Electroweak Phase Transition and the Precision of Higgs Self Coupling

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Based on work with B. Jain and S. Lee 1709.03232
On the validity of the effective potential and the Precision of Higgs Self Coupling

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Various global structure/thermal histories are still plausible

\[ V_h = \frac{1}{2} m_h^2 h^2 + c_3 \frac{1}{6} \left( \frac{3 m_h^2}{v} \right) h^3 + c_4 \frac{1}{24} \left( \frac{3 m_h^2}{v^2} \right) h^4 \]

An interesting physics still allowed is baryogenesis based on strong 1st order EW-phase transition
Strong 1\textsuperscript{st} order
Electroweak Phase Transition

An interesting physics that can be related to the large deviation of the Higgs self coupling
The effective potential is a main tool to examine the thermal history of the Higgs potential:

\[ V_{\text{eff}} = V_{\text{tree}} + V_{\text{CW}}[m_i^2(h) + \Pi_i] + V_T[m_i^2(h) + \Pi_i] \]

\[ V_T = \sum_{i=B,F} (-1)^{F_i} \frac{g_i T^4}{2\pi^2} \int_0^\infty dx \, x^2 \log \left[ 1 + \exp \left( -\sqrt{x^2 + (m_i^2(h) + \Pi_i)^2} / T^2 \right) \right] \]

Obtaining the exact thermal mass is very non-trivial.

What most people do is using

**Truncated Full Dressing (TFD):**

- thermal mass \( \Pi_i \) is still obtained in the high-T approximation

At leading order in temperature, \( \Pi_i \) is mass-independent

\( \rightarrow \) non-decoupling issue

: the related uncertainty has not been well understood in BSM scenarios

Curtin, Meade, Ramani 16' for a recent discussion
The effective potential

In the High-T approximation

\[ V_{eff} = V_{\text{tree}} + V_{CW} + V_T + V_{\text{ring}} \]

\[ V_{CW} = \sum_{i=t,W,Z,h,G,\ldots} (-1)^{F_i} \frac{g_i}{64\pi^2} \left[ m_i^4(h) \left( \log \frac{m_i^2(h)}{m_i^2(v)} - \frac{3}{2} \right) + 2m_i^2(h)m_i^2(v) \right] \]

\[ V_T = \sum_{i=B,F} (-1)^{F_i} \frac{g_i T^4}{2\pi^2} J_{B/F} \left( \frac{m_i^2(h)}{T^2} \right) \]

\[ V_{\text{ring}} = \sum_{i=\text{bosons}} \frac{\bar{g}_i T}{12\pi} \left[ m_i^3(h) - \left( m_i^2(h) + \Pi_i(T) \right)^{\frac{3}{2}} \right] \]

\[ x^2 = \frac{m^2}{T^2} \ll 1 \]

✓ Validity of this approx. carefully has to be checked

We call this Prescription B

1st order PT via thermal effect

\[ J_B(x^2) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12} x^2 - \frac{\pi}{6} x^3 - \frac{1}{32} x^4 \log \left( \frac{x^2}{c_b} \right) \]

\[ J_F(x^2) \approx \frac{7\pi^4}{360} - \frac{\pi^2}{24} x^2 - \frac{1}{32} x^4 \log \left( \frac{x^2}{c_f} \right) \]
On the criteria for strong 1\textsuperscript{st} order phase transition

There has been an ambiguity on how to quantify the strong first order phase transition and this ambiguity can cause $\mathcal{O}(1)$ fluctuation on the precision of Higgs self coupling.

List of sources that can cause $\mathcal{O}(1)$ uncertainty

\[
\frac{m_i^2(v_c)}{T_c^2} = \frac{m^2}{T_c^2} + \text{coupling} \times \frac{v_c^2}{T_c^2} \gtrsim 1
\]

For $v_c \gtrsim T_c$ and $\gtrsim \mathcal{O}(1)$ coupling, integral needs to be exactly evaluated

1. The High-temperature Approximation
2. A Large coupling
3. $\frac{v_c}{T_C}$ vs $\frac{v_N}{T_N}$, and confusion on $\frac{v_c}{T_C} \gtrsim 0.6 - 1.4$

: checks if the potential develops degenerate vacua
: checks if the transition actually happens

The impact on the precision of the Higgs self-coupling of these issues has not been well studied in most literature in the context of BSM physics.
To illustrate the issues we take

**Most commonly considered frameworks**

\[ V_{\text{eff}} = \sum_{i=t,W,Z,h,G,\text{BSM}} V_i \]

Higgs portal $Z_2$-symmetry, e.g. $\langle S \rangle = 0$:
new scalar $S$

**Effective Field Theory:**
higher-dimensional operators

\[ O_H = \left( \partial |H|^2 \right)^2 \quad \text{vs} \quad O_6 = |H|^6 \]

E.g. Strongly coupled theory
PGB vs non-pGB
Higgs Portal

$O_H \ll O_6$ possible

Azatov, Contino, Panico, Son 15’
$O_H \ll O_6$ possible

Azatov, Contino, Panico, Son 15'

E.g. Higgs : pGB

Generic composite state: tuned ...
→ no supurion suppression.

Enhancement by $\frac{g_*^2}{g_{GB}}$

$\bar{c}_H \sim \left(\frac{v}{f}\right)^2 \sim 0.05$

$\bar{c}_6 \sim \left(\frac{v}{f}\right)^2 \frac{g_*^2}{\lambda_4} \sim 3.5 \left(\frac{g_*}{3}\right)^2$

Higgs portal (to strongly coupled sector)

$\mathcal{L} = \lambda |H|^2 O$

$O$ characterized by $\{m_*, g_*\}$

$\bar{c}_H \sim \left(\frac{v}{f}\right)^2 \frac{\lambda^2}{g_*^4}$

$\bar{c}_H / \bar{c}_6 = \frac{\lambda_4}{\lambda}$

$\bar{c}_6 \sim \left(\frac{v}{f}\right)^2 \frac{\lambda^3}{g_*^4 \lambda_4}$

Up to some fine-tuning

Higgs : pGB (SILH basis)

$O_H \sim \left(\partial \mu |H|^2 \right)^2$, $O_6 \sim \frac{g_{GB}}{g_*^2} \times |H|^6$

$\bar{c}_H \sim \bar{c}_6 \sim \left(\frac{v}{f}\right)^2$
Strong 1st order
Electroweak Phase Transition

Higgs Portal
with a singlet scalar

Noble, Perelstein 08’
Katz, Perelstein 14’
Curtin, Meade, Yu 14’
Kurup, Perelstein 17’
Barger, Langacker, McCaskey, Ramsey-Musolf, Shaughness 03’
Espinosa, Konstandin, Riva 11’
Cline, Kaiulainen 12’
Alanne, Tuominen, Vaskonen 14’
Many others ....

Very sorry if I missed your paper
Higgs Portal \[ \text{SM + a singlet scalar with Z2} \]

\[ V_{\text{tree}} = -\frac{1}{2} \mu^2 h^2 + \frac{1}{4} \lambda h^4 + \frac{1}{2} \lambda_{HS} h^2 S^2 + \frac{1}{2} m_0^2 s^2 + \frac{1}{4} \lambda_S S^4 \]

\((\langle h \rangle, \langle s \rangle) = (\nu, 0)\) is a global minimum

Based on naïve criterion, existence of degenerate vacua, with \(\nu_c/T_c > 1\) \hspace{1cm} * Note cutoff \(\lambda_{HS} < 5\) by hand

1. One-step strong 1\textsuperscript{st} phase transition (RED)

\[ V(0, 0) \rightarrow V(\nu, 0) \quad , \langle S \rangle = 0 \]

2. Two-step strong 1\textsuperscript{st} phase transition (GREEN)

\[ V(0, 0) \rightarrow V(0, \nu_s) \rightarrow V(\nu, 0) \]

\[ V(0, \nu_s) > V(\nu, 0) \rightarrow \]

\[ \lambda_s > \lambda_{s}^{\text{min}} \equiv \lambda \frac{m_{0s}^4}{\mu^4} = \frac{2(m_s^2 - \nu^2 \lambda_{HS})^2}{m_h^2 \nu^2} \]

parametrize \(\lambda_s = \lambda_{s}^{\text{min}} + \delta_s\)

Scan:
\(m_s = [100,800] \text{ GeV in steps of 10 GeV}\)
\(\lambda_{HS} = [1,5] \text{ in steps of 0.2}\)

Similar plot in Curtin, Meade, Ramani 14’
Future collider plan can sensitively depend on the exact criteria

E.g. only 100 TeV pp vs various colliders 100 TeV, ILC?

Q. Is there a preferred \( \frac{\nu_c}{T_c} \)?

Jain, Spannowsky, SON in progress
Validity of High-T approximation

One-step PT
High-T approx. fails
(The issue is more pronounced in two step PT)

Hall and Anderson 92’

Low-T approx with \( n = \mathcal{O}(10) \)
= Exact evaluation, e.g. chose \( n = 50 \)

\[
\begin{align*}
J_B(x^2) & \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}x^2 - \frac{\pi}{6}x^3 - \frac{1}{32}x^4 \log \left( \frac{x^2}{c_b} \right) \\
J_F(x^2) & \approx \frac{7\pi^4}{360} - \frac{\pi^2}{24}x^2 - \frac{1}{32}x^4 \log \left( \frac{x^2}{c_f} \right)
\end{align*}
\]

\[
x^2 = \frac{m^2}{T^2} \ll 1: \text{High T approximation}
\]

\[
x^2 = \frac{m^2}{T^2} \gg 1: \text{Low T approximation}
\]
\( O(1) \) fluctuation in Higgs self-coupling

### Prescription A

**Exact \( V_T \)**

One-step strong 1st order phase transition

### Prescription B

**High-T approx \( V_T \)**

One-step strong 1st order phase transition

Similarly for both, the smallest deviation, \( \delta \left( \frac{\lambda_3}{\lambda_{3_{SM}}} \right) \sim [5, 32] \% \) for \( \frac{v_c}{T_c} \geq [0.6, 1.4] \)

The patterns of parameter space in two cases look very different! What's going on?
$O(1)$ fluctuation in Higgs self-coupling

**Exact $V_T$**

- **Prescription A**
  - $\mu_S = [10, 900] \text{GeV}$

- **Prescription B**
  - $\mu_S = [10, 1310] \text{GeV}$

**High-T approx. $V_T$**

- $\mu_S = [10, 1310] \text{GeV}$

High masses do not decouple and screws up Higgs self-coupling

**Decoupling**

High-T approx. does not seem to be appropriate for Higgs portal.
Strong 1st order phase transition
\[ \leftrightarrow \lambda_{HS} \sim \mathcal{O}(1) \text{ for } N_S = 1 \]

The perturbation breaks down? It looks like
\[ \lambda_4 \sim 0.13, \text{ and we assumed } \lambda_S \sim 0, \text{ ignored overall } \frac{m_H^2}{m_S^2} \text{ factor} \]

Resummation
\[ \lambda_{HS} \frac{T^2}{m_S^2} \times \]

1-loop
\[ \sim \lambda_{HS} T^2 \]

(1+1)-loop
\[ \sim \lambda_{HS}^2 \frac{T^3}{m_s} \]

(1+2)-loop
\[ \sim \lambda_{HS}^2 \frac{T^3}{m_s} \left( \frac{\lambda_{HS} T^2}{m_s^2} \right)^{n-1} = \lambda_{HS}^{n+1} \frac{T^{2n+1}}{m_s^{2n-1}} \]

(1+n)-loop

2-loop
\[ \sim \lambda_{HS}^2 T^2 \]

(2+n)-loop
\[ \sim \lambda_{HS}^3 \frac{T^4}{m_s} \]

This issue remains to be clarified
Strong 1st order
Electroweak Phase Transition

EFT approach

Grojean, Gervant, Well 04'
Noble, Perelstein 06'
Delaunay, Grojean, Wells, 08'
Huang, Joglekar, Li, Wagner 15'
Huang, Gu, Yin, Yu, Zhang 16'
Chung, Long, Wang 16'
Gan, Long, Wang 17'
Reichert, Eichhorn, Gies, Pawlowski, Plehn, Scherer 17'
Many others....

Very sorry if I missed your paper
1\textsuperscript{st} order phase transition in EFT approach

I. Only dim-6 operator $\mathcal{O}_6 \sim |H|^6$

** Suitable for the case with $\mathcal{O}_H \ll \mathcal{O}_6$, nevertheless we will present results in SILH basis

& assume Higgs as pGB: NDA in SILH basis: $c_6 \sim \left(\frac{v}{f}\right)^2 \equiv \xi$

$$V_{EFT} = m^2 |H|^2 + \lambda |H|^4 + \frac{c_6}{v^2} \frac{m_h^2}{2 v^2} |H|^6$$

$$\rightarrow \frac{1}{2} m^2 h^2 + \frac{1}{4} \lambda h^4 + \frac{1}{8} \frac{c_6}{v^2} \frac{m_h^2}{2 v^2} h^6$$

At “tree”-level

$$\frac{\lambda_3}{\lambda_3^{SM}} = 1 + c_6$$

$$\frac{\lambda_4}{\lambda_4^{SM}} = 1 + 6 c_6$$

Relation holds only at the level of dimension-six
1st order phase transition in EFT approach

Assuming \( m^2(T) = m^2 + aT^2 \) (keeping only \( T^2 \)-term as thermal effect) the analytic constraint on \( c_6 \) to have 1st order phase transition is possible

\[
\frac{dV_{EFT}}{dh} \bigg|_{h=v_c,T=T_c} = 0 \quad \text{stationary condition}
\]

\[
V(v_c,T_c) = V(0,T_c) \quad \text{degeneracy of the vacua}
\]

\[
v_c^2 = -\frac{4m^2(T_c)}{\lambda} = -\frac{2\lambda v^4}{c_6 m_h^2}
\]

\[
c_6 = \frac{2}{3} \left( \frac{1}{1 - \frac{2v_c^2}{3v^2}} \right)
\]

\[
v_c^2 < v^2 \quad \text{&} \quad \lambda < 0
\]

\[
\frac{2}{3} < c_6 < 2
\]

Either weak or strong

✓ However, already alarming from EFT point of view

EFT description might not make sense
1\textsuperscript{st} order phase transition in EFT approach

II. Resum all $\mathcal{O}_{4+2n} \sim |H|^{4+2n}$

$$V_{EFT} = m^2 |H|^2 + \lambda |H|^4 + \sum \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2 v^2} |H|^{4+2n}$$

$$\rightarrow \frac{1}{2} m^2 h^2 + \frac{1}{4} \lambda h^4 + \sum \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2 v^2} \left( \frac{h^2}{2} \right)^{2+n}$$

NDA in SILH basis: $c_{4+2n} \sim (\frac{v}{f})^{2n}$

$$c_{4+2n} \sim c \left( \frac{v}{f} \right)^{2n} \equiv c \xi^n$$

:can be resumed up to infinite order in Higgs field and

At “tree”-level

$$\frac{\lambda_3}{\lambda_3^{SM}} = 1 + 16c \frac{\xi}{(2 - \xi)^4}$$

$$\frac{\lambda_4}{\lambda_4^{SM}} = 1 + 32c \frac{(6 + \xi)\xi}{(2 - \xi)^5}$$

$$\frac{\lambda_4}{\lambda_4^{SM}} = \frac{\lambda_3}{\lambda_3^{SM}} \frac{2}{2 - \xi} \rightarrow 14 \quad \text{as} \quad \xi \rightarrow 1 \quad \text{(or} \quad f \rightarrow v)$$
1\textsuperscript{st} order phase transition in EFT approach

II. Resum all $\mathcal{O}_{4+2n} \sim |H|^{4+2n}$

$$V_{EFT} = m^2 |H|^2 + \lambda |H|^4 + \sum \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2v^2} |H|^{4+2n}$$

$$\rightarrow \frac{1}{2} m^2 h^2 + \frac{1}{4} \lambda h^4 + \sum \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2v^2} \left(\frac{h^2}{2}\right)^{2+n}$$

NDA in SILH basis: $c_{4+2n} \sim \left(\frac{v}{f}\right)^{2n}$

``EFT can make sense as long as $E < \Lambda_{\text{cutoff}}$``

At “tree”-level

$$\frac{\lambda_3}{\lambda_3^{SM}} = 1 + 16c \frac{\xi}{(2 - \xi)^4}$$

$$\frac{\lambda_4}{\lambda_4^{SM}} = 1 + 32c \frac{(6 + \xi)\xi}{(2 - \xi)^5}$$

$$\frac{\lambda_4}{\lambda_4^{SM}} \frac{\lambda_3}{\lambda_3^{SM}} = 2 \frac{6 + \xi}{2 - \xi} \rightarrow 14 \text{ as } \xi \rightarrow 1 \text{ (or } f \rightarrow v)$$

✓ Not easily matched to a specific UV theory, but might represent the qualitative feature of such a possibility.
Only dim-6 operator

\[ \Delta V_{EFT} = \frac{1}{8} c_6 \frac{m_h^2}{v^2} \frac{m^2}{v^2} h^6 \]

\[ c_6 \sim O(1) : \text{Validity of EFT} \]

- High-T approximation seems to be ok, e.g. no large mass involved
- The uncertainty due to the finite \( v_c/T_c \) is not pronounced
All orders of $|H|^{4+2n}$

$$\Delta V_{EFT} = \sum \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{v^2} \left( \frac{h^2}{2} \right)^{2+n}$$

where $c_{4+2n} \sim c \left( \frac{v}{f} \right)^{2n} \equiv c \xi^n$

![Prescription A](image1.png)

![Prescription B](image2.png)

- High-T approximation seems to be ok, e.g. no large mass involved
- The uncertainty due to the finite $v_c/T_c$ is not pronounced
Strong 1st order
Electroweak Phase Transition

Cubic vs. Quartic
Higgs self-coupling
Cubic vs. Quartic

EFT prefers large Higgs self-couplings

Large quartic coupling might be able to be tested at future collider

Unitarity bound

Perturbativity bound

Electroweak Precision Test
Cubic vs. Quartic

\[ \Delta V = \sum_c c_0 \left( \frac{m_h^2}{2v^2} \right)^{2+n} \]

Unitarity bound

\[ \left| \frac{\lambda_3}{\lambda_3^{SM}} \right| \lesssim 6.5 \quad \left| \frac{\lambda_4}{\lambda_4^{SM}} \right| \lesssim 65 \]

Luzio, Grober, Spannowsky 17'

Electroweak Precision Test

\[ \kappa_\lambda \in [-20, 20] \]

Kribs, Maier, Rzehak, Spannowsky, Waite 17'
Cubic vs. Quartic

\[ V(h) = \frac{1}{2} m_h^2 h^2 + c_3 \frac{1}{6} \left( \frac{3 m_h^2}{v} \right) h^3 + c_4 \frac{1}{24} \left( \frac{3 m_h^2}{v^2} \right) h^4 \]

Higgs Portal

\[ \Delta V_{EFT} = \sum c_{4+2n} \frac{m_h^2}{v^{2n}} \left( \frac{h^2}{2} \right)^{2+n} \]

\[ \Delta V_{EFT} = \frac{1}{8} \frac{c_6 m_h^2}{v^2} \frac{1}{v^2} \]

Purple shaded: Prediction of the case where thermal potential has only \( T^2 \) piece (can be either weak 1\(^{st}\) or strong 1\(^{st}\) order)

Higgs portal with a singlet scalar

\[ \frac{2}{3} < c_6 < 2 \]
Cubic vs. Quartic

\[ V(h) = \frac{1}{2} m_h^2 h^2 + c_3 \frac{1}{6} \left( \frac{3 m_h^2}{v} \right) h^3 + c_4 \frac{1}{24} \left( \frac{3 m_h^2}{v^2} \right) h^4 \]

Higgs Portal

\checkmark \text{very tough to test.}
\checkmark \text{Plausible only at Future colliders}
$\mathcal{O}(1)$ fraction of EFT parameter space can be tested at the HL LHC

Can be tested @ HL LHC with 68% CL

$\Delta V_{EFT} = \sum c_{4+2n} \frac{m_h^2}{v^{2n}} \left( \frac{h^2}{2} \right)^{2+n}$

$\Delta V_{EFT} = \frac{1}{8} \frac{c_6 m_h^2}{v^2} \frac{1}{v^2} \left( \frac{h^2}{2} \right)^{2}$

Higgs portal

$\frac{2}{3} < c_6 < 2$

$\lambda_3 / \lambda_{3,SM}$

$\kappa_t \rightarrow 1$

$c_3 = [0.1, 2.3] \text{ @1}\sigma$

$c_3 = [0.04, 2.9] \text{ U}[4.8, 6.0] \text{ @1}\sigma$

Kim, Sakaki, SON 17’ to appear very soon
Quartic coupling IS useful to distinguish different EFTs!

Can be tested @ HL LHC

**Strong sensitivity on quartic when a large deviation of the cubic is observed

> Isn't it the situation that people in EWPT community are hoping for?
Quartic coupling IS also useful for Higgs portal

\[ \lambda^4 - 1 = d_4 \]

Papaefstathiou, Sakurai 15’

\[ \lambda_3 / \lambda_{3_SM} - 1 = c_3 \]

approximate \( c_3 - d_4 \) exclusion, \( hhh \rightarrow (bb)(bb)(\gamma\gamma) \), pp@100 TeV

Prescription A
One-step strong 1st order phase transition
Summary

- High-T approximation, finite $v/T$, A large coupling
  - Higgs Portal with a singlet scalar with $Z2$
    - Do not use high-T approximation
    - $O(1)$ fluctuation on the precision of Higgs self-coupling due to $v/T$ criteria
    - dramatic impact on future collider plan
  - Effective Field Theory Approach
    - Above issues become mild
    - Large deviation of coupling -> Validity of EFT -> Any reasonable EFT model?

- BSM maps in $(\lambda_3, \lambda_4)$
  - $\lambda_4$ can be used to differentiate different BSM scenarios in a situation when a large $\delta \lambda_3$ is observed
  - The quartic couplings in any BSM scenarios have a good sensitivity @ 100 TeV in that situation