Coherent beam-beam instability in CEPC-H/Z,
Collision with a large Piwinski angle

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Introduction

• Beam-beam simulations have been performed for the design of CEPC and FCC-ee.

• Weak-strong simulations were mainly used in the simulations, especially for collision with a large crossing angle.

• Strong-strong simulations are performed since last year.

• Coherent beam-beam instability with head-tail mode is seen in the simulations.

• Design parameters are required to be revised to overcome the instability in FCC-ee.

• The simulation results are presented for CEPC.

• This instability will be studied experimentally in SuperKEKB this or next year.
# Parameters for CEPC and FCC-ee

<table>
<thead>
<tr>
<th></th>
<th>CEPC-H</th>
<th>CEPC-Z</th>
<th>FCC-ee-H</th>
<th>FCC-ee-Z</th>
<th>FCC-ee-Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumf (km)</td>
<td>61</td>
<td>61</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Energy (GeV)</td>
<td>120</td>
<td>45.5</td>
<td>120</td>
<td>45.5</td>
<td>45.5</td>
</tr>
<tr>
<td>No. bunches $N_b$</td>
<td>107</td>
<td>1100</td>
<td>780</td>
<td>91500</td>
<td>31800</td>
</tr>
<tr>
<td>Bunch pop $N_e (10^{11})$</td>
<td>2.0</td>
<td>0.78</td>
<td>0.8</td>
<td>0.33</td>
<td>1.0</td>
</tr>
<tr>
<td>Emittance $\varepsilon_x (nm)/\varepsilon_y (pm)$</td>
<td>2.05/6.2</td>
<td>0.88/8</td>
<td>0.61/1.2</td>
<td>0.09/1</td>
<td>0.2/1</td>
</tr>
<tr>
<td>Beta at IP $\beta_x (m)/\beta_y (mm)$</td>
<td>0.272/1.3</td>
<td>0.1/1</td>
<td>1/2</td>
<td>1/2</td>
<td>0.5/1</td>
</tr>
<tr>
<td>Synchro. tune $\nu_s$</td>
<td>0.18</td>
<td>0.015</td>
<td>0.056</td>
<td>0.025</td>
<td>0.036</td>
</tr>
<tr>
<td>Bunch length $\sigma_{z,SR}/\sigma_{z,tot} (mm)$</td>
<td>2.7/2.9</td>
<td>3.8/4.0</td>
<td>2.0/2.4</td>
<td>1.6/3.8</td>
<td>1.2/6.7</td>
</tr>
<tr>
<td>Energy spread $\sigma_{\delta,SR}/\sigma_{\delta,tot} (10^{-4})$</td>
<td>12/13</td>
<td>4.8/5.0</td>
<td>10/12</td>
<td>4/9</td>
<td>4/22</td>
</tr>
<tr>
<td>Crossing angle $\phi_c$ (half angle)</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Piwinski angle $\phi_c \sigma_{z,tot}/\sigma_x$</td>
<td>1.88</td>
<td>6.4</td>
<td>1.44</td>
<td>6</td>
<td>10</td>
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<tr>
<td>Damping time $\tau_z/T_0$</td>
<td>41</td>
<td>746</td>
<td>72</td>
<td>1320</td>
<td>1320</td>
</tr>
<tr>
<td>Design Lumi $L (10^{34} \text{ cm}^{-2}\text{s}^{-1})$</td>
<td>3.1</td>
<td>4.48</td>
<td>5.1</td>
<td>90</td>
<td>207</td>
</tr>
</tbody>
</table>

CEPC parameters by D. Wang, 20160918
Strong-strong simulation

• Two colliding bunches are represented by many macro-particles, np=1,000,000.

• The bunches are divided into many pieces, to take into account of hourglass effect and crossing angle.

• Beam averaged position/size luminosity are calculated turn-by-turn.

• The beam-beam parameter calculated by luminosity is used for a index of the beam-beam limit.

\[ L = \frac{N\gamma f_{rep}}{2r_e\beta_y} \xi_y \]
\[ \xi_L = \frac{2r_e\beta_y}{N\gamma f_{rep}} L \]

Equilibrium beam-beam tune shift
=Beam-beam parameter
Strong-strong simulation for Large crossing angle

- Two colliding bunches are divided into many slices, $N_{sl} \sim 10 \times \sigma_z \theta / \sigma_x$.
- Sort slices with their positions $z_i + z_j$, collision order.
- Collision between two slices is evaluated using 3D interaction; 2D interaction plus longitudinal kick by potential slope $d\phi/dz$.
- Slice-by-slice collision is repeated $N_{sl} \times N_{sl}$ for a bunch-by-bunch collision.
Four options of the strong-strong simulation

1. Gaussian approximation using turn-by-turn RMS values.
2. Gaussian approximation using turn-by-turn Gaussian fitting.
3. PIC for core part and Gaussian approximation for slice collision with large offset.
4. Complete PIC using shifted Green function

Example of shifted potential for collision with large offset.

\[ \phi(x) = -\frac{2N r_c}{\gamma} \int d\mathbf{x}' G(\mathbf{x} - \mathbf{x}' - \mathbf{x}_0) \]

Shifted Green function (J. Qiang)
Results of strong-strong simulation

- Beamstrahlung OFF. Use bunch length taking into account of Beamstrahlung.
- Scan Ne(bunch current) with keeping bunch length

- Beam-beam parameter is saturated.
- Luminosity fluctuates high bunch current.
ξ limit for CEPC

• Summarize the relation of $\xi_L$ and bunch current

• Beam-beam parameter is saturated at 0.11 and 0.07 for CEPC-H and Z, respectively.

• Error bar indicates luminosity fluctuation.

• Luminosity is slightly lower than the design for both of H and Z.

• Coherent instability occurs in Z design.
Coherent beam-beam motion

- The luminosity fluctuation is caused by coherent beam-beam instability.
- p mode is seen in CEPC-H, while head-tail mode is seen in CEPC-Z.
- In experience of FCC-ee studies, head-tail mode is serious for larger Piwinski angle.
Animation of x-z motion

It is not simple $m=1$ x-z motion.

Collision frame
Coherent instability and $\beta_x^*$

- FCC-ee simulation showed lower $\beta_x$ suppressed the instability.

- Simulation for changing $\beta_x$ and $\varepsilon_x$ with keeping $\sigma_x$.

- CEPC-H, $\beta_x > 0.5m$ has to be avoid.

- CEPC-Z, $\beta_x$ has to be reduced $\beta_x < 0.075m$. 
Study using simple model I

- Collision of pan-cake and airbag beams.
- The airbag is represented by N macro-particles.
- Linear force due to beam-beam tune shift of projected positron bunch.

\[
\beta_x = 1 \\
\Delta x_p = -p_p s \\
\Delta p_p = -\frac{4\pi \xi}{N} (x_p - x_{e,i}) \\
\Delta x_p = +p_p s
\]

\[
s = (z_{e,i} - z_p)/2 \\
z_{e,i} = \sigma_z \cos(2\pi(v_st + \frac{i}{N}))
\]

\[
\begin{align*}
\Delta x_{e,i} &= z_{e,i} \theta \\
\Delta x_{e,i} &= p_{e,i} s \\
\Delta p_{e,i} &= -4\pi \xi (x_{e,i} - x_p) \\
\Delta x_e &= -p_e s \\
\Delta x_{e,i} &= -z_{e,i} \theta
\end{align*}
\]
Result of the simple model I

- Very strong head-tail motion.
- High order mode \( m \approx 10 \)
- Tune scan showed tune near synchro-beta is worse.

One-Two particle model, which take into account of \( m=1 \) mode, is stable.
K. Ohmi, A. Chao, PRSTAB5, 101001 (2002).
Study using simple model II

- Collision of two airbag beams
- The airbag beam is represented by \( N \) macro-particles.
- Beam-beam force between macro-particles is expressed by Bassetti-Erskine formula.

\[
\begin{align*}
\Delta x_{p,j} &= z_{p,j} \theta \\
\Delta x_{p,j} &= -p_{p,j} s_{ij} \\
\Delta p_{p,j} &= -F_{BE} (x_{p,j} - x_{e,i}) \\
\Delta x_{p,j} &= p_{p,j} s_{ij} \\
\Delta x_{p,j} &= -z_{p,j} \theta \\
\Delta x_{e,i} &= z_{e,i} \theta \\
\Delta x_{e,i} &= p_{e,i} s_{ij} \\
\Delta p_{e,i} &= -F_{BE} (x_{e,i} - x_{p,j}) \\
\Delta x_{e,i} &= -p_{e,i} s_{ij} \\
\Delta x_{e,i} &= -z_{e,i} \theta
\end{align*}
\]
Simulated Beam distribution using simple model II

Lab frame (not collision frame)
Beam-beam study in SuperKEKB

Design of SuperKEKB, $\phi_c \sigma_{z,\text{tot}}/\sigma_x = 20$, $\beta_x = 3\text{cm}$, $\beta_y = 0.3\text{mm}$.

No instability in the design parameters.

In the commissioning stage, beta function at IP is squeezed step-by-step.
Phase II commissioning starts from Autumn 2017.
Target $\beta_x = 24\text{cm}$, $\beta_y = 2.4\text{mm}$ (8x,8x). or $\beta_x = 12\text{cm}$, $\beta_y = 2.4\text{mm}$ (4x,8x).

Simulations with the conditions are performed.
SuperKEKB Phase 2

\[ \beta_x = 8x\beta_{x0}, \quad \beta_y = 8x\beta_{y0} \] and \[ \beta_x = 4x\beta_{x0}, \quad \beta_y = 8x\beta_{y0} \]

\[ I_+ = 1\text{mA}, \quad I_- = 0.8\text{mA}, \quad \text{Crab waist ON/OFF} \]

Coherent beam-beam instability is seen in \( \beta_x = 24\text{cm}, \quad \beta_y = 2.4\text{mm} \) (8x,8x). This is not seen in \( \beta_x = 12\text{cm}, \quad \beta_y = 2.4\text{mm} \) (8x,8x). This fact can be confirmed in the Phase II commissioning.
Summary

• Strong-strong beam-beam simulation is executed to study coherent beam-beam instability in CEPC-H/Z.
• Beam-beam limit is evaluated to be 0.11 and 0.07 for CEPC-H/Z, respectively.
• Coherent dipole/head-tail motion is induced by the beam-beam interaction in H and Z.
• The instability seems safe in CEPC-H, but critical in Z for present parameters (ver Sep. 2016).
• Simplified model showed high order head-tail mode, $m \sim 10$. Simulation results agree with excitation of the high order mode.
• SuperKEKB, $\xi_{x/y} = 0.0028/0.088$ for $\phi_c \sigma_{z,\text{tot}}/\sigma_x = 20$, is stable. CEPC-H, $\xi_{x/y} = 0.04/0.11$ for $\phi_c \sigma_{z,\text{tot}}/\sigma_x = 1.8$, is stable, and CEPC-Z, $\xi_{x/y} = 0.01/0.72$ for $\phi_c \sigma_{z,\text{tot}}/\sigma_x = 6.4$, is critical.
• Increasing $\beta_{xy}^*$ at SuperKEKB, the beam-beam head-tail instability should appear.
• Studies and experiments in SuperKEKB are very important for CEPC/FCC-ee.
Beam-beam limit

- Luminosity
  \[ L = \frac{N^2 f_{rep}}{4\pi \sigma_x \sigma_y} \frac{R(\frac{\sigma_z}{\beta_y}, \frac{\theta_c \sigma_z}{\sigma_x})}{\beta_y} \]  
  \( N=N_+=N_+ \): bunch population
  \( f_{rep} \): collision freq.
  \( \theta_c \): half crossing angle

- \( \frac{\sigma_z}{\beta_y} \): hourglass, \( \frac{\theta_c \sigma_z}{\sigma_x} \): normalized crossing angle (Piwinski angle)

- Tune shift
  \[ \xi_y = \Delta \nu_y = \frac{N r_e \beta_y}{2 \pi \gamma \sigma_y (\sigma_x + \sigma_y)} R\left(\frac{\sigma_z}{\beta_y}, \frac{\theta_c \sigma_z}{\sigma_x}\right) \]

- Increasing N, beam size especially vertical for flat beam increases. Tune shift is saturate at a certain value. Luminosity linearly increases for N, not \( N^2 \). This situation is called Beam-beam limit.

- How large tune shift is achieved in equilibrium? Do simulations predict the beam-beam limit?
\( \xi \) limit for FCC-ee H

No clear difference for (0.54,0.61) from weak-strong \( \xi_{\text{lim}} = 0.2 \).

Big difference for (0.51,0.55), the limit in weak-strong is extremely high \( \xi_{\text{lim}} = 0.6 \) (ws), \( \xi_{\text{lim}} = 0.2 \) (ss).

\( \xi \) limit is weakly dependent of tune in Strong-strong simulation.

It is possible to achieve \( \xi_L = 0.15 \) for tlep-H.
\( \xi \) limit in Z factory

- Error bar is fluctuation due to coherent motion
- The beam-beam parameter is limited 0.06, and coherent oscillation
- Big difference from the weak-strong results \( \xi_{\text{lim}} = 0.2 \) at \((0.51,0.55)\).
- No big difference \( \xi_{\text{lim}} = 0.06(\text{ss}), 0.1(\text{ws}) \) in \((0.54,0.61)\).

\[
\begin{align*}
\xi_x &= 0.13 \\
\xi_{\text{target}} &= 0.17 \\
L_{\text{target}} &= 2.2 \times 10^{36} \text{ cm}^{-2}\text{s}^{-1}
\end{align*}
\]