TESTING NATURALNESS

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Based on
and in-progress work with J. Bernon, J. Hajer, Y. Jiang, I. Low, et. al.
A large discrepancy between two energy scales strongly correlated

Unnatural!
Naturalness Problem in Particle Physics

The electron mass is orders smaller than the EW scale — Unnatural!

EW scale ~ 100 GeV

The electron mass is orders smaller than the EW scale — Unnatural!
Naturalness Problem in Particle Physics

Observations: (1) zero mass limit $\Rightarrow$ chiral symmetry; (2) chiral symmetry breaking $\Rightarrow$ a logarithmically divergent contribution from the cutoff at quantum level

$$m_e \sim m^0_e [1 + 3\alpha/4\pi \ln(\Lambda/m_e)]$$

The t’Hooft statement for “technical naturalness”

If the turning off of an “unnatural” parameter results in an enhanced symmetry which can be (approximately) softly broken, this parameter is "technically" natural.

$\Rightarrow$ The smallness of $m_e$: not natural, but technically natural!

However, not all particles have a mass technically natural in the SM
Naturalness Problem in Particle Physics

\[ \delta m_h^2 \approx \frac{3}{4\pi^2} \left( -\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2 \theta_W} + \Lambda^2 \right) \]

\[ m_h^2 \sim (125 \text{ GeV})^2 \]

- Unnatural!

A hierarchy of 30 orders!
The naturalness problem has driven particle physics for decades. Many technical
Solution I - Fermionic Symmetry (Supersymmetry)
Solution II - Bosonic Symmetry (Little/Twin Higgs)
Some Wisdoms

The underlying symmetry =>

(1) orders a spectrum of ``partner'' particles
(2) predicts a sum rule for canceling quadratic divergence in \( mh^2 \),
either completely or at a leading quantum level

Motivated an amount of searches for ``partner''
particles at, e.g., LEP, Tevatron, LHC, for decades

A must-be-done task post the discovery of any partner-like particle:

Measuring the sum rule
One might ask:

why worry about step 2) when we have seen no empirical sign of a top partner??

The answer:

We live in a unique juncture in history!

As part of the planning for a new generation of particle accelerators, we would like to know the ability of a new hadron machine to unambiguously establish the Naturalness principle, should a top partner-like particle be discovered.
Simplified Model

$\text{SM} + \text{one pair of vector-like (weak isospin singlet) top partners}$

\[
\mathcal{L}_U = u_3^c \left( c_0 f U + c_1 H q_3 + \frac{c_2}{f} H^2 U + \ldots \right) \\
+ U^c \left( \hat{c}_0 f U + \hat{c}_1 H q_3 + \frac{\hat{c}_2}{f} H^2 U + \ldots \right) + \text{h.c.} .
\]
Simplified Model

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$$

<table>
<thead>
<tr>
<th>Model</th>
<th>Coset</th>
<th>SU(2)</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$\hat{c}_0$</th>
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<th>$\hat{c}_2$</th>
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<td>$\frac{SU(3)}{SU(2)^2}$</td>
<td>1</td>
<td>$\lambda_1$</td>
<td>$-\lambda_1$</td>
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<td>Simplest</td>
<td>$\left( \frac{SU(3)}{SU(2)} \right)^2$</td>
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<td>$\lambda$</td>
<td>$-\lambda$</td>
<td>$-\lambda$</td>
<td>$\lambda$</td>
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<td>$-\lambda$</td>
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<tr>
<td>Littlest Higgs</td>
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<td>1</td>
<td>$\lambda_1$</td>
<td>$-\sqrt{2}i\lambda_1$</td>
<td>$-2\lambda_1$</td>
<td>$\lambda_2$</td>
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<td>0</td>
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<td>Custodial</td>
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<td>$y_1$</td>
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<td>$T$-parity invariant</td>
<td>$\frac{SU(3)}{SU(2)^2}$</td>
<td>1</td>
<td>$\lambda$</td>
<td>$-\lambda$</td>
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<td>$-\lambda$</td>
<td>$\lambda$</td>
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<td>$2\lambda$</td>
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<tr>
<td>Mirror twin Higgs</td>
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<td>0</td>
<td>$-\lambda_t$</td>
</tr>
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</table>
Naturalness Sum Rule - Mass Basis Before EWSB

\[ \mathcal{L}_{T'} = m_{T'} T'^c T' + \lambda_{t'} H t'^c t' + \lambda_{T'} H T'^c t' + \frac{\alpha_{t'}}{2m_{T'}} H^2 t'^c T' + \frac{\alpha_{T'}}{2m_{T'}} H^2 T'^c T' \]

\[ + \frac{\beta_{t'}}{6m^2_{T'}} H^3 t'^c t' + \frac{\beta_{T'}}{6m^2_{T'}} H^3 T'^c t' + O(H^4) + \text{h.c.} \]

Quadratically divergent contribution to the \( C-W \) potential from top sector

\[ \frac{1}{16\pi^2} \Lambda^2 \text{tr} \mathcal{M}(H)\dagger \mathcal{M}(H) \]

Require coefficient in \( H^2 \) to vanish ⇒

\[ \alpha_{T'} = - |\lambda_{T'}|^2 - |\lambda_{t'}|^2 \]
Testing the Sum Rule - Traditional Wisdom

\[ \alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2 \]

Traditional wisdom - reconstruct the three couplings
Top quarks and electroweak symmetry breaking in little Higgs models

Maxim Perelstein, Michael E. Peskin, and Aaron Pierce
Phys. Rev. D 69, 075002 – Published 8 April 2004

\[ \alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2 \]

\[ \alpha_{T'} = \lambda_{T'} \frac{m_T}{f} \]

How difficult!

4 Testing the Model at the LHC
4.1 Measuring the parameter \( f \) ................................................................. 16
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Testing the Sum Rule -Traditional Wisdom

Top quarks and electroweak symmetry breaking in little Higgs models

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\[ \alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2 \]

\[ \alpha_{T'} = \lambda_{T'} \frac{m_{T'}}{f} \]

Not representative! E.g., little Higgs with T-parity
Naturalness Sum Rule - Mass Basis After EWSB

\[ \mathcal{L}_T = m_T T^c T + \lambda_t v t^c t + \frac{\lambda_t}{\sqrt{2}} h t^c T + \frac{\lambda_T}{\sqrt{2}} h T^c T + \frac{a_t v}{\sqrt{2} m_T} h t^c T + \frac{a_{T'} v}{\sqrt{2} m_T} h T^c T + \frac{\alpha_t h^2 T^c T}{4 m_T} + \frac{\alpha_{T'} h^2 T^c T}{4 m_T^2} + \frac{b_t v}{4 m_T^2} h^2 t^c T + \frac{b_{T'} v}{4 m_T^2} h^2 T^c T + \mathcal{O} \left( \frac{h^3, v^2}{m_T^2} \right) + \text{h.c.} \]

\[ a_T = \alpha_{T'} + |\lambda_{T'}|^2 \]

\[ a_T = -|\lambda_t|^2 + \mathcal{O} \left( \frac{v^2}{m_T^2} \right) \]

\[ \sum_i a_{T_i} = -|\lambda_t|^2 + \mathcal{O} \left( \frac{v^2}{m_{T_i}^2} \right) \]

- Leading order - *involves diagonal Yukawa couplings only*
- Could be generalized with more top partners introduced:
- No measurement of quartic coupling is needed => a more feasible guideline
With this guideline, we are able to study various benchmark scenarios, e.g., little higgs models without $T$ parity

- Introduce a ``naturalness parameter``

$$\mu = - \frac{\Delta m_H^2 \big|_{\text{NP}}}{\Delta m_H^2 \big|_{\text{SM}}} \implies \mu_t = - \frac{a_T}{\lambda_t^2} + \mathcal{O}\left(\frac{v^2}{m_T^2}\right)$$

$$\mu \big|_{\text{nat}} \equiv 1$$

- Test the sum rule $\iff$ measure the ``naturalness parameter``
TTh Production

(a) 
(b) 
(c) 

$g \rightarrow t, T$
$g \rightarrow tc, Tc$
$q \rightarrow t, T$
$q \rightarrow tc, Tc$

$g \rightarrow t, T$
$g \rightarrow Tc$
$g \rightarrow tc$

$|\mu|$

$m_T$

14 TeV

100 TeV
Not the "Gold" channel for discovery of top partner, but show the effectiveness of the analysis.
``unnaturalness'' hypothesis: exclusion of ``unnatural theories'' against a natural theory

given 30/ab, 10% deviation from ``naturalness'': excluded up to 2.2 TeV
A precision of 10% in measuring mu could be achieved up to ~ 2.5 TeV
The naturalness problem has driven particle physics for several decades.

To establish the Naturalness Principle, it is crucial to measure the naturalness sum rule, post the discovery of any partner-like particle.

For a top sector with fermionic top partners, the naturalness sum rule only depends on flavor-diagonal Yukawa couplings, up to an order $O(v^2/m_T^2)$.

\[
\alpha_T = - |\lambda_t|^2 + \mathcal{O}\left(\frac{v^2}{m_T^2}\right)
\]

At 100 TeV with 30/ab, a precision of 10% for the measurement of the naturalness parameter could be achieved for top partners up to ~2.5 TeV, for the benchmark considered in this analysis.
How to break the degeneracy of the sign in the mu parameter?
In twin Higgs model, how to test the naturalness sum rule at colliders?

Maybe mono-Higgs search can help
How to test the sum rule for supersymmetry at colliders, post the discovery of any superpartner-like particle?

Long journey to go to establish the naturalness principle, but exciting ......
At tree level, we have

Here can be understood as a measure of the de

At loop-level, we have

Thank you!
Simplified Model - Mass Basis Before EWSB

\[ t'^c = \frac{\hat{c}_0 u_3^c - c_0 U^c}{c} \]

\[ T'^c = \frac{\hat{c}_0 U^c + c_0 u_3^c}{c} \]

\[ t' = q_3 \]

\[ T' = U \]

\[ \mathcal{L}_{T'} = m_{T'} T'^c T' + \lambda_{t'} HT'^c T' + \frac{\alpha_{t'}}{2 m_{T'}} H^2 t'^c T' + \frac{\alpha_{T'}}{2 m_{T'}} H^2 T'^c T' \]

\[ + \frac{\beta_{t'}}{6 m_{T'}^2} H^3 t'^c T' + \frac{\beta_{T'}}{6 m_{T'}^2} H^3 T'^c T' + \mathcal{O}(H^4) + h.c. \]

\[ m_{T'} = f c , \]

\[ c = \sqrt{c_0^2 + \hat{c}_0^2} \]

\[ \lambda_{T'} = \frac{c_0 c_1 + \hat{c}_0 \hat{c}_1}{c} , \]

\[ \lambda_{T'} = \frac{c_0 c_1 + \hat{c}_0 \hat{c}_1}{c} , \]

\[ \alpha_{t'} = \hat{c}_0 c_2 - c_0 \hat{c}_2 \]

\[ \alpha_{T'} = c_0 c_2 + \hat{c}_0 \hat{c}_2 \]

\[ \beta_{t'} = (\hat{c}_0 c_3 - c_0 \hat{c}_3) c \]

\[ \beta_{T'} = (c_0 c_3 + \hat{c}_0 \hat{c}_3) c \]
Simplified Model - Mass Basis After EWSB

\[ t^c = t'^c + \mathcal{O} \left( \frac{v^2}{m_{T'}^2} \right), \quad t = t' - T' \frac{v}{m_{T'}} \lambda_{T'}^* + \mathcal{O} \left( \frac{v^2}{m_{T'}^2} \right) \]

\[ T^c = T'^c + \mathcal{O} \left( \frac{v^2}{m_{T'}^2} \right), \quad T = T' + t' \frac{v}{m_{T'}} \lambda_{T'} + \mathcal{O} \left( \frac{v^2}{m_{T'}^2} \right) \]

\[ \mathcal{L}_T = m_T T^c T + \lambda_t v t^c t + \frac{\lambda_t}{\sqrt{2}} h t^c t + \frac{\lambda_T}{\sqrt{2}} h T^c t + \frac{a_t v}{\sqrt{2} m_T} h t^c T + \frac{a_T v}{\sqrt{2} m_T} h T^c T \]

\[ + \frac{\alpha_t}{4 m_T} h^2 t^c T + \frac{\alpha_T}{4 m_T} h^2 T^c T + \frac{b_t v}{4 m_T^2} h^2 t^c t + \frac{b_T v}{4 m_T^2} h^2 T^c t + \mathcal{O} \left( h^3, \frac{v^2}{m_T^2} \right) + \text{h.c.} \]

\[ a_t = \alpha_{T'} + \lambda_{T'}^* \lambda_{T'}, \quad a_T = \alpha_{T'} + |\lambda_{T'}|^2 \]

\[ b_t = \beta_{T'} - \alpha_{T'} \lambda_{T'}, \quad b_T = \beta_{T'} - \alpha_{T'} \lambda_{T'} \]
Outlook I

How to break the degeneracy of the sign in the mu parameter?