Multi-objective Dynamic Aperture Optimization for NSLS-II Ring

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Outline

- Multi-objective genetic algorithm (MOGA)
- Existing optimizations on dynamic aperture
- New efficient method with MOGA
- Applications on NSLS-II storage ring
- Correlation between nonlinear driving terms and dynamic aperture
Genetic Algorithm (GA) mimics the evolution of nature:

**Crossover:** children inherit genetic codes from parents

**Mutation:** change the children’s genetic information

**Selection:** only these “elites” survive and reproduce

“It is not the strongest of the species that survives, nor the most intelligent, but the one most responsive to change.”

-Charles Darwin, 1809
Multi-Objective Optimization

An example for two objectives ranking

Within a same rank, all candidates are equally good

Deb, Kalyanmoy - Natural selection: Non-dominated sorting in N-dimension space
Review of existing methods

- Method 1: minimizing the nonlinear driving terms with specific weights, i.e. MAD

Difficulties:
- Trapped in a local minimum
- How to specify weights?

\[ \sigma = \sum_i w_i h_i \]
Review of existing methods

- Method 2: brute-force MOGA driven by direct tracking
  - L. Yang, Y. Li, et al. (PRST-AB, 2011)
  - M. Borland, integrated to ELEGANT

Works very successfully, but, Difficulties:
- No physics is behind
- Very time-consuming in direct DA tracking, especially when your computer is not powerful, or your ring is big.
Motivation

- A strong correlation between DA and NDTs does exist. (L. Yang & Y. Li @BNL, M. Borland & L. Wang @ANL and SLAC)

Correlation of NDT and DA
Yang and Li, PRST-AB

Figure 3. Automatic reduction of the driving terms after the optimization although DA is set as one of the objectives during the optimization.

M. Borland & L. Wang
An efficient method

- **Using MOGA driven by NDT computing rather than DA tracking**
  - Be efficient: computing NDTs is much cheaper than DA tracking
  - Be of “physics”: having small low order NDTs is an **necessary** condition for larger DA
Parallel Computation and GA

Master
Initialize

Master
Assign tasks

Slaver
Receive task
Compute merit function
Send back

Slaver
Receive task
Compute merit function
Send back

Slaver
Receive task
Compute merit function
Send back

Multiple generations

Slaver
Receive task
Compute merit function
Send back

End

Y

N

Master
Receive data
Generating a new generation (sort/crossover/mutation)
Applications on NSLS-II ring

- Energy: 3GeV
- Emittance: 2nm bare, 1nm with 3x6.8m DWs
- Lattice: 30-standard DBAs (Chasman-Green)

Requirements for DA:

DA >= 15mm at high-beta straight for efficient injection

Energy acceptance >2.5% for sufficient beam lifetime

Tolerate numerous insertion devices and engineering errors
Simply case: Chromaticity
+7/+7

- Purpose: high linear chromaticity to stabilize beam at high stored beam current

- Optimization procedure:
  - Tuning chromatic sextupoles to achieve +7/+7 linear chromaticity
  - Tuning 6 families geometrical sextupoles to optimize DA and energy acceptance
  - Penalty functions: first and second order driving terms: $h_{abcd,e}$, where $a+b+c+d+e = 3$ and 4 (totally ~ 30 terms)
$\xi_{x,y} = +7/+7$ Dynamics: chromaticity

1. Linear chromaticity

\[ \xi_{x,1} = 7.16 \]
\[ \xi_{y,1} = 7.43 \]
Tune dependence on amplitudes

Horizontal

\[ \frac{dw_x}{dx^2} = -111.21 \]
\[ \frac{dw_y}{dx^2} = -0.91 \]

Vertical

\[ \frac{dw_x}{dy^2} = 8.39 \]
\[ \frac{dw_y}{dy^2} = -2472.36 \]
Dynamic aperture and energy acceptance

Correct orbit and optics
Correct coupling

Tune injected beam
Demanding case: Low alpha Lattice

Purpose: to short bunch length by reducing momentum compactor from $5e^{-4}$ to $3.4e^{-06}$

1. Linear chromaticity to $+2/+2$

2. Minimize higher order momentum compactions to have a stable longitudinal motion

3. Have sufficient DA and energy acceptance
Objective 1: chromaticity control

1. Linear chromaticity close to +2/+2
2. Large high-order chromaticities
Objective 2: longitudinal stability

minimize higher order momentum compaction factors to have stable longitudinal motion

\[
\alpha_1 = \frac{1}{\ell_0} \int \frac{\eta_1(s)}{\rho} \, ds \\
\alpha_2 = \frac{1}{\ell_0} \int \left(\frac{\eta_1'(s)}{2} + \frac{\eta_2(s)}{\rho}\right) \, ds \\
\alpha_3 = \frac{1}{\ell_0} \int \eta_1'(s) \eta_2'(s) - \frac{\eta_1(s) \eta_1'(s)^2}{2\rho} + \frac{\eta_3(s)}{\rho} \, ds.
\]
Objective 3: dynamic aperture and energy acceptance

Sufficient dynamic aperture for injection

Sufficient energy acceptance for lifetime
Correlation between DAs and NDTs

1. Having small NDTs is an necessary but insufficient condition for having a large DA

2. Sufficient population per generation is the key parameter to get some good solutions

**A:** small NDTs and large DAs

**B:** small NDTs but small DAs

**C:** large NDTs and small DAs
New characterization with square matrix and sufficient conditions

We may now write this in the form

\[ X = MX_0 \]  \hspace{1cm} (2)

where to 4th order, we define the \(14 \times 1\) monomial array

\[ \tilde{X}_0 = (x_0 \quad p_0 \quad x_0^2 \quad x_0 p_0 \quad p_0^2 \quad \ldots \quad p_0^4) \]  \hspace{1cm} (3)

\[ \tilde{N} = U \tilde{N} U^{-1} \]  \hspace{1cm} (4)

\[ \begin{pmatrix} \tilde{N}_0 & \tilde{N}_1 & \tilde{N}_{-1} & \tilde{N}_2 & \tilde{N}_{-2} & \ldots \end{pmatrix} \]  \hspace{1cm} (5)

\[ \tilde{N}_k = e^{ik\mu} I + \tau^\dagger \]  \hspace{1cm} (6)

with \( k = \pm 1, \pm 2, \ldots \). \( I \) is the identity matrix and \( \tau^\dagger \) the matrix with 1’s just above the diagonal:

\[ \tau^\dagger = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \tau = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \]  \hspace{1cm} (7)
Phase space trajectories

Square matrix

Revisit $\xi = +7/+7$

Driving terms
Summary

- MOGA driven by the nonlinear driving terms is very **efficient**

- Having small low order NDTs is an **necessary**, but **insufficient** condition for having a decent DA.

- The number of populations is the key parameter. **Parallel** computation capability is preferable.

- Tracking simulation is the final criteria to select the best solutions from the last generation.

- New approach of characterization of nonlinear dynamics is under development.