Measuring the Higgs CP property at LHC and CEPC

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Introduction

- After the Higgs was discovered in 2012, understanding its properties, and looking for any possible deviations from the SM prediction, becomes a very important task of LHC.

- CP violation is a necessary condition for baryogenesis, a process leading to matter-antimatter imbalance in the universe. Understanding Higgs’ CP property is one of the important topics that can be done at the LHC or future $e^+e^-$ colliders, but perhaps with better precision for the latter.

  - If Higgs is in a pure CP eigen state: is it CP even or odd?
  - If Higgs is in a CP mixture: gives rise to CP violation. This is a more exciting scenario, as the current known CP violation source (a single complex phase in CKM) is too small to explain the matter-antimatter imbalance.

- Unlike the CP odd Higgs effective coupling to bosons which are dim-6 operators, the CP odd Higgs coupling to fermions is dim-4 and the CP violation effect can be sizable.
LHC Higgs CP test

ATLAS/CMS considered the mixture of SM and BSM CP even/odd in the HVV tensor structure, using either ME-based variables or templates.

\[ L(HVV) \sim a_1 \frac{m_Z^2}{2} HZ^\mu Z_\mu - \frac{\kappa_1}{(\Lambda_1)^2} m_Z^2 HZ_\mu Z^\mu - \frac{1}{2} a_2 HZ^\mu \tilde{Z}_{\mu\nu} - \frac{1}{2} a_3 HZ^\mu \tilde{Z}_{\mu\nu} + \ldots \]

SM \hspace{2cm} BSM CP-even \hspace{2cm} BSM CP odd

The non-SM tensor couplings are consistent with zero for both ATLAS and CMS.
The Optimal Observable (OO) is expected to perform better than $\Delta \Phi$. It is defined as:

$$OO = \frac{2 \text{Re}(M_{SM}^* M_{CP-odd})}{|M_{SM}|^2}$$

with the Matrix Element for VBF production being

$$M = M_{SM} + \tilde{d} \cdot M_{CP-odd}$$

$$|M|^2 = |M_{SM}|^2 + \tilde{d} \cdot 2 \text{Re}(M_{SM}^* M_{CP-odd}) + \tilde{d}^2 \cdot |M_{CP-odd}|^2$$

With all 4-momenta of the final state particles (Higgs and two tagging jets) measured (not possible with $H \rightarrow WW^*$), the LO ME of SM and CP-odd can be calculated from HAWK, and then OO can be calculated per event.
CP test in $H \rightarrow \tau\tau$ decay

- CP-odd Yukawa coupling can enter the Lagrangian at dim-4, thus sensitive at tree-level rather than with the dim-6 operators in HVV

$$-g_\tau (\cos\phi \tau + \sin\phi \imath \gamma_5 \tau) h$$

$\Phi$ is the mixing angle. $\Phi=0$ ($\Phi=\pi/2$) means SM (CP odd)

- CP of $H\tau\tau$ coupling can be distinguished by the transverse tau spin correlations

$$\Gamma(H, A \rightarrow \tau^- \tau^+) \sim 1 - s_T^{-} s_T^{+} \pm s_T^{-} s_T^{+}$$

Sensitive to CP (H vs A)

- For example, with the $\tau \rightarrow \pi \nu$ decay, one can look at the angle between tau decay planes to extract $\Phi$:

$$\frac{d\Gamma(h \rightarrow \tau\tau \rightarrow \pi^+\pi^- + 2\nu)}{d\phi_{CP}} \propto 1 - \frac{\pi^2}{16} \cos(\phi_{CP} - 2\phi)$$

- It is experimentally challenging because the neutrinos are not reconstructed
CP test in $H \rightarrow \tau \tau$ decay

There are two methods to extract CP from $H \rightarrow \tau \tau$ decay:

**Impact Parameter (IP) method:**
- Approximately reconstruct the tau decay plane from its leading track and IP
- Best for the $\tau \rightarrow \pi \nu$ decay. The analyzing power is compromised for other tau decays

**Using the $\tau \rightarrow \rho \nu \rightarrow \pi^\pm \pi^0 \nu$ decay:**
- The tau decay plane can be approximately reconstructed by the track and neutral pion
- However, the relative energy of $\pi^\pm$, $\pi^0$ need to be classified in order to maximize the analyzing power

In order to use the two methods, the tau decay modes (substructure) need to be well differentiated (next few slides)

A few extra references:
EPJC 74 (2014) 3164, Phys. Rev. D88 076009,
Tau substructure in ATLAS

- Efficiency, $\Sigma$ column $\sim 1$
- Purity, $\Sigma$ row $\sim 1$

Good reconstruction of tau mass in different decay modes

Good tau decay classification

In general, non-negligible fraction of $2/1\ \pi^0$ reconstructed as $1/0\ \pi^0$

- With the substructure, a factor of 2 improvement of tau energy w.r.t. the calo-based at low $p_T$ ($\sim 0.16$ for neutral $\pi^0$)

- A factor of 5 improvement in the angular resolution
  - neutral $\pi^0\ \eta: \sim 0.006$
  - neutral $\pi^0\ \Phi: \sim 0.012$
Higgs CP in \( H \to \tau \tau \) decay at LHC

At LHC, the search for Higgs CP mixing, and the reconstruction of the neutrino momenta, is more challenging because

- Missing energy in the Z direction is unknown
- Much harder to resolve \( \pi^0 \) from \( \pi^\pm \) as the tau is highly boosted
- Much worse ditau mass resolution, and hence large background from \( Z \to \tau \tau \)
- The QCD background is roughly as large as \( Z \to \tau \tau \)

Three main tau decay modes are considered

\[
\begin{align*}
- \tau^\pm & \to \pi^\pm \nu \ (\sim 10\%), \\
- \tau^\pm & \to \rho^\pm \nu \to \pi^\pm \pi^0 \nu \ (\sim 25\%), \\
- \tau^\pm & \to a^\pm \nu \to \pi^\pm \pi^\pm \pi^\mp \nu \ (\sim 10\%).
\end{align*}
\]
Higgs CP in $H \to \tau\tau$ decay at LHC

As has been investigated in 1612.00413, the best production mode for $H \to \tau\tau$ CP is not gluon-fusion, but VBF, e.g. CMS result (1708.00373):
Higgs CP in $H \rightarrow \tau\tau$ decay at LHC

As for the $e^+e^-$ case, we reconstruct the neutrino momenta with the mass, MET and impact parameter constraints:

$$
\chi^2 = \left(\frac{m_{\tau\tau} - m_h}{\sigma_h}\right)^2 + \left(\frac{m_{\tau1} - m_{\tau}}{\sigma_{\tau}}\right)^2 + \left(\frac{m_{\tau2} - m_{\tau}}{\sigma_{\tau}}\right)^2 \\
+ \left(\frac{E_{x,\text{fit}} - E_{x}}{\sigma_{\text{mis}}}\right)^2 + \left(\frac{E_{y,\text{fit}} - E_{y}}{\sigma_{\text{mis}}}\right)^2 + \sum_{i} \chi_{\text{IP},i}^2
$$

We assume the IP resolutions are $a \oplus b/p_T$ with $a=8.5$ (13.5) $\mu$m and $b=110$ (200) $\mu$m for $d_0$ ($z_0$) for ATLAS after the Phase II upgrade.

The last term is the contribution from the impact parameters. For 3-prong ($a_1$ axial vector) decays, it is a bit special, since three track can determine the decay vertex and the tau flight direction. We first find the direction by minimizing the IP sum of the 3 tracks, then minimize the above $\chi^2$.

For 1-prong tau, we scan the $\eta/\phi$ of one tau’s neutrino, and repeat by for the other tau. After the global minimal point is obtained, a MINUIT fit is performed for a better estimation.
The impact parameters

- Tracks from taus have broader impact parameter (IP) distributions than the prompt tracks such as the leptons from Z decay.

![Graphs showing impact parameter distributions](image)

- The impact parameters are additional helpful information to reconstruct the neutrinos from tau decay [A. Rouge hep-ex/0505014; D. Jeans arXiv:1507.01700]. Since the resolution of $d_0/z_0$ may not be small, we take a less aggressive approach by treating them as extra constraints.

We first find the intersection of tau flight direction with the track trajectory in the transverse plane, and deduce $z_0$ by $z_0 = L \sinh \eta_\tau - S \sinh \eta_{\text{track}}$. 
The impact parameters

- The collision point (O) can be inside (a) or outside the track path curvature (b, c)

- In the case of (b), two solutions exist and both are tested. In the case of (c), it is assumed to be from resolution effect

- When the fitted impact parameters are in the physical regime, the $\chi^2$ is

$$\chi^2 = \left( \frac{d_0^{\text{fit}} - d_0}{\sigma_d} \right)^2 + \left( \frac{z_0^{\text{fit}} - z_0}{\sigma_z} \right)^2$$

- Otherwise, in the example case of (c), the $\chi^2$ reads (so that the best-fit perigee point for O’ is D)

$$\chi^2 = \left( \frac{d_0^{\text{fit}} + d_0 - 2d_0^C}{\sigma_d} \right)^2 + \left( \frac{z_0^{\text{fit}} - z_0}{\sigma_z} \right)^2$$
Higgs CP in $H \rightarrow \tau \tau$ decay at LHC

- The 3-prong tau direction is reconstructed with a resolution of $\sim 0.007$
- But since tau is highly boosted (small opening angle), this value may not seem as impressive as it looks
Higgs CP in $H \to \tau\tau$ decay at LHC

With some background suppression cuts, the expected event yields roughly scales with the CMS results.

![Event distribution](image.png)

**Table I.** The events for signal (in total and also in each decay mode) and background processes left after all selection cuts at the LHC with 300 fb$^{-1}$ luminosity.

<table>
<thead>
<tr>
<th>Process Signal</th>
<th>$Z \to \tau\tau$</th>
<th>$Z \to \tau\tau$(EW)</th>
<th>QCD-fake(assumed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Events</td>
<td>125</td>
<td>91</td>
<td>7</td>
</tr>
<tr>
<td>$\rho + \rho$</td>
<td>41.3</td>
<td>34.3</td>
<td>1.7</td>
</tr>
<tr>
<td>$a_1 + \rho$</td>
<td>31.0</td>
<td>21.3</td>
<td>2.2</td>
</tr>
<tr>
<td>$\pi + \rho$</td>
<td>30.7</td>
<td>22.5</td>
<td>2.0</td>
</tr>
<tr>
<td>$a_1 + \pi$</td>
<td>11.5</td>
<td>6.2</td>
<td>0.50</td>
</tr>
<tr>
<td>$a_1 + a_1$</td>
<td>5.6</td>
<td>2.8</td>
<td>0</td>
</tr>
<tr>
<td>$\pi + \pi$</td>
<td>4.9</td>
<td>3.4</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Tau cuts:** One or three tracks with unit charge. The leading track has $p_T > 5$ GeV. For the 3-prong tau, $p_T > 2$ GeV for the other tracks. The two taus have opposite charge, and are within $|\eta| < 2.5$. $p_T > 40, 30$ GeV are required on the two taus for trigger. They should also have $|\Delta \phi| < 2.9$ to avoid the back-to-back topology.

**VBF Cut:** $p_T^{j_1} > 50$ GeV, $p_T^{j_2} > 40$ GeV, $|\Delta \eta_{jj}| > 3.8$, $m_{jj} > 500$ GeV, $\eta_{j_1} \times \eta_{j_2} < 0$

**Tau Centrality:** min{$\eta_{j_1}, \eta_{j_2}$} < $\eta_{\tau_1, \tau_2}$ < max{$\eta_{j_1}, \eta_{j_2}$}

**Higgs Mass:** 115 GeV < $m_{\tau\tau}$ < 150 GeV

**Missing Energy:** $E_{proj}^{\text{fit}} - p_T^{\nu_1 + \nu_2} < -6$ GeV
Higgs CP in $H \rightarrow \tau\tau$ decay at LHC

After the signal window cuts, a second fit is done with the Higgs mass constrained to 125 GeV for better neutrino momentum precision.
Higgs CP angle

With all final state particles reconstructed, we can perform a Matrix Element based analysis of the underlying Higgs CP mixing angle $\Phi$. The Higgs decay amplitude can be expressed as

$$|\mathcal{M}|^2 \propto A + B \cos(2\phi) + C \sin(2\phi),$$

$$\propto I_1 \cos^2(\phi) + I_2 \sin(\phi) \cos(\phi) + I_3 \sin^2(\phi)$$

Two observables can be reconstructed per event for the CP test


- ME angle $\Delta\Phi_{ME}$, defined as

$$|\mathcal{M}|^2 \propto A + \sqrt{B^2 + C^2} \cos(\Delta\phi_{ME} - 2\phi)$$

$$\cos(\Delta\phi_{ME}) = \frac{B}{\sqrt{B^2 + C^2}}, \quad \sin(\Delta\phi_{ME}) = \frac{C}{\sqrt{B^2 + C^2}}$$

At low mixing angle values, the two perform similarly, while in high values of $\Phi$, $\Delta\Phi_{ME}$ is better
Higgs CP angle

Explicit expressions of the coefficients in the ME:

\[ A = 2 (k_- \cdot p_-) (k_+ \cdot p_-) - p_-^2 (k_- \cdot k_+) + (p_- \leftrightarrow p_+) \]
\[ B = 2 (g^{\mu \rho} g^{\nu \sigma} + g^{\mu \sigma} g^{\nu \rho} - g^{\mu \nu} g^{\rho \sigma}) k_-^\mu k_+^\nu p_-^\rho p_+^\sigma \]
\[ C = 2 \epsilon_{\mu \nu \rho \sigma} k_-^\mu k_+^\nu p_-^\rho p_+^\sigma \]  

(4)

\[ k_\pm^\mu \equiv 2 (J_\pm \cdot p_\nu) J_\pm^\mu - J_\pm^2 p_\nu \]

\[ J_\pm^\mu (\tau^\pm \rightarrow \pi^\pm \nu) = p_{\pi^\pm}^\mu \]
\[ J_\pm^\mu (\tau^\pm \rightarrow \pi^\pm \pi^0 \nu) = p_{\pi^\pm}^\mu - p_{\pi^0}^\mu \]
\[ J_\pm^\mu (\tau^\pm \rightarrow \pi^\pm \pi^\pm \pi^\mp \nu) = F^{13} (q_1^\mu - q_3^\mu - G^{13} Q^\mu) + (1 \leftrightarrow 2) \]

where \( Q^\mu = q_1^\mu + q_2^\mu + q_3^\mu \), \( G^{i3} = \frac{Q \cdot (q_i - q_3)}{Q^2} \) and \( F^{i3} \) are the form factors for \( a_1 \) channel

Form factors verified with the TauDecay library (Hagiwara et al.)
Higgs CP in $H \rightarrow \tau \tau$ decay at LHC

Angular observable distributions

$\alpha_1 \nu + \pi \nu$

combined
Higgs CP in $H \rightarrow \tau\tau$ decay at LHC

CP mixing angle determination:

A combined precision of $6.9^\circ$ can be achieved at LHC with 3 ab$^{-1}$
At a $e^+e^-$ collider, the Higgs can be produced via Zh or VBF productions.

We assume a 250 GeV collision energy where the Higgs is mainly produced by the Zh mode. This corresponds to low-energy ILC running.

Three main decay channels are investigated:

<table>
<thead>
<tr>
<th>Mode</th>
<th>BR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_\tau\nu_l$</td>
<td>35.04</td>
</tr>
<tr>
<td>$\nu_\tau\pi^\pm$</td>
<td>10.77</td>
</tr>
<tr>
<td>$\nu_\tau\pi^\pm\pi^0$</td>
<td>25.37</td>
</tr>
</tbody>
</table>

Encouraged by the tau substructure techniques from ATLAS, it is assumed that $\pi^0$ can be resolved with a 10% energy resolution in this analysis. It is further assumed that no cross talk between different modes.
Refined Higgs momentum

Compared with a hadron collider, the e\(^+\)e\(^-\) collider has the advantage to resolve the Higgs momentum in z-axis by the recoil of Z, but subject to the ISR photons.

With the known Higgs mass, the fraction of momentum carried away by the collinear photon can be solved, subject to a two-fold ambiguity:

\[
x = \frac{E_{\text{CM}}^2 - 2E_{\text{CM}}E + m^2 - m_h^2}{\pm E_{\text{CM}}^2 \mp E_{\text{CM}}E + E_{\text{CM}}p_z}
\]

E, m and \(p_z\) are for the recoiling Z boson. \(E_{\text{CM}}=250\) GeV.
Refined Higgs momentum

To resolve the ambiguity, collinear approximation (neutrinos from the tau are collinear with the visible products) is used and the following $\chi^2$ is minimized

$$\chi^2 = \sum_{i=0}^{3} \left( \frac{p_{h,i} - p_{h,RC,i}}{0.5} \right)^2 + \left( \frac{m_Z - 91.2}{2.5} \right)^2 + \left( \frac{f_{i1} - 1}{0.06} \right)^2 + \left( \frac{f_{i2} - 1}{0.06} \right)^2$$

$p_h$ and $p_{h,RC}$ are Higgs 4-momentum from collinear calculation and Z recoil respectively. The $f_{1,2}$ are correction factors for the jets from Z decay.

After minimization, not only the $x$ ambiguity is resolved, but also the Higgs recoil momentum is improved.
Neutrino momentum
Cleaning cuts

The combined efficiencies after objects selection (due to jet resolution and neutrino pair, the lepton+Z→jj modes are not considered):

<table>
<thead>
<tr>
<th></th>
<th>ℓ + π</th>
<th>ℓ + ρ</th>
<th>π + π</th>
<th>π + ρ</th>
<th>ρ + ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z → ee/μμ</td>
<td>31.4%</td>
<td>27.2%</td>
<td>19.2%</td>
<td>18.5%</td>
<td>15.7%</td>
</tr>
<tr>
<td>Z → jj</td>
<td>34.8%</td>
<td>30.8%</td>
<td>24.5%</td>
<td>21.3%</td>
<td>18.9%</td>
</tr>
</tbody>
</table>

A sequence of cuts are applied to suppress the background, and to purify well reconstructed signal events

<table>
<thead>
<tr>
<th>Z → ℓℓ</th>
<th>Z → jj</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_Z &gt; 70 GeV</td>
<td>m_Z ≤ 105 GeV</td>
</tr>
<tr>
<td>m_{h}^{RC} &gt; 120 GeV</td>
<td>m_{h}^{RC} &gt; 110 GeV</td>
</tr>
<tr>
<td>m_{h,fit}^{RC} ≥ 122 GeV</td>
<td>80 GeV &lt; m_Z^{fit} &lt; 100 GeV</td>
</tr>
<tr>
<td>120 GeV &lt; m_h &lt; 130 GeV</td>
<td>1.5 GeV &lt; m_τ &lt; 2.0 GeV</td>
</tr>
<tr>
<td>m_ρ &gt; 0.3 GeV (for channels with ρ)</td>
<td></td>
</tr>
</tbody>
</table>

With 5 ab⁻¹ of data, expect to have about 1519 (133) signal (background) events.
Higgs CP

The OO and $\Delta \Phi_{\text{ME}}$ distributions in the $\pi^+\rho$ and $\rho^+\rho$ channels for CP even and $\Phi=0.16$
Higgs CP

- The OO or $\Delta \Phi_{\text{ME}}$ is better than the other observables such as $\Delta \Phi_{\text{IP}}$ and $\Delta \Phi_{\text{CP}}$

\[
\Phi = \arccos\left(\hat{p}_{d1} \cdot \hat{p}_{d2}\right) \times \text{sgn}\left(\hat{p}_{m1} \cdot (\hat{p}_{d1} \times \hat{p}_{d2})\right)
\]

For $\Delta \Phi_{\text{IP}}$, $(\hat{p}_{m1}, \hat{p}_{d1}, \hat{p}_{m2}, \hat{p}_{d2}) = (p_{\pi^+} + p_{\pi_0^+}, n^+, p_{\pi^-} + p_{\pi_0^-}, n^-)$

For $\Delta \Phi_{\text{CP}}$, $(\hat{p}_{m1}, \hat{p}_{d1}, \hat{p}_{m2}, \hat{p}_{d2}) = (p_{\pi^+}, p_{\pi_0^+}, p_{\pi^-}, p_{\pi_0^-})$

\[(c)\]
Template PDF functions for different CP mixing angle hypotheses are prepared and fit to the pseudo-data. The difference (w.r.t. the minimum) of the Negative Log Likelihood ($\Delta NLL$) is plotted for different $\Phi$, from which the 1\(\sigma\) confidence interval can be found.

With 5 (2) ab\(^{-1}\) of data, a precision of 2.9\(^{\circ}\) (5.2\(^{\circ}\)) can be reached for the Higgs CP mixing angle measurement.
Summary

Testing the CP nature of the Higgs is one of the important tasks after its discovery. This needs a large and pure Higgs signal events with rich decay products, and can be achieved with a high precision at future $e^+e^-$ colliders

At the LHC, the $H \rightarrow \tau\tau$ CP is best studied in the VBF channel. The estimated precision is $6.9^\circ$ with 3 ab$^{-1}$ of HL-LHC data for each experiment (ATLAS/CMS)

The $H \rightarrow \tau\tau$ decay is an ideal channel for probing Higgs CP angle for possible effect of CP violation. Our study, based on three tau decay modes, show that with 5 (2) ab$^{-1}$ of data, a precision of $2.9^\circ$ ($5.2^\circ$) can be reached for the CP angle measurement. Further improvement is expected if the 3-prong tau decay mode is added