Phenomenology of a Family Model

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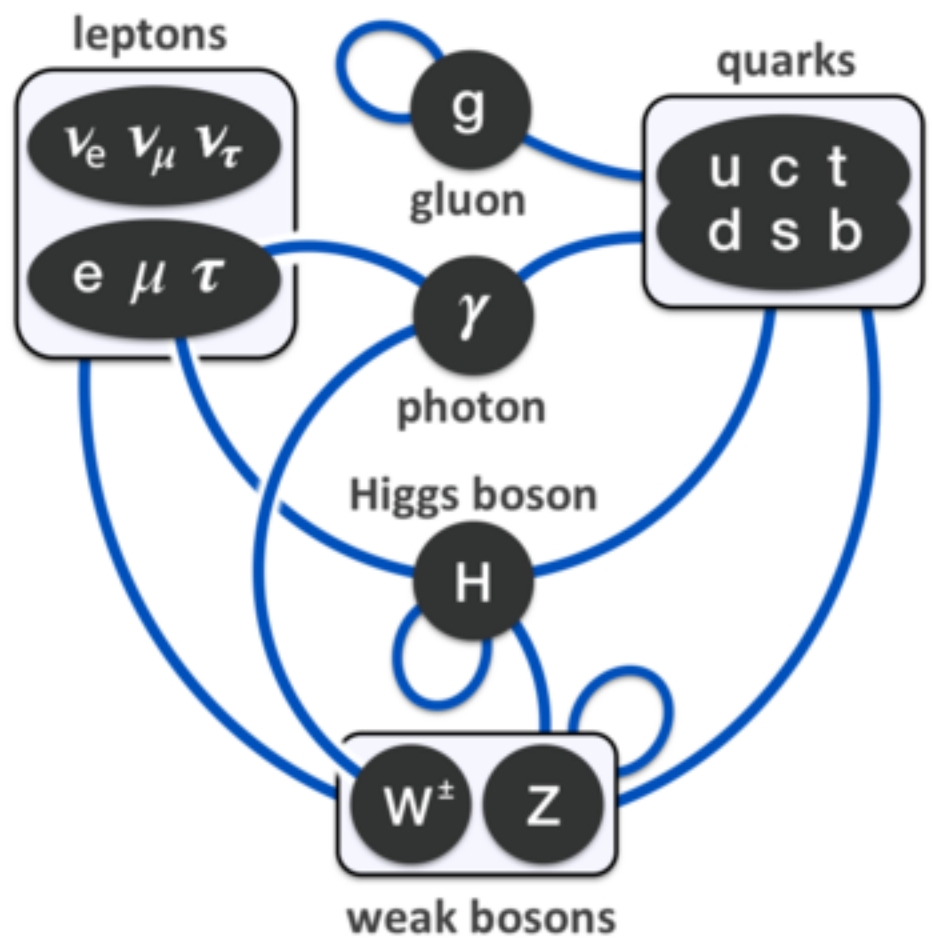
Outline

- Introduction
- Setup
- Interactions in the model
- Constraining the model
- Summary

Introduction

SM



From
https://en.wikipedia.org/wiki/Standard_Model

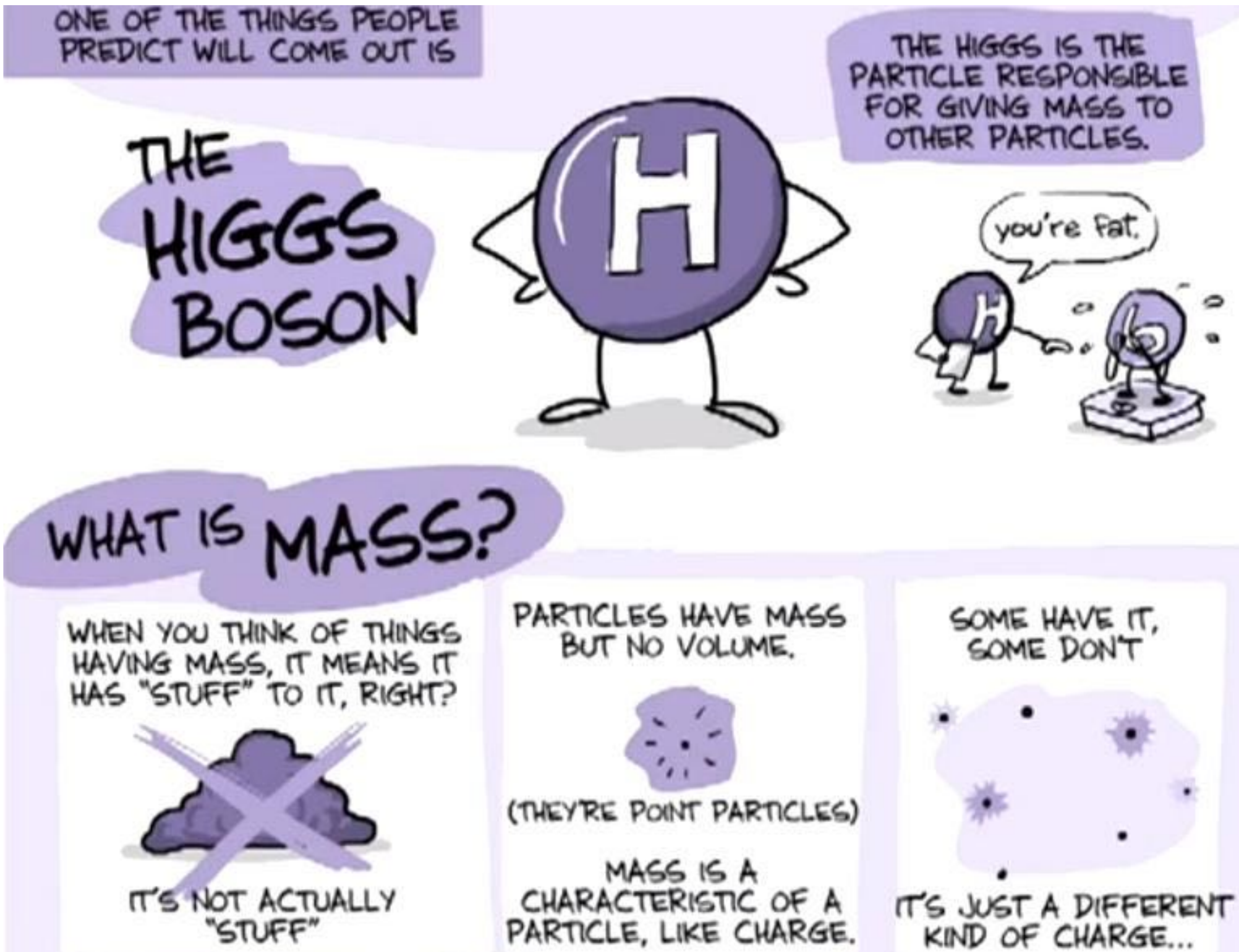
- Quite successful
- W/ mysteries
- Fermion matter fields: quarks and leptons
- Gauge interactions define quarks and leptons

Introduction

SM

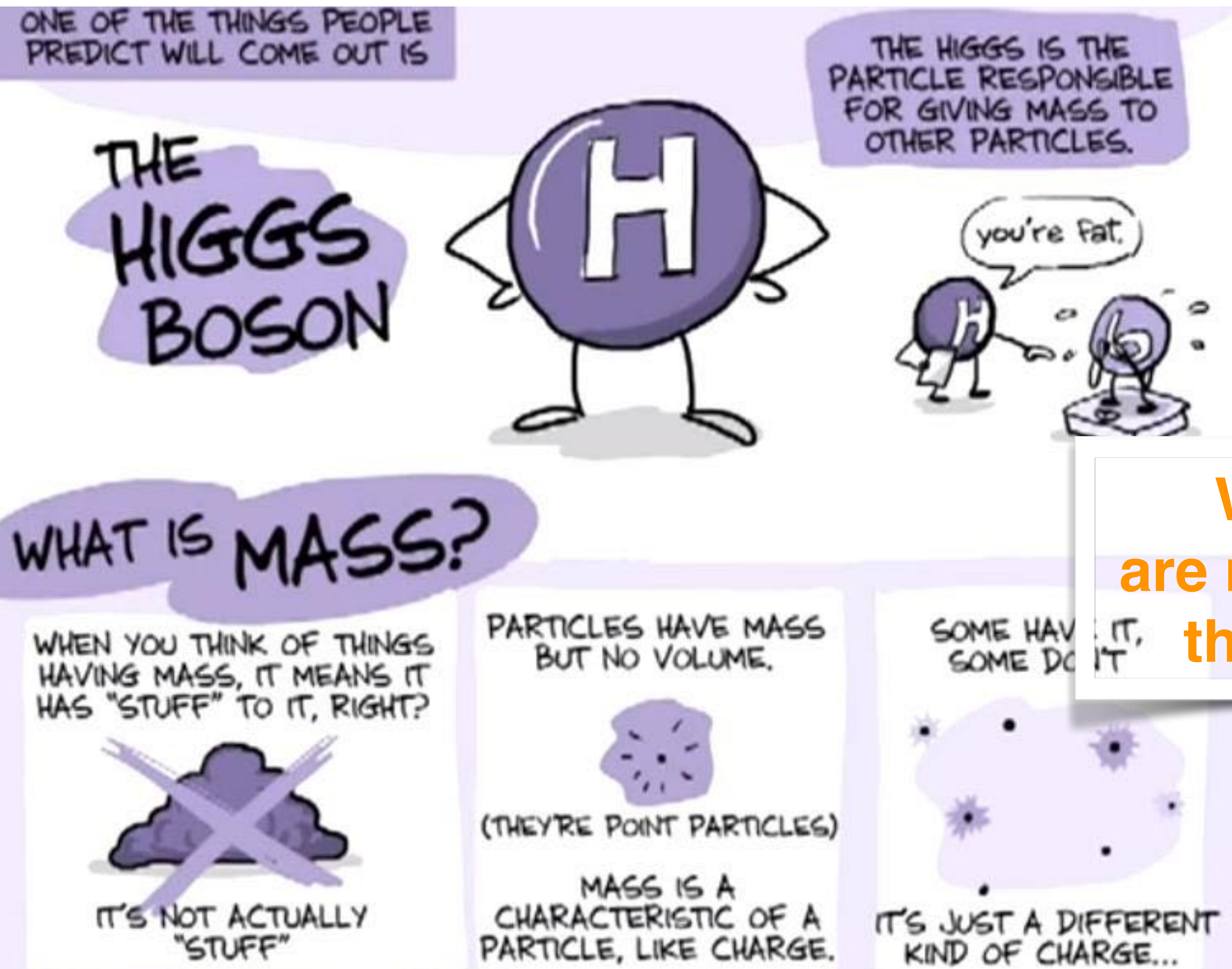
- Gauge interactions are generation (flavor) blind
- What distinguishes different generations/flavors?
- — Mass
- Origin of mass?
- — Higgs sector

Introduction



From X-bit Labs Community

Introduction



SM

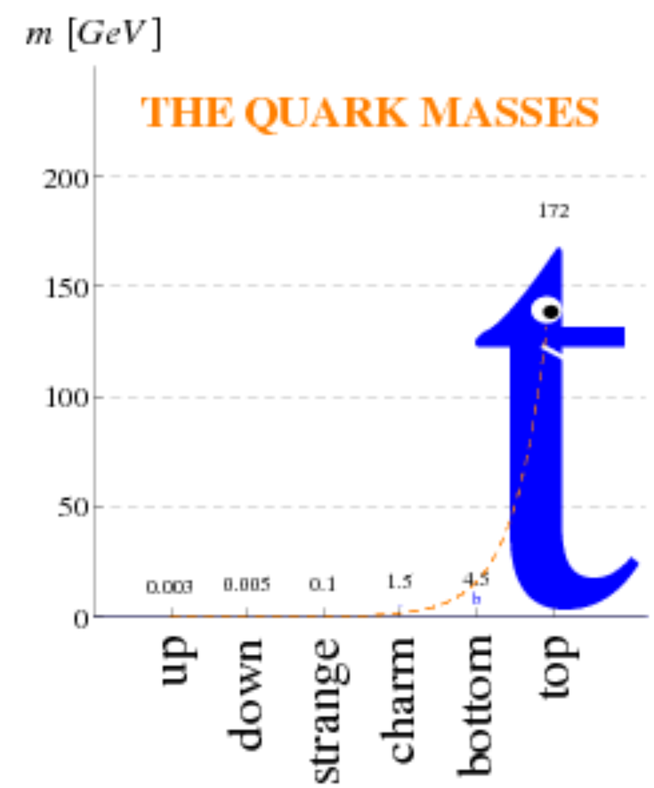
Why some are much heavier than others?

From X-bit Labs Community

Introduction

SM

- Large mass hierarchies
- Highly non-universal flavor structure
- Why?



From arXiv:1112.6387 [hep-ph]

Introduction

SM

- Flavor Changing Neutral Currents
- No tree-level, loop level suppressed by loop factors and small CKM matrix elements (GIM)
- Seems to be good w/ experimental data

$\mathrm{BR}(t \to Hc) = 10^{-15}$

well below

ATLAS: $\mathrm{BR}(t \to Hc) < 0.46\%$

CMS: $\mathrm{BR}(t \to Hc) < 0.56\%$

Introduction

But SM does not address flavor problems:

- Why 3 generations?
- **Why fermion mass hierarchy? Why top is so heavy?**

This talk

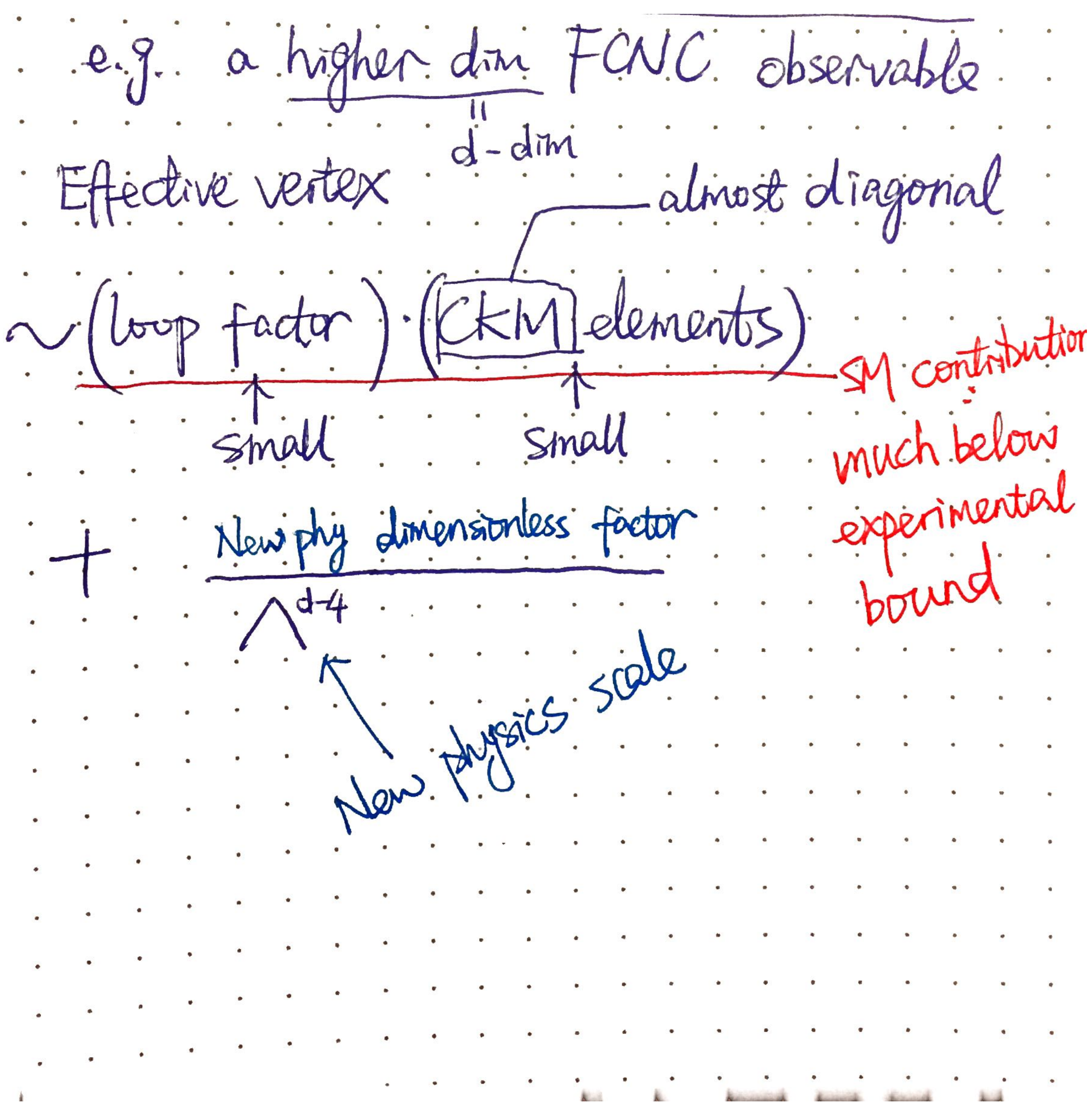
- etc

Introduction

- Other problems SM does not explain: DM, DE, EW hierarchy, gravity, etc
- —> New physics needed
- How to probe new physics?
- One important probe: FCNC

Introduction

- Low energy FCNC observables are highly sensitive to new physics due to the absence of SM tree-level contribution



Sizable but experimental -compatible FCNC: probe and strong signature of new physics

Sp. tree-level FCNC

↓

This talk

Introduction

Scenarios to address those flavor problems:

- Warped extra dimension
- **Extended EW sector** **Focus**
- etc

Introduction

Extended EW sector

- + **scalars** ⟶ e.g. 2HDM, invisible axion DFSZ/KSVZ, Georgi-Machacek model, etc

- + scalars and gauge bosons

Introduction

Extended EW sector

- + scalars
- + **scalars and gauge bosons**

↓

e.g. extra gauge bosons may be from a larger group,
or from higher dim

Introduction

Family models

Different generations have different charges.

- General case: 1 SU(2) gauge group for 1 generation

Li & Ma, 81

- A less complicated case: the first 2 generations charged under 1 SU(2), the 3rd generation charged under the other SU(2)

Our focus

Chivukula, Simmons & Terning, 94;
Chivukula & Simmons, 02;
Muller & Nandi, 96;
Malkawi, Tait & Yuan, 96;
Batra, Delgado & Kaplan & Tait, 04;
Chivukula, He, Howard & Simmons, 04

Chiang, Deshpande, He & Jiang, 09;
Chiang, He & Valencia, 16

Setup

- EW gauge sector:

$$SU(2)_1 \times SU(2)_2 \times U(1)_Y$$

- Scalar sector: **1 doublet under each SU(2)**, and **1 bifundamental simultaneously under both SU(2)**

Setup

SM fermions

No new fermions
No anomaly introduced

	$SU(2)_1$	$SU(2)_2$	$U(1)_Y$
Q_{aL}	$\mathbf{2}$	$\mathbf{1}$	$\frac{1}{6}$
Q_{3L}	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{6}$
U_{iR}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{3}$
D_{iR}	$\mathbf{1}$	$\mathbf{1}$	$-\frac{1}{3}$
L_{aL}	$\mathbf{2}$	$\mathbf{1}$	$-\frac{1}{2}$
L_{3L}	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
E_{iR}	$\mathbf{1}$	$\mathbf{1}$	-1
Φ_1	$\mathbf{2}$	$\mathbf{1}$	$\frac{1}{2}$
Φ_2	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$
η	$\mathbf{2}$	$\mathbf{2}$	X

Table 1. EW Spectrum

Flavor indices:

$a = 1,\ 2,$

$i = 1,\ 2,\ 3.$

Symmetry breaking

$$\mathcal{L}_k = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 + \mathrm{Tr}(|D_\mu \eta|^2) + \bar{\psi}\, i\gamma_\mu D^\mu \psi,$$

$$SU(2)_1 \times SU(2)_2 \times U(1)_Y$$

$$< \eta >= \frac{1}{\sqrt{2}} \begin{pmatrix} u & 0 \\ 0 & u \end{pmatrix}$$

↓

$$SU(2)_L \times U(1)_Y$$

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}$$

↓

SM EW group

$$Q = Y + T_1^3 + T_2^3$$

	$SU(2)_1$	$SU(2)_2$	$U(1)_Y$
Q_{aL}	**2**	**1**	$\frac{1}{6}$
Q_{3L}	**1**	**2**	$\frac{1}{6}$
U_{iR}	**1**	**1**	$\frac{2}{3}$
D_{iR}	**1**	**1**	$-\frac{1}{3}$
L_{aL}	**2**	**1**	$-\frac{1}{2}$
L_{3L}	**1**	**2**	$-\frac{1}{2}$
E_{iR}	**1**	**1**	-1
Φ_1	**2**	**1**	$\frac{1}{2}$
Φ_2	**1**	**2**	$\frac{1}{2}$
η	**2**	**2**	X

Table 1. EW Spectrum

Scalar masses

$$V = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \frac{1}{4} \sum_{a,b=1}^{2} \lambda_{ij} \Phi_a^\dagger \Phi_a \Phi_b^\dagger \Phi_b$$
$$+ M^2 \mathrm{Tr}(\eta^\dagger \eta) + \mathrm{Tr}(\tilde{M} \tilde{\eta} \eta^\dagger + h.c.) + \frac{1}{4} \lambda' [\mathrm{Tr}(\eta^\dagger \eta)]^2 + \frac{1}{4} \left(\tilde{\lambda}' [\mathrm{Tr}(\tilde{\eta} \eta^\dagger)]^2 + h.c. \right)$$
$$+ \mathrm{Tr}(\eta^\dagger \eta) \mathrm{Tr}(\tilde{f} \tilde{\eta} \eta^\dagger + h.c.) + \frac{1}{4} \lambda'' \mathrm{Tr}([\eta \eta^\dagger]^2) + \frac{1}{4} \tilde{\lambda}'' \mathrm{Tr}([\eta \tilde{\eta}^\dagger]^2)$$
$$+ \frac{1}{2} \mathrm{Tr}(\eta^\dagger \eta) \sum_{a=1}^{2} f_a (\Phi_a^\dagger \Phi_a) + p_1 \Phi_1^\dagger \eta \eta^\dagger \Phi_1 + p_2 \Phi_2^\dagger \eta^\dagger \eta \Phi_2$$
$$+ \tilde{p}_1 \Phi_1^\dagger \tilde{\eta} \eta^\dagger \Phi_1 + \tilde{p}_2 \Phi_2^\dagger \tilde{\eta}^\dagger \eta \Phi_2 + (t' \Phi_1^\dagger \eta \Phi_2 + h.c.) + (\tilde{t} \Phi_1^\dagger \tilde{\eta} \Phi_2 + h.c.),$$

$$\tilde{\eta} = \sigma_2 \eta^\star \sigma_2$$

Consider all coefficients real

$$<\eta>= \frac{1}{\sqrt{2}} \begin{pmatrix} u & 0 \\ 0 & u \end{pmatrix}$$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + \mathrm{Re}\phi_1^0 + i \mathrm{Im}\phi_1^0}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 e^{i\xi} + \mathrm{Re}\phi_2^0 + i \mathrm{Im}\phi_2^0}{\sqrt{2}} \end{pmatrix}$$

Scalar masses

- To extract Goldstones eaten by gauge bosons, we perform an SU(2) rotation

$$\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} = \mathcal{O}\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix},$$

$$\Phi_1 = \begin{pmatrix}\phi_1^+\\ \frac{v_1+\mathrm{Re}\phi_1^0+i\mathrm{Im}\phi_1^0}{\sqrt{2}}\end{pmatrix}, \quad \Phi_2 = \begin{pmatrix}\phi_2^+\\ \frac{v_2e^{i\xi}+\mathrm{Re}\phi_2^0+i\mathrm{Im}\phi_2^0}{\sqrt{2}}\end{pmatrix}$$

$$\downarrow$$

$$c_\beta = \cos\beta = \frac{v_1}{v}, \quad s_\beta = \sin\beta = \frac{v_2}{v},$$

$$\mathcal{O} = \begin{pmatrix}c_\beta & s_\beta\\ -s_\beta & c_\beta\end{pmatrix},$$

$$\Psi_1 = \begin{pmatrix}G^+\\ \frac{v+h+iG^0}{\sqrt{2}}\end{pmatrix}, \quad \Psi_2 = \begin{pmatrix}H^+\\ \frac{H^0+iA^0}{\sqrt{2}}\end{pmatrix}$$

charged complex Higgs H^+

neutral Higgs H^0 neutral CP-odd Higgs A^0

direction h to get a VEV v

assumed $t < 0$ (correspondingly $\cos\xi = 1$)

3 real Goldstone modes (G^0, G^+)

Scalar masses

- In the new scalar basis

$$M^2_{G^+,\,H^+} = M^2_{G^0,\,A^0} = \mathcal{O} M^2_{\phi^+} \mathcal{O}^T = \begin{pmatrix} 0 & 0 \\ 0 & \frac{|t|u}{s_\beta c_\beta} \end{pmatrix}$$

$M^2_{h,\,H^0}$ Off-diagonal entries

$$-\frac{\lambda_1}{2} v^2 c^3_\beta s_\beta + \frac{\lambda_2}{2} v^2 s^3_\beta c_\beta + \frac{\lambda_{12}}{2} v^2 c_\beta s_\beta c_{2\beta}$$

In the limit: $\lambda_1 = \lambda_2 = \lambda_{12}$ $\quad M^2_{h,\,H^0} = \begin{pmatrix} \frac{\lambda v^2}{2} & 0 \\ 0 & -\frac{tu}{s_\beta c_\beta} \end{pmatrix}$ No h-H mixing

Yukawa interactions

Quark Yukawa

$$\mathcal{L}_Y^Q = f_{ia}^u \bar{U}_{iR} \tilde{\Phi}_1^\dagger Q_{aL} + g_{i3}^u \bar{U}_{iR} \tilde{\Phi}_2^\dagger Q_{3L} + f_{ia}^d \bar{D}_{iR} \Phi_1^\dagger Q_{aL} + g_{i2}^d \bar{D}_{iR} \Phi_2^\dagger Q_{3L} + h.c.,$$

$$\tilde{\Phi} = i\sigma_2 \Phi^\star$$

can be re-written as

$$\begin{aligned}\mathcal{L}_Y^Q = & -\bar{U}_R \hat{M}^u U_L (1 + \frac{h}{v}) - \bar{D}_R \hat{M}^d D_L (1 + \frac{h}{v}) \\ & - \bar{U}_R \lambda^u U_L (H^0 + iA^0) - \bar{D}_R \lambda^d D_L (H^0 - iA^0) \\ & + \sqrt{2} \bar{U}_R \lambda^u S_U^\dagger T_D D_L H^+ - \sqrt{2} \bar{D}_R \lambda^d S_D^\dagger T_U U_L H^- + h.c.,\end{aligned}$$

Tree-level FCNC

$$V_{KM} = T_U^\dagger T_D$$

$$S_U^\dagger M^u T_U = \mathrm{diag}\{m_u, m_c, m_t\} \equiv \hat{M}^u, \quad S_D^\dagger M^d T_D = \mathrm{diag}\{m_d, m_s, m_b\} \equiv \hat{M}^d$$

λ^u λ^d off-diagonal matrices

Yukawa interactions

Quark Yukawa

Quark mass

$$\mathcal{L}_Y^Q = -\bar{U}_R \hat{M}^u U_L(1+\frac{h}{v}) - \bar{D}_R \hat{M}^d D_L(1+\frac{h}{v})$$
$$- \bar{U}_R \lambda^u U_L(H^0 + iA^0) - \bar{D}_R \lambda^d D_L(H^0 - iA^0)$$
$$+ \sqrt{2}\bar{U}_R \lambda^u S_U^\dagger T_D D_L H^+ - \sqrt{2}\bar{D}_R \lambda^d S_D^\dagger T_U U_L H^- + h.c.,$$

λ^u λ^d

off-diagonal

$$S_U^\dagger M^u T_U = \mathrm{diag}\{m_u, m_c, m_t\} \equiv \hat{M}^u, \quad S_D^\dagger M^d T_D = \mathrm{diag}\{m_d, m_s, m_b\} \equiv \hat{M}^d \qquad V_{KM} = T_U^\dagger T_D$$

$$\lambda_1^u = -\begin{pmatrix} f_{11}^u & f_{12}^u & 0 \\ f_{21}^u & f_{22}^u & 0 \\ f_{31}^u & f_{32}^u & 0 \end{pmatrix}, \quad \lambda_2^u = -\begin{pmatrix} 0 & 0 & g_{13}^u \\ 0 & 0 & g_{23}^u \\ 0 & 0 & g_{33}^u \end{pmatrix}, \quad M^u = \frac{v}{\sqrt{2}}(c_\beta \lambda_1^u + s_\beta \lambda_2^u)$$

Special case $S_U = T_U = S_D = 1$

3rd generation: Top mass ~ s_β

1st, 2nd generation: u, c masses ~ c_β

Large top mass: large beta, or large VEV v2 compared to v1

Gauge interactions

$$|D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 + \mathrm{Tr}(|D_\mu \eta|^2)$$

$$iD^\mu \Phi_1 = (i\partial^\mu + g_1 W_1^\mu + g' Y_{\Phi_1} B^\mu)\Phi_1,$$
$$iD^\mu \Phi_2 = (i\partial^\mu + g_2 W_2^\mu + g' Y_{\Phi_2} B^\mu)\Phi_2,$$
$$iD^\mu \eta = (i\partial^\mu + g_1 W_1^\mu - g_2 W_2^\mu)\eta,$$

$$< \eta >= \frac{1}{\sqrt{2}} \begin{pmatrix} u & 0 \\ 0 & u \end{pmatrix}$$

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}$$

In the final basis:

massless photon, light W boson, light Z boson, heavy W boson, heavy Z boson

Masses of massive gauge bosons in terms of leading term + corrections characterized by new physics parameter $\epsilon^2 = \frac{v^2}{u^2}$

Gauge interactions

Interactions with fermions

$$\mathcal{L}_k \ni \mathcal{L}_{neutral} = \bar{\psi}\gamma_\mu(g'BY + g_1 W_1^3 T_1^3 + g_2 W_2^3 T_2^3)^\mu \psi$$

Due to the light and heavy Z boson mixing, there exist **tree-level FCNCs**

$$\left(-\frac{g}{2c_E s_E} Z_h^\mu + \frac{\epsilon^2 g c_E^2 Z_l^\mu}{4c_W}\right)\left(\bar{U}_L T_U^\dagger \gamma_\mu \Delta T_U U_L - \bar{D}_L T_D^\dagger \gamma_\mu \Delta T_D D_L\right)$$

$$\Delta = \mathrm{diag}\,(0,\, 0,\, 1)$$

Small for lighter Z

$$s_E = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad c_E = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}.$$

EW precision tests

- Assume: $e^{SM} = e, \quad G_F^{SM} = G_F, \quad m_Z^{SM}(\text{also denoted as } m_Z) = m_{Z_l}$

- Express quantities in the model as corresponding SM values plus corrections

$$v = v_0\left(1 + \frac{\epsilon^2}{4}\right) \qquad x = x_0\left[1 + \frac{1-x_0}{1-2x_0} f_E \epsilon^2\right] \qquad g = g_0\left[1 - \frac{1}{2}\frac{1-x_0}{1-2x_0} f_E \epsilon^2\right]$$

$$f_E = \frac{1 - c_E^4}{2}$$

s_W^2

- Found: custodial sym approx preserved

$$\rho = \frac{m_{W_l}^2}{m_{Z_L}^2 c_W^2} = 1 + \frac{\epsilon^4 s_W^2 c_E^2 s_E^2 (c_E^2 s_\beta^2 - s_E^2 c_\beta^2)^2}{4 c_W^2} + \mathcal{O}(\epsilon^6)$$

EW precision tests

Universality constraints

non-universality in the charged lepton decays $\tau \to \mu\bar{\nu}_\mu\nu_\tau$ and $\tau \to e\bar{\nu}_e\nu_\tau$,

violation of universality between $\tau \to \mu\bar{\nu}_\mu\nu_\tau(\tau \to e\bar{\nu}_e\nu_\tau)$ and $\mu \to e\bar{\nu}_e\mu$

$$\frac{G_{\tau e}^2}{G_F^2} = 1.0029 \pm 0.0046, \quad \frac{G_{\tau\mu}^2}{G_F^2} = 0.981 \pm 0.018.$$

$$\frac{G_{\tau e}^2}{G_F^2} = \frac{G_{\tau\mu}^2}{G_F^2} = \left(1 - \frac{\epsilon^2}{2} + \mathcal{O}(\epsilon^4)\right)^2 = 1 - \epsilon^2 + \mathcal{O}(\epsilon^4).$$

EW precision tests

- Fit w/ data: EWP observables with universality constrains

- Found: best fit favors a small ϵ

  insensitive to the value of the mixing angle c_E

EW precision tests

insensitive to the value of the mixing angle c_E

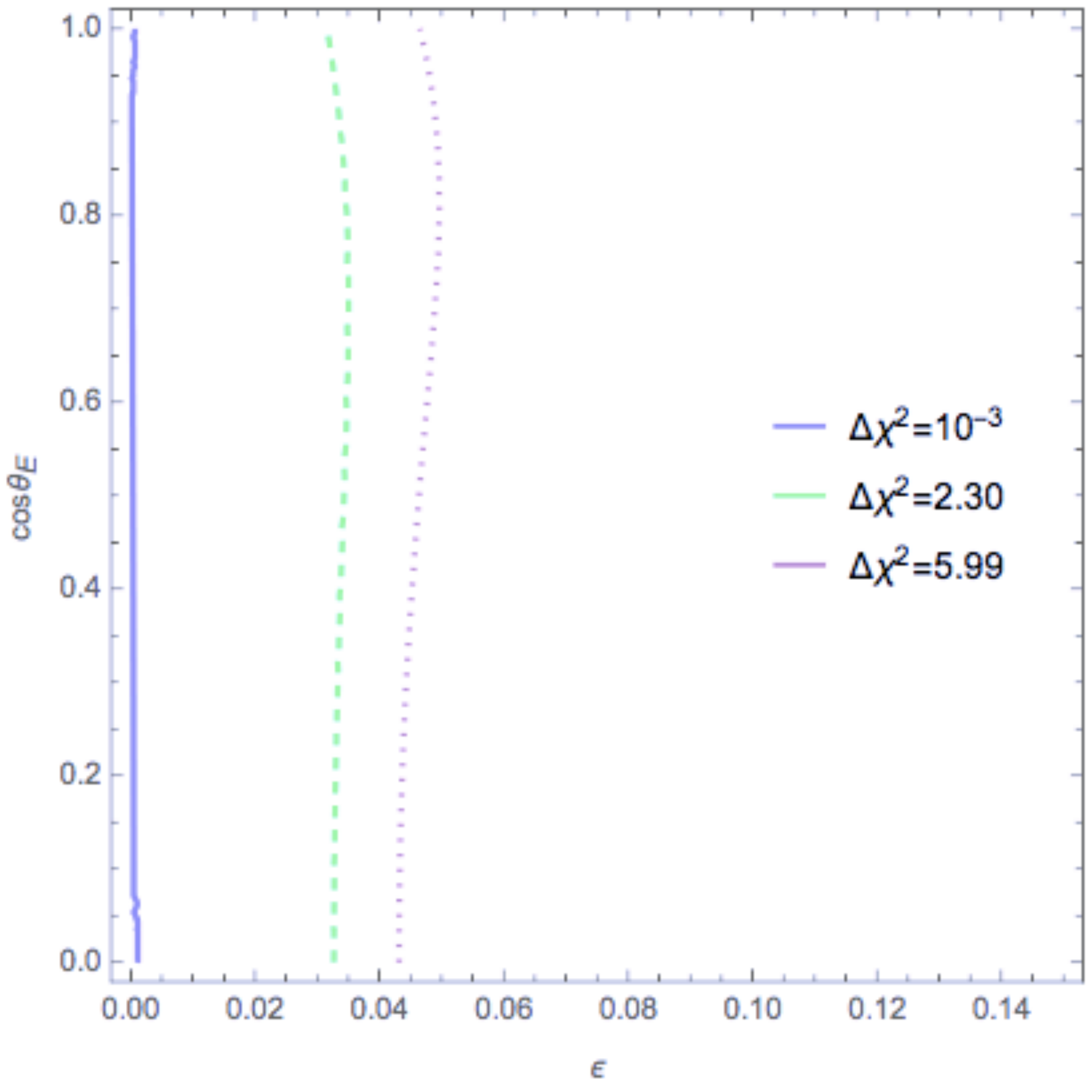


Figure 1. Contours of the χ^2 fit based on the EW observables and the non-universality constraints. The blue, green and purple curves indicate the χ^2 values away from the mean by 10^{-3}, 2.30 (1σ standard deviation) and 5.99 (2σ standard derivation), respectively.

$\Delta\chi^2$	ϵ	$m^{min}_{Z_h/W_l}$ GeV
0.001	0.001	226132
2.30 (1σ)	0.035	6460.28
5.99 (2σ)	0.049	4614.04

Min heavy gauge boson masses at

$c_E^2 = \frac{1}{2}$

Matching with Higgs Data

- Relevant tree-level Higgs couplings include corrections due to **corrections to the VEV v, to the light W boson mass and to the fermion masses**

$$
\begin{aligned}
g_{hff} &= \frac{m_f}{v}, \\
g_{hZZ} &= \frac{2m_{Z_l}^2}{v} \\
g_{hWW} &= \frac{2m_{W_l}^2}{v} \\
g_{hhZZ} &= \frac{m_{Z_l}^2}{v^2}, \\
g_{hhWW} &= \frac{m_{W_l}^2}{v^2}.
\end{aligned}
$$

- SM Higgs decay: bb, WW, gg, tautau, cc, ZZ, diphoton, Z gamma, mumu

Matching with Higgs data

- Parameters characterized new physics: **quantities in terms of SM values plus powers of those parameters**

$\epsilon^2 = \frac{v^2}{u^2}$ $\qquad c_\beta$. (expect beta is large but not too large)

$$\mu_i^f = \frac{\sigma_i \cdot \mathrm{B}^f}{(\sigma_i)_{\mathrm{SM}} \cdot (\mathrm{B}^f)_{\mathrm{SM}}} = \mu_i \cdot \mu^f. \qquad i = gg\mathrm{F}, \mathrm{VBF}, \qquad f = ZZ, WW, \gamma\gamma, \tau\tau,$$

- Fit with Higgs signal strength

Matching with Higgs data

- 3 cases:
- 1) w/o h to charged Higgs coupling, w/o h-H mixing

mainly in diphoton and Z gamma loops

- 2) w/ h to charged Higgs coupling, w/o h-H mixing
- 3) w/ h to charged Higgs coupling, w/ h-H mixing (in progress)

Matching with Higgs data

- Fit in 3 cases:
- 1) w/o h to charged Higgs coupling, w/o h-H mixing
- 2) w/ h to charged Higgs coupling, w/o h-H mixing
- 1) and 2) consistent with $\epsilon^2 = \frac{v^2}{u^2}$ EWP fit, c_β. approaching 0.1
- In 2), h to H^+ coupling mostly constrained from diphoton signal strength

Doing/To do

- Higgs signal strength fit in 3): w/ h to charged Higgs coupling, w/ h-H mixing
- Tree-level FCNC constraints: h-H mixing, heavy Higgs H, heavy Z

Summary

- We analyze a family model with $SU(2)_1 \times SU(2)_2 \times U(1)_Y$ EW gauge group.
- Fermion mass **hierarchy** is understood as a **hierarchy btw VEVs of scalars charged under different SU(2) gauge group**.
- The model has **tree-level FCNCs**. Those being compatible with experiment are signatures of new physics.
- EWP, Higgs data and FCNC constrain the parameter space.

Thank you!

Backup

Gauge interactions

Interactions with fermions

$$\mathcal{L}_k \ni \mathcal{L}_{charged} = \bar{Q}_L(g_1 W_1^\mu + g_2 W_2^\mu)\gamma_\mu Q_L$$

$$\begin{aligned}\mathcal{L}_{charged} \approx\ & \frac{g}{\sqrt{2}} W_h^{+\mu}\left[-\frac{\epsilon^2 c_E^3 s_E}{2}\bar{U}_L\gamma_\mu V_{KM} D_L - \bar{U}_L\gamma_\mu T_U^\dagger N T_D D_L\right] \\ & + \frac{g}{\sqrt{2}} W_l^{+\mu}\left[\bar{U}_L\gamma_\mu V_{KM} D_L - \frac{\epsilon^2 c_E^3 s_E}{2}\bar{U}_L\gamma_\mu T_U^\dagger N T_D D_L\right] \\ & + h.c.\end{aligned}$$

$$s_E = \frac{g_1}{\sqrt{g_1^2+g_2^2}}, \quad c_E = \frac{g_2}{\sqrt{g_1^2+g_2^2}}.$$