The Physics of the Cosmological Collider
-- Non-Gaussianity in the Post-Planck Era

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Collaboration with
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X. Chen & M. H. Namjoo 1509.03930, 1601.06228, 1608.01299
DAWN OF TIME

tiny fraction of a second

inflation

380,000 years

13.7 billion years
Review of Non-G before Planck
Inflationary \((a \sim e^{Ht})\) correlation functions

\[
\langle \zeta_{k_1} \zeta_{k_2} \cdots \zeta_{k_n} \rangle
\]

\(\zeta\): curvature fluctuation on uniform density slices

\[
\zeta \leftrightarrow \frac{\delta T}{T} \text{ (CMB)}, \quad \zeta \leftrightarrow \frac{\delta \rho}{\rho} \text{ (LSS)}
\]
How to calculate \( <\zeta_{k_1} \zeta_{k_2} \ldots \zeta_{k_n}> \) ?

in-in formalism

\[
\langle \Omega | Q(\tau) | \Omega \rangle = \langle 0 | \left[ \mathcal{T} e^{i \int_{\tau_0}^{\tau} H_I(\tau') d\tau'} \right] Q^I(\tau) \left[ \mathcal{T} e^{-i \int_{\tau_0}^{\tau} H_I(\tau') d\tau'} \right] | 0 \rangle
\]

expansion order by order ~ “Feynman” diagrams
Inflationary \((a \sim e^{Ht})\) correlation functions

\[ \langle \zeta_{k_1} \zeta_{k_2} \cdots \zeta_{k_n} \rangle \]

\(\zeta\): curvature fluctuation on uniform density slices

\[ \zeta \Leftrightarrow \frac{\delta T}{T} \text{ (CMB)}, \quad \zeta \Leftrightarrow \frac{\delta \rho}{\rho} \text{ (LSS)} \]

\[ \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^7 \delta^3 (k_1 + k_2 + k_3) \frac{P^2_\zeta}{k_1^2 k_2^2 k_3^2} \mathcal{F}(k_1/k_3, k_2/k_3) \]

\[ P_\zeta \sim 2\text{pt} \quad \mathcal{F} \left\{ \begin{array}{l}
\text{size of non-G: } \sim f_{NL} \\
\text{shape: shape of non-G}
\end{array} \right. \]
Local shape non-G:
Example of local non-G: the curvaton scenario:

Sasaki, Valiviita, Wands 2006

assuming same decay product
entropy perturbation becomes adiabatic

curvaton density catches up
perturbation starts to gravitate

curvaton has entropy perturbation
inflaton perturbation assumed small

\[ f_{NL} = \frac{5}{4r} \left( 1 + \frac{g g''}{g'^2} \right) - \frac{5}{3} - \frac{5r}{6} \]

\[ r = \frac{3\Omega_{\chi,\text{dec}}}{4 - \Omega_{\chi,\text{dec}}} = \left. \frac{3\rho_\chi}{3\rho_\chi + 4\rho_r} \right|_{t_{\text{dec}}} \]
Equilateral and orthogonal shapes of non-G:
Example of equilateral non-G: modified sound speed

\[ \mathcal{L} = P(\dot{\phi}, X) \]

\[ S_2 = \int dt d^3 x \left[ \alpha^3 \frac{\epsilon}{c_s^2} \dot{\zeta}^2 - a_\epsilon (\partial \zeta)^2 \right] \]

\[ c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}} \]

\[ \mathcal{F} = \left( \frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma} \right) \frac{3k_1k_2k_3}{2(k_1 + k_2 + k_3)^3} \]
\[ + \left( \frac{1}{c_s^2} - 1 \right) \left[ \frac{k_1^2k_2^3 + k_1^2k_3^3 + k_2^2k_3^3 + k_2^2k_1^3 + k_3^2k_1^3 + k_3^2k_2^3}{k_1k_2k_3(k_1 + k_2 + k_3)} \right] + \frac{k_1^2k_2^3 + k_1^2k_3^3 + k_2^2k_3^3 + k_2^2k_1^3 + k_3^2k_1^3 + k_3^2k_2^3}{2k_1k_2k_3(k_1 + k_2 + k_3)^2} \]
\[ + \frac{k_1^3 + k_2^3 + k_3^3}{8k_1k_2k_3} \]

Of order \( f_{NL} \)

Chen, Huang, Kachru, Shiu 2006
Many many many similar stories 2000s – 2013.
Planck 2013 results. XXIV. Constraints on primordial non-Gaussianity

Planck Collaboration

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Received: 22 March 2013
Accepted: 16 December 2013

Abstract

The Planck nominal mission cosmic microwave background (CMB) maps yield unprecedented constraints on primordial non-Gaussianity (NG). Using three optimal bispectrum estimators, separable template-fitting (KSW), binned, and modal, we obtain consistent values for the primordial local, equilateral, and orthogonal bispectrum amplitudes, quoting as our final result $f_{NL}^{\text{local}} = 2.7 \pm 5.8$, $f_{NL}^{\text{equil}} = -42 \pm 75$, and $f_{NL}^{\text{orth}} = -25 \pm 39$ (68% CL statistical). Non-Gaussianity is detected in the data; using skew-$C_l$ statistic, we find a nonzero bispectrum from residual point sources, and the integrated-Sachs-Wolfe-lensing bispectrum at a level expected in the $\Lambda$CDM scenario. The results are based on comprehensive cross-validation of these estimators on Gaussian and non-Gaussian simulations, are stable across component separation techniques, pass an extensive suite of tests, and are confirmed by skew-$C_l$, wavelet bispectrum and Minkowski functional estimators. Beyond estimates of individual shape amplitudes, we present robust, independent, three-dimensional reconstructions of the Planck CMB bispectrum and thus derive constraints on early-Universe scenarios that generate primordial NG, including general single-field models of inflation, excited initial states (non-Bunch-Davies vacua), and directionally-dependent vector models. We provide an initial survey of dependence-dependent feature and resonance models. These results bound both general single-field and multi-field model parameter ranges, such as the speed of sound, $c_s \geq 0.02$ (95% CL), in an effective field theory parametrization, and the curvaton decay fraction $r_D \geq 0.15$ (95% CL). The Planck data significantly limit the viable parameter space for ekpyrotic/cyclic scenarios. The amplitude of the four-point function in the local model $f_{NL} \leq 2800$ (95% CL). Taken together, these constraints represent the highest precision tests to date of physical mechanisms for the origin of cosmic structure.

\[ f_{NL}^{\text{local}} = 2.7 \pm 5.8, \quad f_{NL}^{\text{equil}} = -42 \pm 75, \quad \text{and} \quad f_{NL}^{\text{orth}} = -25 \pm 39 \quad (68\% \text{ CL statistical}) \]
Planck 2015 results
XVII. Constraints on primordial non-Gaussianity

Planck Collaboration
P. A. R. Ade97, N. Aghanim63, M. Arnaud79, F. Arroja71,85, M. Ashdown75,6, J. Aumont63, C. Baccigalupi95, M. Ballardini51,53,34, A. J. Banday109,10, R. B. Barreiro70, N. Bartolo33,71*, S. Basak95, E. Battaner110,111, K. Benabed64,108, A. Benoit61, A. Benoit-Lévy26,64,108, L.-P. Bernard109,10, M. Bersanelli37,52, P. Bialowas89,10,95, 1 1, A. Bock72,12 A

Received: 6 February 2015
Accepted: 27 January 2016

Abstract
The Planck full mission cosmic microwave background (CMB) temperature and E-mode polarization maps are analysed to obtain constraints on primordial non-Gaussianity (NG). Using three classes of optimal bispectrum estimators – separable template-fitting (KSW), binned, and modal – we obtain consistent values for the primordial local, equilateral, and orthogonal bispectrum amplitudes, quoting as our final result from temperature alone \( f_{\text{local}}^{\text{NL}} = 2.5 \pm 5.7 \), \( f_{\text{equil}}^{\text{NL}} = -16 \pm 70 \), and \( f_{\text{ortho}}^{\text{NL}} = -34 \pm 32 \) (68% CL, statistical). Combining temperature and polarization data we obtain \( f_{\text{local}}^{\text{NL}} = 0.8 \pm 5.0 \), \( f_{\text{equil}}^{\text{NL}} = -4 \pm 43 \), and \( f_{\text{ortho}}^{\text{NL}} = -26 \pm 21 \) (68% CL, statistical). The result based on comprehensive cross-validation of these estimators on Gaussian and non-Gaussian simulations, are stable across component separation techniques, pass an extensive suite of tests, and are consistent with estimators based on measuring the Minkowski functionals of the CMB. The effect of time-domain de-glitching systematics on the bispectrum is negligible. In spite of these test outcomes we conservatively label the results including polarization data as preliminary, owing to a known mismatch in the noise model in simulations and the data. Beyond estimates of individual shape amplitudes, we present model-independent, three-dimensional reconstructions of the foreground-free CMB bispectrum and derive constraints on early universe scenarios that generate primordial NG, including general single-field models of inflation, axion inflation, initial modifications, models producing parity-violating tensor bispectra, and directionally dependent vector models. We present a wide survey of scale-dependent features in resonance models, accounting for the “look elsewhere” effect in estimating the statistical significance of features. We also look for isocurvature NG, and find no signal, but obtain constraints that improve significantly with the inclusion of polarization. The primordial trispectrum amplitude in the local model is constrained to be \( g_{\text{local}}^{\text{NL}} = \left( \times 10^{-4} \right) \) (68% CL statistical), and we perform an analysis of trispectrum shapes beyond the local case. The global picture that emerges is one of consistency with the premises of the \( \Lambda \)CDM cosmology, namely that the structure we observe today was sourced by adiabatic, passive, Gaussian, and primordial seed perturbations.

\[ f_{\text{local}}^{\text{NL}} = 0.8 \pm 5.0, \quad f_{\text{equil}}^{\text{NL}} = -4 \pm 43, \quad \text{and} \quad f_{\text{ortho}}^{\text{NL}} = -26 \pm 21 \quad (68\% \text{ CL, statistical}) \]
COBE (1990s) $\Delta f_{NL}\sim 2000$

WMAP 1yr (2003) $\Delta f_{NL}\sim 100$
7yr (2010) $\Delta f_{NL}\sim 20$

Planck (2013) $\Delta f_{NL}\sim 5$

LSS $\Delta f_{NL}\sim 0.5$

21cm: $\Delta f_{NL} \sim 10^{-3}$
CMB distortion: $\Delta f_{NL} \sim 10^{-3}$
In ~ 5-10 years $\Delta f_{NL} \sim 0.5$ (e.g. SPHEREx) (And $\Delta f_{NL} \sim 10^{-3}$ in the very distant future.) What is the implication if $|f_{NL}| < 1$?
In ~ 5-10 years $\Delta f_{NL} \sim 0.5$ (e.g. SPHEREx)

What is the implication if $|f_{NL}| < 1$?

- Local: Curvaton will be very unlikely.
- Equilateral: $c_s \sim 1$, up to small corrections.
In ~ 5-10 years $\Delta f_{NL} \sim 0.5$ (e.g. SPHEREx)

What is the implication if $|f_{NL}| < 1$?
- Local: Curvaton will be very unlikely.
- Equilateral: $c_s \sim 1$, up to small corrections.

What is the motivation for future study?
(Before addressing motivation of $|f_{NL}| < 1 \ldots$)

History of particle physics experiments:
- Early stage: studying external particle
  - $\alpha$ particle scattering
  - $\mu$ from cosmic rays
  - deep inelastic scattering
  - …
- Nowadays: study internal particle
  - Higgs
  - BSM
  - …
(Before addressing motivation of $|f_{NL}| < 1 \ldots$)

external particle $\rightarrow$ internal particle

Cosmological non-G: is there a similarity?

Curvaton, $c_s, \ldots$ : external particle ($\zeta$)
If seen: pin down inflation model.

What about internal particles?
Internal particles:
Quasi-single field inflation

Xingang Chen & YW 2009

m ~ H fields
Example: quasi-single field inflation
\[ S_m = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} (\tilde{R} + \sigma)^2 g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V_{sr}(\theta) - V(\sigma) \right] \]

\[ \delta \mathcal{L}_2 = 2a^3 R \dot{\theta}_0 \delta \sigma \delta \theta , \quad \mathcal{L}_3 = -\frac{1}{6} a^3 V''' \delta \sigma^3 \]
\[ S_m = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} (\tilde{R} + \sigma)^2 g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V_{sr}(\theta) - V(\sigma) \right] \]

\[ \delta \mathcal{L}_2 = 2a^3 R\dot{\theta}_0 \delta \sigma \delta \theta , \quad \mathcal{L}_3 = -\frac{1}{6} a^3 V''' \delta \sigma^3 \]
\[
- 12c_2^3 c_3 u_{p_1}^*(0) u_{p_2}(0) u_{p_3}(0)
\times \text{Re} \left[ \int_{-\infty}^{0} d\tilde{\tau}_1 \ a^3(\tilde{\tau}_1) v_{p_1}^*(\tilde{\tau}_1) u_{p_1}'(\tilde{\tau}_1) \int_{-\infty}^{\tilde{\tau}_1} d\tilde{\tau}_2 \ a^4(\tilde{\tau}_2) v_{p_1}(\tilde{\tau}_2) v_{p_2}(\tilde{\tau}_2) v_{p_3}(\tilde{\tau}_2) \right]
\times \int_{-\infty}^{0} d\tau_1 \ a^3(\tau_1) v_{p_2}^*(\tau_1) u_{p_2}'(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 \ a^3(\tau_2) v_{p_3}^*(\tau_2) u_{p_3}'(\tau_2) \right]
\times (2\pi)^3 \delta^3(\sum_i p_i) + 9 \text{ other similar terms}
+ 5 \text{ permutations of } p_i.
\]
quasi-single field \( (m = 3H/2) \)

quasi-single field \( (m = 1.414H) \)

quasi-single field \( (m = H) \)

quasi-single field \( (m = H/2) \)
\[ \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \]
Distant future 21cm forecast

\[ \mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}} \]

\[ v = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \]

\( \mu = 0.7 \) (purple), \( \mu = 1.0 \) (blue), \( \mu = 3.0 \) (red)

\( v = 0.8 \) (purple), \( v = 1.0 \) (blue), \( v = 1.4 \) (red)

Meerburg, Munchmeyer, Munoz, Chen 2016
Cosmological double slit experiment

Arkani-Hamed, Maldacena 2015
Contributions to correlation functions:

<table>
<thead>
<tr>
<th>type</th>
<th>meaning</th>
<th>analytic in k?</th>
<th>integrate out?</th>
<th>suppression at large $\mu$</th>
<th>suppression at large $x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>local</td>
<td>vacuum correlation</td>
<td>Yes</td>
<td>Yes</td>
<td>$1/\mu^2$</td>
<td>vanish outside lightcone</td>
</tr>
<tr>
<td>non-local</td>
<td>thermal particle production</td>
<td>No</td>
<td>No</td>
<td>$e^{-\pi \mu}$</td>
<td>non-vanishing</td>
</tr>
</tbody>
</table>
massive $\rightarrow$ time dependent phase $e^{imt} \sim (-\tau)^{im/H}$

curvature mode $\sim e^{ik\tau}$, at resonance record the phase
\begin{align*}
\frac{\langle \zeta \zeta \zeta \rangle}{\langle \zeta \zeta \rangle_{\text{short}} \langle \zeta \zeta \rangle_{\text{long}}} & \sim \epsilon e^{-\pi \mu} |c(\mu)| \left[ e^{i\delta(\mu)} \left( \frac{k_{\text{long}}}{k_{\text{short}}} \right)^{3/2+i\mu} + e^{-i\delta(\mu)} \left( \frac{k_{\text{long}}}{k_{\text{short}}} \right)^{3/2-i\mu} \right] P_s(\cos \theta) \\
\mu &= \sqrt{\frac{m^2}{H^2}} - \frac{9}{4}
\end{align*}
Correlation between the density fluctuation and a clock
Observational consequence:

scale-independent

shape-dependent

oscillations on shape of non-Gaussianities
The Precision Era of Non-Gaussianity
<table>
<thead>
<tr>
<th>Era</th>
<th>Pre-Planck</th>
<th>Post-Planck</th>
</tr>
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<tr>
<td>Observable</td>
<td>CMB</td>
<td>LSS</td>
</tr>
<tr>
<td>NonG size</td>
<td>$f_{NL} &gt; O(1)$</td>
<td>$f_{NL} &lt; O(1)$</td>
</tr>
<tr>
<td>Physics</td>
<td>Curvaton, EFT, …</td>
<td>Massive states</td>
</tr>
<tr>
<td>Interest</td>
<td>External particles</td>
<td>Internal particles</td>
</tr>
<tr>
<td>Toolkit</td>
<td>In-in formalism</td>
<td>+ EdS, $O_{12}$, nEFT, …</td>
</tr>
</tbody>
</table>
Standard Model background of the

Cosmological Collider

X. Chen, YW, Z. Z. Xianyu
SM background contains two questions:

1. Mass spectrum of the SM particles
2. Their contributions to $\langle \zeta_{k_1} \zeta_{k_2} \ldots \zeta_{k_n} \rangle$
How to extract mass of SM particles?

Aren’t they known already?
For example, $M_h = 125\text{GeV}$?

During inflation, roughly:

$$h \sim T \sim H, \quad \lambda h^4 \supset \lambda\langle h^2\rangle h^2, \quad m_{\text{eff}}^2 \sim \lambda\langle h^2\rangle$$

Similarly for W, Z. However,

(curvature radius) $\sim T \sim H$,

thus flat space thermal field theory is not enough.
How to extract mass of SM particles?

We extract the SM mass spectrum from cosmological correlation functions, calculated using in-in formalism.
How to extract mass of SM particles?

Method 1:
calculate IR growth
+ DRG resummation

rolling speed
time dependence in the IR

\[ 2\text{pt} \sim (-\tau)^s \]

\[ s = \frac{2m^2}{3H^2} \] (from tree level)

s calculable for loop (and DRG)

Then mass obtained.
How to extract mass of SM particles?

Method 2: mass renormalization on a sphere

Example:

\[ S_{\text{toy}} = -\frac{1}{2} \int d^D x \sqrt{-g} \left[ (\partial_{\mu} \phi)^2 + (\partial_{\mu} \chi)^2 + M_{\phi}^2 \phi^2 + M_{\chi}^2 \chi^2 + \lambda \phi \chi^2 \right] \]

mass renormalization \( \sim \frac{\lambda^2 \mu_R^{4-D}}{2} \int d\Omega d\Omega' \phi(x) \phi(x') G_{\chi}(x, x')^2 \)
can set $\phi = \text{constant}$

$$
\frac{\lambda^2 \mu_R^{4-D}}{2} \int d\Omega d\Omega' \phi(x) \phi(x') G_{\chi}(x, x')^2
$$

$$
= \frac{\lambda^2 \mu_R^{4-D} H^{2D-4}}{2} \int d\Omega d\Omega' \sum_{\tilde{L}, \tilde{M}} \frac{1}{\lambda_L \lambda_M} Y_{\tilde{L}}(x) Y_{\tilde{L}}^*(x') Y_{\tilde{M}}^*(x) Y_{\tilde{M}}(x')
$$

$$
= \frac{\lambda^2 \mu_R^{4-D} H^{D-4}}{2} \int d\Omega \sum_{\tilde{L}} \frac{1}{\lambda_L^2} Y_{\tilde{L}}(x) Y_{\tilde{L}}^*(x)
$$

$$
= - \frac{\lambda^2 \mu_R^{4-D} H^{D-2}}{2} \frac{\partial}{\partial m^2} \int d\Omega \sum_{\tilde{L}} \frac{1}{\lambda_L} Y_{\tilde{L}}(x) Y_{\tilde{L}}^*(x)
$$

$$
= - \frac{\lambda^2 \mu_R^{4-D}}{2} [\partial_{m^2} G(x, x)]_{m^2 = M_{\chi}^2} \int d\Omega,
$$

$$
\lambda_L = (L + d/2 + \mu)(L + d/2 - \mu)
$$

$$
\mu = \sqrt{(d/2)^2 - (m/H)^2}
$$
Thus

\[ \delta M^2_\phi = \frac{3 \lambda^2 H^4}{8 \pi^2 M^2_\chi} + \mathcal{O}(M^0_\chi) \]
How to extract mass of SM particles?

**Method 3: for Higgs only – non-perturbative PI**

\[ S = - \int d^4x \sqrt{-g} \left[ f_H(X, \phi) H^\dagger H + f_{DH}(X, \phi) |D_\nu H|^2 - \int d^4x \sqrt{-g} \xi RH^\dagger H \right] \]

\[ M_{H_0}^2 = \frac{12\xi H^2 + f_H(X_0, \phi_0)}{1 + f_{DH}(X_0, \phi_0)} \]

\[ \langle h^2 \rangle = \frac{\int d^N h h^2 \exp[-V_D(m_0^2 h^2/2 + \lambda h^4/4)]}{\int d^N h \exp[-V_D(m_0^2 h^2/2 + \lambda h^4/4)]} = \frac{2}{\sqrt{V_D} \lambda} \frac{\tilde{F}_1 \left( \frac{N+2}{4}; \frac{1}{2}; z^2 \right) - z \tilde{F}_1 \left( \frac{N+4}{4}; \frac{3}{2}; z^2 \right)}{\tilde{F}_1 \left( \frac{N}{4}; \frac{1}{2}; z^2 \right) - z \tilde{F}_1 \left( \frac{N+2}{4}; \frac{3}{2}; z^2 \right)} \]

\[ M_{H}^2 = \frac{\sqrt{\lambda}}{V_D} \frac{4 \left[ 1 - \sqrt{\pi} ze^2 \text{Erfc}(z) \right]}{-2z + \sqrt{\pi}(1 + 2z^2)e^2 \text{Erfc}(z)} \]

For \( z = 0 \) : \[ M_{H}^2 = \sqrt{\frac{6\lambda}{\pi^3}} H^2 \]
The Higgs mass: tree vs quantum-corrected
The full SM spectrum depending on inflation models
3pt based on the SM mass spectrum

\[
\langle \delta \phi(k_1) \delta \phi(k_2) \delta \phi(k_3) \rangle'_\alpha
= 4f'^2(X_0) \sum_{a,b=\pm} ab \int_{-\infty}^{0} \frac{d\tau'}{(H\tau')^2} \int_{-\infty}^{0} \frac{d\tau''}{(H\tau'')^2} \left[ - \partial_{\tau''}G_{ba}(k_3, \tau, \tau'') \partial_{\tau''}\phi_0 \right] \\
\times \left[ - \partial_{\tau'}G_{+a}(k_1, \tau, \tau') \partial_{\tau'}G_{+a}(k_2, \tau, \tau') - k_1 \cdot k_2 G_{+a}(k_1, \tau, \tau')G_{+a}(k_2, \tau, \tau') \right] \\
\times \int d^3 X e^{-ik_L \cdot X} \langle \mathcal{O}_\alpha(\tau', x') \mathcal{O}_\alpha(\tau'', x'') \rangle_{ab}
\]

No UV divergence in the non-local part of loop.
\[ S_H = \frac{f_H^2(X_0) \dot{\phi}_0^2}{\pi^4} \left[ C_H(\mu_h) \left( \frac{k_L}{2k_S} \right)^{2-2\mu_h} + (\mu_h \to -\mu_h) \right], \]

\[ S_{DH} = \frac{f_{DH}^2(X_0) H^4 \dot{\phi}_0^2}{4\pi^4} \left[ C_{H4}(\mu_h) \left( \frac{k_L}{2k_S} \right)^{2-2\mu_h} + (\mu_h \to -\mu_h) \right], \]

\[ S_{\Psi} = \frac{f_\Psi^2(X_0) H^4 \dot{\phi}_0^2 \mu_{1/2}^6}{2\pi^4} \left[ C_{\Phi}(\mu_{1/2}) \left( \frac{k_L}{k_S} \right)^{1+2i\mu_{1/2}} + \text{c.c.} \right], \]

\[ S_A = \frac{27 f_A^4(X_0) H^8 \dot{\phi}_0^2}{16\pi^4 M_A^4} \left[ C_A(\mu_1) \left( \frac{k_L}{2k_S} \right)^{2-2\mu_1} + (\mu_1 \to -\mu_1) \right]. \]
SM spectra is affected by coupling to inflaton.

E.g. gauge sector is affected by the following:

\[
S \supset - \int d^4x \sqrt{-g} \left[ f_{DH}(X, \phi) |D_\mu H|^2 + \frac{1}{4} f_W(X, \phi) W_\mu^a W^{\mu a} + \frac{1}{4} f_B(X, \phi) B_\mu B^{\mu} + \cdots \right].
\]

\[
g^2 = \frac{g_{SM}^2}{1 + f_W(X_0, \phi_0)},
\]

\[
g'^2 = \frac{g'^2_{SM}}{1 + f_B(X_0, \phi_0)}.
\]

\[
M^2_W = \frac{3g^2 H^4}{8\pi^2 M^2_H}, \quad M^2_Z = \frac{3g^2 H^4}{8\pi^2 M^2_H \cos^2 \theta_W}.
\]
SM spectra is affected by coupling to inflaton.

But still we can make predictions:

\[
\frac{d \ln \tan^2 \theta_W}{d \ln k} = \frac{\pi (\eta - 2\epsilon)}{3 \sqrt{3} P_\zeta \sin^2 \theta_W} \left[ \frac{M_W^2}{H^2} \sqrt{\frac{f_{NL}^W}{N_W |C_A(\mu_W)|}} - \frac{M_Z^2}{H^2} \sqrt{\frac{f_{NL}^Z}{N_Z |C_A(\mu_Z)|}} \right]
\]
Predictions for BSM physics on the cosmological collider?
Expansion History of the Primordial Universe

X. Chen, M. H. Namjoo, YW 2015/16
Possibilities:

\[ a = t^p \]
Inflation is the most natural paradigm.  
But alternative scenarios are also being studied.  

Nice to distinguish those scenarios observationally.

The question is equivalent to --
How to measure the evolution history of the primordial universe?
It is a difficult job.

There is no structure in the primordial universe.

Only $10^{-5}$ fluctuations, quantum to classical.
How to measure the evolution history of the primordial universe?

1. Primordial gravitational waves (GWs)
2. Quantum Primordial Standard Clocks (massive fields)
Quantum Primordial Standard Clock
massive $\rightarrow$ curvature, tells physical time

curvature mode, tells conformal time at $k\tau = \frac{M}{H}$

$$a(t) = a_0 \left( \frac{t}{t_0} \right)^p$$
inflation (fast expansion, $|p|>1$)

matter contraction (fast contraction, $0<p\sim O(1)<1$)

slow expansion ($-1\ll p<0$)

Ekpyrosis (slow contraction, $0<p\ll 1$)
Summary: massive fields during inflation

Cosmological collider:

SM studied

BSM? Strings? Black holes?

Quantum primordial standard clock

Probing the expansion history
Thank you!