

Precision QCD Study at Future e^+e^- Colliders

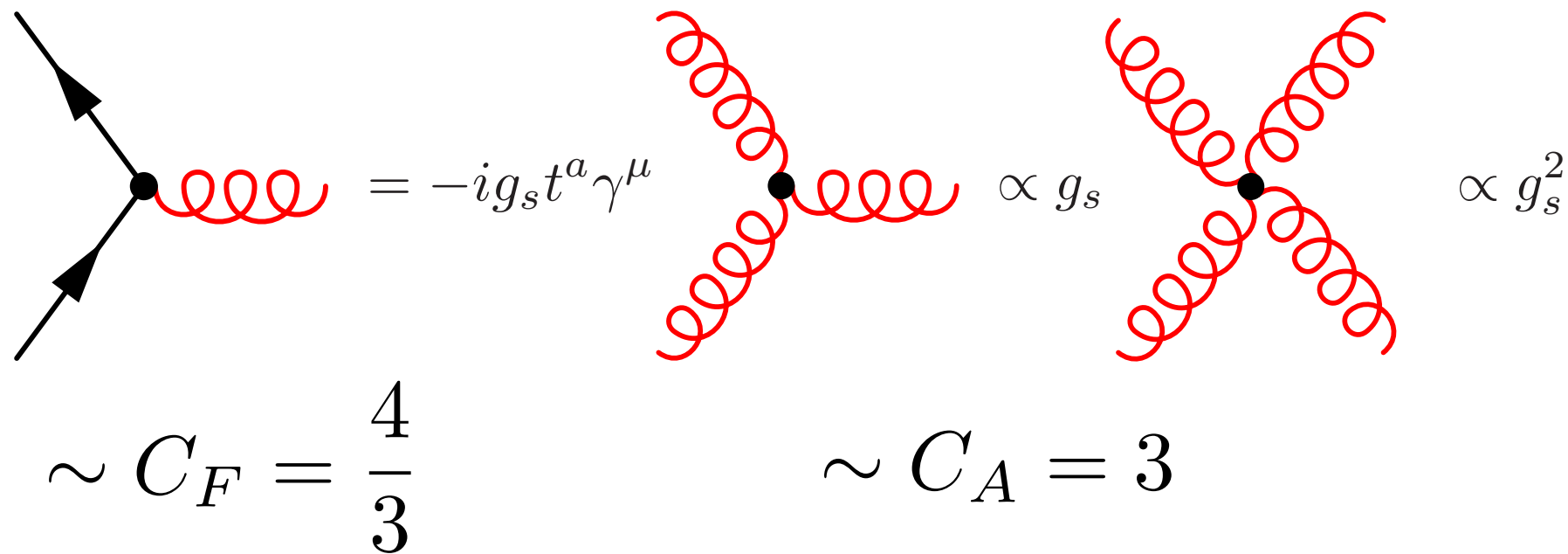
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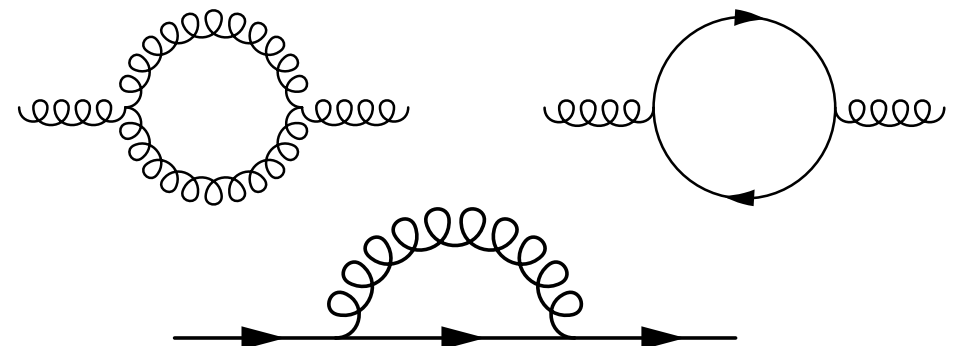
Motivation

- ★ QCD is a **non-abelian Yang-Mills theory** with very rich phenomena:
 - Asymptote freedom, confinement, jet formation, quark gluon plasma, color superconductivity ...
- ★ Yet it's definition is **simple**:



The diagram shows three Feynman interaction vertices. The first vertex on the left shows two black lines (quarks) meeting at a point with a red curly line (gluon) attached. It is labeled with the equation $= -ig_s t^a \gamma^\mu$ and below it $\sim C_F = \frac{4}{3}$. The middle vertex shows three red curly lines (gluons) meeting at a point, labeled $\propto g_s$ and below it $\sim C_A = 3$. The right vertex shows four red curly lines (gluons) meeting at a point, labeled $\propto g_s^2$.

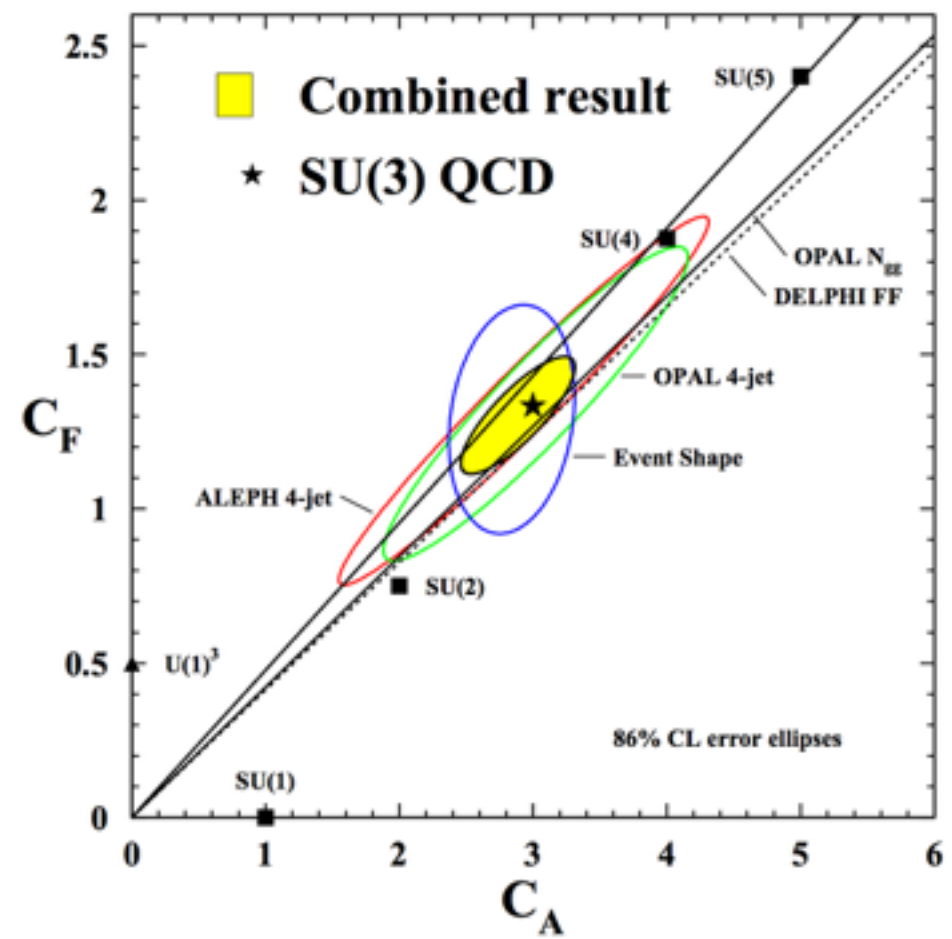
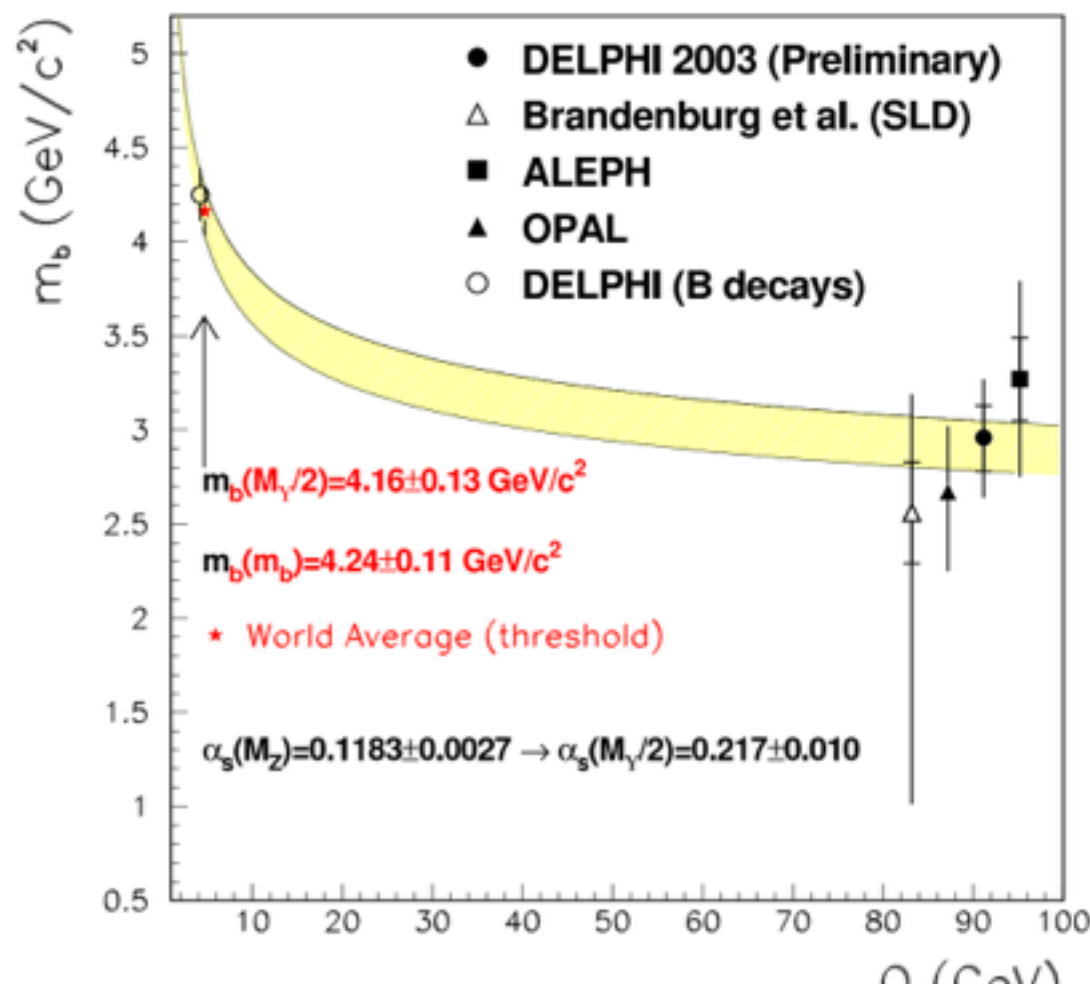
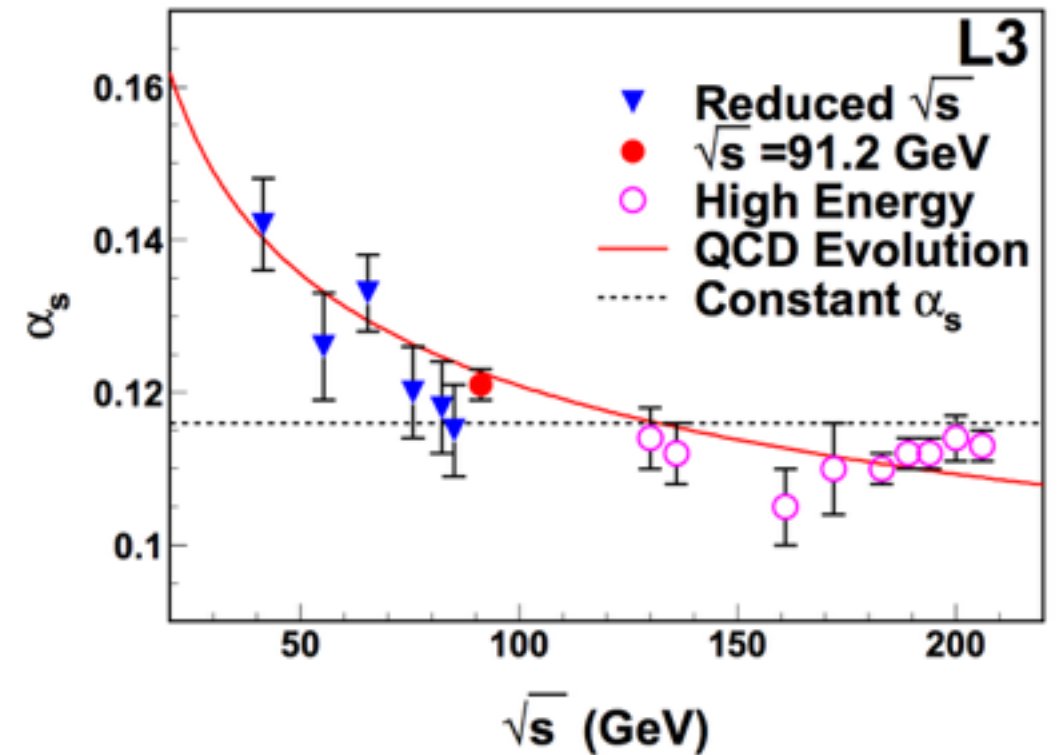
- ★ For example, the non-abelian gauge interaction leads to
 - **running of strong coupling**
 - **running of heavy quark mass**



Legacy of LEP from 14 to 209 GeV

★ QCD as the correct theory of strong interaction undoubtedly established from many different measurements at LEP:

- Running of α_s through **event shapes** and **jet rates** measurement
- Running of b-quark mass through **3-jet rate** in b-quark event
- Combined Fits of QCD **gauge structure constant**



QCD at future e⁺e⁻ study

- ★ While e⁺e⁻ collider is not a QCD machine, it has unique advantage over hadron colliders for understanding QCD at a more precise level
 - Fixed initial state energy
 - Absent of underlying events
- ★ A future e⁺e⁻ machine will support a rich program of precision QCD study
 - More precision in α_s measurement
 - Understanding of non-global observable
 - Finer structure of multi-jet events
 - Testing models of hadronization
 - Monte-Carlo Tuning
 - Fragmentation function
 - Interplay with EW, Higgs, BSM
 -

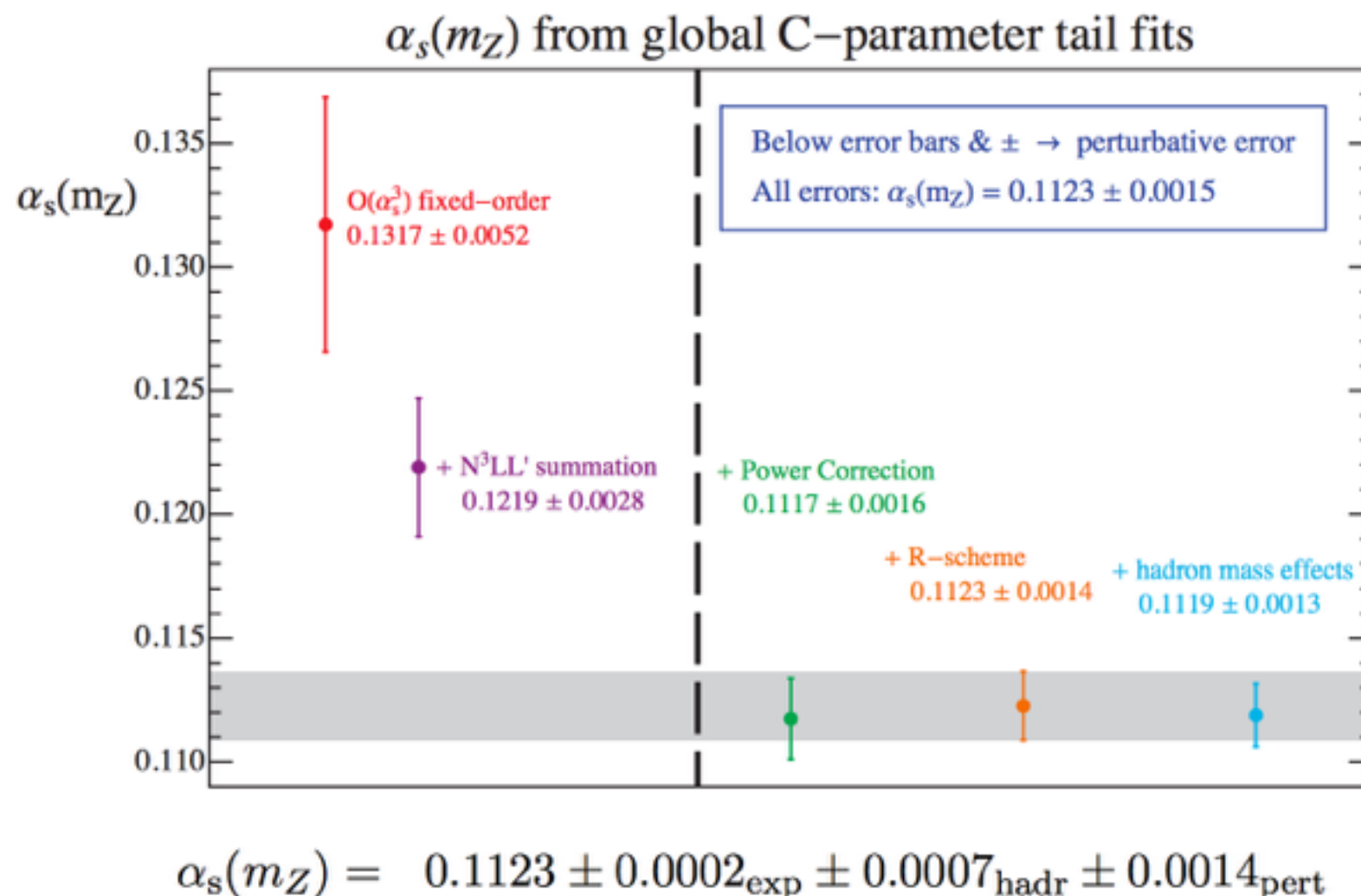
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Event shape

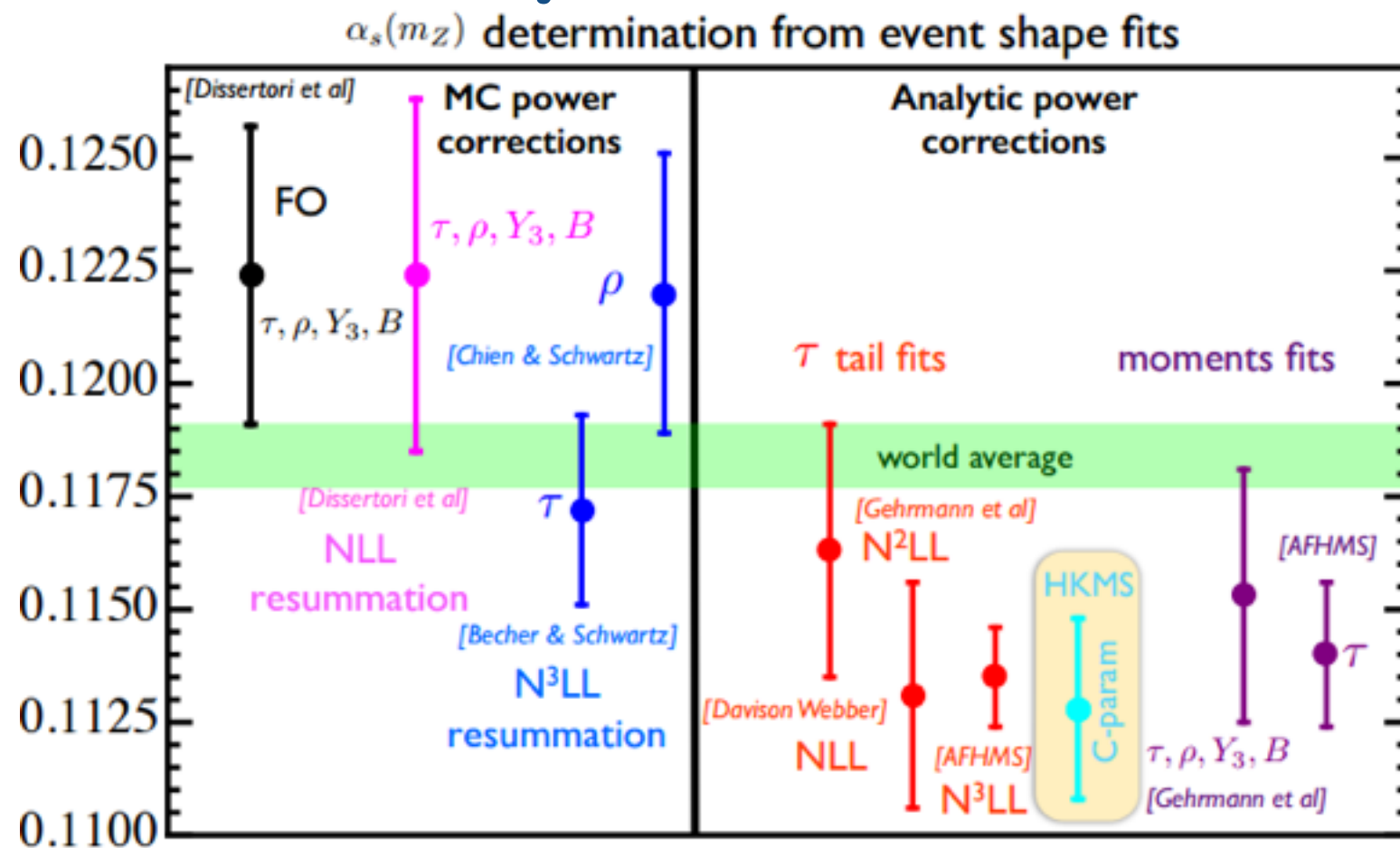
- ★ Remarkable theoretical progress over the last decade for event shape calculation. The state of the arts:
 - NNLO QCD + NLO EW for e+e- to 3 jets
- ★ N3LL resummation for thrust, heavy jet mass, C parameter form
Soft-Collinear Effective Theory

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



- ★ **Simultaneous** fit of α_s and non-perturbative corrections (Analytic power corrections). **Consistent** between thrust, heavy jet mass and C parameter
- ★ The biggest effect is the **-8% corrections** when adding the non-perturbative fit corrections

Issue with nonperturbative modeling



Mateu, ICHEP 2014

- ★ **MC power corrections:** estimate NP effects from difference of hadron and parton level MC
- ★ **Significant difference** between MC power corrections and analytic power corrections
- ★ While α_s fit using MC power corrections tend to agree better with world average, one should be careful that MC use **LO matrix** in its prediction
- ★ Future e⁺e⁻ collider with **different/higher energy** will be important to address this issue

Next-to-Leading Power logs in event shape

$$\frac{d\sigma}{d\tau} \sim \frac{\ln^m \tau}{\tau} (\text{Leading Power}) + \ln^m \tau (\text{Next-to-Leading Power}) + \dots$$

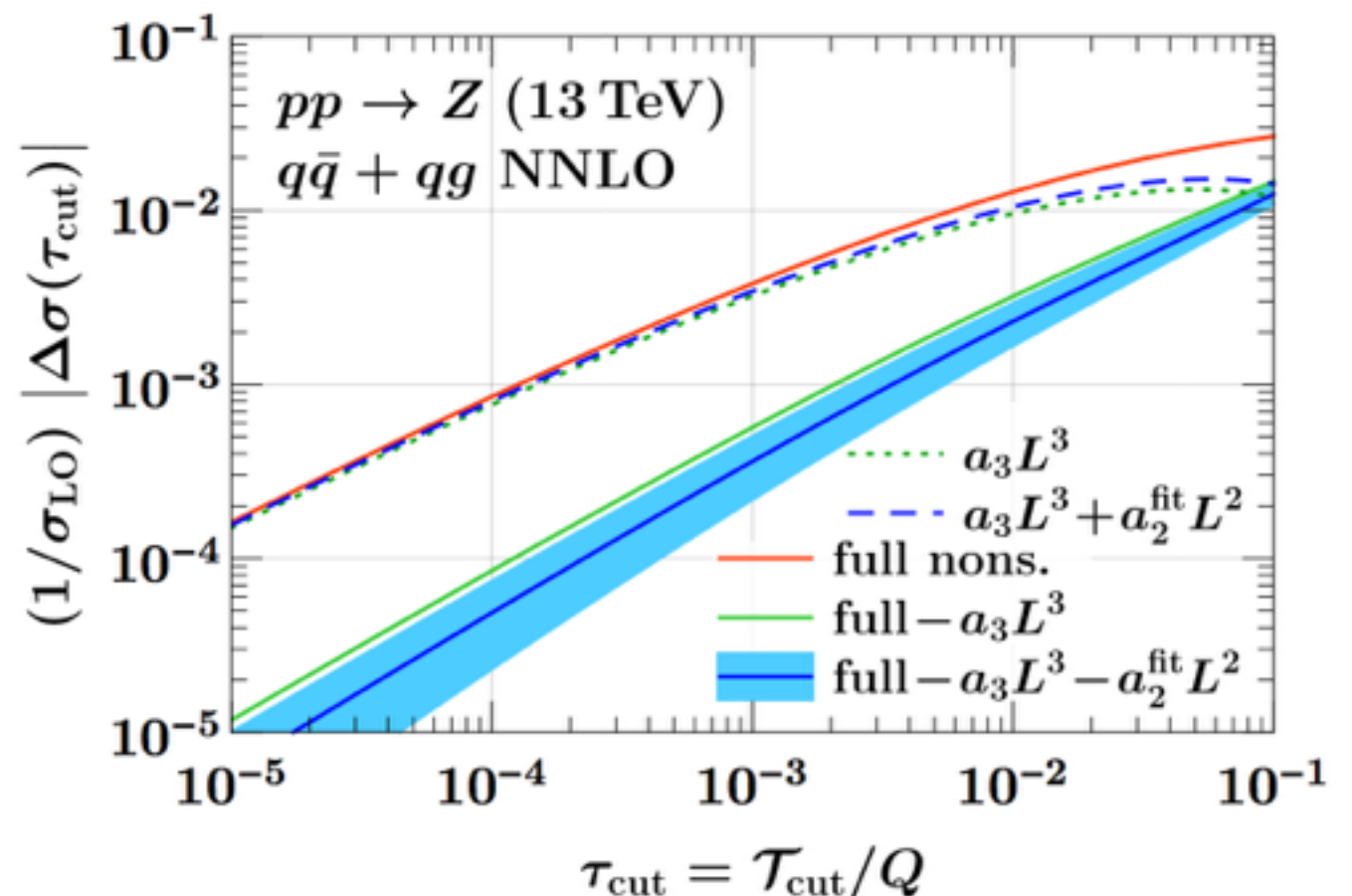
- ★ While the leading power large logarithms are under good control, the **subleading power logarithms** are far from understood
- ★ Recently the full set of operators for NLP corrections have been obtained in SCET

[Kolodrubetz, Moul, Stewart, 16];
Feige, Kolodrubetz, Moul, Stewart, forthcoming]

- ★ First $O(\alpha_s^2)$ analytic result for leading logarithms at NLP for thrust

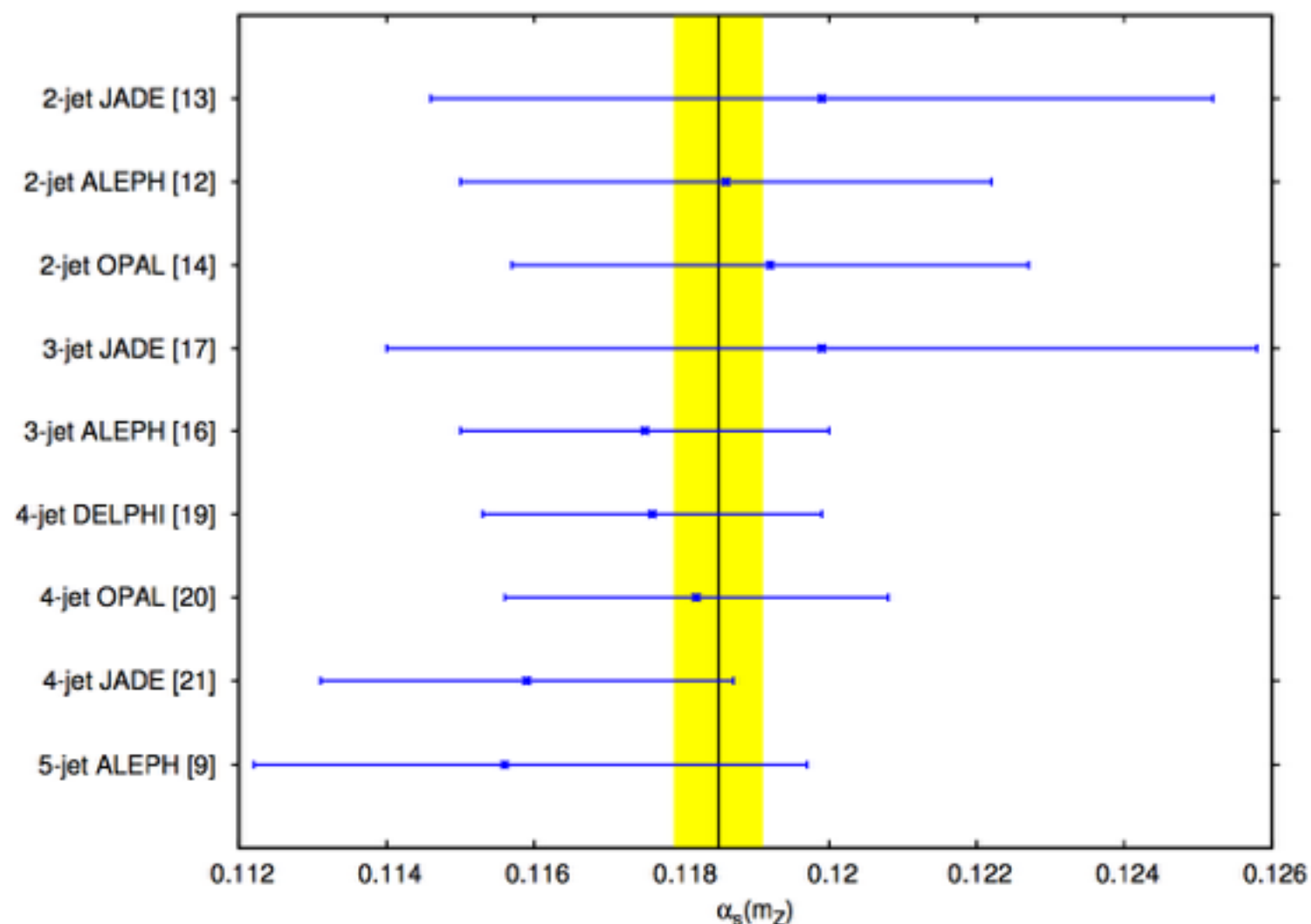
$$\begin{aligned} \frac{1}{\sigma_0} \frac{d\sigma^{(2,2)}}{d\tau} &= \left[-32C_F^2 + 8C_F(C_F + C_A) \right] \ln^3 \tau \\ &= 8C_F(C_A - 3C_F) \ln^3 \tau. \end{aligned}$$

- ★ (Right Fig.) Comparison of analytic and numerical NLP log for beam thrust at hadron collider
- ★ **Resummation** of NLP logarithms may be important to further reduced theoretical uncertainties in event shape fit



Jet rates extraction: NNLO frontier

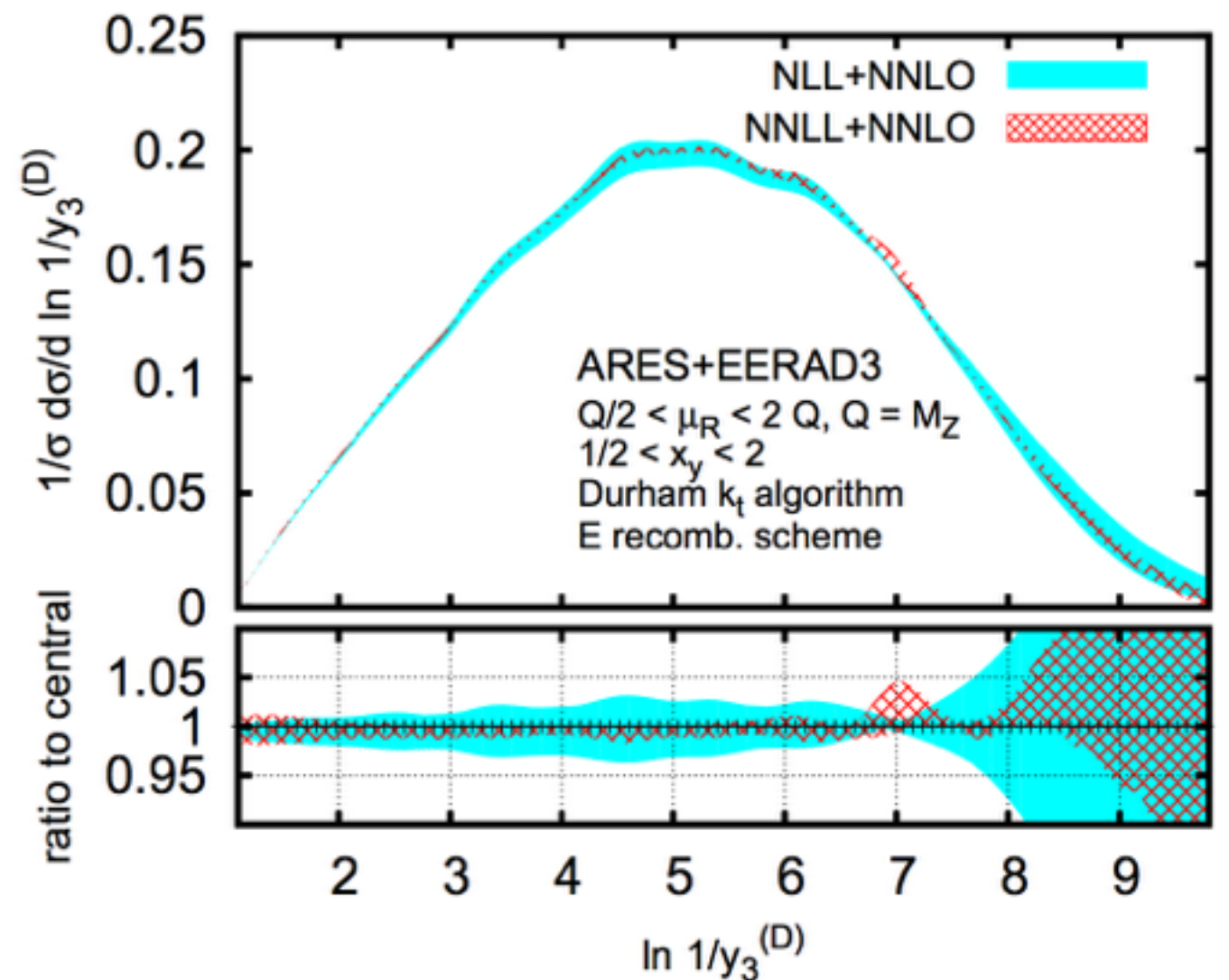
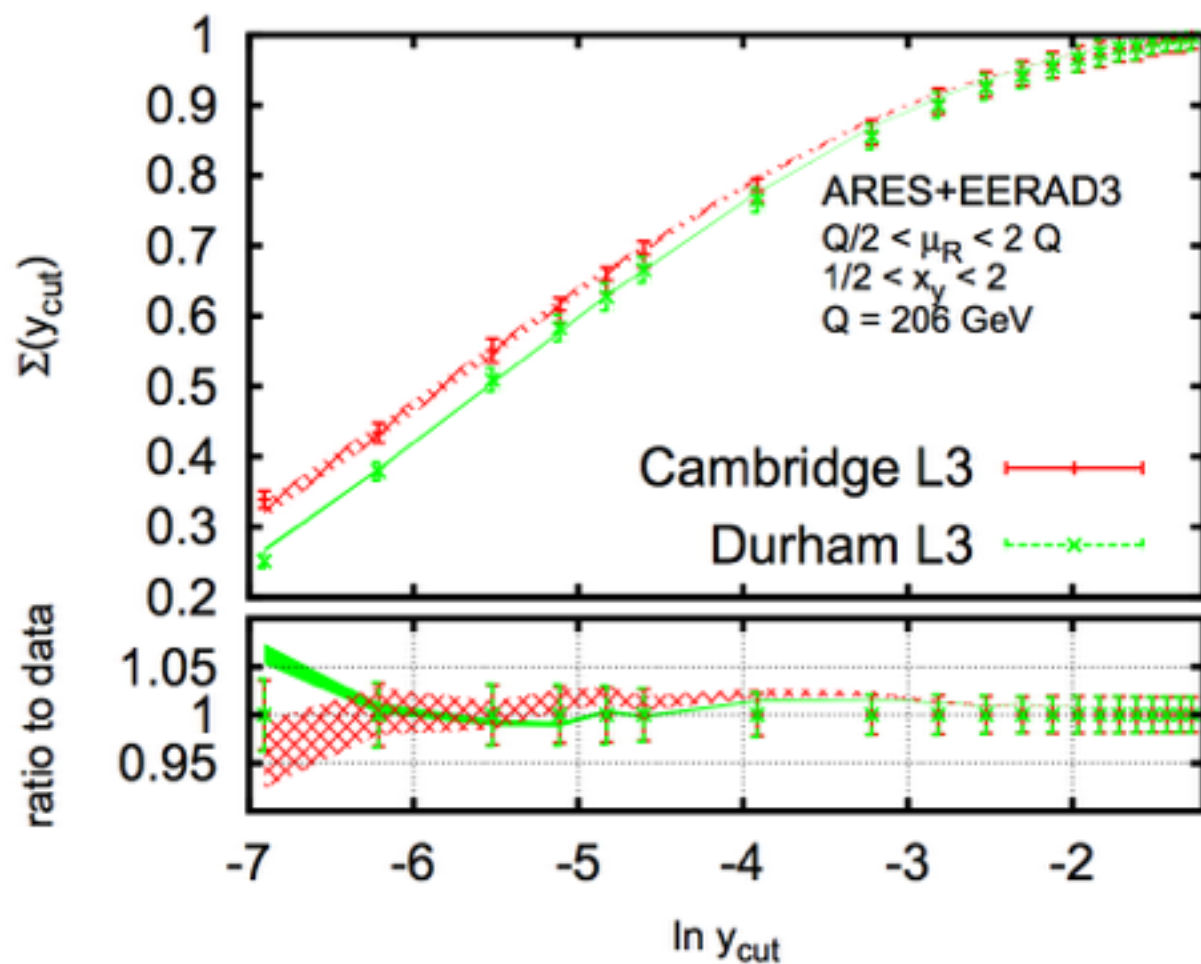
- ★ The fixed order revolution in the past decade make extraction of **five-jet rate at NLO** and **three-jet rate at NNLO** possible
- ★ For jet rates, parton level MC predictions agree well with pQCD calculation, indicating that **MC power corrections** gives sensible estimate
- ★ Dominant theoretical uncertainties from **scale variations**: call for NNLO/N3LO calculation and resummation



High-precision α_s measurements from LHC to FCC-ee, 1512.05194

Resummation two-jet rate

- ★ For a long time two-jet rate can only be resummed at **NLL accuracy**
- ★ Recent progress: resumming two-jet rate at **NNLL** with **ARES method**
Banfi, McAslan, Monni, Zanderighi, 2016

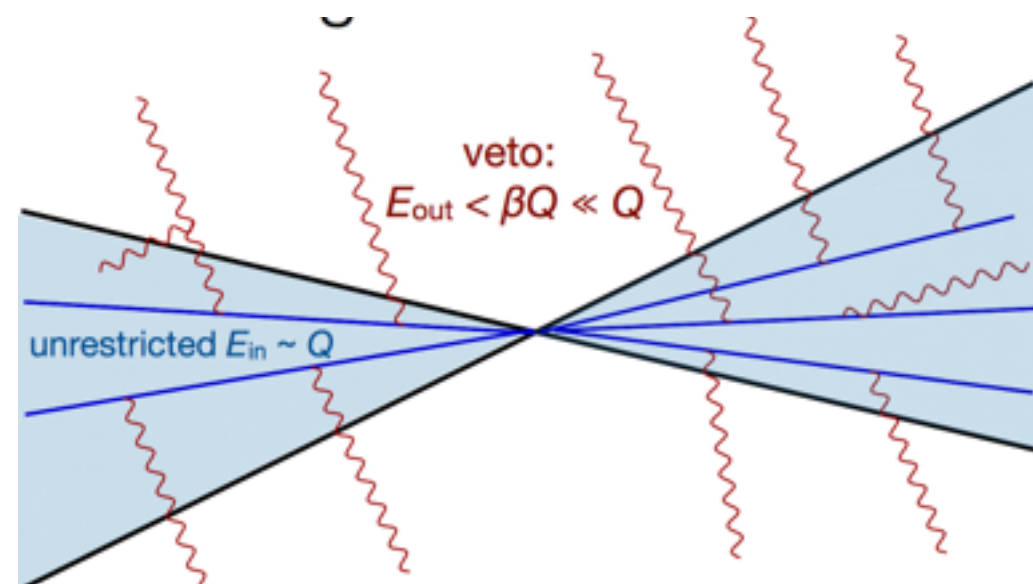


- ★ NNLL resummation leads to **sizable reduction of scale uncertainties** for two-jet rate with Durham algorithm. Promising for precision determination of α_s .

Towards better understand of non-global logarithms

- ★ Non-global logarithms (**Dasgupta, Salam, 2002**): observable only sensitive to **a portion of phase space**. Example: Serman-Weinberg jet cross section
- ★ Many more examples:
 - light-jet mass, narrow jet broadening, exclusive jet mass, jet veto, gaps between jet, jet substructure, isolated photon

First reason why they are interesting!



Large logarithms $\alpha_s^n \log^m \beta$
do not exponentiate

- ★ Leading non-global logarithms at large N_c govern by non-linear evolution equation (Banfi, Marchesini, Smye, 2002)

$$\partial_L G_{kl}(L) = \int \frac{d\Omega(n_j)}{4\pi} \frac{p_k \cdot p_l}{(p_k \cdot \hat{p}_j)(\hat{p}_j \cdot p_l)} \left[\Theta_{\text{in}}^{n\bar{n}} G_{kj}(L) G_{jl}(L) - G_{kl}(L) \right]$$

- ★ If $\Theta_{\text{in}}=1$, this reduced to the famous Balitsky/Korchevov equation. **The possibility of observing BFKL dynamics in jet physics is another reason why they are interesting.** e+e- collider provides the cleanest environment for discover
- ★ Significant renew interests since 2010: Banfi, Becher, Caron-Huot, Chien, Delenda, Dusgupta, Hagiwara, Hatta, Hornig, Kelley, Khelifa-Kerfa, Larkoski, Lee, Lorena, Mout, Neill, Neubert, Pecjak, Schabinger, Schwartz, Shao, Spannowsky, Stewart, Ueda, von Manteuffel, Walsh, HXZ, Zuberi ...

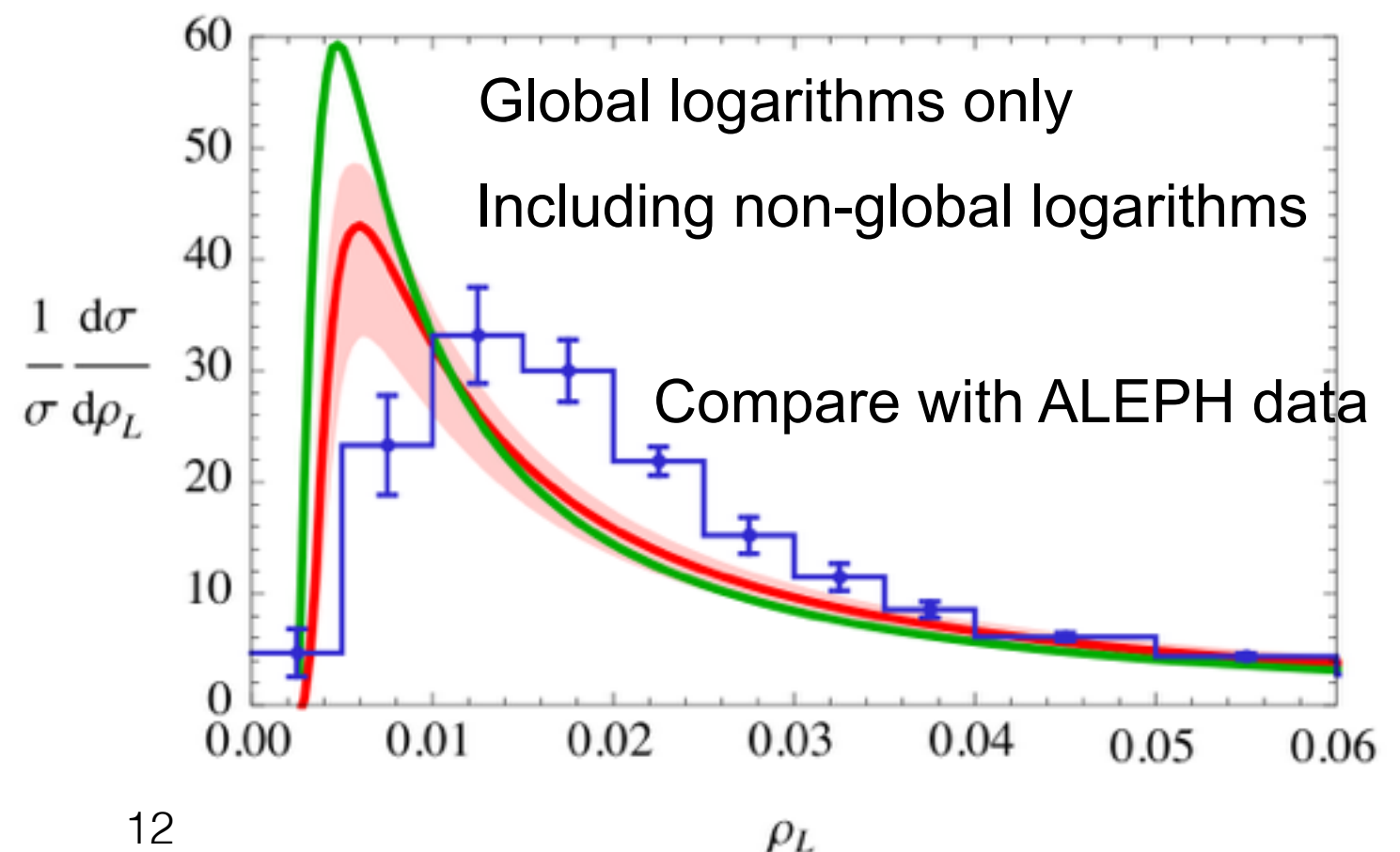
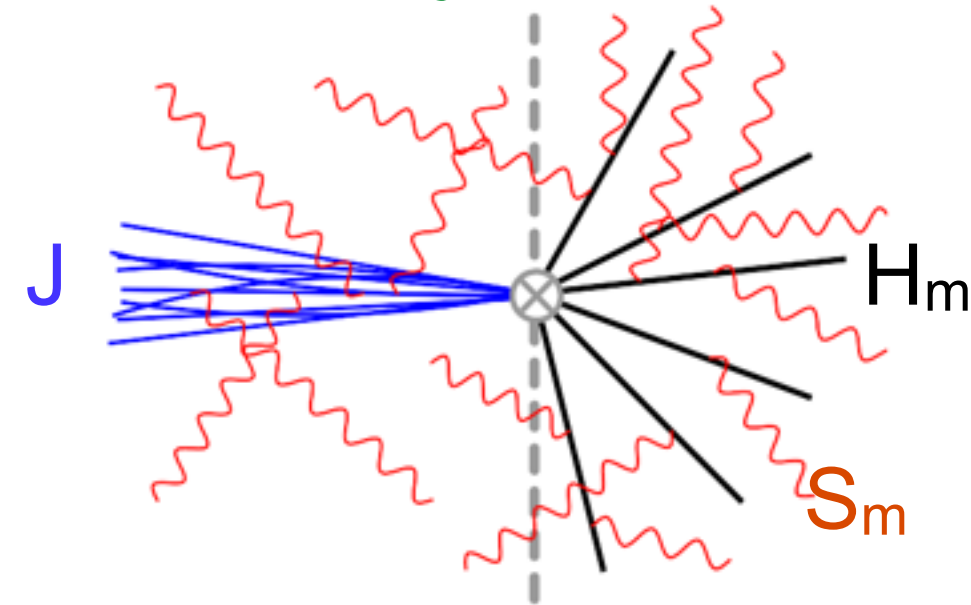
First result for an realistic observable: light jet mass

Becher, Pecjak, Shao, 2016

- ★ Highly non-trivial “**factorization**” theorem
- ★ Infinite many soft operator S_m , mixed under renormalization
- ★ Non-trivial **convolution** between **soft and hard function**

$$\sigma(M_L^2) \sim \int_0^\infty d\omega_L J(M_L^2 - Q\omega_L) \otimes \sum_{m=1}^{\infty} \langle \mathcal{H}_m^i(\{n\}, Q) \otimes \mathcal{S}_m(\{n\}, \omega_L) \rangle$$

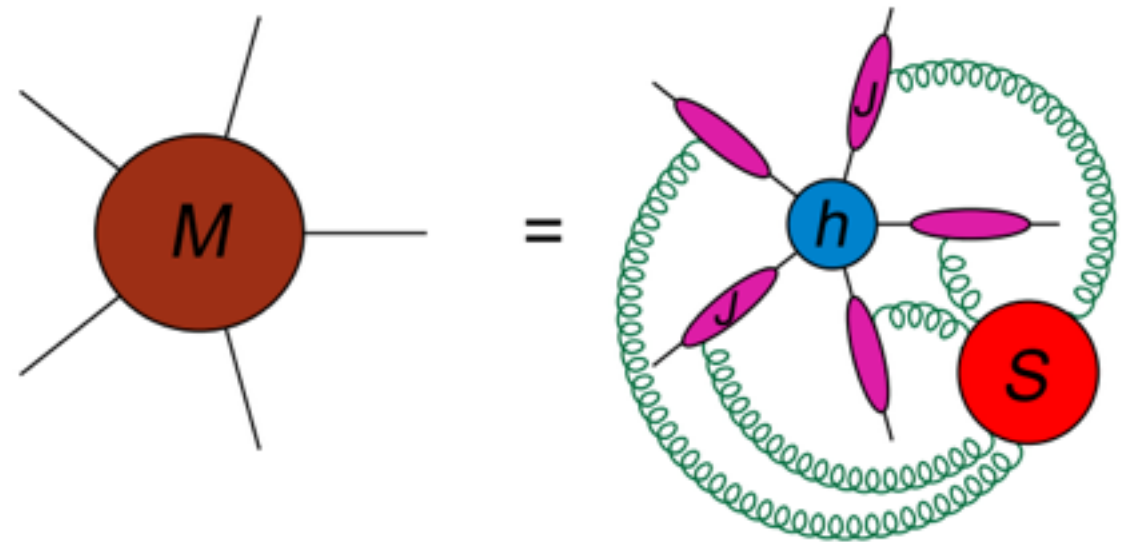
- ★ Including non-global logarithms leads to better agreement with data
- ★ Substantial **non-perturbative corrections** expected based on experience with heavy jet mass: shift the peak



Revealing the finer structure of QCD amplitudes

- ★ Scattering amplitudes: the **most perfect microscopic structures** in the universe (**Dixon, 1105.0771**)

Infrared singularities of n -particle on-shell scattering amplitudes factorize into a hard function, n collinear jet function, and a soft function which depends only on the **four velocity** of the jets.



- ★ For a two-jet event at e^+e^- , $Q \rightarrow p_1 p_2$, the structure is **trivial** $((\mu^2/2p_1 \cdot p_2)^{n\epsilon})$
- ★ It's only from **three or more jets events** $Q \rightarrow p_1 p_2 p_3 \dots$ that one can see the non-trivial structures through the Lorentz invariants constructed from $p_1 p_2 p_3 \dots$
- ★ Early resummation study on three-jet event shape: thrust minor, D parameter. **[Banfi, Dokshitzer, Marchesini, Zanderighi, 2001]**
- ★ A general N -jet event shape: N -jettiness (**Tackmann, Waalewijn, Stewart, 2009**)

3-jettiness in e+e-

- ★ The simplest example having non-trivial structure is 3-jettiness in e+e- collider

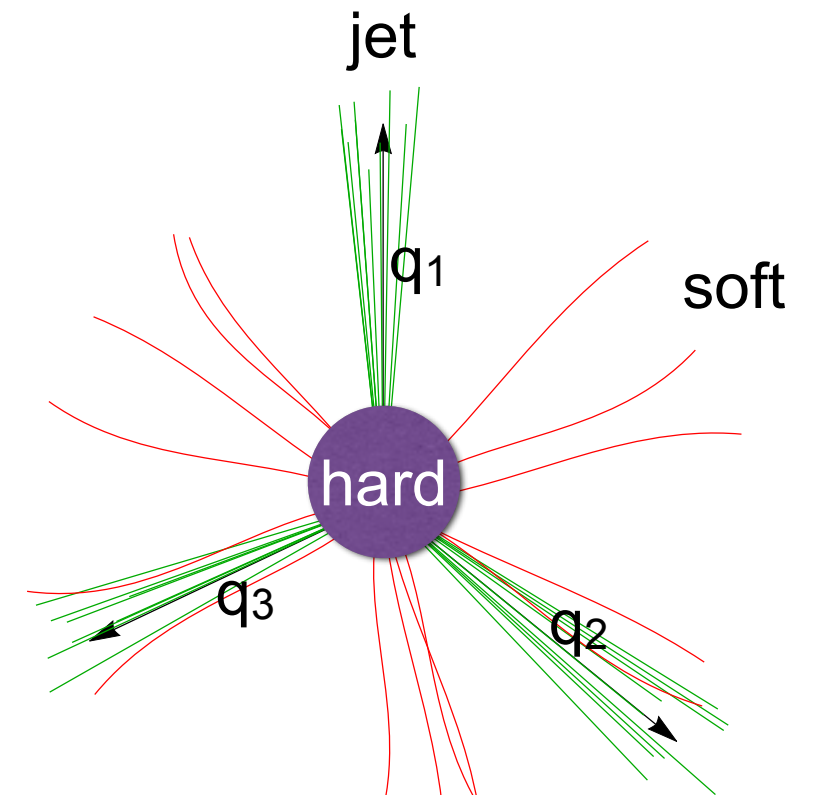
$$\tau \equiv \sum_k \min_{\{q_i\}} \frac{2q_i \cdot p_k}{Q^2}$$

$$\sum_i^3 q_i^\mu = (Q, 0, 0, 0) \quad q_i^2 = 0$$

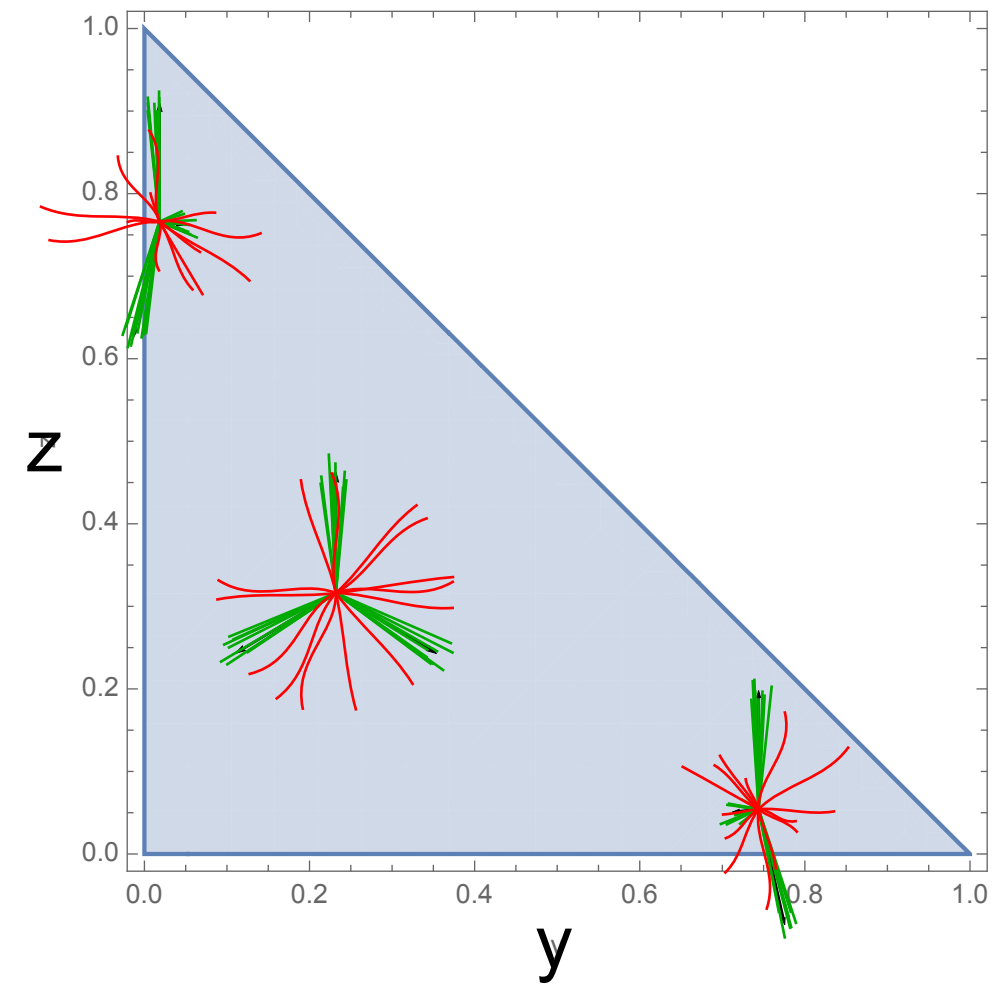
- ★ The born configuration has only three particles in the final state, and 3-jettiness **vanishes**
- ★ The born configuration can be parameterized by **two dimensionless variable** ranged from 0 to 1

$$y = \frac{q_1 \cdot q_3}{Q^2}, \quad z = \frac{q_2 \cdot q_3}{Q^2}$$

- ★ The event is **dijet like** in the boundary of y-z triangle

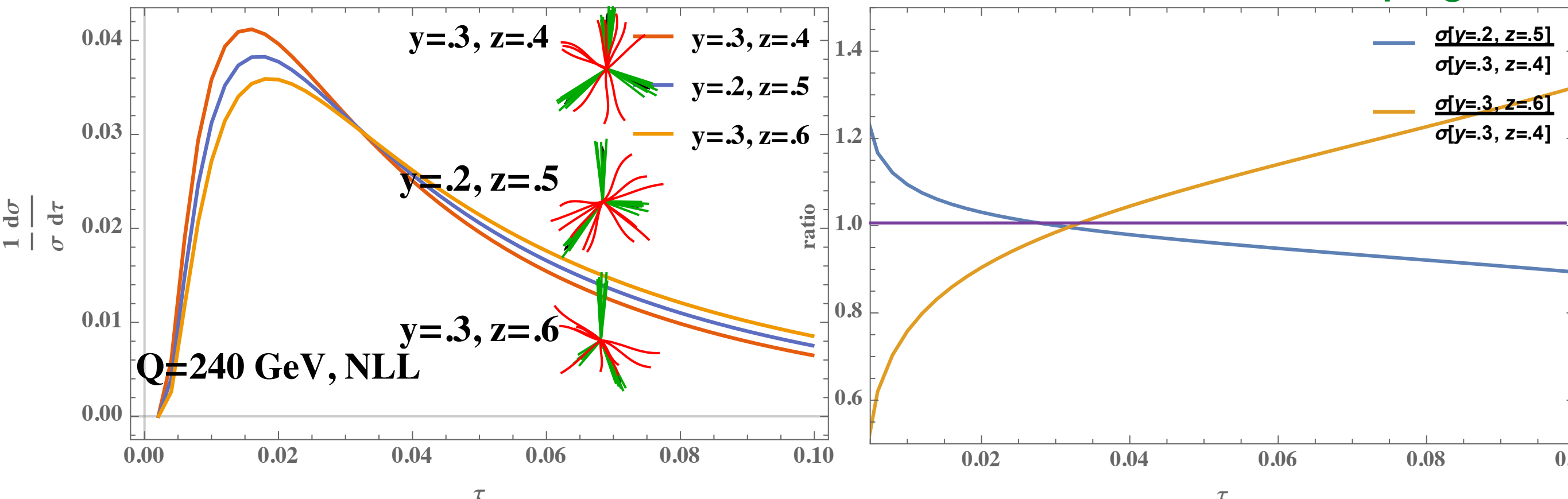


$$\frac{d\sigma}{d\tau} \sim H \cdot J_1 \otimes J_2 \otimes J_3 \otimes S$$

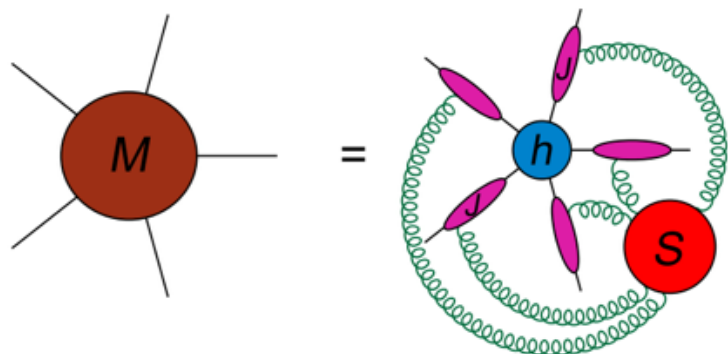


Seeing the IR structure of QCD amplitudes

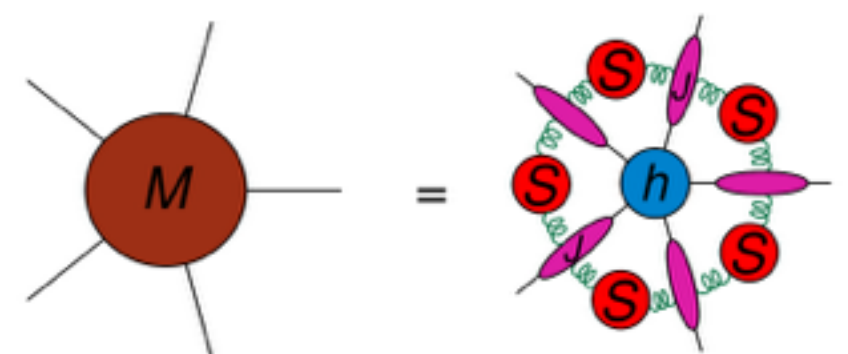
J. Gao, HXZ, work in progress



- ★ NLL 3-jettiness distribution for **different** born configuration, and **ratio** to Benz configuration, assuming **100% quark-gluon discrimination**
- ★ At Leading log, the normalized cross section is **insensitive** to the shape of born configuration.
- ★ At NLL, the born sensitivities enter through **large angle soft gluon radiation**

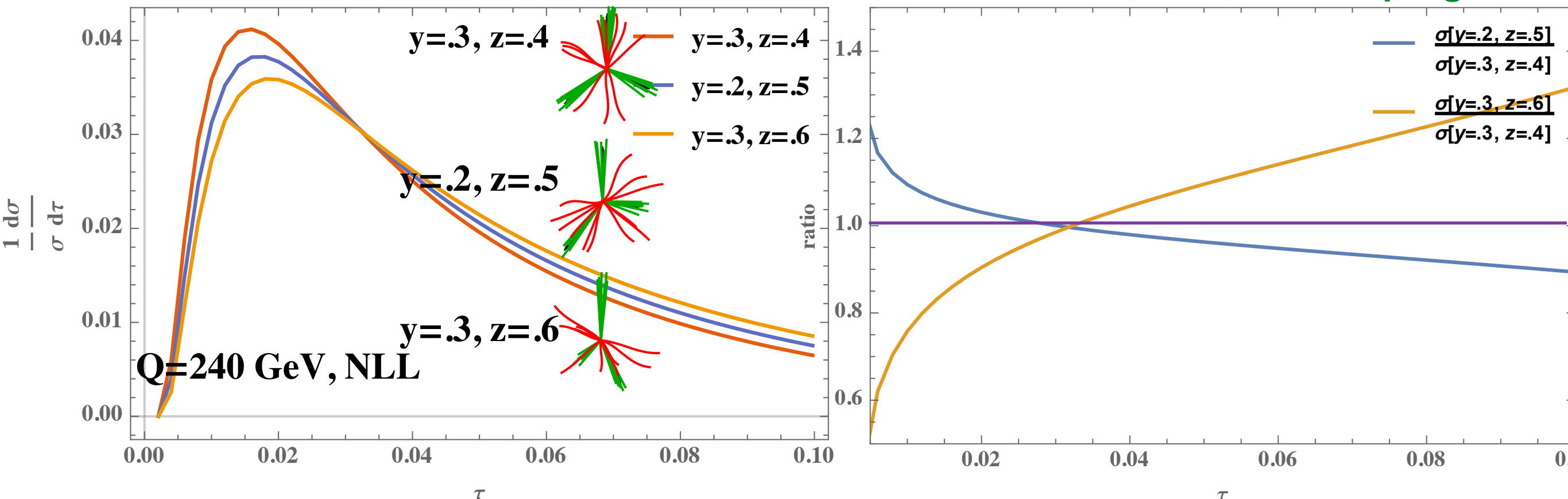


Soft correlation become dipole-like at one and two loop due to conformal symmetry in the IR



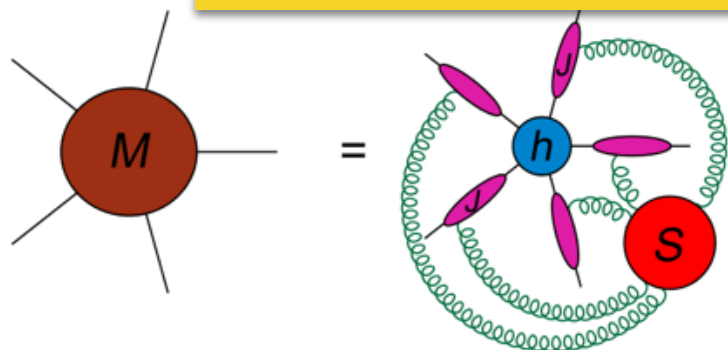
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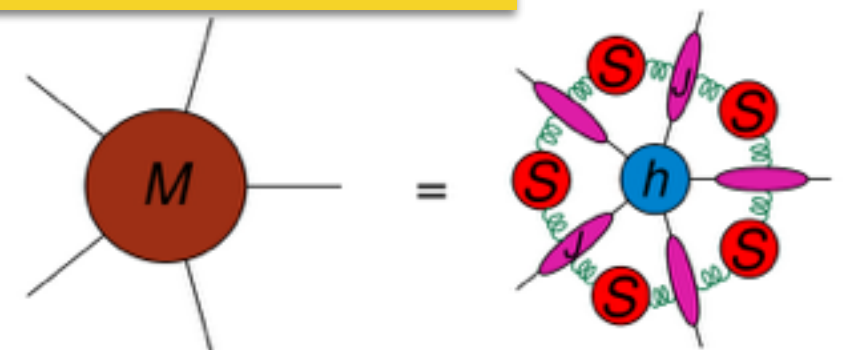


- ★ NLL 3-jettiness distribution for different born configuration, and ratio to Benz configuration. assuming 100% quark-gluon discrimination

- ★ At LHC conf
 - ★ At N
- Measuring the deviation from 1 will be a direct proof of the non-trivial IR dynamics, and will be useful to constraint Monte-Carlo tools



Soft correlation become dipole-like at one and two loop due to conformal symmetry in the IR



Summary

- ★ I have talked about a number of theoretical QCD topics in e+e- colliders that is closest to my heart
 - Precision determination of α_s from event shape and jet rates
 - Non-global observables
 - More differential observables that can test the finer structure of QCD
- ★ The field is healthy: a lot of interesting **unsolved** problems; is evolving: **new observables/techniques** to be and will be invented to help us understanding our real world QFT at colliders. Stay tuned.

Thank you for your attention!