Leptonic non-unitarity is a “smoking gun” signal of (“heavy”) new physics in the neutrino sector to explain the observed neutrino masses.

Circular lepton colliders like the CEPC are very sensitive to such signals, and can thereby help to clarify how to extend the Standard Model …
Outline

Main part:

- Introduction: Origin of non-unitary lepton mixing, effective field theory treatment, phenomenological consequences
- Present constraints
- Improved sensitivities to leptonic non-unitarity at the CEPC

At the end of my talk I also like to make a quick comment ...

- ... on the motivation for SUSY at the SppC: “Predicting the sparticle spectrum from GUTs ...”
Neutrino masses: How to extend the SM?

See-saw (type I)

See-saw (type II)

Dirac neutrinos

Radiative mechanisms

Effective theory: d=5 operator

\[ \delta \mathcal{L}^{d=5} = \frac{1}{2} c_{\alpha \beta}^{d=5} \left( \overline{L} \tilde{\phi}^* \right) \left( \tilde{\phi}^\dagger L \right) + h.c. \]
A comparatively model-independent consequence of new physics introduced to generate the observed neutrino masses: Non-unitarity of the leptonic mixing matrix ...
Non-unitary leptonic mixing

- Typical situation, intuitively:

  (Effective) mixing matrix of light neutrinos is submatrix of a larger unitary mixing matrix (mixing with additional heavy particles)

  \[ U_{PMNS} \equiv N \]

 Examples with possible large non-unitarity: 'inverse' seesaw or 'multiple' seesaw at TeV energies, SUSY with R-parity violation, large extra dimensions, ...

Langacker, London ('88)
Non-unitary leptonic mixing

Lagrangian in the mass basis ...

\[ \mathcal{L}^{\text{eff}} = \frac{1}{2} (\bar{\nu}_i \phi \nu_i - \bar{\nu}^c_i m_i \nu_i + \text{h.c.}) - \frac{g}{2\sqrt{2}} (W^+_\mu \, \bar{l}_\alpha \gamma_\mu (1 - \gamma_5) \, (N^\dagger N)_{\alpha i} \nu_i + \text{h.c.}) 
- \frac{g}{2 \cos \theta_W} (Z_\mu \, \bar{\nu}_i \gamma^\mu (1 - \gamma_5) \, (N^\dagger N)_{ij} \nu_j + \text{h.c.}) + \ldots \]

+ modification in neutral current interaction in minimal schemes (MUV), to be explained later ...
Non-unitary leptonic mixing

... now when we change to the flavour basis:

Non-unitarity of the leptonic mixing matrix corresponds to non-canonical kinetic terms in the flavour basis!
Non-unitary leptonic mixing

- There is a unique gauge invariant $d=6$ effective operator which leads to non-canonical kinetic terms only for the neutrinos:

\[
\delta \mathcal{L}^{d=6} = c_{\alpha\beta}^{d=6} \left( \overline{L}_\alpha \tilde{\phi} \right) i\tilde{\phi} \left( \tilde{\phi}^\dagger L_\beta \right)
\]

- After EW symmetry breaking it results in a non-unitary leptonic mixing matrix with:

\[
|N N^\dagger - 1|_{\alpha\beta} = \frac{\nu^2}{2} |c^{d=6}|_{\alpha\beta}
\]

- + modification of the NC interaction shown earlier ...

De Gouvea, Giudice, Strumia, Tobe ('01), Broncano, Gavela, Jenkins ('02)

S.A., Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon ('06)
Non-unitary leptonic mixing in the MUV scheme

A minimal way to introduce neutrino masses and non-unitary leptonic mixing thus consists in adding a $d=5$ and a $d=6$ operator to the SM:

\[ \mathcal{L}^{\text{eff}} = \mathcal{L}_{SM} + \delta \mathcal{L}^{d=5} + \delta \mathcal{L}^{d=6} + \ldots \]

**Neutrino masses** (violates $L$)

\[ \delta \mathcal{L}^{d=5} = \frac{1}{2} c^{d=5}_{\alpha \beta} \left( \overline{L}_\alpha \phi^* \right) \left( \phi^\dagger L_\beta \right) + h.c. \]

**Non-unitarity** (conserves $L$)

\[ \delta \mathcal{L}^{d=6} = c^{d=6}_{\alpha \beta} \left( \overline{L}_\alpha \phi \right) i\phi \left( \phi^\dagger L_\beta \right) \]

not necessarily suppressed by the smallness of the neutrino masses

MUV scheme: Minimal Unitarity Violation

S.A., Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon ('06)
Generic scenario: SM + heavy neutral leptons

Integrating out the heavy neutral leptons (= sterile neutrinos) generates dominantly the two effective operators:

**Neutrino mass operator (d=5)**

\[ \delta L^{d=5} = \frac{1}{2} c_{\alpha \beta}^{d=5} \left( \bar{L}_\alpha \phi^* \right) \left( \phi^\dagger L_{\beta} \right) + h.c. \]

\[ c_{\alpha \beta}^{d=5} = (Y_N^T)_{\alpha I} (M_N)_{IJ}^{-1} (Y_N)_{J \beta} \]

\[ (m_\nu)_{\alpha \beta} = -\frac{v_{EW}^2}{2} c_{\alpha \beta}^{d=5} \]

**Effective (d=6) operator which modifies the kin. terms of the neutrinos**

\[ \delta L^{d=6} = c_{\alpha \beta}^{d=6} \left( \bar{L}_\alpha \phi^* \right) i\phi \left( \phi^\dagger L_{\beta} \right) \]

\[ c_{\alpha \beta}^{d=6} = \sum_I (Y_N^\dagger)_{\alpha I} (M_N)_{IJ}^{-2} (Y_N)_{I \beta} \]
**EW interactions modified**

$W^- \approx N_{\alpha i}$

$Z \approx (N^+ N)_{ij}$

**Definition:**
Non-unitarity parameters

$$(NN^\dagger)_{\alpha\beta} = (1_{\alpha\beta} + \varepsilon_{\alpha\beta})$$ (Hermitean matrix)

- Theory predictions for various observables which involve weak interactions get modified!
Two comments ...

- Within the MUV scheme, one can show that the off-diagonal non-unitarity parameters satisfy the following inequality:

\[ |\varepsilon_{\alpha\beta}| \leq \sqrt{|\varepsilon_{\alpha\alpha}| |\varepsilon_{\beta\beta}|} \]

This can provide an interesting consistency check of the MUC scheme!

- Furthermore, the diagonal non-unitarity parameters are always \( \leq 0 \):

\[ \varepsilon_{\alpha\alpha} \leq 0 \]

Constraints on Leptonic Non-Unitarity: 
Most relevant processes

- Electroweak precision observables (EWPOs)
- Universalitiy tests (at low E and via W decays)
- Charged LFV decays
**Electroweak precision observables**

- Remark: As usual, one expresses the predictions for the EWPOs in terms of the very well measured quantities

\[
\begin{align*}
\alpha(m_Z)^{-1} & = 127.944(14), \\
G_F & = 1.1663787(6) \times 10^{-5}\text{GeV}^{-2}, \\
m_Z & = 91.1875(21),
\end{align*}
\]

- However: \(G_F\) is measured in muon decays ... and thus gets modified due to the non-unitary mixing \(N:\)

\[
G^2_{\mu} = G^2_F (NN^\dagger)_{\mu\mu} (NN^\dagger)_{ee}
\]

→ affects many observables
Summary of modified EWPO predictions

<table>
<thead>
<tr>
<th>Prediction in MUV</th>
<th>Prediction in the SM</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>([R_\ell]<em>{SM} (1 - 0.15(\varepsilon</em>{ee} + \varepsilon_{\mu\mu})))</td>
<td>20.744(11)</td>
<td>20.767(25)</td>
</tr>
<tr>
<td>([R_b]<em>{SM} (1 + 0.03(\varepsilon</em>{ee} + \varepsilon_{\mu\mu})))</td>
<td>0.21577(4)</td>
<td>0.21629(66)</td>
</tr>
<tr>
<td>([R_c]<em>{SM} (1 - 0.06(\varepsilon</em>{ee} + \varepsilon_{\mu\mu})))</td>
<td>0.17226(6)</td>
<td>0.1721(30)</td>
</tr>
<tr>
<td>(\sigma_{had}^0_{SM} (1 - 0.25(\varepsilon_{ee} + \varepsilon_{\mu\mu}) - 0.27\varepsilon_{\tau\tau})))</td>
<td>41.470(15) nb</td>
<td>41.541(37) nb</td>
</tr>
<tr>
<td>([R_{inv}]<em>{SM} (1 + 0.75(\varepsilon</em>{ee} + \varepsilon_{\mu\mu}) + 0.67 \varepsilon_{\tau\tau}))</td>
<td>5.9721(10)</td>
<td>5.942(16)</td>
</tr>
<tr>
<td>([M_W]<em>{SM} (1 - 0.11(\varepsilon</em>{ee} + \varepsilon_{\mu\mu})))</td>
<td>80.359(11) GeV</td>
<td>80.385(15) GeV</td>
</tr>
<tr>
<td>([\Gamma_{lept}]<em>{SM} (1 - 0.59(\varepsilon</em>{ee} + \varepsilon_{\mu\mu})))</td>
<td>83.966(12) MeV</td>
<td>83.984(86) MeV</td>
</tr>
<tr>
<td>((s_{\ell,lep}^W)<em>{SM} (1 + 0.71(\varepsilon</em>{ee} + \varepsilon_{\mu\mu})))</td>
<td>0.23150(1)</td>
<td>0.23113(21)</td>
</tr>
<tr>
<td>((s_{\ell,\text{had}}^W)<em>{SM} (1 + 0.71(\varepsilon</em>{ee} + \varepsilon_{\mu\mu})))</td>
<td>0.23150(1)</td>
<td>0.23222(27)</td>
</tr>
</tbody>
</table>

Sensitivity via $G_F$!

S.A., O. Fischer (1407.6607)
Other sensitive probes: Universality tests

Consider ratios of observables of the form:

\[ R_{\alpha \beta} = \sqrt{\frac{(NN^\dagger)_{\alpha \alpha}}{(NN^\dagger)_{\beta \beta}}} \approx 1 + \frac{1}{2} (\varepsilon_{\alpha \alpha} - \varepsilon_{\beta \beta}) \]

<table>
<thead>
<tr>
<th>Process</th>
<th>Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{\mu e}^\ell )</td>
<td>[ \frac{\Gamma(\tau \rightarrow \nu_\tau \mu \bar{\nu}<em>\mu)}{\Gamma(\tau \rightarrow \nu</em>\tau e \bar{\nu}_e)} ] 1.0018(14)</td>
</tr>
<tr>
<td>( R_{\tau \mu}^\ell )</td>
<td>[ \frac{\Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}<em>e)}{\Gamma(\mu \rightarrow \nu</em>\mu e \bar{\nu}_e)} ] 1.0006(21)</td>
</tr>
<tr>
<td>( R_{e \mu}^W )</td>
<td>[ \frac{\Gamma(W \rightarrow e \bar{\nu}<em>e)}{\Gamma(W \rightarrow \mu \bar{\nu}</em>\mu)} ] 1.0085(93)</td>
</tr>
<tr>
<td>( R_{\tau \mu}^W )</td>
<td>[ \frac{\Gamma(W \rightarrow \tau \bar{\nu}_\tau)}{\Gamma(W \rightarrow \mu \bar{\nu}_e)} ] 1.032(11)</td>
</tr>
<tr>
<td>( R_{\mu e}^\pi )</td>
<td>[ \frac{\Gamma(\pi \rightarrow \mu \bar{\nu}_\mu)}{\Gamma(\pi \rightarrow e \bar{\nu}_e)} ] 1.0021(16)</td>
</tr>
<tr>
<td>( R_{\tau \mu}^\pi )</td>
<td>[ \frac{\Gamma(\pi \rightarrow \nu_\tau \pi)}{\Gamma(\pi \rightarrow \mu \bar{\nu}_\mu)} ] 0.9956(31)</td>
</tr>
<tr>
<td>( R_{\tau \mu}^K )</td>
<td>[ \frac{\Gamma(\tau \rightarrow K \nu_\tau)}{\Gamma(K \rightarrow \mu \bar{\nu}_\mu)} ] 0.9852(72)</td>
</tr>
<tr>
<td>( R_{\tau e}^K )</td>
<td>[ \frac{\Gamma(\tau \rightarrow K \nu_\tau)}{\Gamma(K \rightarrow e \bar{\nu}_e)} ] 1.032(11)</td>
</tr>
</tbody>
</table>
Bounds on LFV $\mu$ and $\tau$ decays $l_i \rightarrow l_j \gamma$ (and on $\mu \rightarrow 3e$ and $\mu \rightarrow e$ conversion in nuclei) lead to constraints on the $|\varepsilon_{\alpha\beta}|$:

$$\frac{\Gamma(l_\alpha \rightarrow l_\beta \gamma)}{\Gamma(l_\alpha \rightarrow \nu_\alpha l_\beta \bar{\nu}_\beta)} = \frac{3\alpha}{32\pi} \left| \sum_k N_{\alpha k} N_{k\beta}^\dagger F(x_k) \right|^2$$

where:

$$x_k \equiv \frac{m_k^2}{M_W^2}$$

$m_k$: light neutrinos' masses

Example diagram for $l_\alpha \rightarrow l_\beta + \gamma$
Present constraints

- We performed a global MCMC fit of the six MUV parameters

\[ \varepsilon_{ee}, \varepsilon_{\mu\mu}, \varepsilon_{\tau\tau}, |\varepsilon_{e\mu}|, |\varepsilon_{e\tau}|, |\varepsilon_{\mu\tau}| \]

to the most relevant 34 observables (including also tests of CKM matrix unitarity, ...).
Results of a global fit to the present data

S.A., O. Fischer (1407.6607)

- Highest posterior density intervals at 90% Bayesian C.L.:

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{ee}$</td>
<td>$-0.0021$</td>
<td>$-0.0002$</td>
</tr>
<tr>
<td>$\varepsilon_{\mu\mu}$</td>
<td>$-0.0004$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\varepsilon_{\tau\tau}$</td>
<td>$-0.0053$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

- $|\varepsilon_{e\mu}| < 1.0 \times 10^{-5}$
- $|\varepsilon_{e\tau}| < 2.1 \times 10^{-3}$
- $|\varepsilon_{\mu\tau}| < 8.0 \times 10^{-4}$

→ treated as constraints

constraints from charged LFV
Results of a global fit to the present data

S.A., O. Fischer (1407.6607)

- Highest posterior density intervals at 90% Bayesian C.L.:

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon_{ee}$</th>
<th>$\varepsilon_{\mu\mu}$</th>
<th>$\varepsilon_{e\mu}$</th>
<th>$\varepsilon_{e\tau}$</th>
<th>$\varepsilon_{\mu\tau}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower</td>
<td>$-0.0021$</td>
<td>$-0.0004$</td>
<td>$-0.0004$</td>
<td>$-0.0053$</td>
<td>$-0.0053$</td>
</tr>
<tr>
<td>Upper</td>
<td>$-0.0002$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

→ treated as constraints from charged LFV

Interesting: Non-zero best-fit point for $\varepsilon_{ee}$

$\varepsilon_{ee} = -0.0012$

(but of course only a mild indication of non-zero $\varepsilon_{ee}$, at the level of $\sim 2\sigma$)
How much can the CEPC improve the sensitivity to leptonic non-unitarity?
CEPC: Very sensitive tests of universality with $W$ decays

- **Blue line**: Present constraints
- **Orange line**: Planned experimental sensitivity of universality tests at low $E$ [MOLLER, TRIUMF, PSI, NA62, Tau/Charmed factories]
- **Green line**: Sensitivity (statistical uncertainties only) with $10^8 W$ bosons [as possible at the CEPC]

Universality tests probe the differences between the $\varepsilon_{\alpha\alpha}$.

S.A., O. Fischer (1407.6607)
CEPC: Greatly improved precision for EWPOs

With $10^{11}$ Z bosons produced at CEPC

<table>
<thead>
<tr>
<th>Observable</th>
<th>LEP precision</th>
<th>CEPC CDR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_W$ [MeV]</td>
<td>33</td>
<td>3</td>
</tr>
<tr>
<td>$\sin^2 \theta_W^{\text{eff}}$</td>
<td>0.07%</td>
<td>0.01%</td>
</tr>
<tr>
<td>$R_b$</td>
<td>0.3%</td>
<td>0.08%</td>
</tr>
<tr>
<td>$R_c$</td>
<td>0.3%</td>
<td>0.07%</td>
</tr>
<tr>
<td>$R_{inv}$</td>
<td>0.27%</td>
<td>$8.9 \times 10^{-4}$</td>
</tr>
<tr>
<td>$R_\ell$</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>$\Gamma_\ell$</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>$\sigma^0_h$ [nb]</td>
<td>$8.9 \times 10^{-4}$</td>
<td>$1 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

- Values taken from the CEPC pre-CDR where available (otherwise estimates were used)
Sensitivities of future EWPO measurements

For Yukawa couplings $y_\alpha = O(1)$, e.g., the CEPC can probe sterile neutrinos with masses up to $M \sim 12$ TeV!

[Using connection to sterile neutrino parameters in minimal symmetry protected seesaw scenarios (SPSS):

$$\varepsilon_{\alpha\beta} = -y_\alpha^* y_\beta v_{EW}^2/(2M^2)$$]

The EWPOs are very sensitive to the sum $|\varepsilon_{ee} + \varepsilon_{\mu\mu}|$

cf. S.A., O. Fischer (1407.6607)
Non-unitarity of the leptonic mixing matrix is a generic (and rather model-independent) consequence of new physics in the neutrino sector to generate the observed neutrino masses.

Effective field theory: MUV ("Minimal Unitarity Violation") represents a large class of models.

EWPOs and universality tests at the CEPC (with pre-CDR parameters) can probe leptonic non-unitarity with very high sensitivity (i.e. sums and differences of $\varepsilon_{aa}$ up to $O(10^{-4})$).

... as announced, I have a few more slides
A different topic (in 2 slides):

Comment on the motivation for SUSY at the SppC
Predicting the SUSY spectrum from GUTs

- GUTs not only predict gauge coupling unification, but also unification of quarks and leptons in joint multiplets at high energy

- Predictions for the Yukawa coupling ratios, e.g. $y_{\tau}/y_{b} = 1$ or $3/2$, $y_{\mu}/y_{s} = 3$, $9/2$ or $6$, $y_{e}/y_{d} = 1/2$ or $1/3$

See e.g.: Georgi, Jarlskog ('79), S.A., Spinrath ('09)
S.A., King, Spinrath ('13)

- Performing the RG running to low energies and including the SUSY threshold corrections, the predictions can be compared to the experimental data on quark and lepton masses

- Link to the sparticle spectrum
Predicted spectrum
(1σ HPD intervals: example SU(5) GUT scenario)


Range can be probed at a 100 TeV pp collider

New tool: SusyTC

Note: No constraints from present SUSY searches applied. Sparticles out of present LHC reach is a result!
Thanks for your attention!
Extra slides
**Future sensitivities to the $\varepsilon_{\alpha\beta}$ from LFV**

<table>
<thead>
<tr>
<th>Process</th>
<th>MUV Prediction</th>
<th>Bound</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Br_{\tau e}$</td>
<td>$4.3 \times 10^{-4}</td>
<td>\varepsilon_{\tau e}</td>
<td>^2$</td>
</tr>
<tr>
<td>$Br_{\tau \mu}$</td>
<td>$4.1 \times 10^{-4}</td>
<td>\varepsilon_{\tau \mu}</td>
<td>^2$</td>
</tr>
<tr>
<td>$Br_{\mu e e e}$</td>
<td>$1.8 \times 10^{-5}</td>
<td>\varepsilon_{\mu e}</td>
<td>^2$</td>
</tr>
<tr>
<td>$R^T_{\mu e}$</td>
<td>$1.5 \times 10^{-5}</td>
<td>\varepsilon_{\mu e}</td>
<td>^2$</td>
</tr>
</tbody>
</table>

Note: A measurement of LFV would guarantee non-zero $\varepsilon_{\alpha\alpha}$

- Also very sensitive at the CEPC: cLFV $Z$ decays $Z \rightarrow l_1 l_2$
  
  cf. Abada, Dr Romeri, Monteil, Orloff, Teixeira (1412.6322)

S.A., O. Fischer (1407.6607)
Possible sensitivity also to the phases of $\varepsilon_{\alpha\beta}$ at a neutrino factory

Interplay of (tau-sensitive) near and far detectors could provide information on the phase of the MUV parameters $\varepsilon_{\tau\mu}$ and $\varepsilon_{\tau e}$ (i.e. on the phases of $-\theta^{*}_\tau \theta_\mu$ and $-\theta^{*}_\tau \theta_e$)

Example: Precision vs. direct tests in the “Symmetry Protected Seesaw Scenario (SSMS)”

**Symmetry Protected low scale Seesaw Scenario (SSPS)**

Minimal: Two sterile neutrinos forming a pseudo-Dirac pair

\[ \mathcal{L}_N = - \overline{N_R}^1 M \overline{N_R}^2 - y_\alpha \overline{N_R}^1 \phi \dagger L^\alpha + \text{H.c.} \]

+ possible additional “decoupled” sterile neutrinos, which do not mix with the active neutrinos in the symmetry limit

\[ M_N = \begin{pmatrix} 0 & M \\ M & \mu \end{pmatrix} \]

E.g.: Sterile neutrinos with masses \( \sim \) EW scale

Similar: “inverse” seesaw, “linear” seesaw

→ Assumption: The two sterile neutrinos \( N_R^1 \) and \( N_R^2 \) dominate phenomenology
Sterile neutrinos with masses $\sim$ EW scale

Symmetry Protected low scale Seesaw Scenario (SSPS)

In the symmetry limit:

$$\mathcal{L}_N = - \overline{N_R}^1 M_1 N_R^c \phi \bar{\phi}^\dagger L^\alpha + H.c.$$  

The leptonic mixing matrix to leading order in the active-sterile mixing parameters:

$$U = \begin{pmatrix} N_{1e} & N_{1\mu} & N_{1\tau} & -\frac{i}{\sqrt{2}} \theta_e & \frac{1}{\sqrt{2}} \theta_e \\ N_{2e} & N_{2\mu} & N_{2\tau} & -\frac{i}{\sqrt{2}} \theta_\mu & \frac{1}{\sqrt{2}} \theta_\mu \\ N_{3e} & N_{3\mu} & N_{3\tau} & -\frac{i}{\sqrt{2}} \theta_\tau & \frac{1}{\sqrt{2}} \theta_\tau \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\theta_e^* & -\theta_\mu^* & -\theta_\tau^* & \frac{1}{\sqrt{2}}(1 - \frac{1}{2} \theta^2) & \frac{1}{\sqrt{2}}(1 - \frac{1}{2} \theta^2) \end{pmatrix}$$

Active-sterile neutrino mixing parameters:

$$\theta_\alpha = \frac{y_\alpha^*}{\sqrt{2}} \frac{v_{EW}}{M}, \quad \alpha = e, \mu, \tau$$

Parameters:
- $M$, $y_\alpha$, ($\alpha=1,2,3$)
- or equivalently $M$, $\theta_\alpha$, ($\alpha=1,2,3$)

E.g.:

- Sterile neutrinos with masses $\sim$ EW scale
- Similar: "inverse" seesaw, "linear" seesaw
At low energy: Relation to parameters of the effective theory (MUV)

<table>
<thead>
<tr>
<th></th>
<th>$y_{\nu_{\alpha}}$</th>
<th>$\theta_{\alpha}$</th>
<th>$\varepsilon_{\alpha\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{\nu_{\alpha}} =$</td>
<td>$-$</td>
<td>$\frac{\sqrt{2M}}{v_{EW}} \theta_{\alpha}^*$</td>
<td>$-\frac{\sqrt{2M}}{v_{EW}} \varepsilon_{\beta\alpha}/\sqrt{-\varepsilon_{\beta\beta}}$</td>
</tr>
<tr>
<td>$\theta_{\alpha} =$</td>
<td>$\frac{v_{EW}}{\sqrt{2M}} y_{\nu_{\alpha}}^*$</td>
<td>$-$</td>
<td>$-\varepsilon_{\beta\alpha}/\sqrt{-\varepsilon_{\beta\beta}}$</td>
</tr>
<tr>
<td>$\varepsilon_{\alpha\beta} =$</td>
<td>$-\frac{v_{EW} y_{\nu_{\alpha}}^* y_{\nu_{\beta}}}{2M^2}$</td>
<td>$-\theta_{\alpha}^* \theta_{\beta}$</td>
<td>$-$</td>
</tr>
</tbody>
</table>
Comparison: Non-unitarity (precision) vs direct probes


Direct searches
- $Z$ pole search $\pm 2\sigma$: $|y| = \sqrt{\sum_{\alpha} |y_{\nu_{\alpha}}|^2}$, $\Theta^2 = \sum_{\alpha} |\theta_{\alpha}|^2$
- Higgs $\rightarrow$ WW $\pm 1\sigma$: $|y| = \sqrt{\sum_{\alpha} |y_{\nu_{\alpha}}|^2}$, $\Theta^2 = \sum_{\alpha} |\theta_{\alpha}|^2$
- $e^+ e^- \rightarrow h + \text{ME(T)}$ $\pm 1\sigma$: $|y| = |y_{\nu_e}|$, $\Theta^2 = |\theta_e|^2$
- $e^+ e^- \rightarrow l\nu l^* + \text{ME(T)}$ $\pm 1\sigma$: $|y| = |y_{\nu_e}|$, $\Theta^2 = |\theta_e|^2$

Other
- Precision constraints: $|y| = \sqrt{|y_{\nu_e}|^2 + |y_{\nu_{\mu}}|^2}$, $\Theta^2 = |\theta_e|^2 + |\theta_{\mu}|^2$
- "Unprotected" type-I seesaw