λ
T-parity invariant	$\frac{SU(5)}{SO(5)}$	[19]	**1**	λ	$-\sqrt{2}i\lambda$	-2λ	$-\lambda$	$-\sqrt{2}i\lambda$	2λ
Mirror twin Higgs	$\frac{SU(4)\,U(1)}{SU(3)\,U(1)}$	[24]	**1**	0	$i\lambda_t$	0	λ_t	0	$-\lambda_t$

Naturalness Sum Rule - Mass Basis Before EWSB

$$\mathcal{L}_{T'} = m_{T'} T'^c T' + \lambda_{t'} H t'^c t' + \lambda_{T'} H T'^c t' + \frac{\alpha_{t'}}{2m_{T'}} H^2 t'^c T' + \frac{\alpha_{T'}}{2m_{T'}} H^2 T'^c T' + \frac{\beta_{t'}}{6m_{T'}^2} H^3 t'^c t' + \frac{\beta_{T'}}{6m_{T'}^2} H^3 T'^c t' + \mathcal{O}\left(H^4\right) + \text{h.c.}$$

Quadratically divergent contribution to the C-W potential from top sector

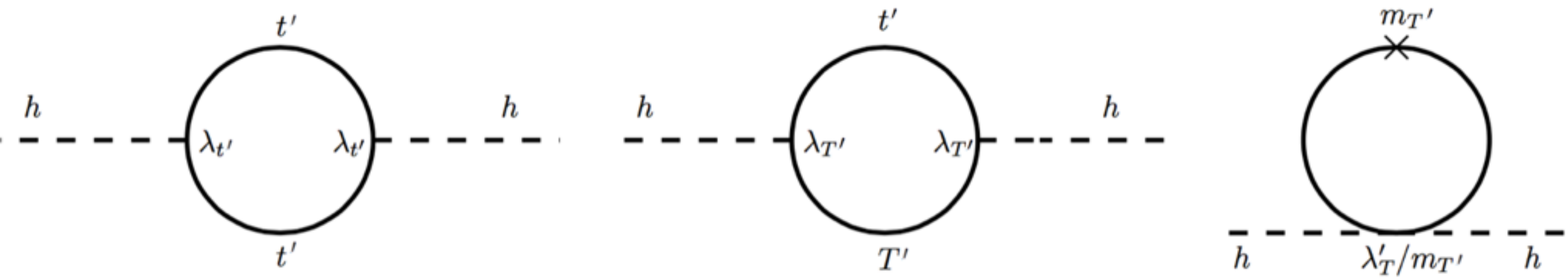
$$\frac{1}{16\pi^2} \Lambda^2 \, \mathrm{tr}\, \mathcal{M}(H)^\dagger \mathcal{M}(H)$$

Require coefficient in H^2 to vanish =>

$$\boxed{\alpha_{T'} = -\left|\lambda_{T'}\right|^2 - \left|\lambda_{t'}\right|^2}$$

Testing the Sum Rule -Traditional Wisdom

$$\alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2$$



(a) (b) (c)

$$\mathcal{L}_{T'} = m_{T'}T'^cT' + \lambda_{t'}Ht'^ct' + \lambda_{T'}HT'^ct' + \frac{\alpha_{t'}}{2m_{T'}}H^2t'^cT' + \frac{\alpha_{T'}}{2m_{T'}}H^2T'^cT'$$
$$+ \frac{\beta_{t'}}{6m_{T'}^2}H^3t'^ct' + \frac{\beta_{T'}}{6m_{T'}^2}H^3T'^ct' + \mathcal{O}\left(H^4\right) + \text{h.c.}$$

Traditional wisdom - reconstruct the three couplings

Testing the Sum Rule -Traditional Wisdom

Top quarks and electroweak symmetry breaking in little Higgs models

Maxim Perelstein, Michael E. Peskin, and Aaron Pierce
Phys. Rev. D **69**, 075002 – Published 8 April 2004

$$\alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2$$

$$\alpha_{T'} = \lambda_{T'} \frac{m_T}{f}$$

How difficult!

Testing the Sum Rule -Traditional Wisdom

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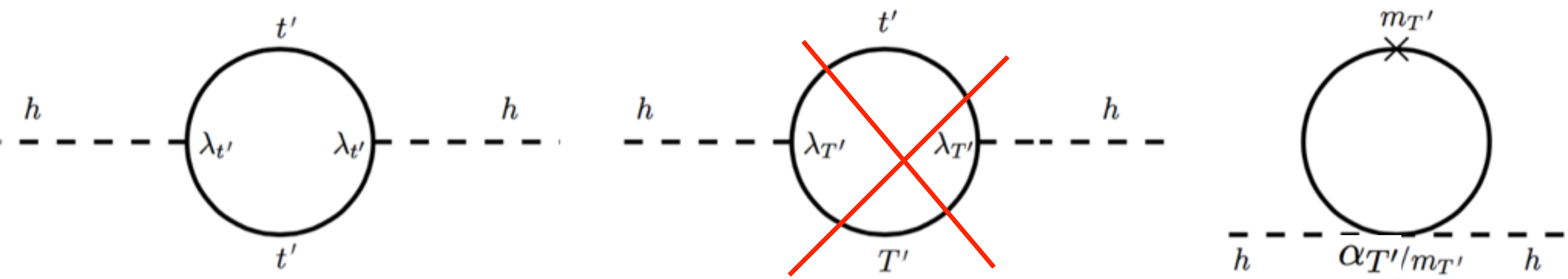


Not representative! E.g., little Higgs with T-parity

Naturalness Sum Rule - Mass Basis After EWSB

$$\mathcal{L}_T = m_T T^c T + \lambda_t v t^c t + \frac{\lambda_t}{\sqrt{2}} h t^c t + \frac{\lambda_T}{\sqrt{2}} h T^c t + \frac{a_t v}{\sqrt{2} m_T} h t^c T + \frac{a_T v}{\sqrt{2} m_T} h T^c T$$

$$+ \frac{\alpha_t}{4 m_T} h^2 t^c T + \frac{\alpha_T}{4 m_T} h^2 T^c T + \frac{b_t v}{4 m_T^2} h^2 t^c t + \frac{b_T v}{4 m_T^2} h^2 T^c t + \mathcal{O}\left(h^3, \frac{v^2}{m_T^2}\right) + \text{h.c.}$$

$$\alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2$$

$$a_T = \alpha_{T'} + |\lambda_{T'}|^2$$

$$a_T = -|\lambda_t|^2 + \mathcal{O}\left(\frac{v^2}{m_T^2}\right)$$

- Leading order - involves diagonal Yukawa couplings only
- Could be generalized with more top partners introduced:

$$\sum_i a_{T_i} = -|\lambda_t|^2 + \mathcal{O}\left(\frac{v^2}{m_{T_i}^2}\right)$$

- No measurement of quartic coupling is needed => a more feasible guideline

Collider Strategy - Colored Top Partners

- With this guideline, we are able to study various benchmark scenarios, e.g., little higgs models without T parity

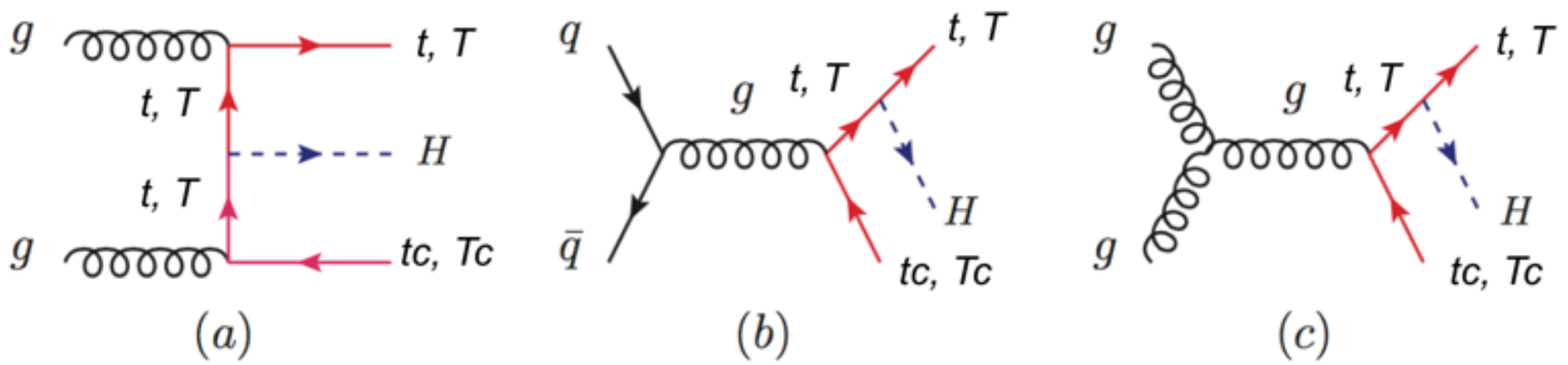
- Introduce a ``naturalness parameter"

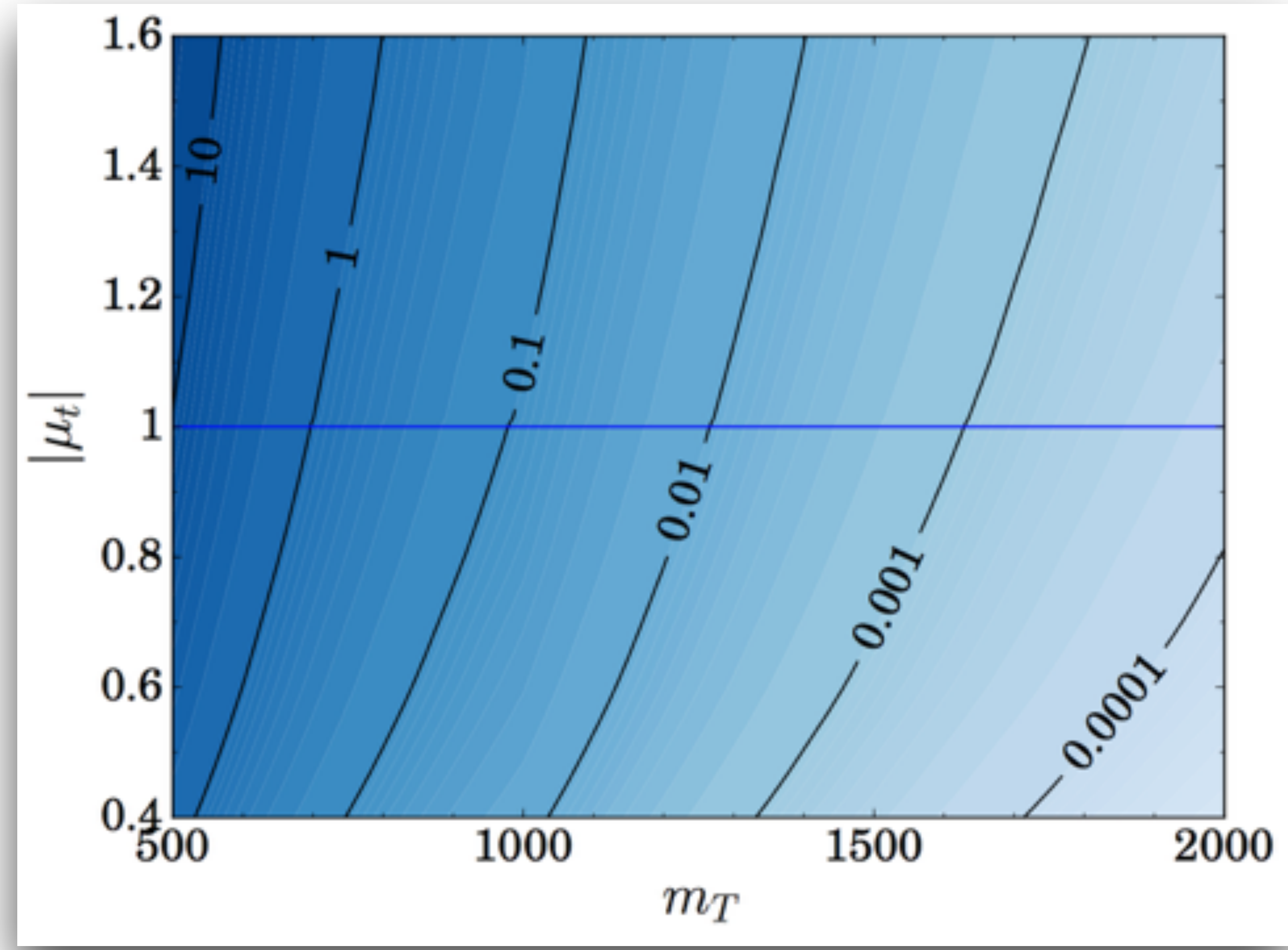
$$\mu = -\frac{\Delta m_H^2|_{\rm NP}}{\Delta m_H^2|_{\rm SM}} \quad => \quad \mu_t = -\frac{a_T}{\lambda_t^2} + \mathcal{O}\Big(\frac{v^2}{m_T^2}\Big)$$

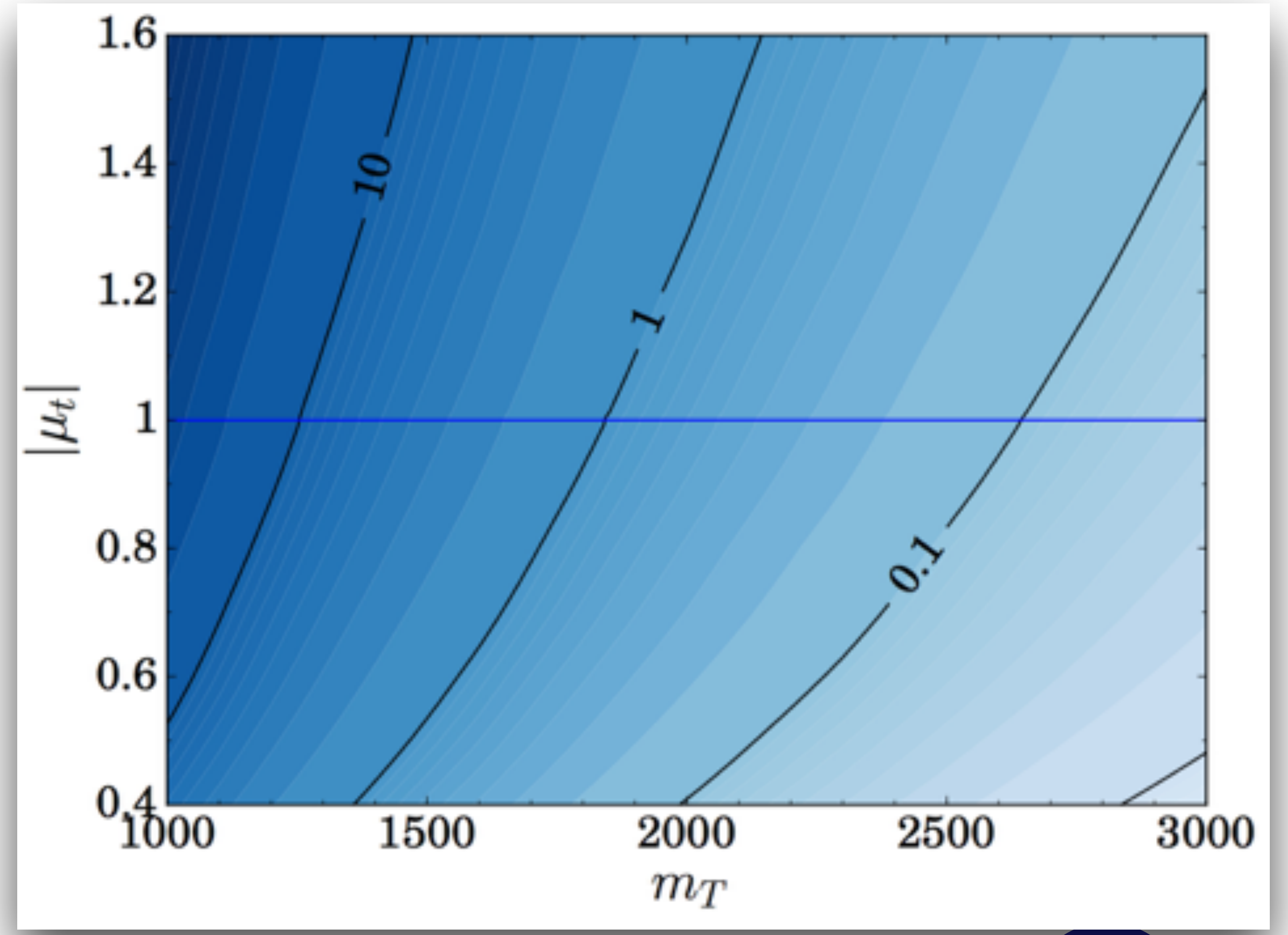
$$\mu|_{\rm nat} \equiv 1$$

- Test the sum rule <=> measure the ``naturalness parameter"

TTh Production



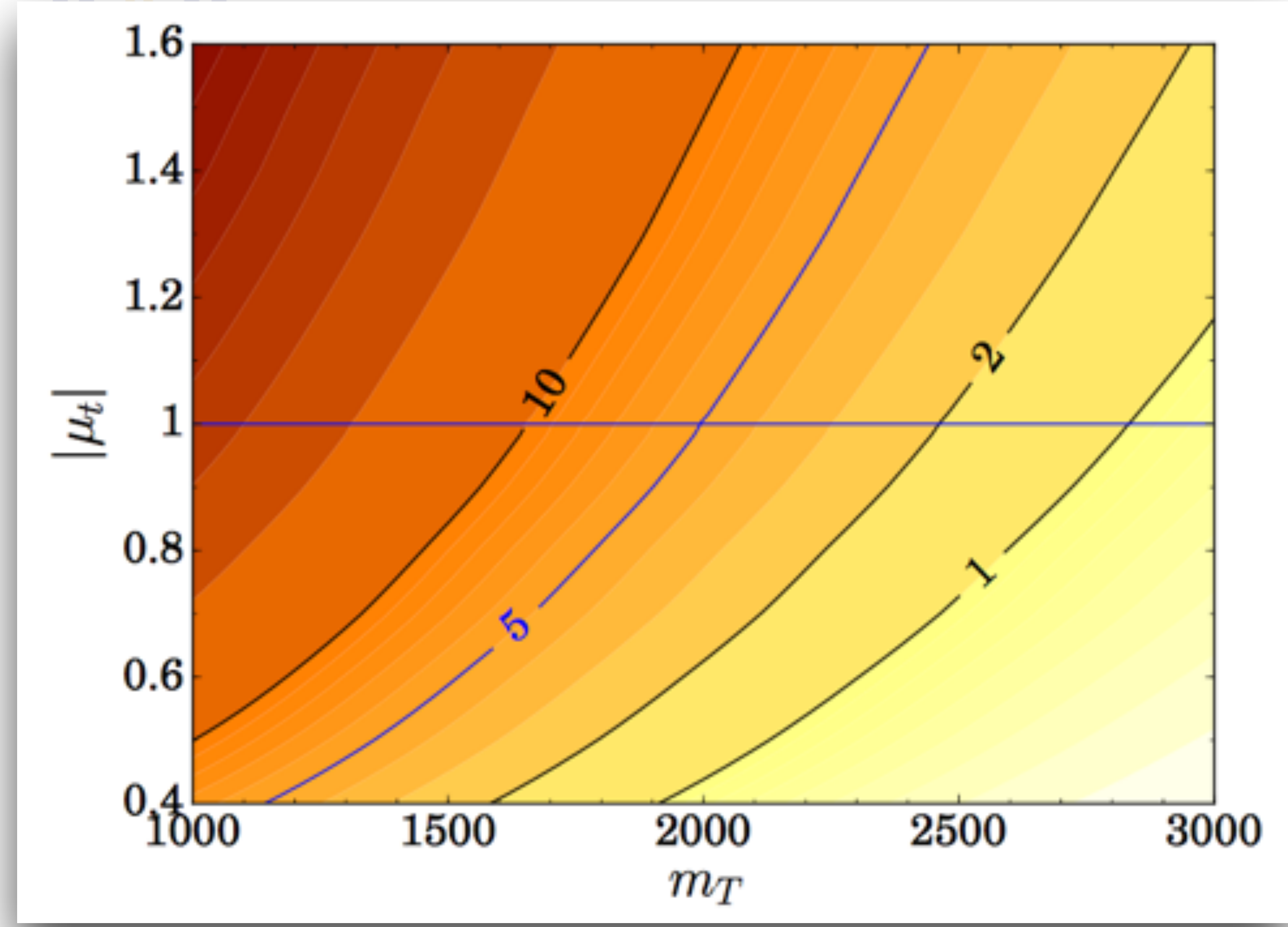


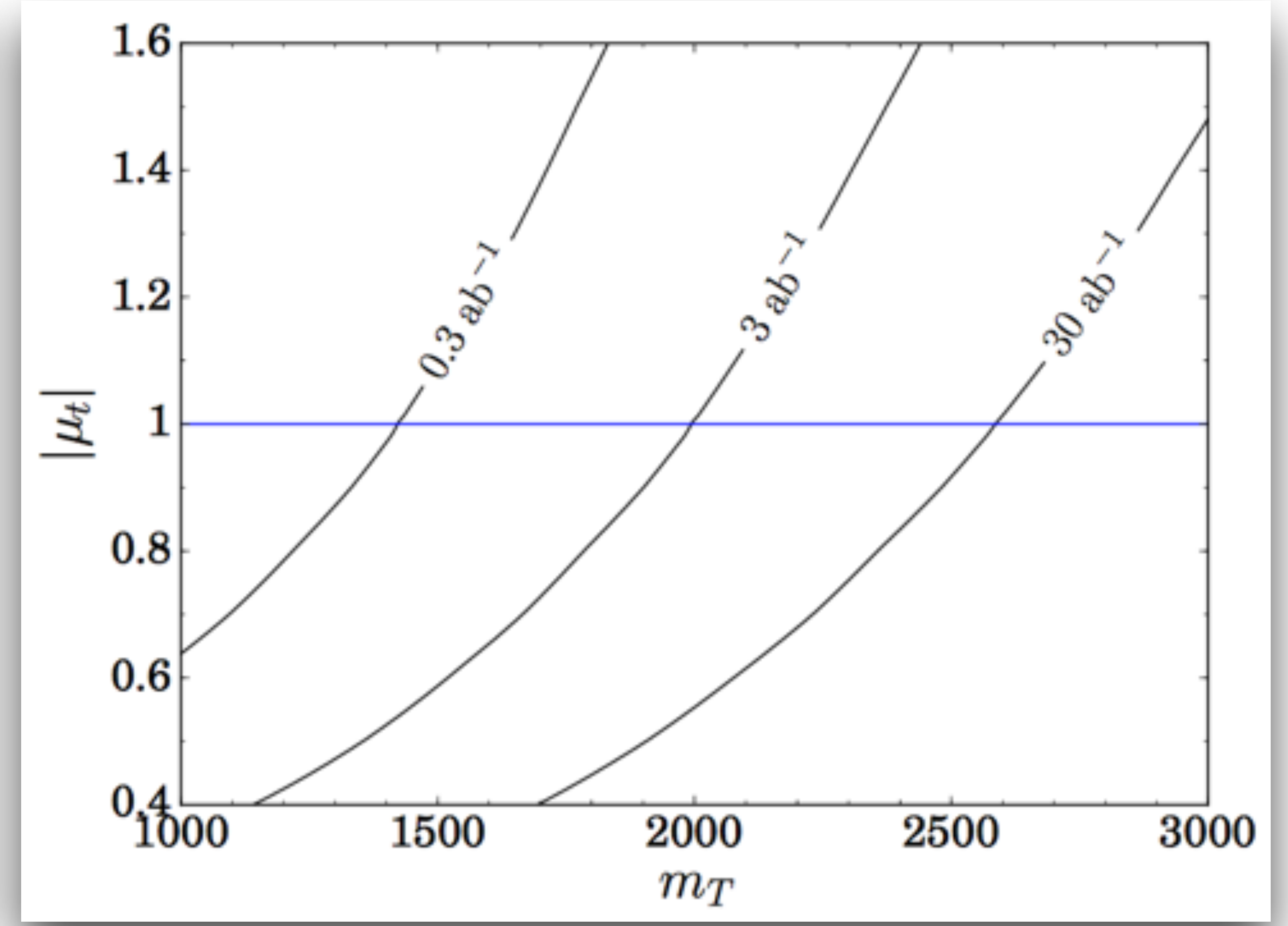


14 TeV

100 TeV

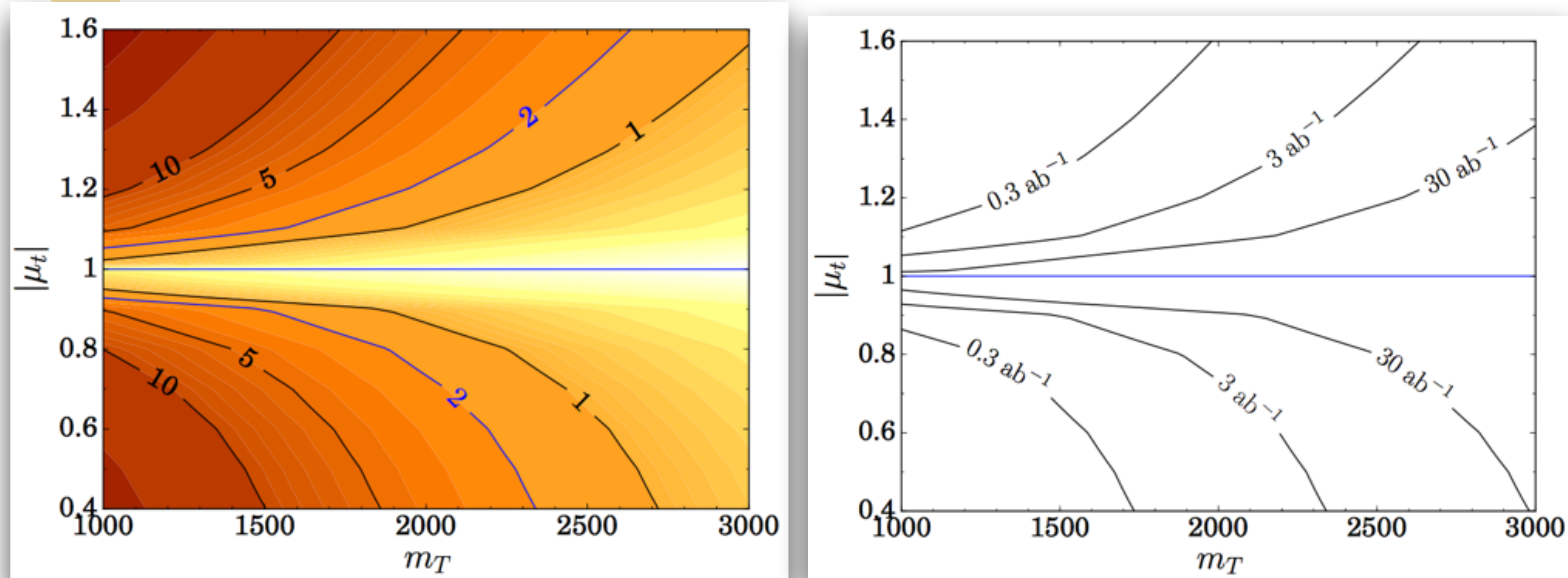
Discovery Potential of Top Partner at 100 TeV





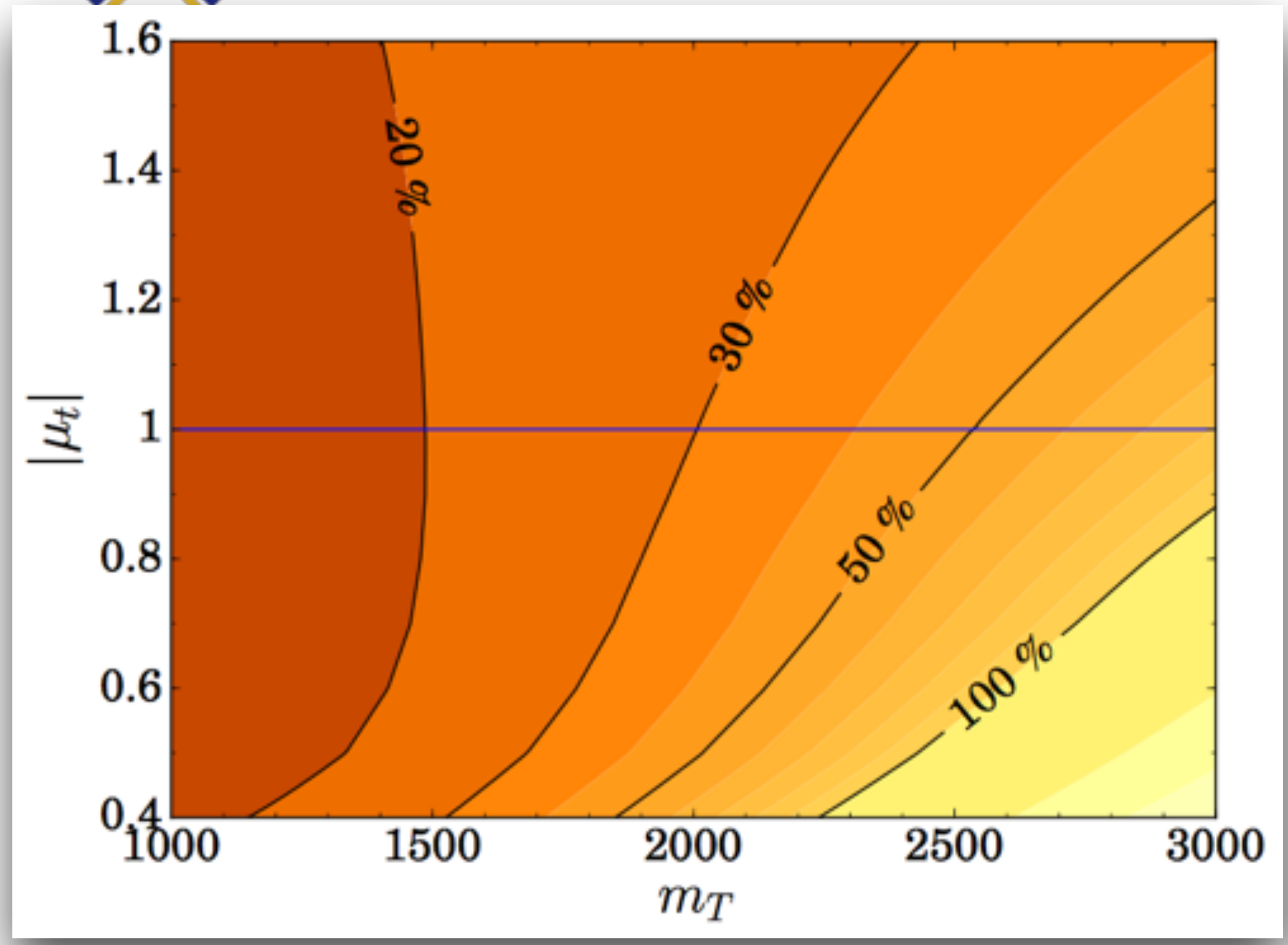
- Not the ``Gold" channel for discovery of top partner, but show the effectiveness of the analysis

Exclusion of Unnatural Theories at 100 TeV

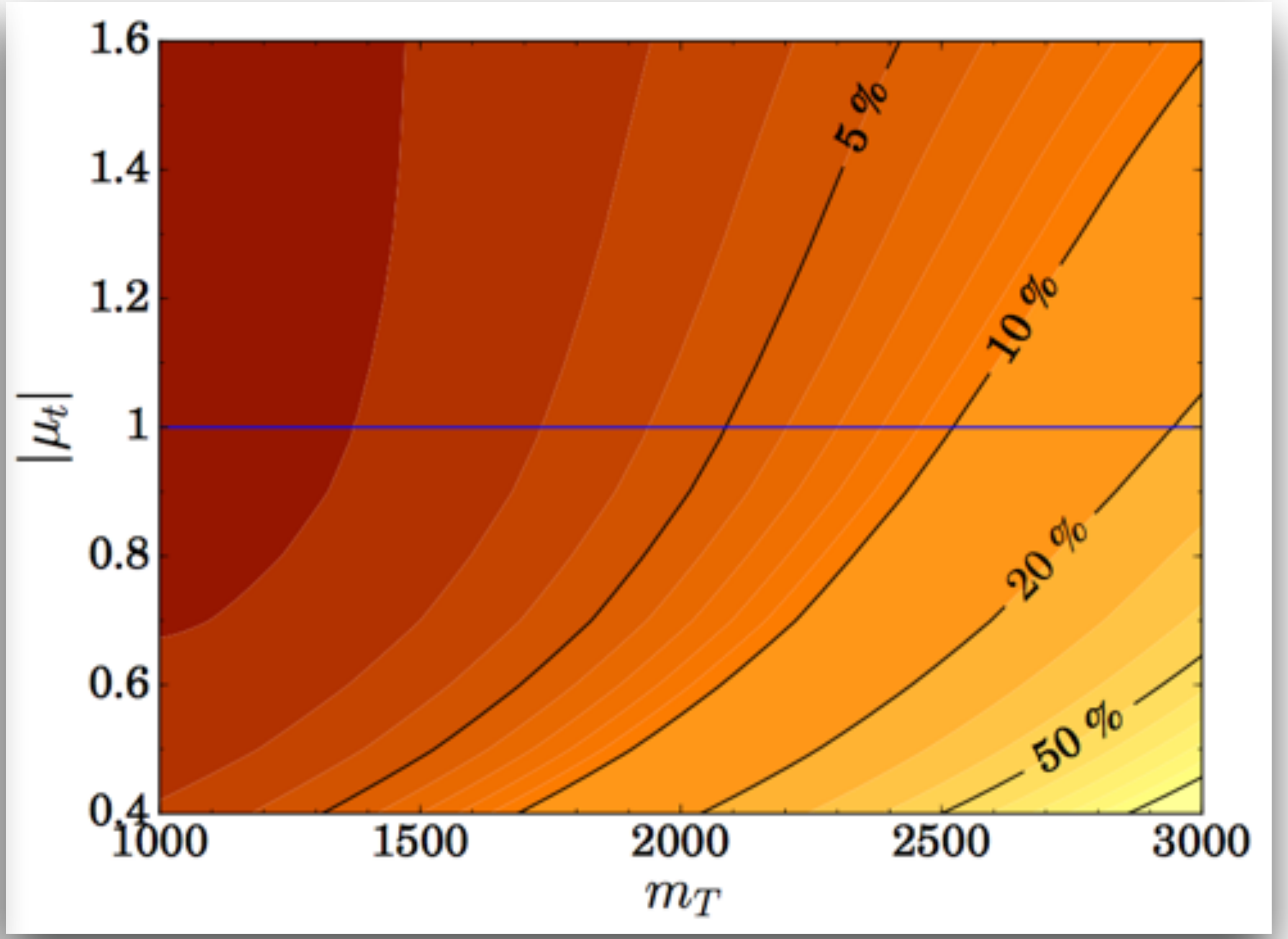


- ``unnaturalness'' hypothesis: exclusion of ``unnatural theories'' against a natural theory
- given 30/ab, 10% deviation from ``naturalness'': excluded up to 2.2TeV

Precision of Measuring Naturalness Parameter at 100 TeV



delta_lambda_t ~10% (HL-LHC)
+ delta_aT (3/ab, 100TeV)



delta_lambda_t ~1% (30/ab, 100TeV)
+ delta_aT (30/ab, 100TeV)

- A precision of 10% in measuring mu could be achieved up to ~ 2.5TeV

$$\delta\mu = \sqrt{\left(-\frac{1}{\lambda_t^2}\delta a_T\right)^2 + \left(2\frac{a_T}{\lambda_t^3}\delta\lambda_t\right)^2}$$

Summary

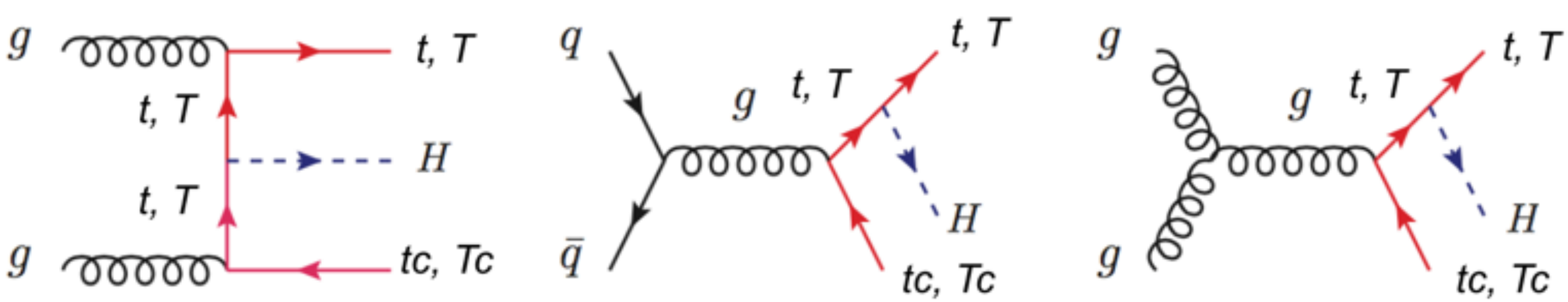
- The naturalness problem has driven particle physics for several decades
- To establish the Naturalness Principle, it is crucial to measure the naturalness sum rule, post the discovery of any partner-like particle
- For a top sector with fermionic top partners, the naturalness sum rule only depends on flavor-diagonal Yukawa couplings, up to an order O(v^2/mT^2)

$$a_T = -|\lambda_t|^2 + \mathcal{O}\left(\frac{v^2}{m_T^2}\right)$$

- At 100 TeV with 30/ab, a precision of 10% for the measurement of the naturalness parameter could be achieved for top partners up to ~2.5TeV, for the benchmark considered in this analysis

Outlook I

How to break the degeneracy of the sign in the mu parameter?

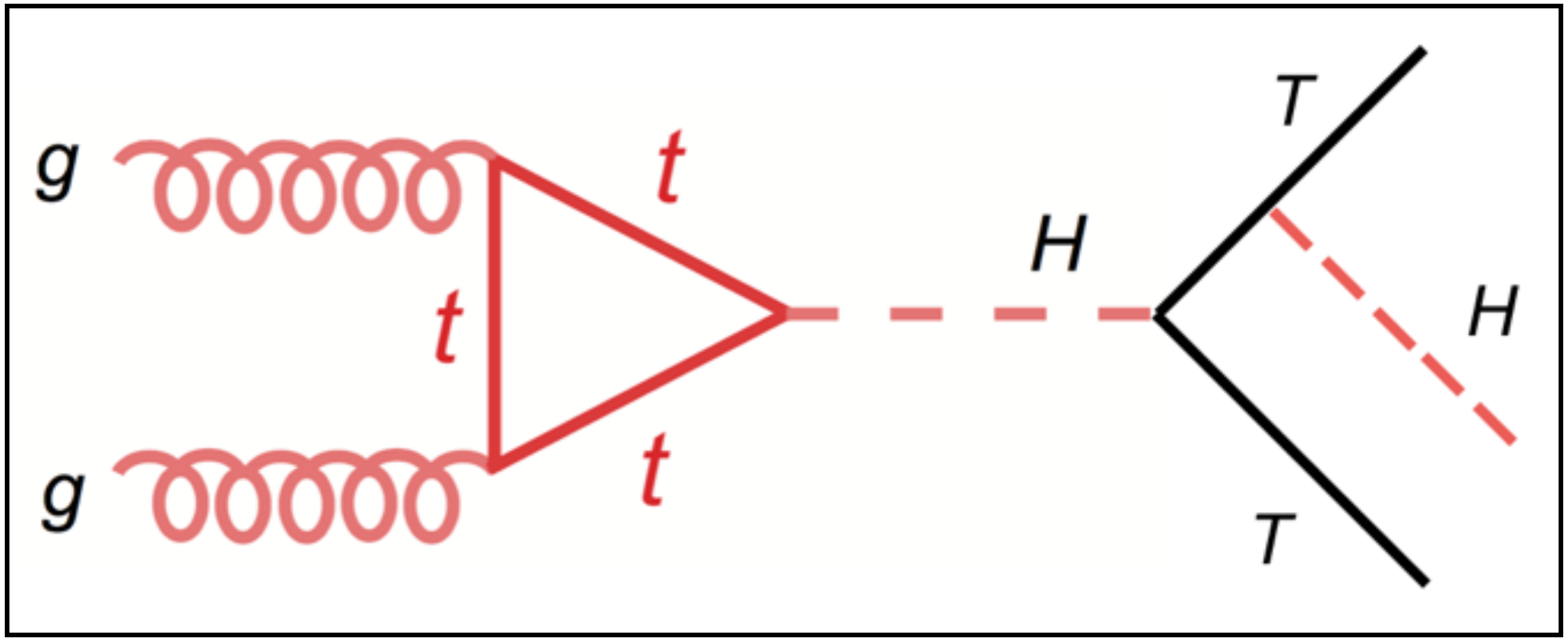


(a) (b) (c)

Outlook II

In twin Higgs model, how to test the naturalness sum rule at colliders?

Maybe mono-Higgs search can help



Outlook III

How to test the sum rule for supersymmetry at colliders, post the discovery of any superpartner-like particle?

Long journey to go to establish the naturalness principle, but exciting ……


Thank you!

Simplified Model - Mass Basis Before EWSB

$$t'^c = \frac{\widehat{c}_0 u_3^c - c_0 U^c}{c} \qquad t' = q_3$$

$$T'^c = \frac{\widehat{c}_0 U^c + c_0 u_3^c}{c} \qquad T' = U$$

$$\mathcal{L}_{T'} = m_{T'} T'^c T' + \lambda_{t'} H t'^c t' + \lambda_{T'} H T'^c t' + \frac{\alpha_{t'}}{2 m_{T'}} H^2 t'^c T' + \frac{\alpha_{T'}}{2 m_{T'}} H^2 T'^c T' + \frac{\beta_{t'}}{6 m_{T'}^2} H^3 t'^c t' + \frac{\beta_{T'}}{6 m_{T'}^2} H^3 T'^c t' + \mathcal{O}\left(H^4\right) + \text{h.c.}$$

$$m_{T'} = f c\,, \qquad c = \sqrt{c_0^2 + \widehat{c}_0^2}$$

$$\lambda_{t'} = \frac{\widehat{c}_0 c_1 - c_0 \widehat{c}_1}{c}\,, \qquad \lambda_{T'} = \frac{c_0 c_1 + \widehat{c}_0 \widehat{c}_1}{c}\,,$$

$$\alpha_{t'} = \widehat{c}_0 c_2 - c_0 \widehat{c}_2\,, \qquad \alpha_{T'} = c_0 c_2 + \widehat{c}_0 \widehat{c}_2\,,$$

$$\beta_{t'} = \left(\widehat{c}_0 c_3 - c_0 \widehat{c}_3\right) c\,, \qquad \beta_{T'} = \left(c_0 c_3 + \widehat{c}_0 \widehat{c}_3\right) c$$

Simplified Model - Mass Basis After EWSB

$$t^c = t'^c + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right) \,, \qquad t = t' - T'\frac{v}{m_{T'}}\lambda_{T'}^* + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right)$$

$$T^c = T'^c + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right) \,, \qquad T = T' + t'\frac{v}{m_{T'}}\lambda_{T'} + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right)$$

$$\mathcal{L}_T = m_T T^c T + \lambda_t v t^c t + \frac{\lambda_t}{\sqrt{2}} h t^c t + \frac{\lambda_T}{\sqrt{2}} h T^c t + \frac{a_t v}{\sqrt{2} m_T} h t^c T + \frac{a_T v}{\sqrt{2} m_T} h T^c T$$

$$+ \frac{\alpha_t}{4 m_T} h^2 t^c T + \frac{\alpha_T}{4 m_T} h^2 T^c T + \frac{b_t v}{4 m_T^2} h^2 t^c t + \frac{b_T v}{4 m_T^2} h^2 T^c t + \mathcal{O}\left(h^3, \frac{v^2}{m_T^2}\right) + \text{h.c.}$$

$$a_t = \alpha_{t'} + \lambda_{T'}^* \lambda_{t'} \,, \qquad \boxed{a_T = \alpha_{T'} + |\lambda_{T'}|^2}$$

$$b_t = \beta_{t'} - \alpha_{t'} \lambda_{T'} \,, \qquad b_T = \beta_{T'} - \alpha_{T'} \lambda_{T'}$$

Outlook I

How to break the degeneracy of the sign in the mu parameter?