Theoretical Issues in Precision Physics at Future e^+e^- Colliders

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INFN, Sezione di Pavia

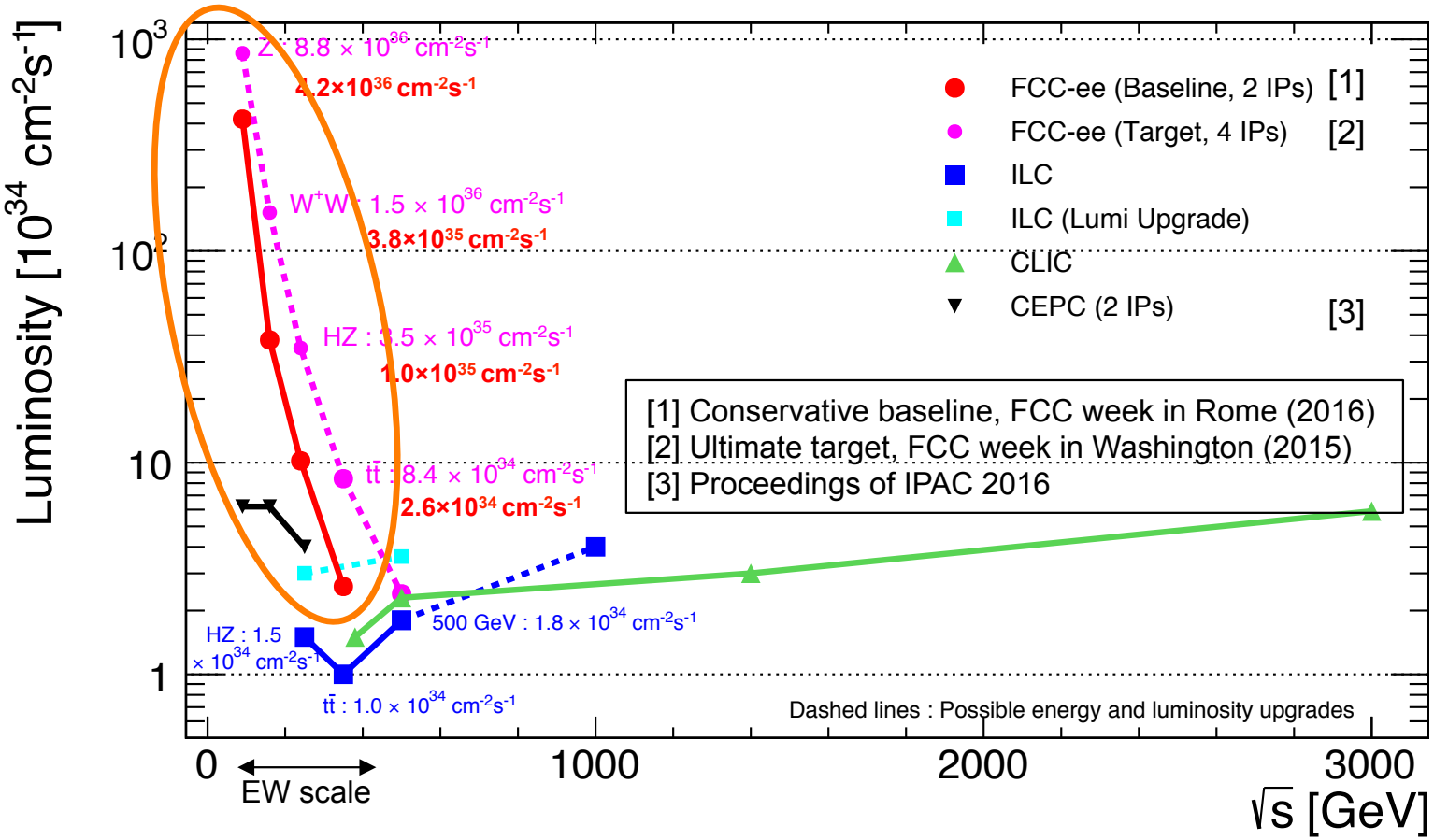
High Energy Physics Conference, IAS, Hong Kong, 23/01/2017

much more material at the CERN 1st FCC Physics Workshop
https://indico.cern.ch/event/550509/timetable/

in particular talks by

- P. Azzurri
- J. de Blas
- A. Blondel
- J. Gluza
- S. Heynemeier
- B.F.L. Ward

Future e^+e^- machine projects

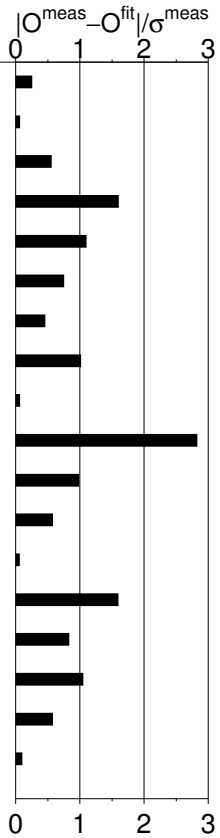


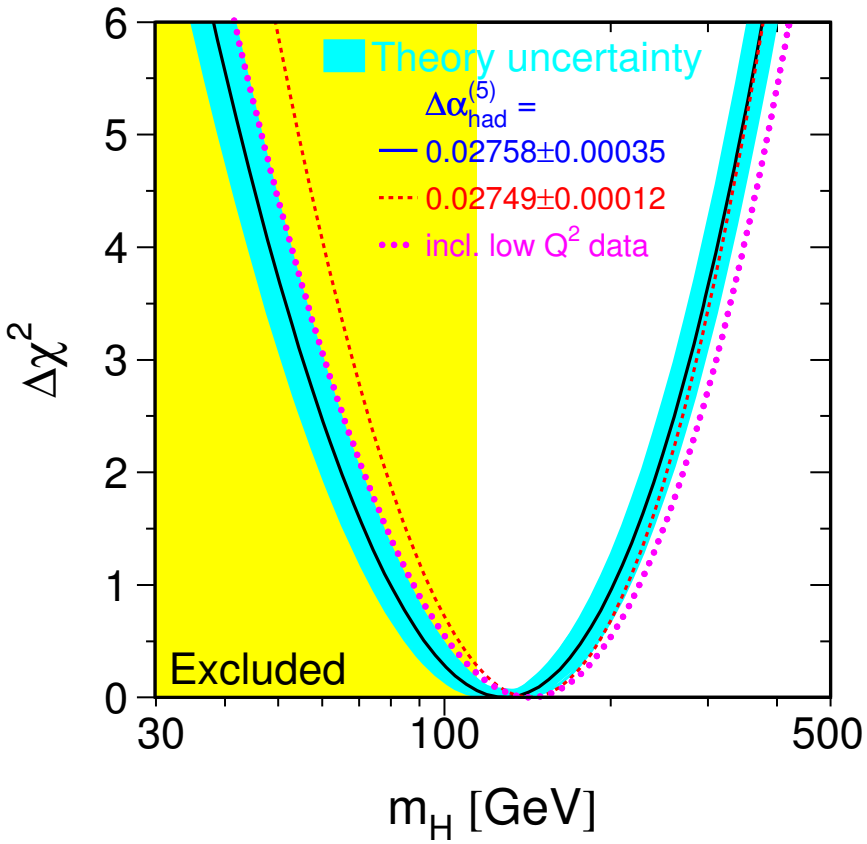
P. Janot, eeFACT16, DAresbury, 24-27 October 2016

- $\sqrt{s} \simeq M_Z$ and $\sqrt{s} \simeq WW$ threshold already investigated at LEP luminosities lower by several orders of magnitude $(\sim 10^{31}\ \mathrm{cm}^{-2}\mathrm{s}^{-1})$
- HZ and $t\bar{t}$ thesholds never investigated at a leptonic collider

LEP/SLC legacy

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ^0_{had} [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21562
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1037
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.389
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.5

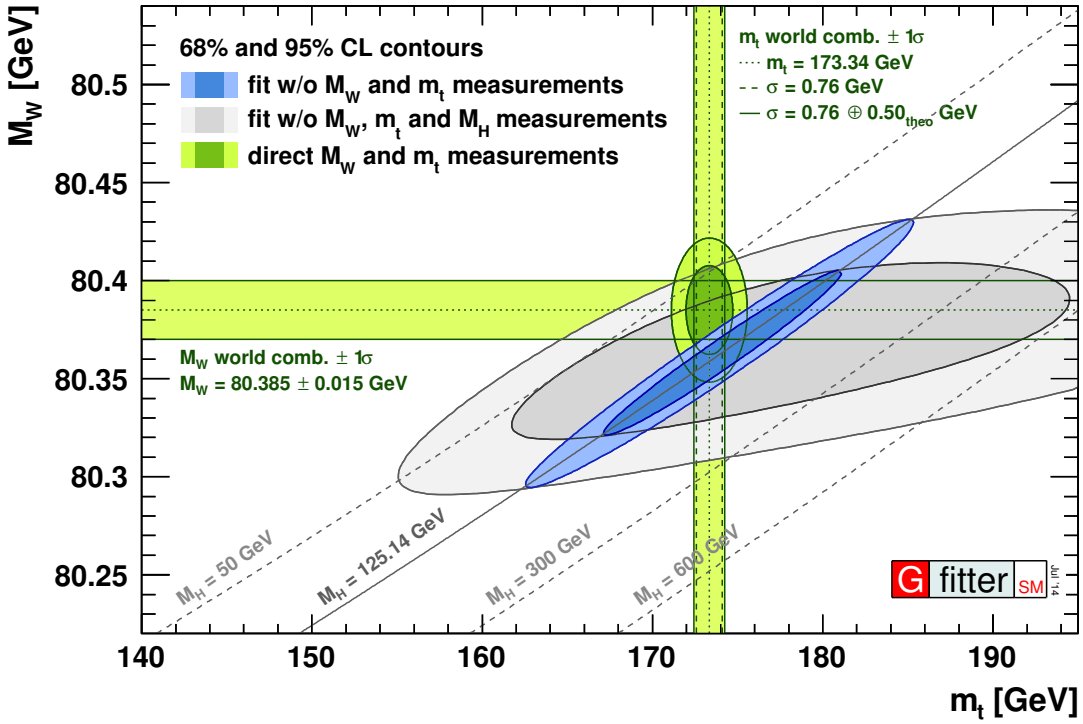




LEP EWWG, SLD WG, ALEPH, DELPHI, L3, OPAL, Phys. Rept. 427 (2006) 257

- crucial role played by theoretical uncertainties
  - intrinsic uncertainties (higher orders)
  - parametric uncertainties (input parameters)

adding recent results, mostly from Tevatron and LHC

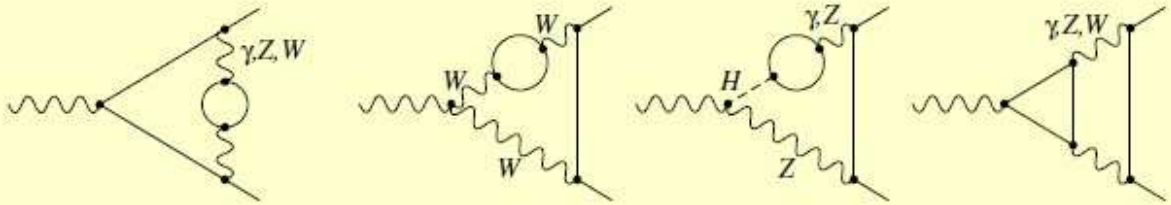


http://project-gfitter.web.cern.ch/project-gfitter/Standard_Model/

- electroweak fit based on *derived (pseudo-)*observables (allow easy combination among experiments and easy comparison data/theory within and beyond SM)
- primary measured observables: cross section and asymmetries

present knowledge of pseudo-observables (EWPO)

Known corrections to Δr, $\sin^2\theta^f_{\text{eff}}$, g_{Vf}, g_{Af}:



- Complete NNLO corrections (Δr, $\sin^2\theta^\ell_{\text{eff}}$) Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02; Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
- "Fermionic" NNLO corrections (g_{Vf}, g_{Af}) Czarnecki, Kühn '96
  Harlander, Seidensticker, Steinhauser '98
  Freitas '13,14
- Partial 3/4-loop corrections to ρ/T-parameter
  $\mathcal{O}(\alpha_t\alpha_s^2)$, $\mathcal{O}(\alpha_t^2\alpha_s)$, $\mathcal{O}(\alpha_t\alpha_s^3)$ Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05; Chetyrkin et al. '06
  Boughezal, Czakon '06

$(\alpha_t \equiv \frac{y_t^2}{4\pi})$

A. Freitas, 2016

- two-loop bosonic EW corrections to $\sin^2\vartheta^b_{eff}$

$$\sin^2\vartheta^b_{eff} = \left(1-\frac{M_W^2}{M_Z^2}\right)(1+\Delta k_b)$$

$$\Delta k_b^{\alpha^2,\mathrm{bos}} = -0.9855\times 10^{-4}$$

I. Dubovyk et al., PLB762 (2016) 184

Intrinsic uncertainties:

Quantity	current experimental unc.	current intrinsic unc.	
M_W [MeV]	15	4	$(\alpha^3, \alpha^2\alpha_s)$
$\sin^2\theta^\ell_{\mathrm{eff}}$ [10^{-5}]	16	4.5	$(\alpha^3, \alpha^2\alpha_s)$
Γ_Z [MeV]	2.3	0.5	$(\alpha^2_{\mathrm{bos}}, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$
R_b [10^{-5}]	66	15	$(\alpha^2_{\mathrm{bos}}, \alpha^3, \alpha^2\alpha_s)$
R_l [10^{-3}]	25	5	$(\alpha^2_{\mathrm{bos}}, \alpha^3, \alpha^2\alpha_s)$

Parametric uncertainties:

Quantity	$\delta m_t = 0.9$ GeV	$\delta(\Delta\alpha_{\mathrm{had}}) = 10^{-4}$	$\delta M_Z = 2.1$ MeV
$\delta M_W^{\mathrm{para}}$ [MeV]	5.5	2	2.5
$\delta\sin^2\theta^{\ell,\mathrm{para}}_{\mathrm{eff}}$ [10^{-5}]	3.0	3.6	1.4

S. Heynemeier, FCC 1st Physics Workshop, CERN

From measured observables to pseudo-observables

$$\sigma_{\rm T}(s) = \int_{z_0}^{1} dz H(z;s)\hat{\sigma}_{\rm T}(zs)$$

$$A_{FB}(s) = \frac{\pi\alpha^2 Q_e^2 Q_f^2}{\sigma_{\rm tot}} \int_{z_0}^{1} dz \frac{1}{(1+z)^2} H_{\rm FB}(z;s)\,\hat{\sigma}_{\rm FB}(zs)$$

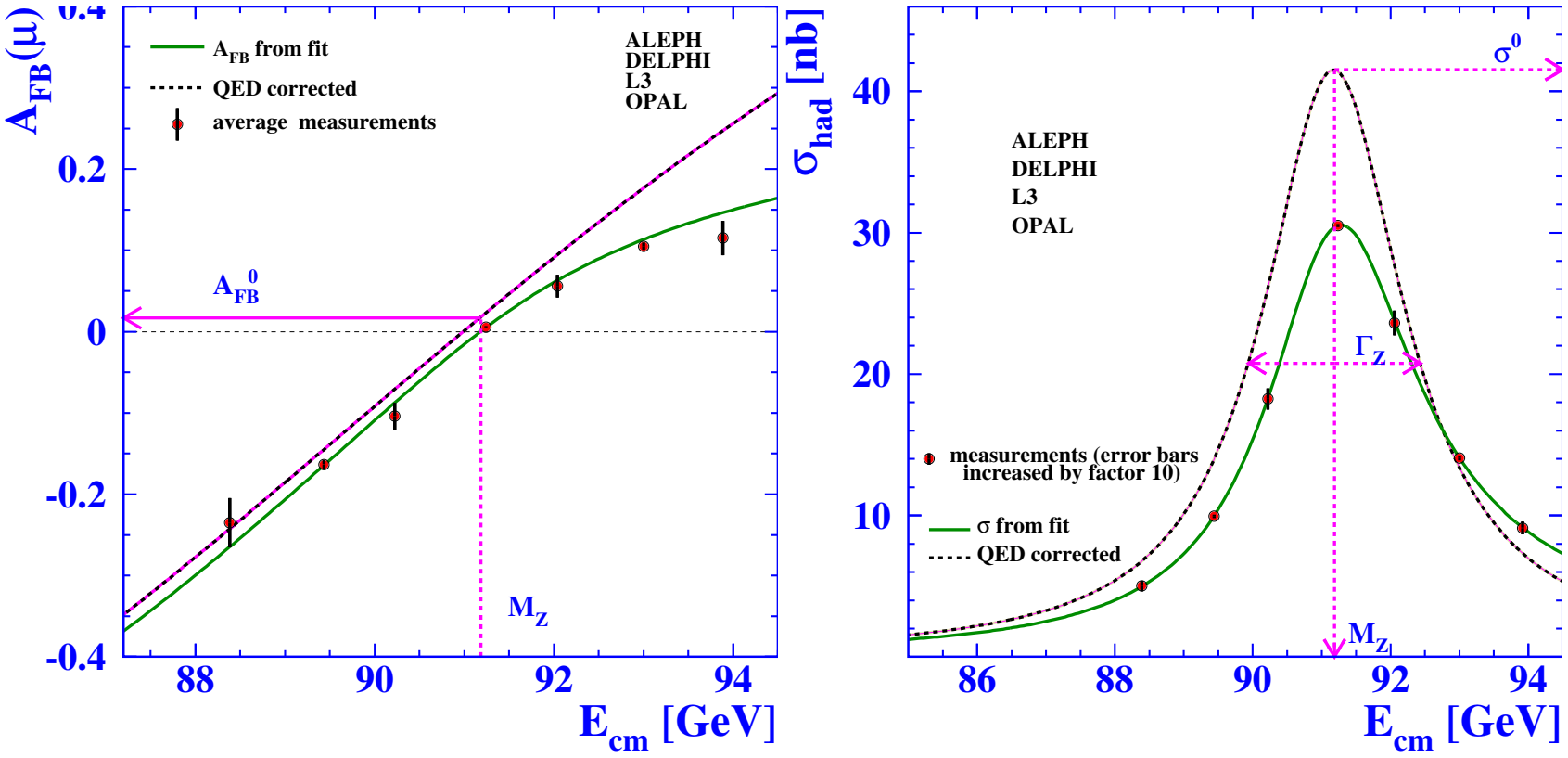
- Radiator function known up to $\mathcal{O}(\alpha^3)$
  1. additive form

     G. Montagna, O. Nicrosini, F.P., PLB 406, (1997) 243
  2. factorized form

     S. Jadach, M. Skrzypek, B.F.L. Ward, PLB257 (1991) 173, M. Skrzypek, APPB23 (1992) 135
- $H_{\rm FB}$ known up to $\mathcal{O}(\alpha^2)$

Theoretical control on the derived observables with a certain precision requires the process of "deconvolution" of ISR and FSR with the same level of precision

Effect of QED ISR deconvolution



LEP EWWG, SLD WG, ALEPH, DELPHI, L3, OPAL, Phys. Rept. 427 (2006) 257

Deconvolution performed at LEP by means of

- TOPAZ0

G. Montagna, O. Nicrosini, G. Passarino, F.P., R. Pittau, 1993, 1996, 1999

- ZFITTER

D. Bardin et al., 1989, 1991, 1992, 1994, 2001

Uncertainty on the ISR deconvolution

- by comparison of additive and factorized form of the radiator

	LEP 1 energy in GeV				
	$M_Z - 3$	$M_Z - 1$	M_Z	$M_Z + 1$	$M_Z + 3$
$10^4 \times$ (fact/add-1)					
σ_μ	0.44 0.88	0.63 0.63	0.61 0.68	0.72 0.72	0.49 0.49
$\sigma_{\rm had}$	0.58 0.61	0.58 0.62	0.64 0.67	0.73 0.76	0.59 0.62
$10^5 \times$ (fact-add)					
A^μ_{FB}	1.00 -4.00	1.00 -2.00	0.00 0.00	0.00 1.00	-1.00 1.00

D.Y. Bardin, M. Grünewald, G. Passarino, hep-ph/9902452

- The level of agreement between TOPAZ0 and ZFITTER around the Z peak is below the 0.01% level $\longrightarrow$ analysis at the 0.1% level on the derived observables are robust
- going from 0.1% to 0.01% (or even more) precision requires an improvement of the deconvolution process

model-independent parameterization of $e^+e^- \to f\bar{f}$

$$A_{\rm SM} = A_\gamma + A_Z + \text{non-factorizable}$$

- aim: write the Z-line shape in a model independent way

Borrelli, Consoli, Maiani, Sisto, NPB333 (1990) 357

$$\sigma^Z_{f\bar{f}} = \sigma^{\rm peak}_{f\bar{f}} \frac{s\Gamma_Z^2}{(s-M_Z)^2 + s^2\Gamma_Z^2/M_Z^2}$$

$$\sigma^{\rm peak}_{f\bar{f}} = \frac{\sigma^0_{f\bar{f}}}{R_{\rm QED}}; \qquad \sigma^0_{f\bar{f}} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

- what is not factorizable on the Z-exchange tree-level is subtracted at fixed SM parameters
- $\longrightarrow$ model independence is lost. At LEP the remainders show dependence on the SM Lagrangian parameters well below 0.1%
- with higher luminosities at FCC-ee/CEPC it will be important to control the subtraction with off-peak data points
- also, the M_Z definition should be changed in favour of the complex mass pole

N_ν from Z invisible width

$$R^0_{\rm inv} = \frac{\Gamma_{\rm inv}}{\Gamma_{ll}} = \sqrt{\frac{12\pi R^0_l}{\sigma^0_{\rm had} m_Z^2}} - R^0_l - (3+\delta_\tau)$$

- assuming lepton universality

$$\left(R^0_{\rm inv}\right)_{\rm exp} = N_\nu \left(\frac{\Gamma_{\nu\bar\nu}}{\Gamma_{ll}}\right)_{\rm SM}$$

- from LEP Z-peak measurements

$$\begin{aligned} N_\nu &= 2.9840 \pm 0.0082 \quad \text{hint for right handed neutrinos?} \\ \delta N_\nu &\simeq 10.5\frac{\delta n_{\rm had}}{n_{\rm had}} \oplus 3.0\frac{\delta n_{\rm lept}}{n_{\rm lept}} \oplus 7.5\frac{\delta\mathcal{L}}{\mathcal{L}} \\ \frac{\delta\mathcal{L}}{\mathcal{L}} &= 0.061\% \Longrightarrow \delta N_\nu = 0.0046 \end{aligned}$$

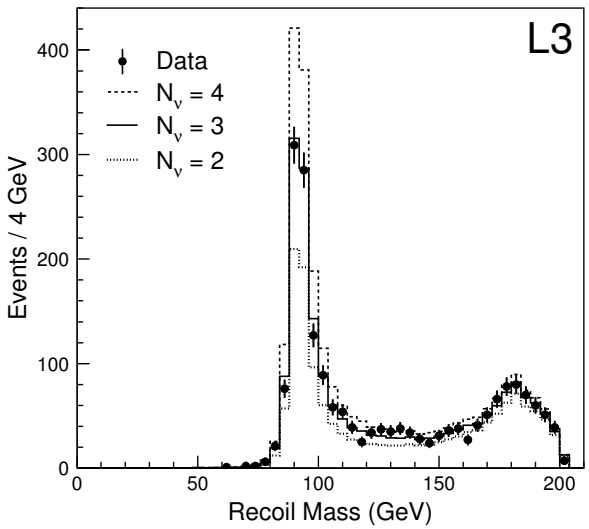
ADLO, SLD and LEPEWWG, Phys. Rept. 427 (2006) 257, hep-ex/0509008

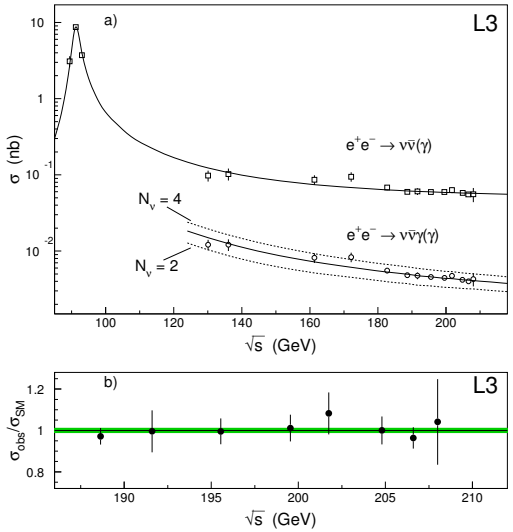
- δN_ν severely afftected by luminosity uncertainty through σ_0 (theory dominated at LEP $\Longrightarrow$ see later)

Independent way for ν count: $\nu\bar{\nu}\gamma$ and LEP2

- no need to tune the collider energy at the Z peak (radiative return)
- provided large enough luminosity is available to be competitive with $\Gamma_{\rm inv}$ method

190 GeV $\leq \sqrt{s} \leq 208$ GeV, $\mathcal{L} \sim 600$ pb^{-1}





L3 Collab., P. Achard et al., CERN-EP/2003-068 (2003)

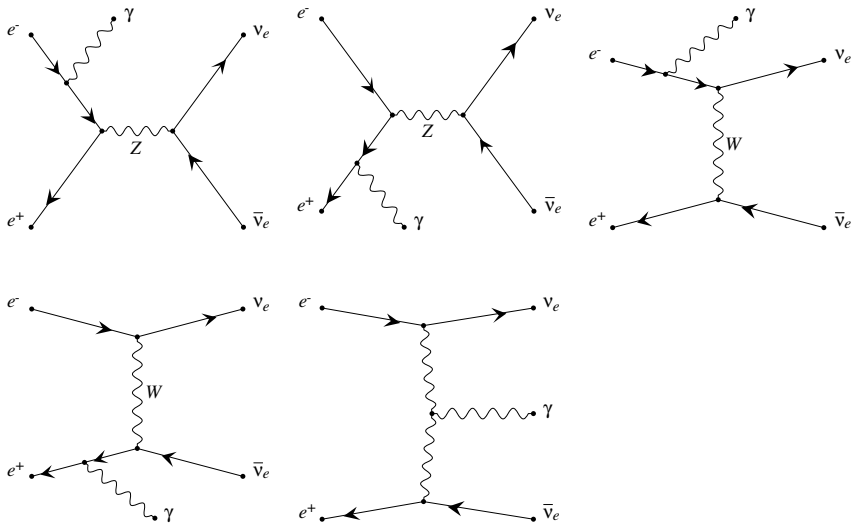
- agreement of data with SM predictions at % level
- $N_\nu = 2.98 \pm 0.05 \pm 0.04$ (L3) (important but not competitive with the $\Gamma_{\rm inv}$ method)
- similar results for ALEPH, DELPHI and OPAL

$\nu\bar{\nu}\gamma$: ratio measurements

- a factor $10^3/10^4$ of improvement in luminosity @FCC/CEPC w.r.t. LEP allows to exploit the ratios

$$\frac{d\sigma(e^+e^- \to \nu\bar{\nu}\gamma)}{d\sigma(e^+e^- \to \mu^+\mu^-\gamma)}$$

in order to cancel common systematics (such as luminosity)



- $\mu^+\mu^-$ only $s-$channel but ISR and FSR
- ν_μ and ν_τ f.s.: only $s-$channel ISR
- ν_e f.s.: ISR with $t-$channel
- ν_e f.s.: also W radiation

- QED and EW corrections do not cancel completeley in the ratio
- but now the technology for full $2 \to 3$ EW one-loop calculations is available

Luminosity: theoretical systematics on σ normalization

- theoretical error in small angle Bhabha process at LEP1

Type of correction/error	(%)	(%)	updated (%)
missing photonic $O(\alpha^2 L)$	0.100	0.027	0.027
missing photonic $O(\alpha^3 L^3)$	0.015	0.015	0.015
vacuum polarization	0.040	0.040	0.040
light pairs	0.030	0.030	0.010
Z-exchange	0.015	0.015	0.015
total	0.110	0.061	0.054

I column: S. Jadach, O. Nicrosini et al. Physics at LEP2 YR 96-01, Vol. 2
A. Arbuzov et al., Phys. Lett. B389 (1996) 129
II column: B.F.L. Ward, S. Jadach, M. Melles, S.A. Yost, hep-ph/9811245
III column: G. Montagna et al., Nucl. Phys. B547 (1999) 39

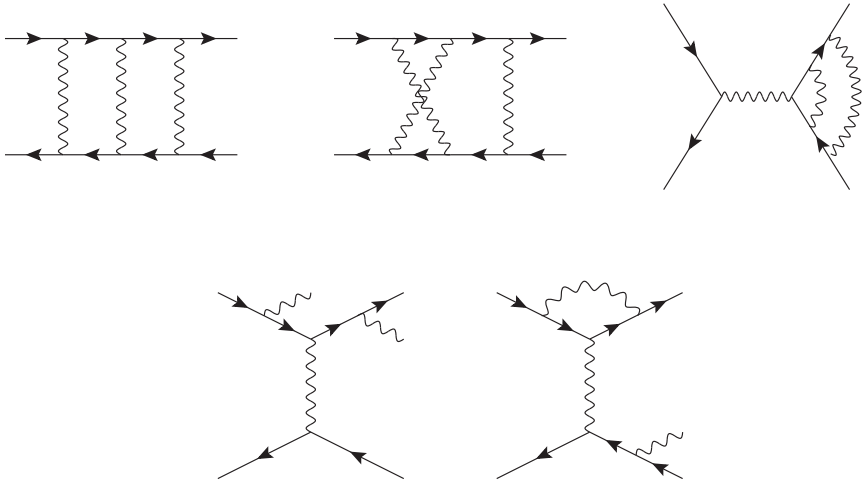
- after LEP, progress in complete two-loop pure photonic contributions to QED Bhabha scattering (see following two slides)
  - $\Longrightarrow$ building blocks available for MC programs with th. precision below 0.1% on the perturbative side
    - this has already been achieved for large angle Bhabha at flavour factories, with a validation of the BabaYaga event generator below the 0.1% (for the QED higher orders)

G. Balossini et al., NPB758 (2006) 227; C.M. Carloni Calame et al., JHEP 1107 (2011) 126

  - vacuum polarization issue, both for Bhabha as well as for Z observ.

NNLO Bhabha calculations (I)

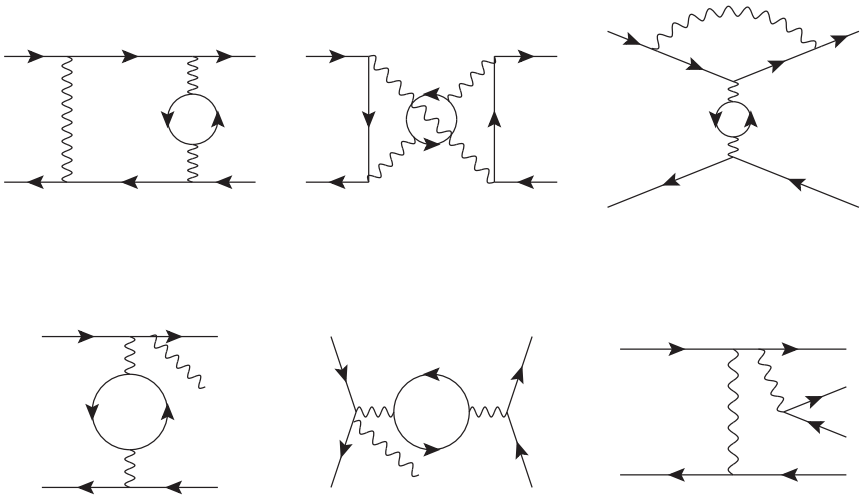
- Photonic corrections A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185



- Electron loop corrections

R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280

S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26

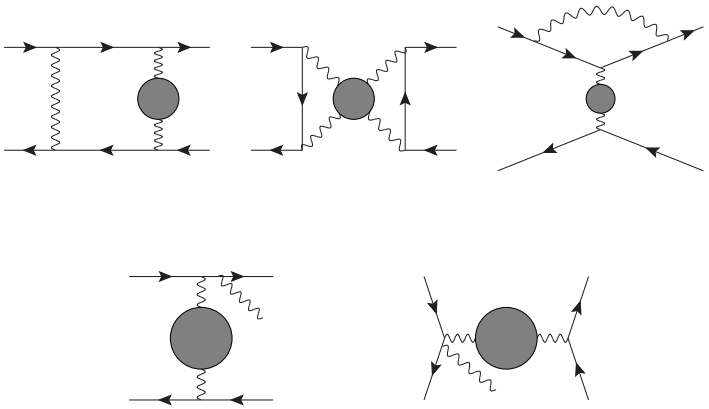


NNLO Bhabha calculations (II)

- Heavy fermion and hadronic loops

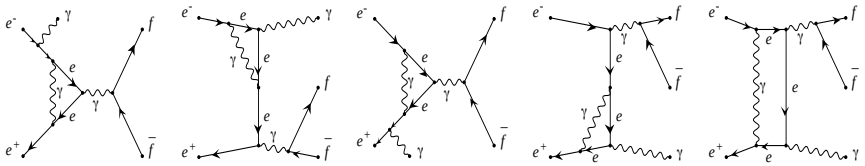
R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601
S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602

J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300



- One-loop soft+virtual corrections to single hard bremsstrahlung

S. Actis, P. Mastrolia and G. Ossola, Phys. Lett. **B682** (2010) 419



Vacuum Polarization: bottleneck for future precision

- $\alpha \to \alpha(q^2) \equiv \frac{\alpha}{1-\Delta\alpha(q^2)}$ $\qquad \Delta\alpha(q^2) = \Delta\alpha_{e,\mu,\tau,\text{top}}(q^2) + \Delta\alpha^{(5)}_{\text{had}}(q^2)$
- $\Delta\alpha^{(5)}_{\text{had}}$ is an intrinsically non-perturbative contribution. It can be calculated from $e^+e^- \to hadrons$ data using dispersion relations

$$\Delta\alpha^{(5)}_{\text{had}}(q^2) = -\frac{q^2\alpha}{3\pi}\left[\mathcal{P}\!\!\int_{4m_\pi^2}^{E^2_{cut}} \frac{R^{data}_{had}(s)}{s(s-q^2)}ds + \mathcal{P}\!\!\int_{E^2_{cut}}^{\infty} \frac{R^{pQCD}_{had}(s)}{s(s-q^2)}ds\right]$$

- it is affected by an uncertainty, due to low energy data on $\sigma_{had}(s)$ $\Longrightarrow$ it reflects on Bhabha predictions $\Longrightarrow$ Z-observables
- an historical perspective on the evolution of the error
  - $\Delta\alpha(M_Z^2) = 0.0280 \pm 0.0007 \Longrightarrow \alpha^{-1}(M_Z^2) = 128.89 \pm 0.09$
    H. Burkhardt and B. Pietrzyk, Phys. Lett. B356 (1995) 398
  - $\Delta\alpha(M_Z^2) = 0.02750 \pm 0.00033$ H. Burkhardt and B. Pietrzyk, Phys. Rev. D84 (2011) 037502
  - $\Delta\alpha(M_Z^2) = 0.027498 \pm 0.000135 [0.027510 \pm 0.000218]$
    F. Jegerlehner, arXiv:1107.4683
  - $\Delta\alpha(M_Z^2) = 0.02757 \pm 0.0001 \Longrightarrow \alpha^{-1}(M_Z^2) = 128.952 \pm 0.014$
    Davier, Hoecker, Malaescu, Zhang, arXiv:1010.4180
  - $\Delta\alpha(M_Z^2) = 0.027626 \pm 0.000138$
    T. Teubner et al., Nucl. Phys. Proc. Suppl. 225 (2012) 282

a comprehensive compilation by F. Jegerlehner

Authors	data	pQCD	sum
Jegerlehner 1985:	247.37± 7.	38.63±0.37	286. ± 7.
Lynn et al. 1985:	145. ±13.	129. ±1.	274. ±13.
Burkhardt et al. 1989:	164.32± 8.2	123.68±3.71	288. ± 9.
Martin, Zeppenfeld 1994:	51.5 ± 1.1	221.7 ±4.1	273.2 ± 4.2
Swartz 1995:	232.56± 4.6	42.64±0.10	275.2 ± 4.6
Eidelman, Jegerlehner 1995:	237.55± 6.43	42.82±0.10	280.37± 6.43
Burkhardt, Pietrzyk 1995:	159. ± 7.	121. ±0.2	280. ± 7.
Adel, Yndurain 1995:	45.99± 0.85	226.6 ±4.0	272.59± 4.09
Alemany, Davier, Höcker 1997:	238.01± 6.3	42.82±0.10	280.9 ± 6.3
Kühn, Steinhauser 1998:	82.9 ± 1.40	194.45±0.96	277.43± 1.70
Davier, Höcker 1998:	56.53± 0.83	219.77±1.40	276.3 ± 1.6
Erler 1998 :	56.9 ± 1.1	220.8 ±1.5	277.7 ± 1.9
Burkhardt, Pietrzyk 2001:	155.8 ± 3.6	120.3 ±0.2	276.1 ± 3.6
Hagiwara et al 2004:	150.18± 2.3	125.32±0.15	275.5 ± 2.3
Jegerlehner 2006 direct:	106.07± 2.24	115.66±0.11	276.07± 2.25
Jegerlehner 2006 Adler :	73.69± 0.98	201.83±1.03	275.52± 1.42
Hagiwara et al 2011:	138.70± 1.37	137.56±0.16	276.26± 1.38
Davier et al 2011:	80.57± 1.00	195.33±0.09	275.90± 1.00
Jegerlehner 2016 direct:	126.86± 1.78	149.57±0.05	276.43± 1.78
Jegerlehner 2016 Adler:	60.49± 0.66	214.48±1.00	275.04± 1.19

possible alternative to Bhabha scattering: $\mathrm{e}^+\mathrm{e}^- \to \gamma\gamma$

- $e^+e^- \to \gamma\gamma$ could be used to cross-check independently $\mathcal{L}$ measurements
  - ⋆ At present, its theoretical accuracy is similar to Bhabha (NLO + h.o.)

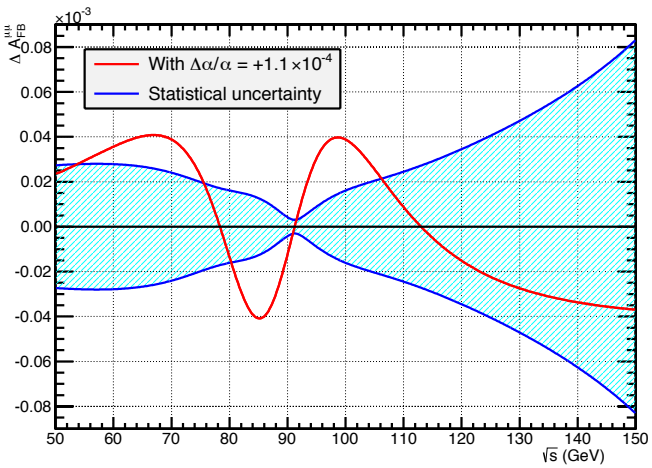
    G. Balossini et al., Phys.Lett. B663 (2008) 209
  - ⋆ Advantages
    - no Z exchange diagrams (at LO)
    - no photon VP corrections (up to NNLO)
  - ⋆ Disadvantages
    - lower x-section
    - efficiency in detecting $\gamma\gamma$ events

recent new proposal

P. Janot, JHEP 1602 (2016) 053

- idea: measuring $\alpha(M_Z^2)$ from a measurement of A_{FB} below and above peak

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu} - A_{\rm FB,0}^{\mu\mu}} \times \frac{\mathcal{Z}+\mathcal{G}}{\mathcal{Z}-\mathcal{G}} \simeq \frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu}} \times \frac{\mathcal{Z}+\mathcal{G}}{\mathcal{Z}-\mathcal{G}},$$



- preliminary investigations show that an accuracy of the order of 10^{-5} could be reached
- first studies on the impact of QED corrections, in particular initial-final interference

S. Jadach, talk at FCC Week, Rome, 12 April, 2016

Summary

some theoretical issues for Z peak energy runs at future e^+e^- colliders

- the exceptional recent progress in the calculation of higher order corrections for LHC makes it plausible thinking that future progress in higher order electroweak corrections can meet the projected experimental accuracy
- for a successful physics program, theoretical improvements needed for
  - QED corrections and their unfolding in $e^+e^- \to f\bar{f}$
  - luminosity determination
- common issue given by the uncertainty in the hadronic contribution to the vacuum polarization
- recent promising proposal to determine $\alpha(M_Z^2)$ from A_{FB} below and above the Z peak