

# Polarization studies for beam parameters of CEPC and FCCee

Ivan Koop, BINP, 630090 Novosibirsk

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# Outline

- Polarization specificity of CEPC and FCC-ee
- Simplified spin tracking code
- Attainable self-polarization degree estimations
- **Resonant depolarization** -
- **Coherent free precession** – two complementary techniques to determine the resonant spin precession frequency.
- Conclusion.

# Polarization: CEPC and FCCee specificity

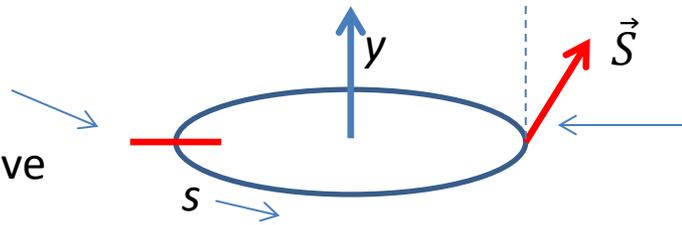
- Beam emittances in CEPC and FCCee are so small that all intrinsic resonances with betatron oscillation frequencies are suppressed and can be neglected:

$$\nu_0 \cdot |\sigma_{y'}| \sim 2 \cdot 10^{-5} \quad (\text{even for } E=80 \text{ GeV})$$

- Therefore, only **static vertical orbit distortions** and the **longitudinal magnetic fields** with nonzero integrals can affect the spin motion!
- Precession frequency modulation by the synchrotron oscillations is very important! The relevant parameter is:  
 $\xi = \nu_0 \sigma_\delta / Q_s$  Must prefer  $\xi < 1$ , means:  $Q_s$  is as high as possible! At **LEP** was  $Q_s=0.083$  - comfortable for  $E=45$  GeV!

# Simplified spin tracking code

Spin perturbation,  $w$   
Is localized at  $s=0$ .  
Random jumps of relative  
energy deviation  $\delta$  are  
localized also here.



Spin precession around the  
 $y$ -axis with  $\nu \sim \delta$ .  
Radiation damping of  $\delta$  is  
taken into account!

These jumps + SR damping produce some equilibrium  
energy spread  $\sigma_\delta$ :  $\sigma_\delta = 0.66 \cdot 10^{-3}$  at  $E=80$  GeV

The synchrotron motion and energy diffusion can be modeled without invoking of any information about the lattice of a ring! Essential input parameters are only: the equilibrium energy spread  $\sigma_\delta$ , damping time  $\tau_\delta$ , synchrotron tune  $Q_s$ .

Parent resonances (due to vertical orbit distortions, mainly) are sitting on integers of the spin tune  $\nu_0$ , which is defined as:

$$\nu_0 = \bar{\gamma} a = \bar{E}(\text{GeV})/0.44064846.$$

Due to energy modulation by the synchrotron oscillations the higher order spin resonances are powered. They are spaced from the parent resonance ( $\nu_0 = n$ ) by the integer number of synchrotron tunes:  $\nu_0 = n + m \cdot Q_s$

Their strength is proportional to  $J_m(\xi)$  and raises promptly with the increase of  $\xi$ .

# 1. Equilibrium beam polarization degree simulation

The equilibrium polarization degree can be calculated as:

$$P = 92.6(\%) / (1 + \tau_{ST} / \tau_{dep})$$

where  $\tau_{ST}$  is the Sokolov-Ternov polarization time, while  $\tau_{dep}$  is the obtained by the spin tracking code depolarization time.

The harmonic spin matching, if applied as it was done at LEP and HERA, can minimize to some extent the strengths of few nearby resonances.

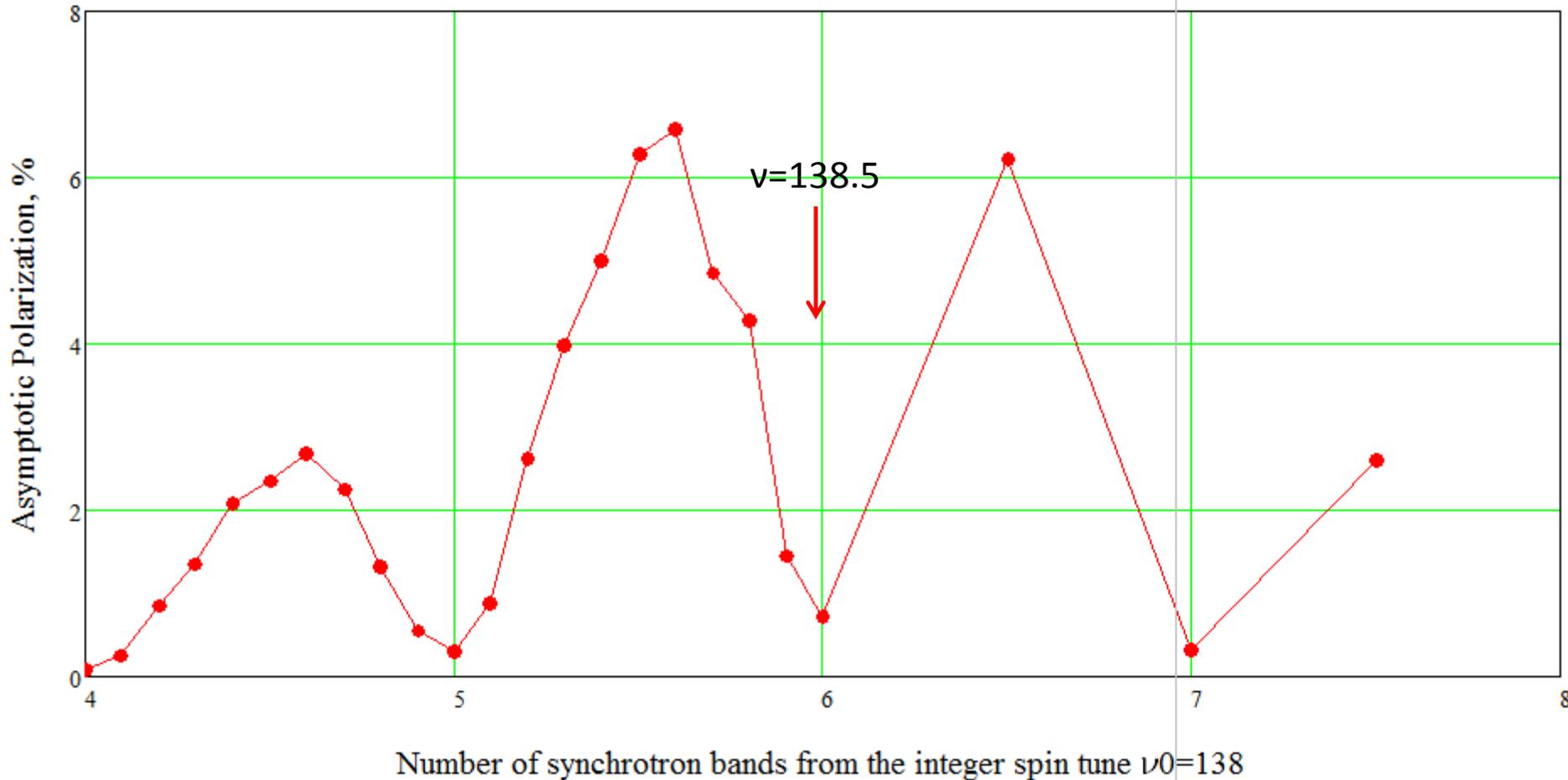
We rely on data from LEP at 61 GeV, where some polarization level, say about 6%, was observed (see R.Assmann et al. , “**Spin dynamics in LEP with 40–100 GeV beams**”, AIP Conference Proceedings **570**, 169 (2001); doi: 10.1063/1.1384062).

This translates to estimation of some residual uncompensated spin perturbation:  $w = 0.0015$ , which we will use further as a reference.

# Equilibrium polarization for LEP at 61 GeV, $Q_s=0.0833$

The parent and the side band resonances are induced by the local spin rotation around the longitudinal axis by the angle  $\varphi = w \cdot 2\pi$ . Here  $w = 0.0015$  was chosen to explain the polarization level observed at LEP experimentally. At such relatively high value of  $Q_s = 0.0833$  dips at  $m \cdot Q_s$  detunings from the parent resonance are quite pronounced.

$C=26.7$  km,  $E=61$  GeV,  $Q_s=0.0833$ ,  $\sigma_\delta=0.000939$  ( $\sigma_E=57.3$  MeV),  $\lambda=154$  turns,  $\xi=1.56$



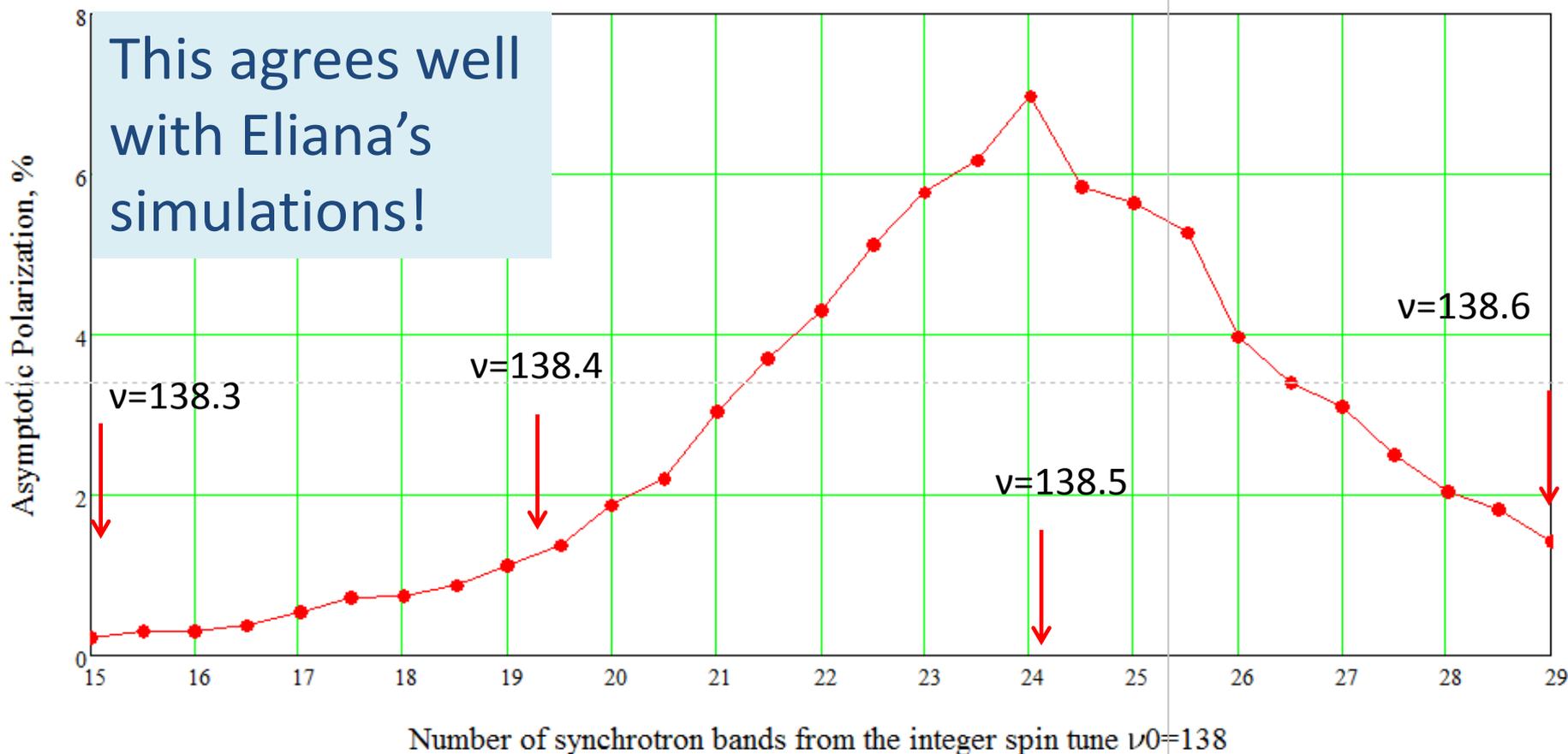
# Equilibrium polarization for LEP at 61 GeV, $Q_s=0.02073$

Here  $w = 0.0015$ ,  $Q_s = 0.02073$ . Dips at integer detunings  $m \cdot Q_s$  from the parent resonance  $\nu = n$  disappear because of high  $m$ .

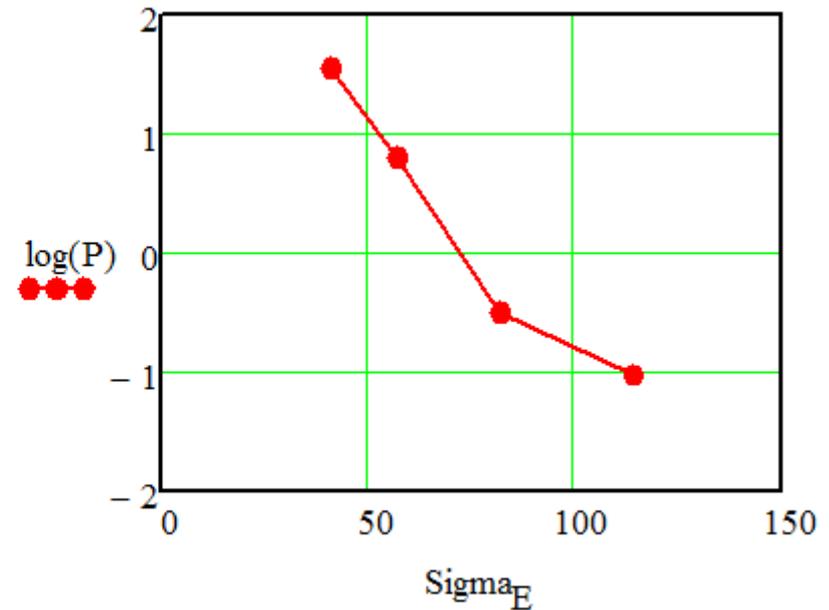
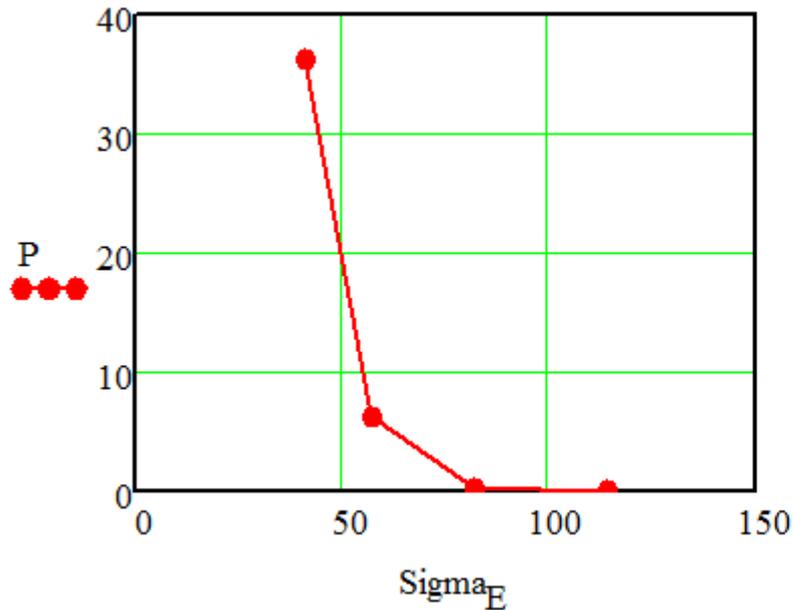
It is remarkable that polarization survives near half-integer spin tune values! Siberian Snake?

$J_m(\xi)$  is a rather small for  $m=20$ ,  $\xi = 6$ :  $J_{20}(6) = 10^{-9}$ .

C=26.7 km, E=61 GeV,  $Q_s=0.02073$ ,  $\sigma_\delta=0.000939$  ( $\sigma_E=57.3$  MeV),  $\lambda=154$  turns,  $\xi=6.274$



# Polarization dependence on energy diffusion rate



Lessons from this study:

- 1) No strong influence of Qs on the attainable polarization level. Synchrotron modulation not too much important!
- 2) Only the value of beam energy spread is really important. Recommendation given from the LEP experience:  $\sigma_E < 52 \text{ MeV}$  is confirmed by these simulations.

## 2. Resonant depolarization studies

RF flipper/depolarizer frequency:  $\nu = f_{DP}/f_0$ , with  $f_0$  – revolution frequency

RF flipper/depolarizer strength – spin rotation around the longitudinal or transverse beam axis:

$$w = \begin{cases} B_{\parallel} l / 2\pi B\rho \\ \nu_0 \cdot B_{\perp} l / 2\pi B\rho \end{cases} \quad (\nu_0 = \gamma a = 103.5 \text{ at } Z)$$

Frequency scanning rate:  $\delta\nu = d\nu/dn$ . Here  $n$  – the number of turns

Typical values:  $w = 3 \cdot 10^{-5} \div 5 \cdot 10^{-4}$   
 $\delta\nu = 1 \cdot 10^{-9} \div 1 \cdot 10^{-8}$

Froisart-Stora formula for reversing of polarization (dynamical depolarization!):

$$P(t \rightarrow \infty) = P(0) \cdot (2e^{-J} - 1) \quad J = \pi^2 w^2 / \delta\nu$$

My simulations will show, that to depolarize a beam we shall adjust parameters to such values, that:

$$w^2 / \delta\nu \cong 1 \quad (\text{then } J \cong 10)$$

# Scaling of resonant depolarization parameters

Assuming that we can narrow the energy search interval to  $\pm 1 \cdot 10^{-5}$ , we may choose:

Reasonable spin tune interval to scan:  $\Delta\nu_0 = \pm \nu_0 \cdot 10^{-5} = \pm 0.001$  ( at Z and twice larger at W)

Frequency increment per turn:  $\varepsilon' \equiv d\nu = 2\Delta\nu_0 / (f_0 \cdot \Delta t) = 0.002 / (3300 \cdot 300) = \mathbf{2 \cdot 10^{-9}}$  ( $6.7 \cdot 10^{-9}$  Hz/s)

Now let evaluate the reasonable strength  $w$  of the depolarizer (tune provided by transverse RF-field). But first, let's define the parameter called as index of synchrotron modulation:

$$\xi = \nu_0 \sigma_\delta / Q_s \quad (\text{With } \nu_0 = 182.5, \sigma_\delta = .00066, Q_s = 0.075 \rightarrow \xi = 1.6)$$

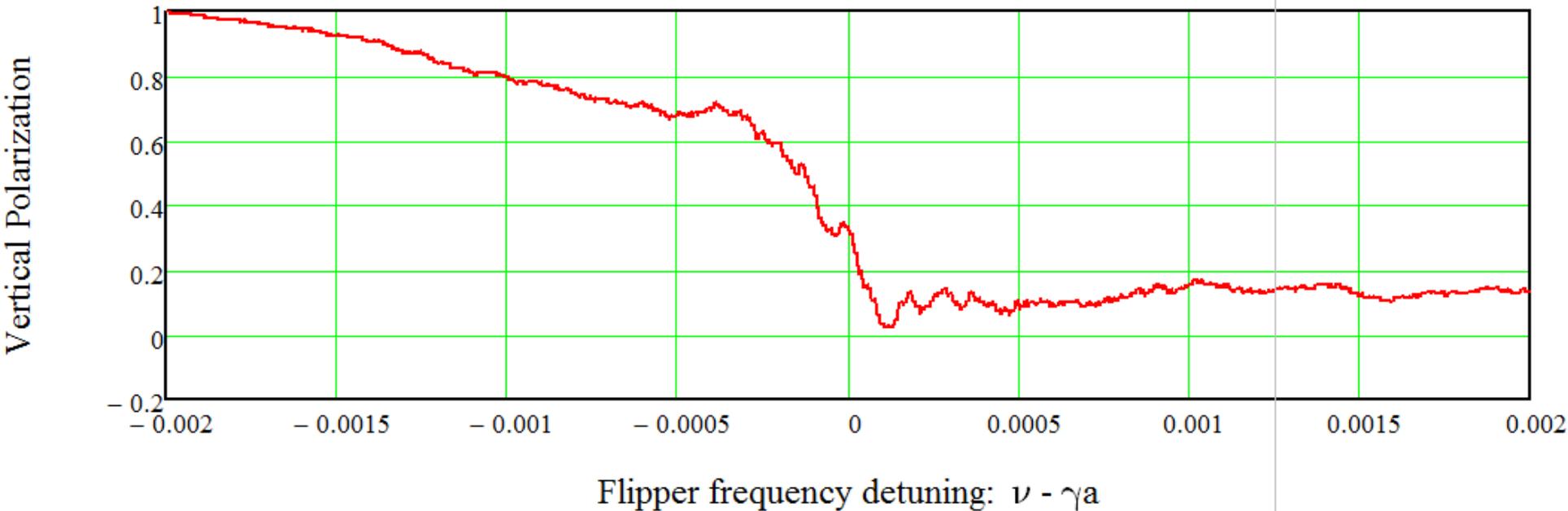
If this index is small  $\xi < 1$ , then minimal  $w$  is:  $w^2 / \delta\nu \cong 1 \rightarrow w = 4.5 \cdot 10^{-5}$

If this index is large, say  $\xi = 1.6$ , then:  $(w \cdot J_0(\xi))^2 / \delta\nu \cong 1 \rightarrow w = \mathbf{1.0 \cdot 10^{-4}}$

In fact, beam has a spread of amplitudes of synchrotron oscillations, therefore the depolarizer acts on different particles differently, according to spread of synchrotron modulation index. Still, fluctuations due to SR emission make random work and every particle is subjected to more or less equal depolarization effect. Here we shall remind that the width of the resonance zone will be crossed during many radiation damping times . At W  $\tau_\delta := 235$  turns:  $w / \delta\nu \cong 5 \cdot 10^4$  turns - this is about  $\mathbf{200 \tau_\delta}$

# FCC-ee at 80.41 GeV, $Q_s=0.100$ , $w=10^{-4}$ , $d\nu=0.5\cdot 10^{-8}$

$C=97.75$  km, 80.41 GeV,  $Q_s=0.100$ ,  $\sigma_\delta=0.00066$ ,  $w=1\cdot 10^{-4}$ ,  $\varepsilon'=0.5\cdot 10^{-8}$



$$\nu_0 = 182.481$$

$$w^2/\varepsilon' = 2$$
$$(w \cdot J_0(\xi))^2/\varepsilon' = 0.9$$

$$\xi = \nu_0 \sigma_\delta / Q_s = 0.87$$

$$\text{Scan time: } T = 260.8 \text{ s}$$

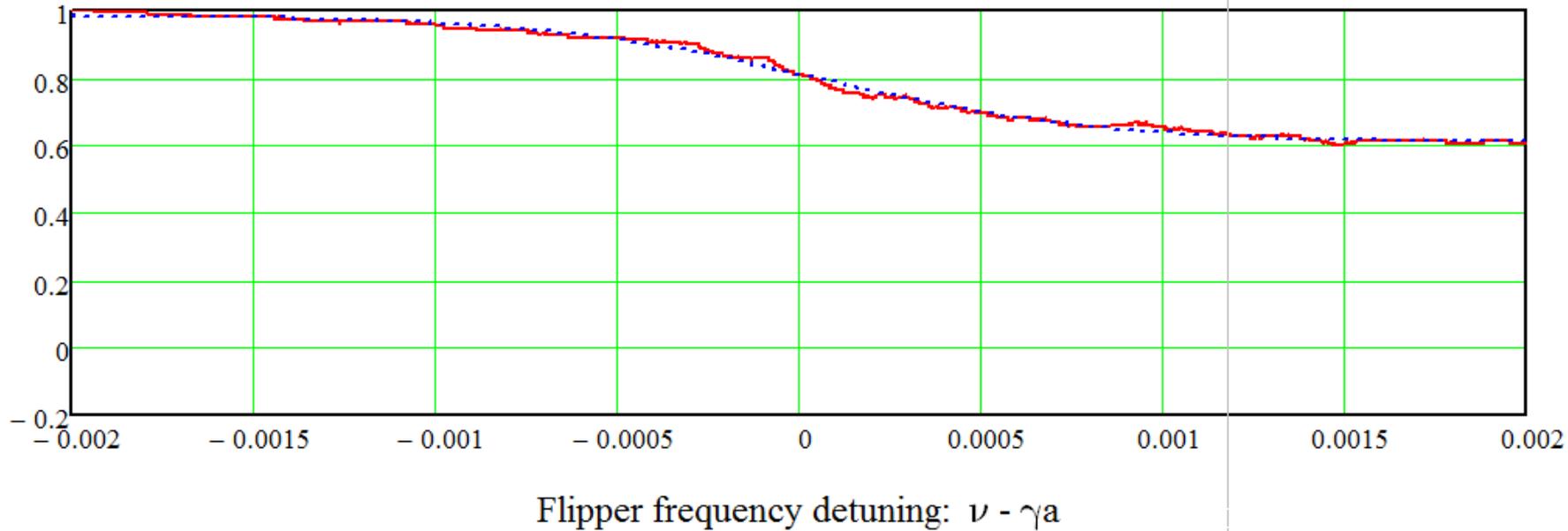
With such, relatively high value of the synchrotron tune, the localization of the resonance frequency is quite accurate.

Remind: the wanted accuracy is  $\Delta\nu=0.0001$

# FCC-ee at 80.41 GeV, $Q_s=0.075$ , $w=0.5 \cdot 10^{-4}$ , $d\nu=0.5 \cdot 10^{-8}$

$C=97.75$  km, 80.41 GeV,  $Q_s=0.075$ ,  $\sigma_\delta=0.00066$ ,  $w=0.5 \cdot 10^{-4}$ ,  $\epsilon'=0.5 \cdot 10^{-8}$

Vertical Polarization



$$\nu_0 = 182.481$$

$$w^2/\epsilon' = 0.5$$
$$(w \cdot J_0(\xi))^2/\epsilon' = 0.1$$

$$\xi = \nu_0 \sigma_\delta / Q_s = 0.87$$

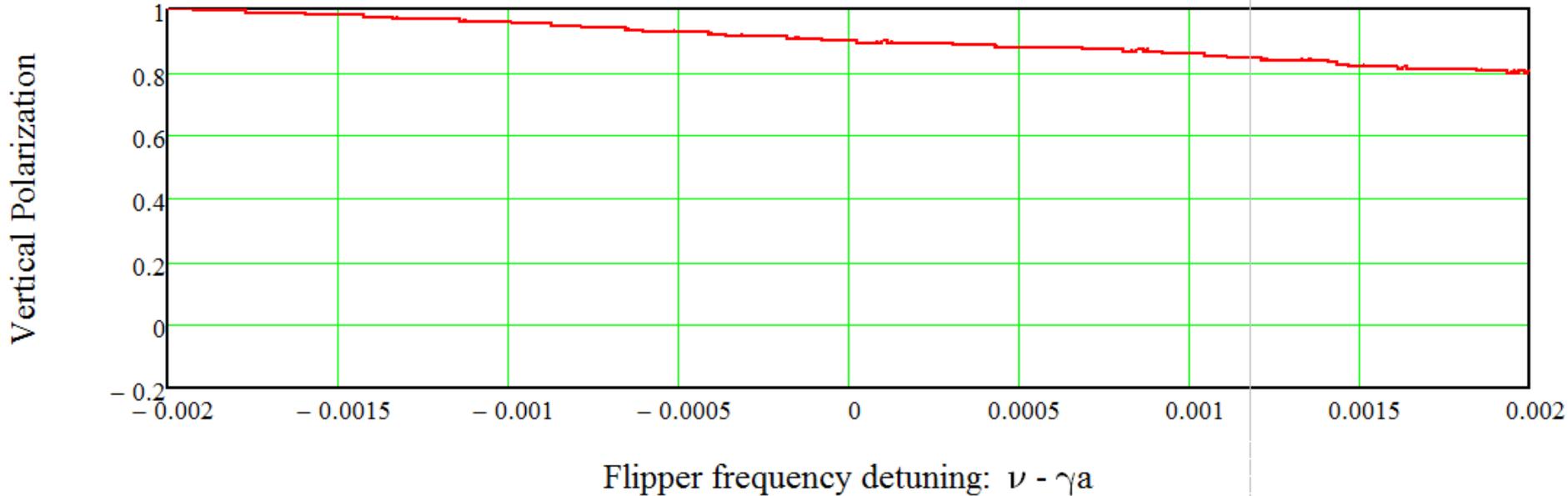
$$\text{Scan time: } T = 260.8 \text{ s}$$

With such, relatively low value of the depolarizer strength  $w=0.5 \cdot 10^{-4}$  and, subsequently, with low adiabaticity parameter  $(w \cdot J_0(\xi))^2/\epsilon'=0.1$ , a beam became only partially depolarized. A jump in polarization is about -37%.

Still, the applied fit by hyperbolic tangent (blue dots at the plot) shows small error  $\Delta\nu=0.00005$  in determination of the crossing the resonance frequency point. Looks acceptable?

# FCC-ee at 80.41 GeV, $Q_s=0.05$ , $w=.5 \cdot 10^{-4}$ , $dv=0.5 \cdot 10^{-8}$

$C=97.75$  km, 80.41 GeV,  $Q_s=0.050$ ,  $\sigma_\delta=0.00066$ ,  $w=0.5 \cdot 10^{-4}$ ,  $\varepsilon'=0.5 \cdot 10^{-8}$



$$\nu_0 = 182.481$$

$$w^2/\varepsilon' = 0.5$$
$$(w \cdot J_0(\xi))^2/\varepsilon' = 0!$$

$$\xi = \nu_0 \sigma_\delta / Q_s = 2.4$$

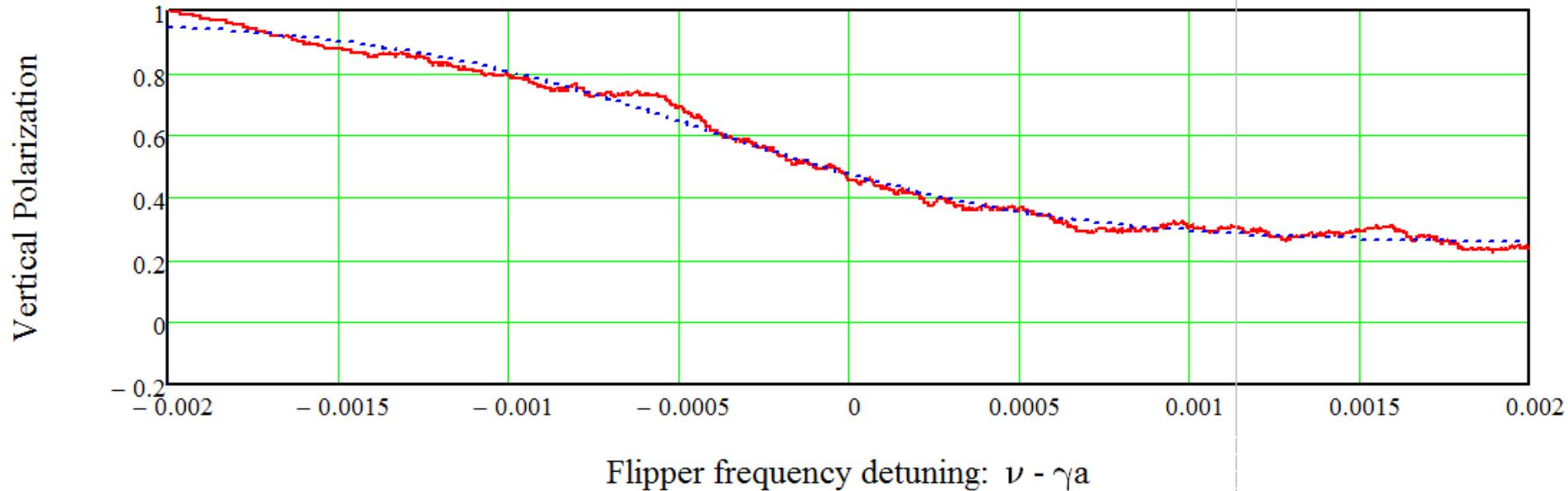
Scan time:  $T = 260.8$  s

With low values of the synchrotron tune  $Q_s=0.05$  and of the average value of Froissart – Stora parameter  $(w \cdot J_0(\xi))^2/\varepsilon' = 0!$ , depolarization process does not show any resonance!

This is due to high value of the average synchrotron modulation index:  $\xi = \nu_0 \sigma_\delta / Q_s = 2.4$ . Remind:  $J_0(2.405) = 0$ .  
**Non-acceptable!**

# FCC-ee at 80.41 GeV, $Q_s=.05$ , $w=1.41 \cdot 10^{-4}$ , $dv=0.5 \cdot 10^{-8}$

$C=97.75$  km, 80.41 GeV,  $Q_s=0.050$ ,  $\sigma_\delta=0.00066$ ,  $w=1.41 \cdot 10^{-4}$ ,  $\epsilon'=0.5 \cdot 10^{-8}$



$$\nu_0 = 182.481$$

$$w^2 / \epsilon' = 4$$

$$(w \cdot J_0(\xi))^2 / \epsilon' = 0!$$

$$\xi = \nu_0 \sigma_\delta / Q_s = 2.4$$

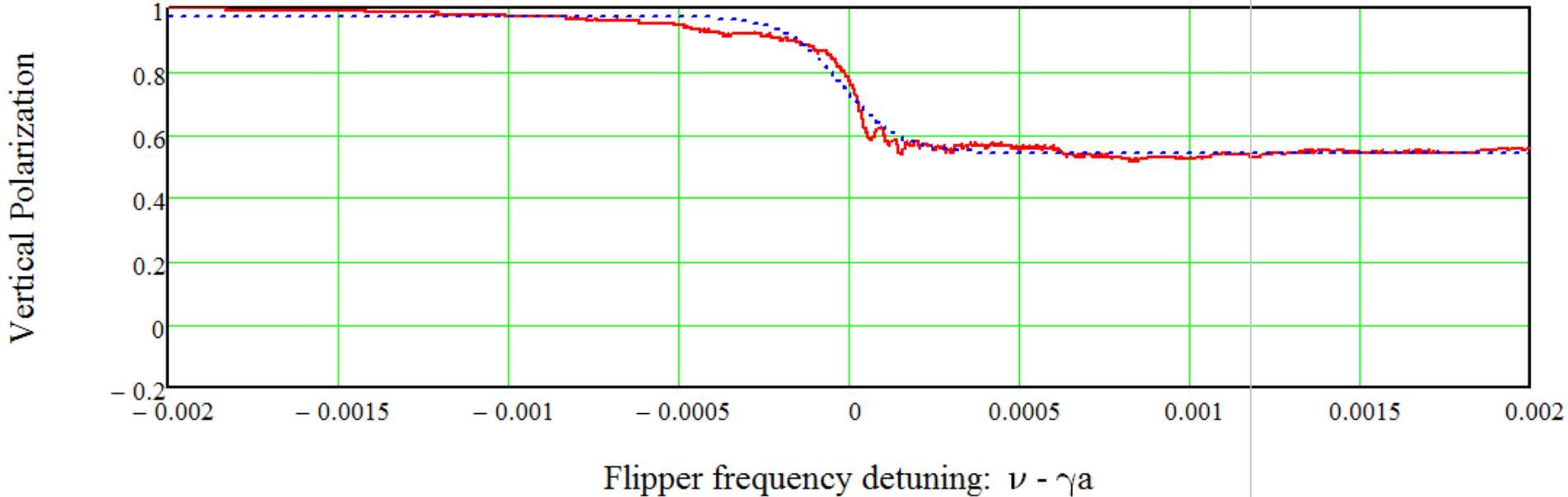
$$\text{Scan time: } T = 260.8 \text{ s}$$

Try to increase the depolarizer strength up to  $w=1.41 \cdot 10^{-4}$ .  
But not clear, trustable picture we see. Simple fit gives the resonance frequency with an error  $\Delta\nu = -0.0004$ .

I think, the last two plots show that the synchrotron tune at W should be made much higher. Its minimal acceptable value is  $Q_s=.075$ , or even higher!

# FCC-ee at 45.6 GeV, $Q_s=.025$ , $w=0.5 \cdot 10^{-4}$ , $d\nu=0.5 \cdot 10^{-8}$

$C=97.75$  km, 45.59 GeV,  $Q_s=0.025$ ,  $\sigma_\delta=0.00038$ ,  $w=0.5 \cdot 10^{-4}$ ,  $\epsilon'=0.5 \cdot 10^{-8}$



$$\nu_0 = 103.461$$

$$w^2/\epsilon' = 0.5$$

$$(w \cdot J_0(\xi))^2/\epsilon' = 0.115$$

$$\xi = \nu_0 \sigma_\delta / Q_s = 1.556$$

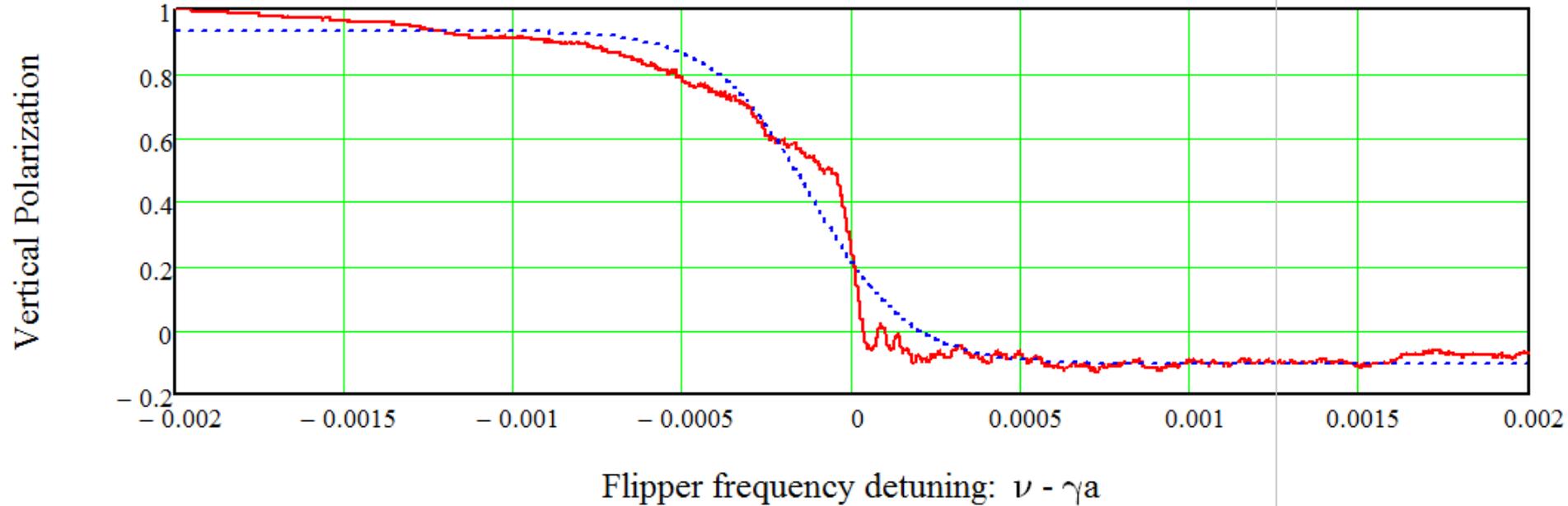
Scan time:  $T=260.8$  s

With nominal  $Q_s=.025$  at Z. Due to high value of the synchrotron modulation index  $\xi=1.555$  the effective value of Froissart-Stora parameter became too small  $(w \cdot J_0(\xi))^2/\epsilon'=0.115$  and jump in depolarization level is only 40%.

My fit gives the resonance frequency with an error  $\Delta\nu= -0.00002$ .

# FCC-ee at 45.6 GeV, $Q_s=.025$ , $w=1\cdot 10^{-4}$ , $d\nu=0.5\cdot 10^{-8}$

$C=97.75$  km, 45.59 GeV,  $Q_s=0.025$ ,  $\sigma_\delta=0.00038$ ,  $w=1\cdot 10^{-4}$ ,  $\varepsilon'=0.5\cdot 10^{-8}$



$$\nu_0 = 103.461$$

$$w^2/\varepsilon' = 2$$

$$(w \cdot J_0(\xi))^2/\varepsilon' = 0.46$$

$$\xi = \nu_0 \sigma_\delta / Q_s = 1.556$$

$$\text{Scan time: } T = 260.8 \text{ s}$$

With nominal  $Q_s=.025$  at Z. And with strong depolarizer  $w=1\cdot 10^{-4}$ .

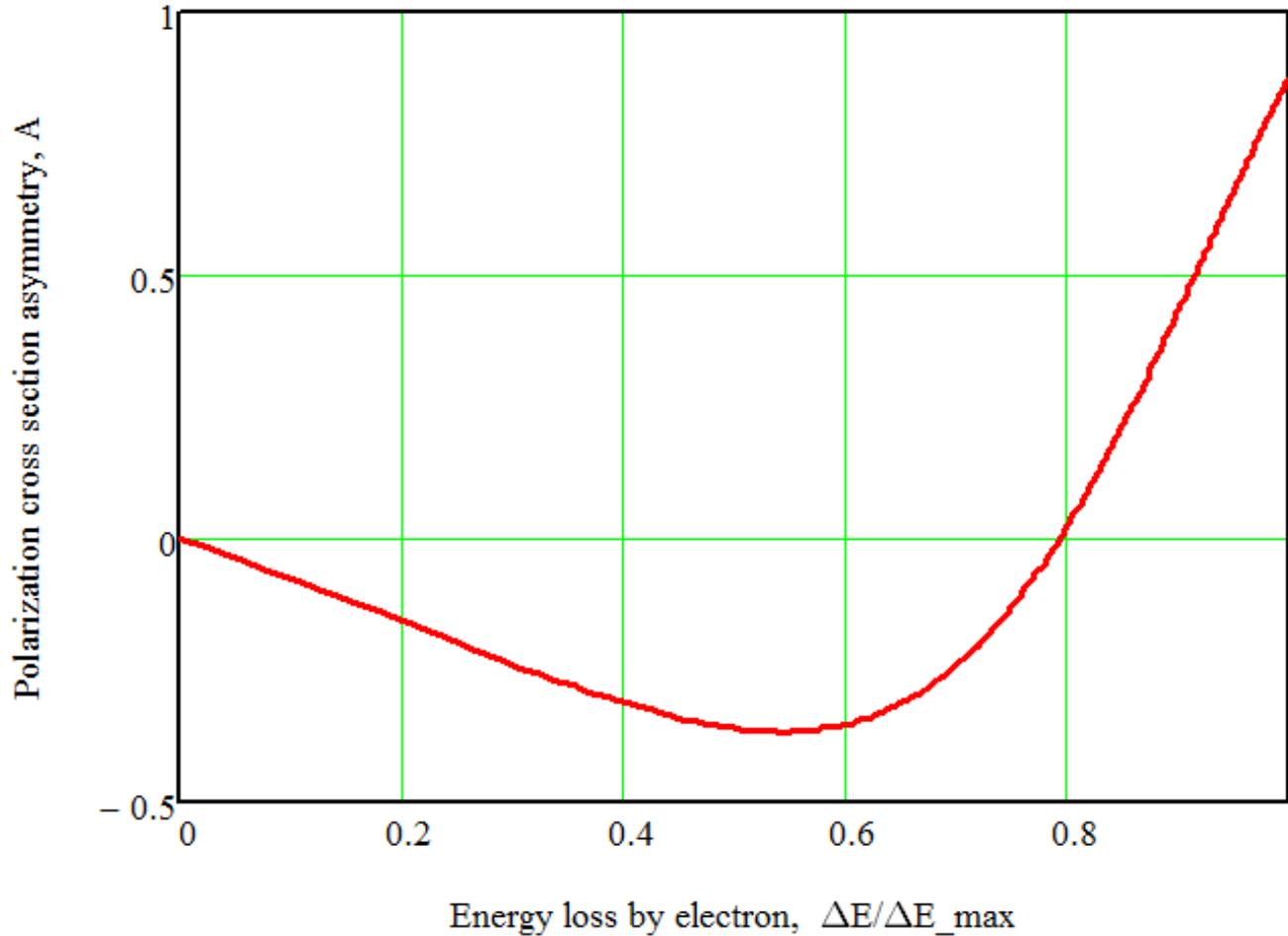
My simple fit gives the resonance frequency with an error  $\Delta\nu = -0.00011$ . But, in fact, the transition zone here is very narrow and is centered to the right spin tune value very well.

# Conclusions on the resonant depolarization:

- At **Z** peak with the last set of beam and lattice parameters there is no obstacles to perform the Resonant Depolarization, even with such small synchrotron tune as  $Q_s=.025$ .
- At **W** threshold there is a serious problem with the choice of too low  $Q_s=.023$  value. RD works, if  $Q_s$  will be increased to  $0.075$ , at least.

# Longitudinal Compton Polarimeter

Compton cross section asymmetry for scattering of circularly polarized light on 80 GeV electron.



For polarization measurements is very beneficial to detect lost energy electrons instead of backscattered gammas!

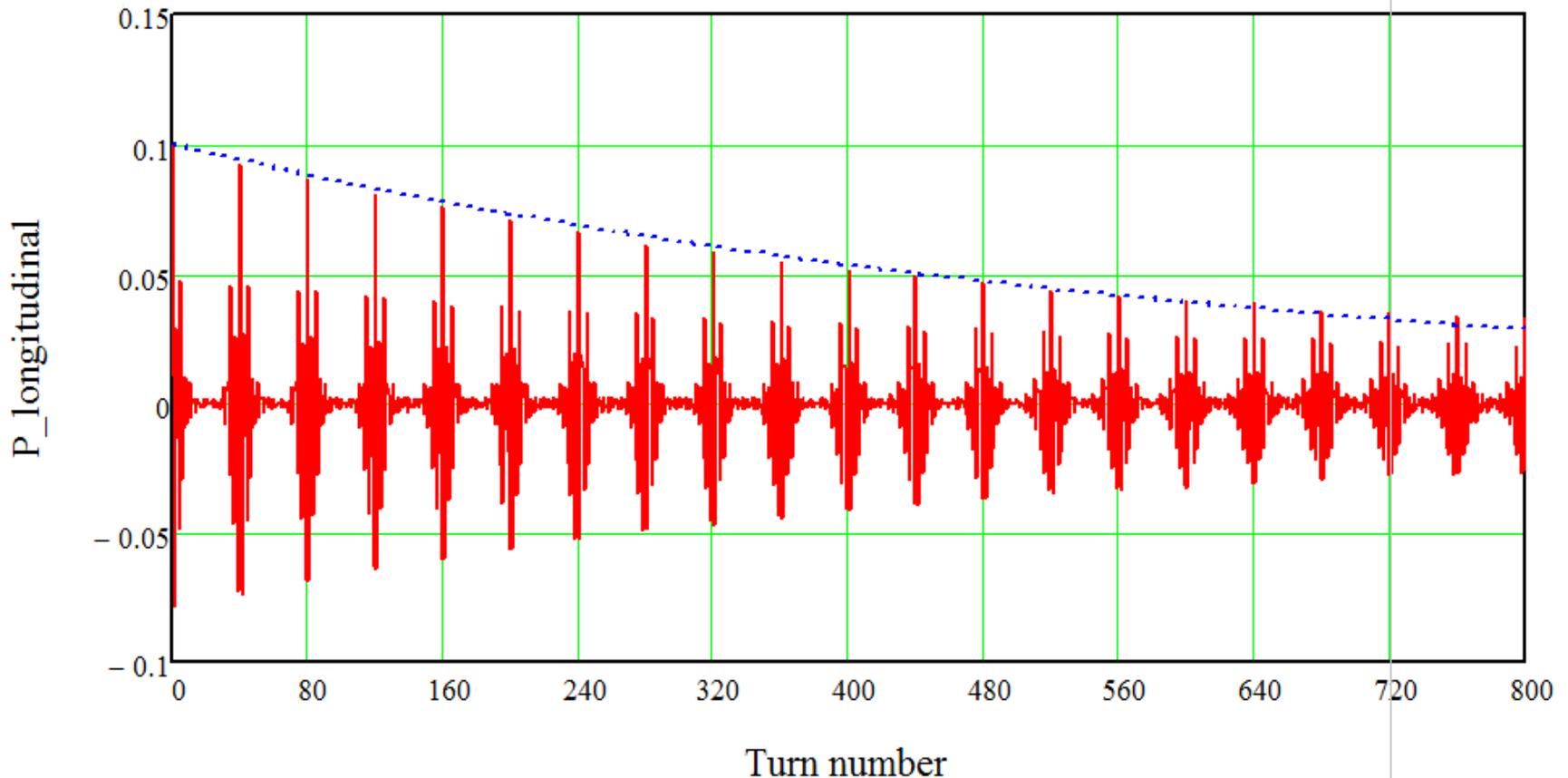
We plan use the 2d-pixelized detector.

According to N. Muchnoi estimations of the counting rate, about 700 events /turn is expected with moderate laser power: 50 mJ/pulse, 3500 Hz repetition frequency,  $10^{10}$  electrons/bunch. This translates to 12% of statistical noise for polarization.

### 3. Precession approach. Example with 45 GeV spin ensemble.

Turn by turn plot for the longitudinal polarization component: beam energy  $E=45.563$  GeV. One can see polarization echoes at integer numbers of the synchrotron periods – each 40-th turn. The dotted line is the exponential fit with  $\tau=640$  turns, which describes the long term decoherence. Initial polarization level (in the horizontal plane) is  $P=0.1$ .

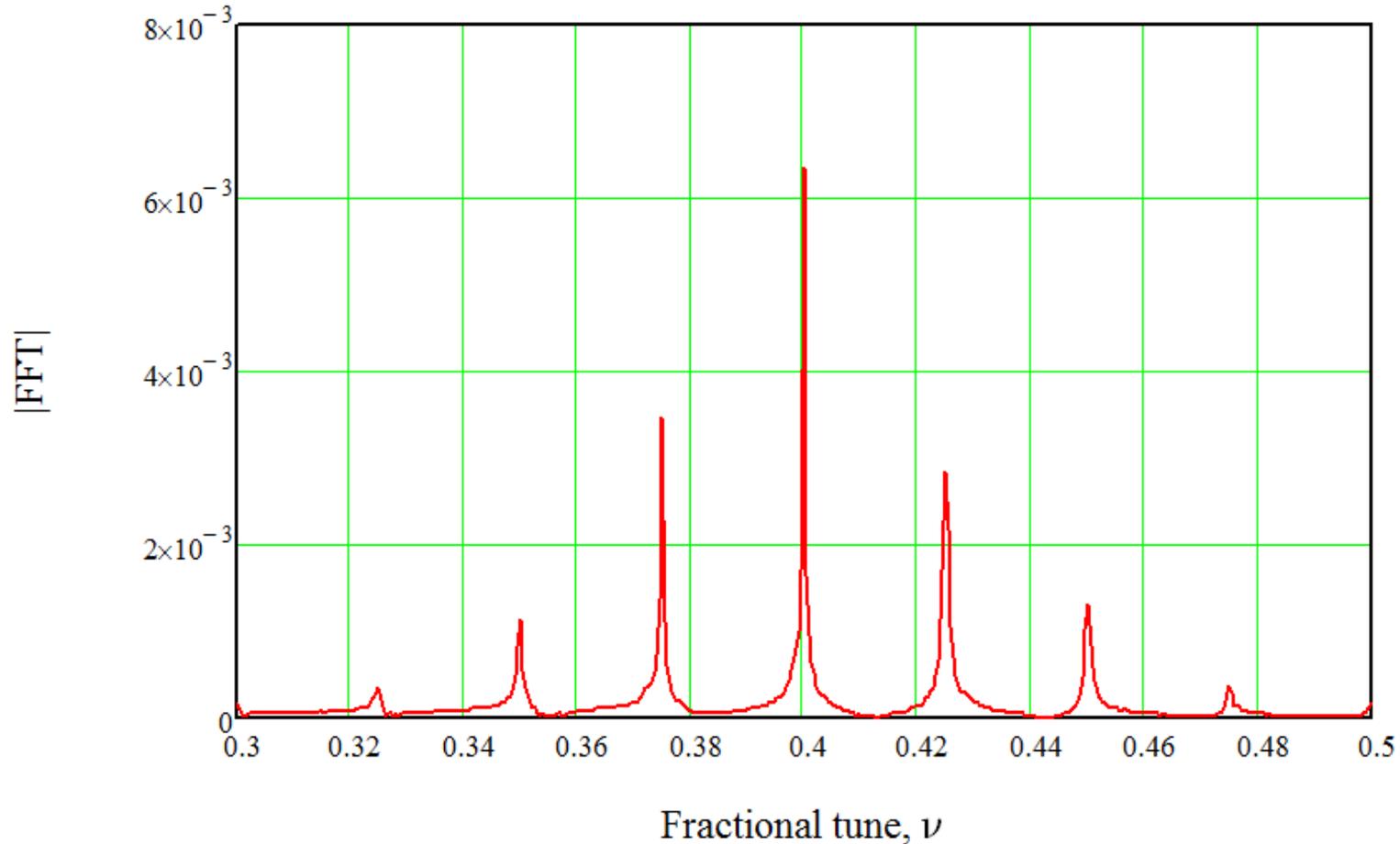
45.563 GeV,  $Q_s=0.025$ ,  $\sigma_\delta=0.000376$ ,  $\xi=1.555$ , Noise=0



# FFT of 45 GeV beam free precession

Fast Fourier Transform (FFT) for beam energy  $E=45$  GeV,  $Q_s=0.025$ ,  $N=2048$  turns,  $N_p=4000$  particles, as it will be recorded by the ideal longitudinal polarimeter (means - no statistical noise in measuring of the polarization level).

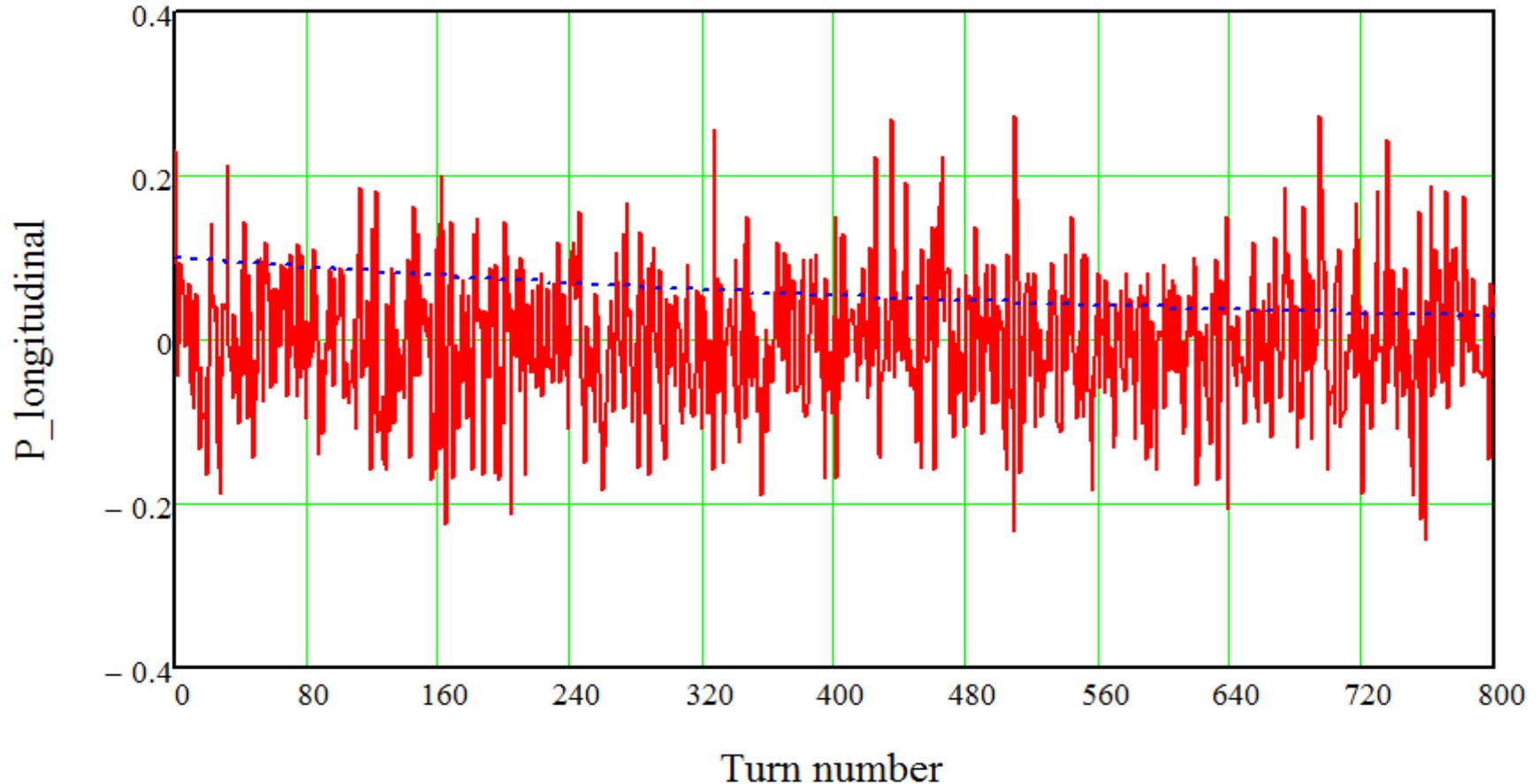
45.563 GeV,  $Q_s=0.025$ ,  $\sigma_\delta=0.000376$ ,  $\xi=1.555$ , Noise=0



# Precession signal with the statistical noise added

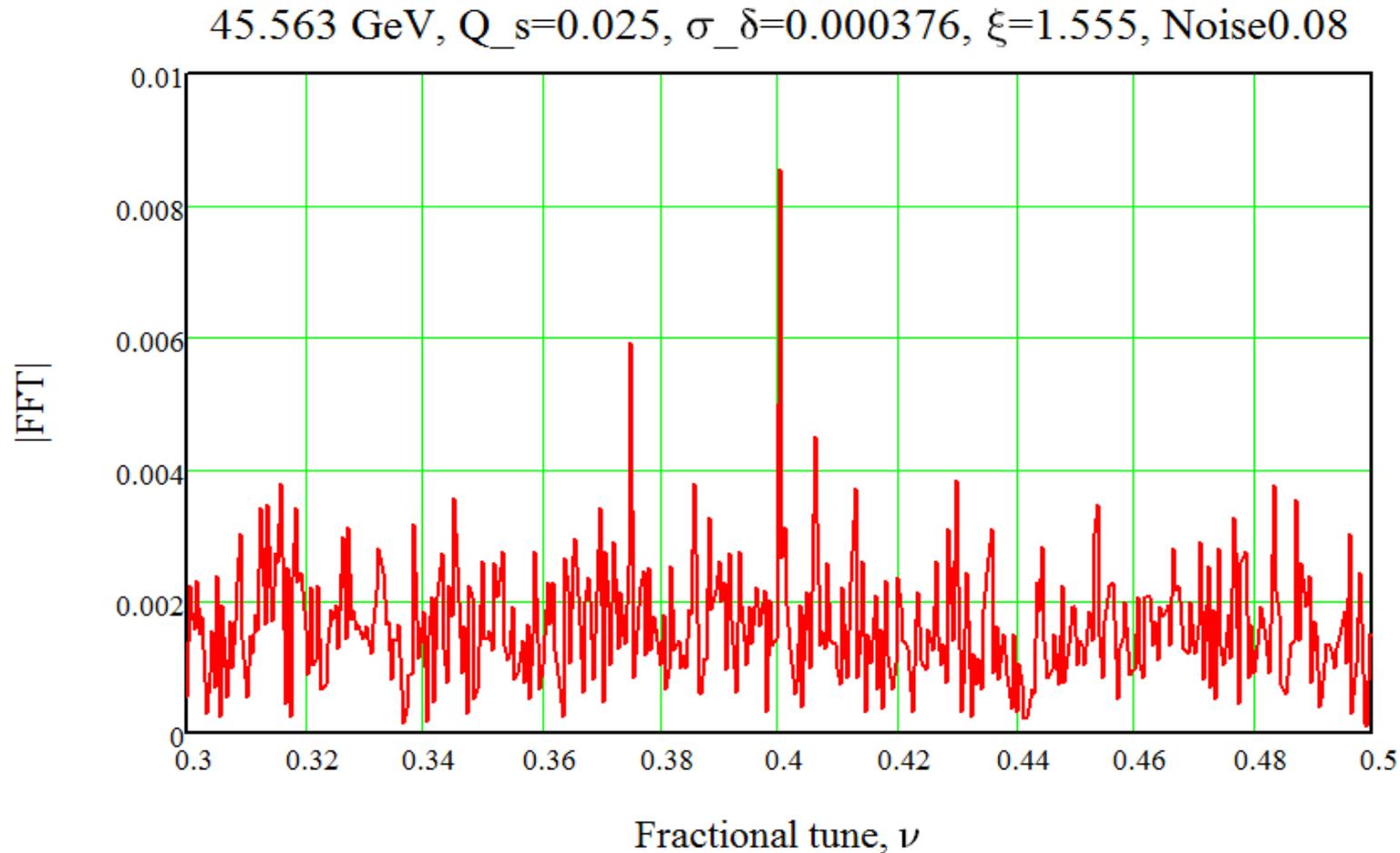
Turn by turn plot of the longitudinal polarization component for a beam energy  $E=45$  GeV with polarimeter statistical noise  $\sigma_{\text{noise}}=0.08$  added to a signal.

45.563 GeV,  $Q_s=0.025$ ,  $\sigma_\delta=0.000376$ ,  $\xi=1.555$ , Noise0.08



# Spectrum of 45 GeV signal with a noise

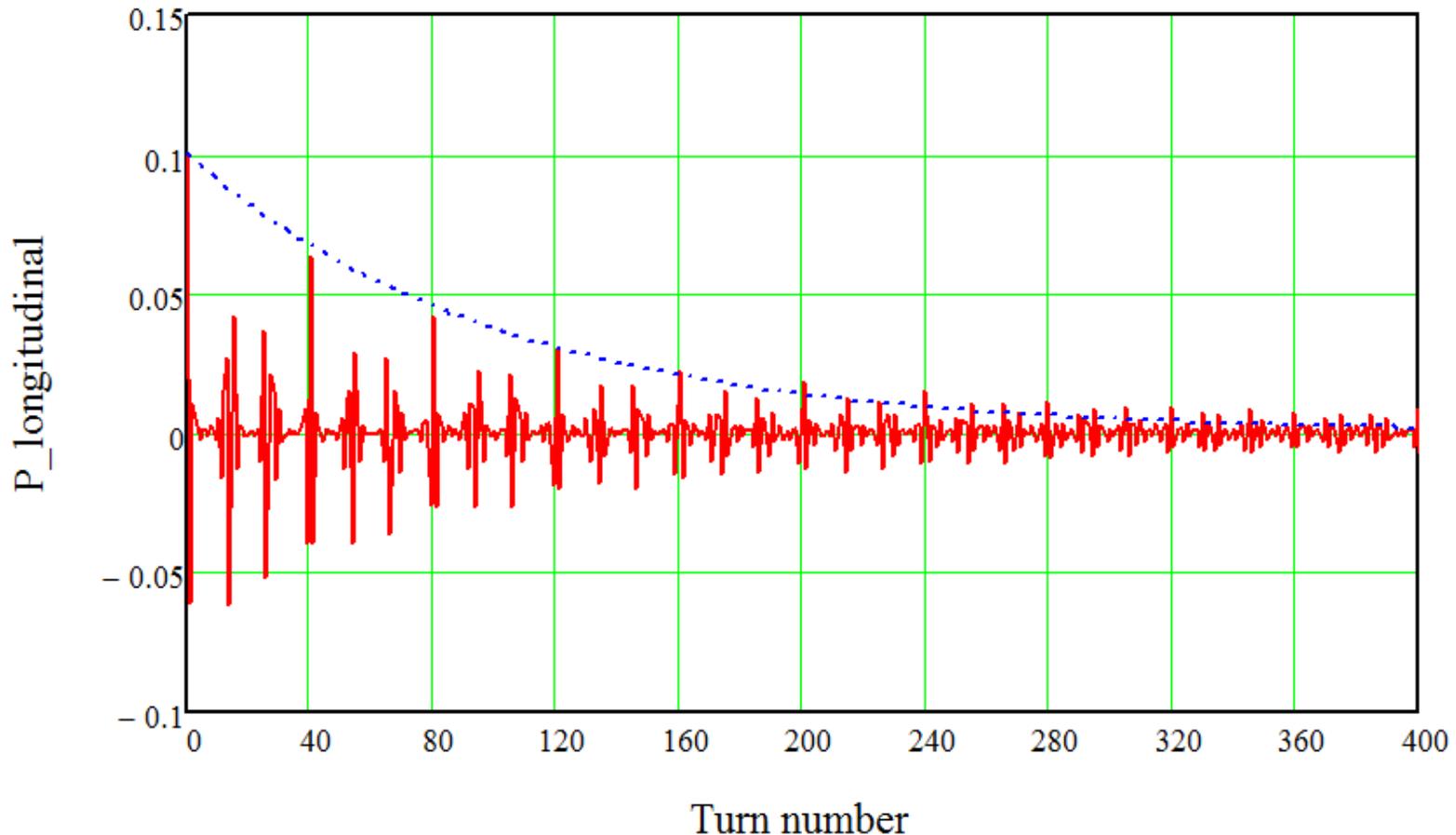
Fast Fourier Transform for a beam energy  $E=45.563$  GeV,  $Q_s=0.025$ ,  $N=2048$  turns,  $N_p=4000$  particles. A polarimeter statistical noise  $\sigma_{\text{noise}}=0.08$  is added to a signal. Still the wanted signal peak  $\nu=0.400$  is seen here almost at a right position.



# Free precession signal of 80 GeV beam

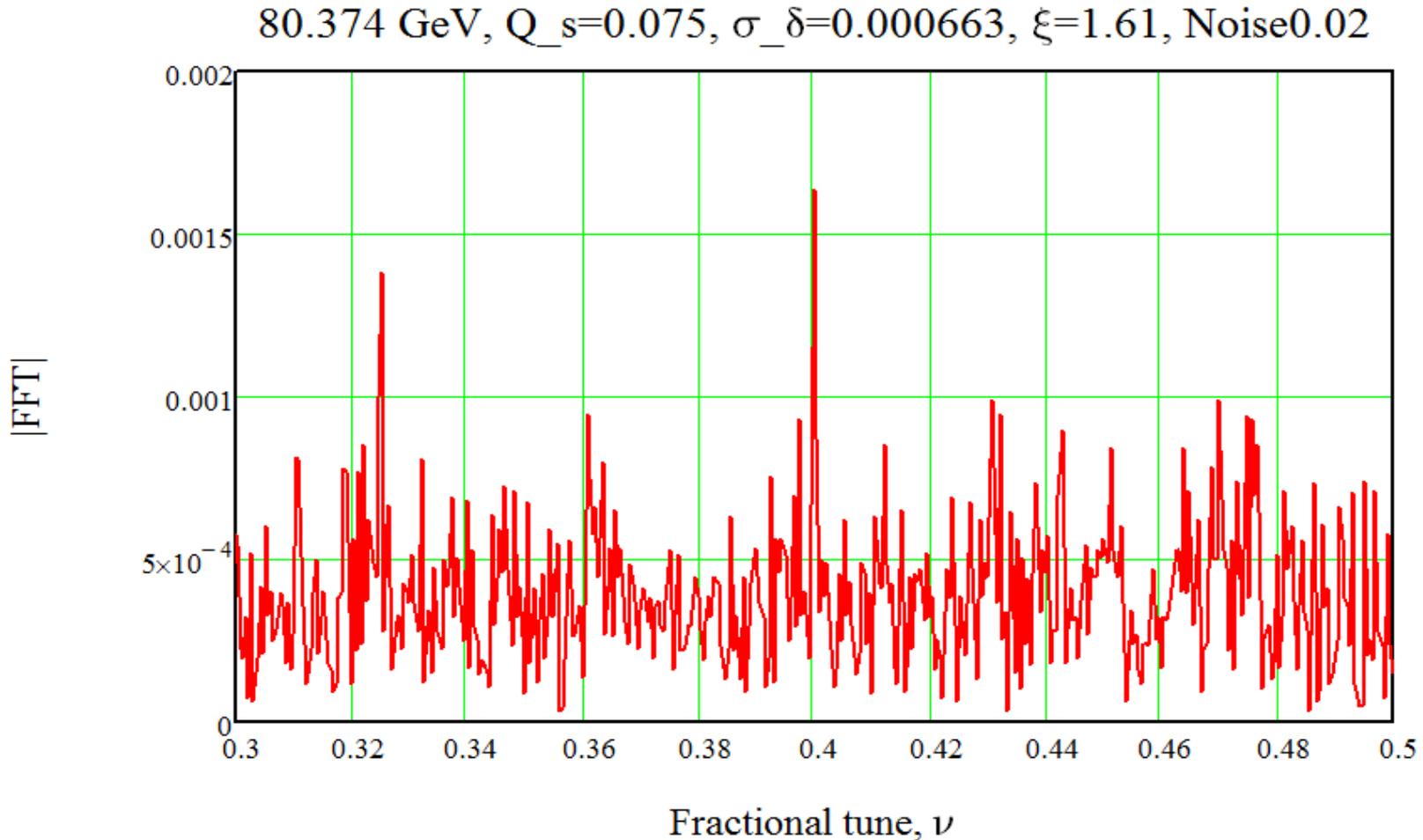
Spin precession echoes for the case  $E=80$  GeV,  $Q_s=0.075$  and  $\sigma_\delta=0.000663$ . **No any statistical noise is added here!** Free precession starts from a level  $P=0.1$ . The long decoherence time is equal to  $\tau=100$  turns, only.

80.374 GeV,  $Q_s=0.075$ ,  $\sigma_\delta=0.000663$ ,  $\xi=1.61$ , Noise0



# Spectrum of 80 GeV beam with a noise

Fast Fourier Transform for a beam energy  $E=80.374$  GeV,  $Q_s=0.075$ ,  $N=2048$  turns,  $N_p=4000$  particles,  $\sigma_{\text{noise}}=0.02$ . The precession peak is located at  $\nu=0.4$ . A synchrotron side band at  $\nu=0.325$  is clearly visible, but not so at  $\nu=0.475$ .



# Summary on free precession approach:

| Beam energy, E (GeV) | Spin tune, $\nu_0$ | Relative energy spread, $\sigma_\delta$ | Synchro-tron tune, $Q_s$ | Modulation index $\xi = \nu_0 \sigma_\delta / Q_s$ | Spin De-coherence time, $\tau$ (turns) | Statistical noise limit, $\sigma_{\text{noise}}$ |
|----------------------|--------------------|---|--------------------------|--|--|--|
| 45.5                 | 103.4              | 0.000376                                | 0.025                    | 1.555  | 640                                    | 0.08   |
| 45.5                 | 103.4              | 0.000376                                | 0.050                    | 0.777  | 2560                                   | 0.15   |
| 45.5                 | 103.4              | 0.000376                                | 0.075                    | 0.518  | 5760                                   | 0.50   |
| 80.4                 | 182.4              | 0.000663                                | 0.050                    | 2.419  | 40                                     | 0.01   |
| 80.4                 | 182.4              | 0.000663                                | 0.075                    | 1.613  | 100                                    | 0.02   |
| 80.4                 | 182.4              | 0.000663                                | 0.100                    | 1.210  | 145                                    | 0.05   |

To be able to measure the spin free precession frequency using the longitudinal Compton backscattering polarimeter the last should provide at 80 GeV very good sensitivity to single turn polarization level: better than 1-2 %.  $Q_s$  should be as large as **0.075**.

**Thank you for your attention!**