Testing the Non-Unitarity of the Leptonic Mixing Matrix at the CEPC

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Leptonic non-unitarity is a “smoking gun” signal of (“heavy”) new physics in the neutrino sector to explain the observed neutrino masses

$\Updownarrow$

Circular lepton colliders like the CEPC are very sensitive to such signals, and can thereby help to clarify how to extend the Standard Model …

Outline

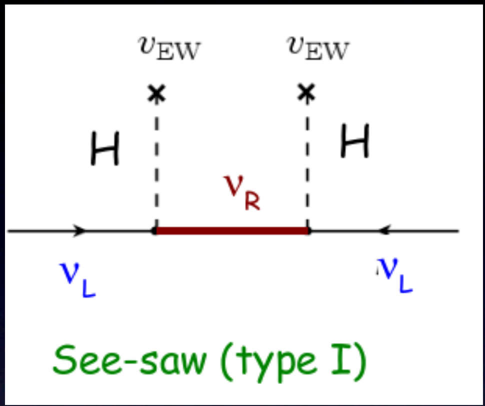
Main part:

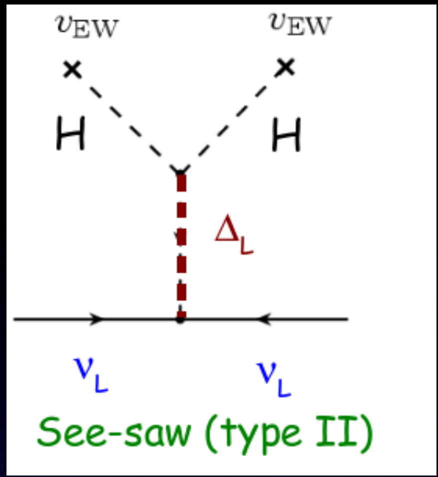
- Introduction: Origin of non-unitary lepton mixing, effective field theory treatment, phenomenological consequences
- Present constraints
- Improved sensitivities to leptonic non-unitarity at the CEPC

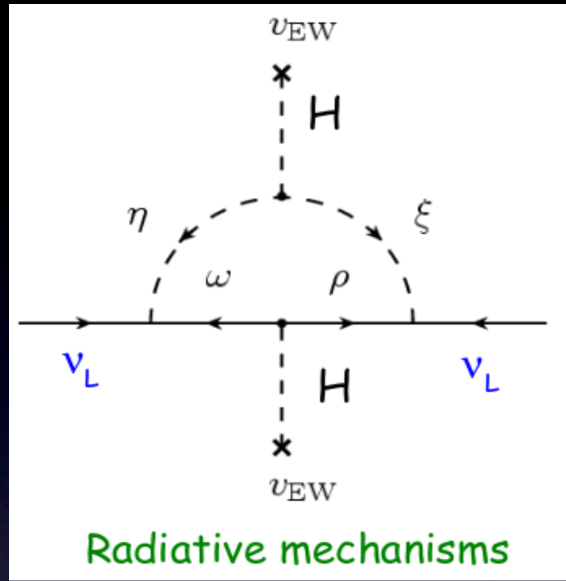
At the end of my talk I also like to make a quick comment ...

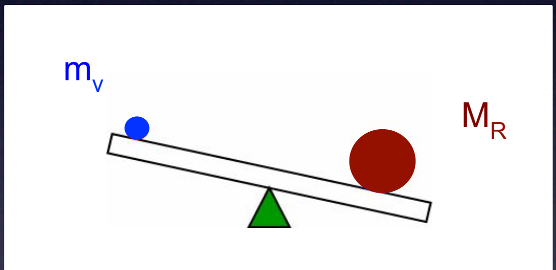
- … on the motivation for SUSY at the SppC: “Predicting the sparticle spectrum from GUTs …”

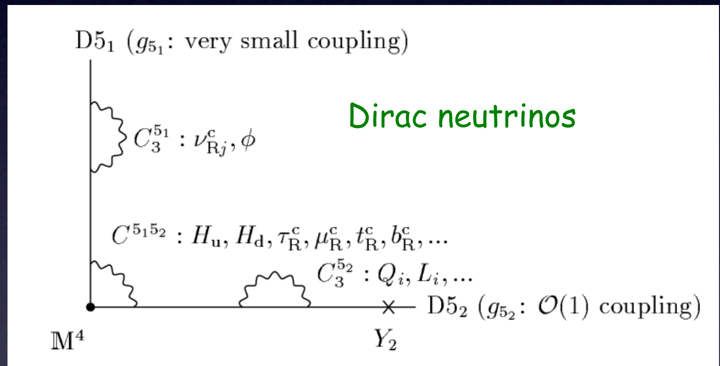
Neutrino masses:
How to extend the SM?

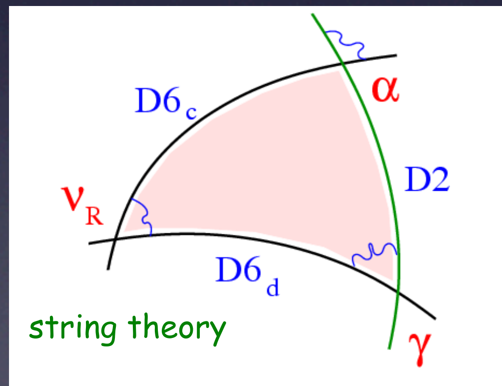




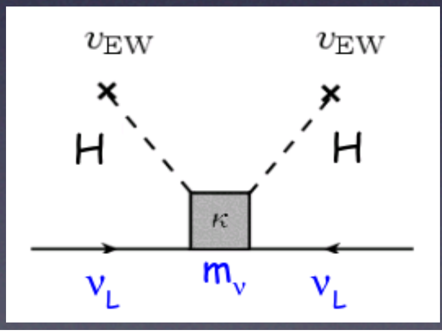


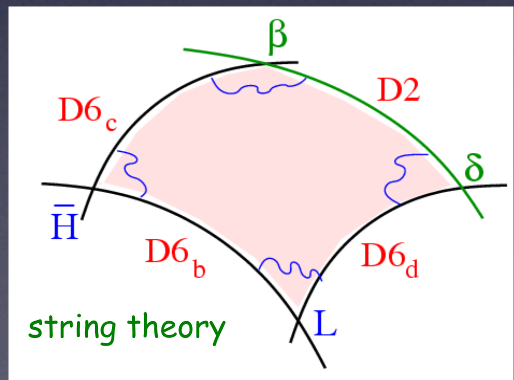






Effective theory:
d=5 operator





$$\delta\mathcal{L}^{d=5} = \frac{1}{2}\, c^{d=5}_{\alpha\beta} \left(\overline{L^c}_\alpha \tilde{\phi}^*\right)\left(\tilde{\phi}^\dagger L_\beta\right) + h.c.$$

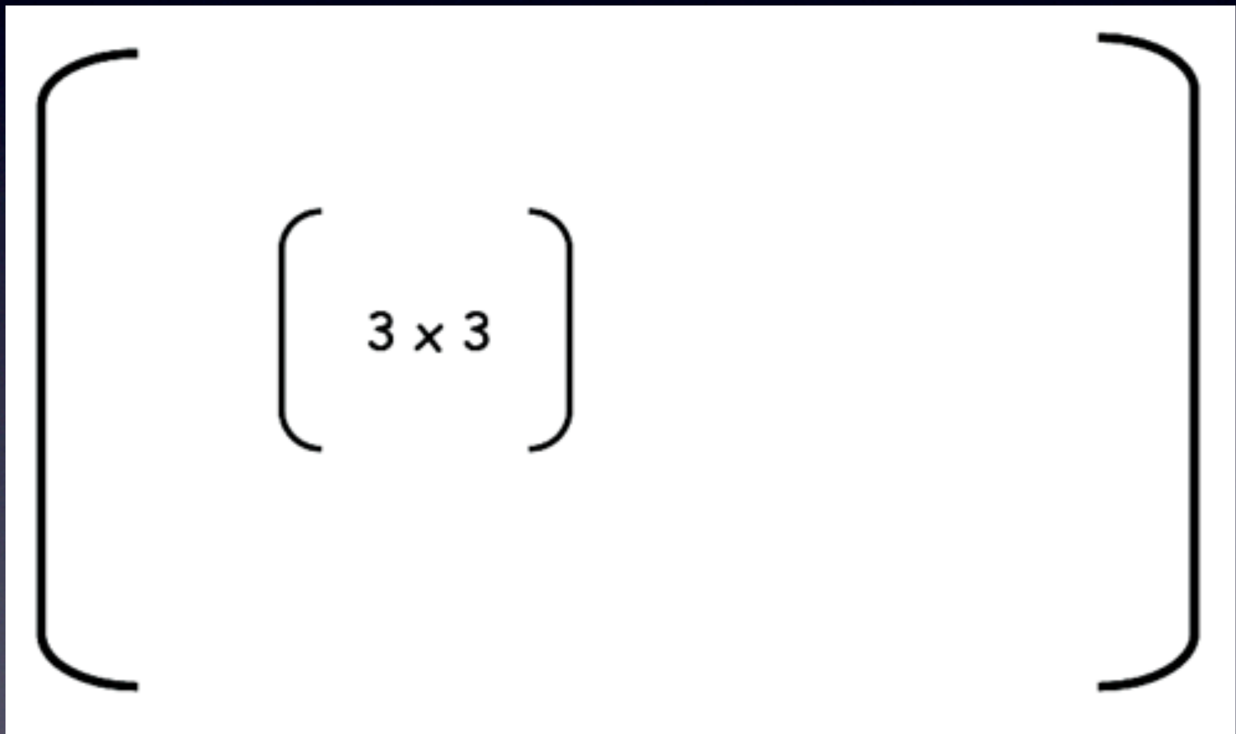
A comparatively model-independent consequence of new physics introduced to generate the observed neutrino masses:

Non-unitarity of the leptonic mixing matrix ...

Non-unitary leptonic mixing

- Typical situation, intuitively:

(Effective) mixing matrix of light neutrinos is submatrix of a larger unitary mixing matrix (mixing with additional heavy particles)



Langacker, London ('88)

$\Rightarrow U_{PMNS} \equiv N$ is non-unitary

Examples with possible large non-unitarity: 'inverse' seesaw or 'multiple' seesaw at TeV energies, SUSY with R-parity violation, large extra dimensions, ...

Non-unitary leptonic mixing

- Lagrangian in the mass basis ...

kinetic term

neutrino
mass term

charged
current
interaction

non-unitary
mixing matrix N

$$\mathcal{L}^{eff} = \frac{1}{2}\left(\bar{\nu}_i i \not\partial \nu_i - \overline{\nu^c}_i m_i \nu_i + h.c.\right) - \frac{g}{2\sqrt{2}}\left(W^+_\mu \bar{l}_\alpha \gamma_\mu (1-\gamma_5) N_{\alpha i} \nu_i + h.c.\right)$$
$$- \frac{g}{2\cos\theta_W}\left(Z_\mu \bar{\nu}_i \gamma^\mu (1-\gamma_5) (N^\dagger N)_{ij} \nu_j + h.c.\right) + \dots$$

+ modification in neutral current interaction in minimal schemes (MUV), to be explained later ...

Non-unitary leptonic mixing

- ... now when we change to the flavour basis:

non-canonical kinetic terms

$$\begin{aligned}\mathcal{L}^{eff} \;=\; & \boxed{\frac{1}{2}\left(i\,\bar{\nu}_\alpha\;\partial\!\!\!/\,(NN^\dagger)^{-1}_{\alpha\beta}\;\nu_\beta\right.} - \overline{\nu^c}_\alpha\,[(N^{-1})^t m N^{-1}]_{\alpha\beta}\,\nu_\beta + h.c.\Big) \\ & - \frac{g}{2\sqrt{2}}\left(W^+_\mu\,\bar{l}_\alpha\,\gamma^\mu\,(1-\gamma_5)\,\nu_\alpha + h.c.\right) \\ & - \frac{g}{2\cos\theta_W}\left(Z_\mu\,\bar{\nu}_\alpha\,\gamma^\mu\,(1-\gamma_5)\,\nu_\alpha + h.c.\right) + \ldots\,,\end{aligned}$$

Non-unitarity of the leptonic mixing matrix corresponds to non-canonical kinetic terms in the flavour basis!

Non-unitary leptonic mixing

- There is a unique gauge invariant d=6 effective operator which leads to non-canonical kinetic terms only for the neutrinos:

$$\boxed{\delta\mathcal{L}^{d=6} = c^{d=6}_{\alpha\beta}\,\left(\overline{L}_\alpha\tilde{\phi}\right) i\partial\!\!\!/ \left(\tilde{\phi}^\dagger L_\beta\right)}$$

- After EW symmetry breaking it results in a non-unitary leptonic mixing matrix with:

De Gouvea, Giudice, Strumia, Tobe ('01), Broncano, Gavela, Jenkins ('02)

$$\boxed{|NN^\dagger - 1|_{\alpha\beta} = \frac{v^2}{2}\,|c^{d=6}|_{\alpha\beta}}$$

S.A., Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon ('06)

- + modification of the NC interaction shown earlier ...

Non-unitary leptonic mixing in the MUV scheme

- A minimal way to introduce neutrino masses and non-unitary leptonic mixing thus consists in adding a d=5 and a d=6 operator to the SM:

$$\boxed{\mathcal{L}^{eff} = \mathcal{L}_{SM} + \delta\mathcal{L}^{d=5} + \delta\mathcal{L}^{d=6} + \ldots}$$

MUV scheme: Minimal Unitarity Violation

S.A., Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon ('06)

Neutrino masses (violates L)

$$\delta\mathcal{L}^{d=5} = \frac{1}{2}\, c^{d=5}_{\alpha\beta} \left(\overline{L^c}_\alpha \tilde{\phi}^*\right) \left(\tilde{\phi}^\dagger L_\beta\right) + h.c.$$

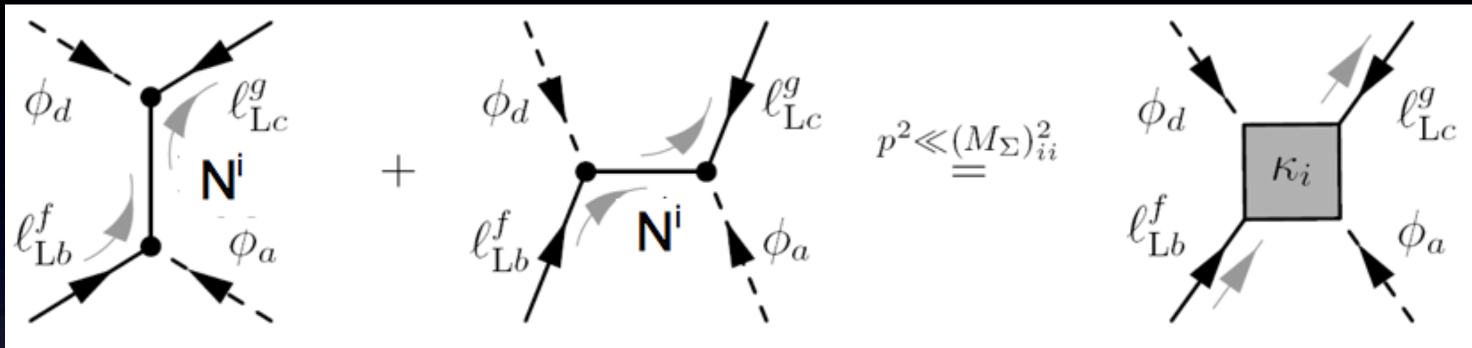
Non-unitarity (conserves L)

$$\delta\mathcal{L}^{d=6} = c^{d=6}_{\alpha\beta} \left(\overline{L}_\alpha \tilde{\phi}\right) i\partial\!\!\!/ \left(\tilde{\phi}^\dagger L_\beta\right)$$

not necessarily suppressed by the smallness of the neutrino masses

Generic scenario: SM + heavy neutral leptons

Integrating out the heavy neutral leptons (= sterile neutrinos) generates dominantly the two effective operators:

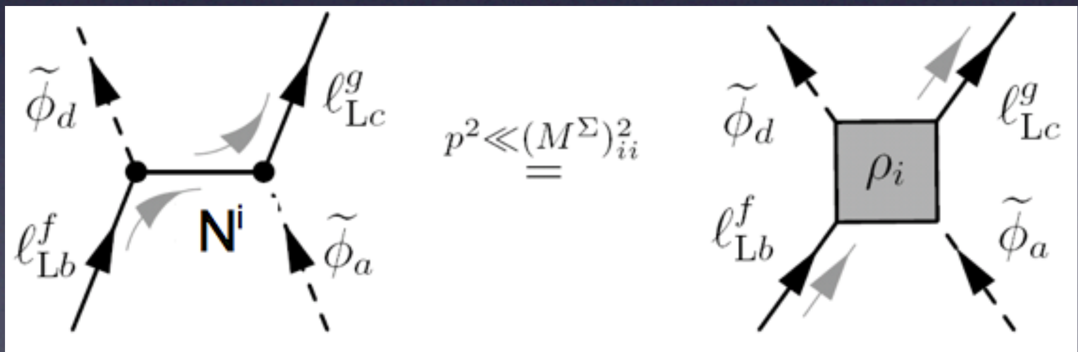


Neutrino mass operator (d=5)

$$\delta\mathcal{L}^{d=5} = \frac{1}{2}\, c^{d=5}_{\alpha\beta} \left(\overline{L^c}_\alpha \tilde{\phi}^*\right) \left(\tilde{\phi}^\dagger L_\beta\right) + h.c.$$

$$c^{d=5}_{\alpha\beta} = (Y_N^T)_{\alpha I} (M_N)^{-1}_{IJ} (Y_N)_{J\beta}$$

$$(m_\nu)_{\alpha\beta} = -\frac{v_{\mathrm{EW}}^2}{2} c^{d=5}_{\alpha\beta}$$

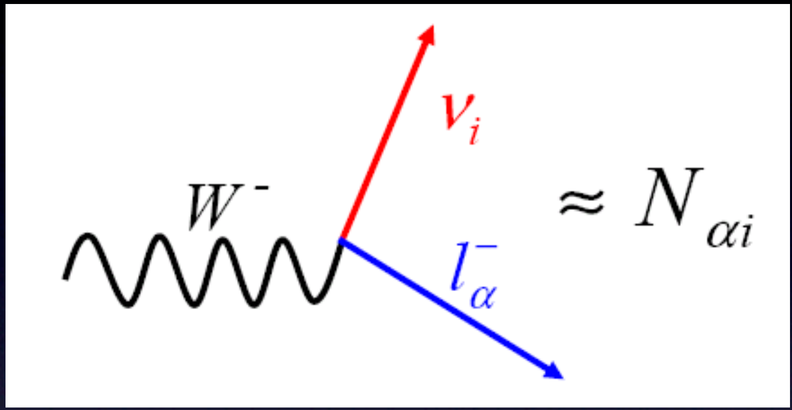


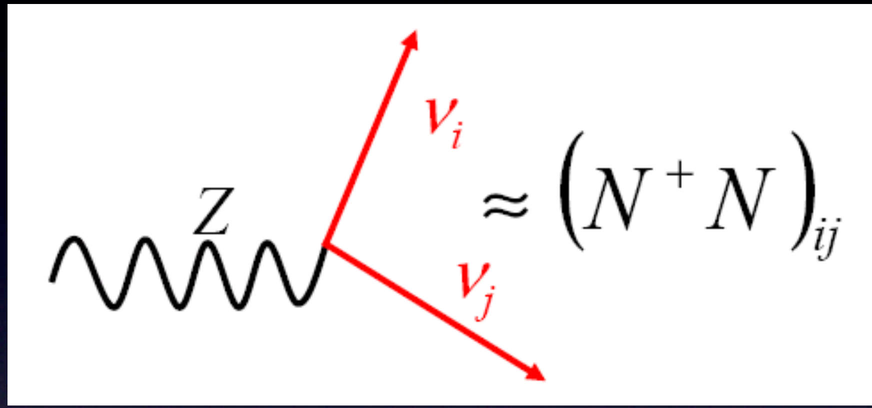
Effective (d=6) operator which modifies the kin. terms of the neutrinos

$$\delta\mathcal{L}^{d=6} = c^{d=6}_{\alpha\beta} \left(\overline{L}_\alpha \tilde{\phi}\right) i\partial\!\!\!/ \left(\tilde{\phi}^\dagger L_\beta\right)$$

$$c^{d=6}_{\alpha\beta} = \sum_I (Y_N^\dagger)_{\alpha I} (M_N)^{-2}_{II} (Y_N)_{I\beta}$$

EW interactions modified





Definition:
Non-unitarity parameters

$$\boxed{(NN^\dagger)_{\alpha\beta} = (1_{\alpha\beta} + \varepsilon_{\alpha\beta})}$$

(Hermitean matrix)

- Theory predictions for various observables which involve weak interactions get modified!

Two comments ...

- Within the MUV scheme, one can show that the off-diagonal non-unitarity parameters satisfy the following inequality:

$$|\varepsilon_{\alpha\beta}| \leq \sqrt{|\varepsilon_{\alpha\alpha}||\varepsilon_{\beta\beta}|} .$$

S.A., E. Fernandez-Martinez, J. Baumann (arXiv:0807.1003)

This can provide an interesting consistency check of the MUC scheme!

- Furthermore, the diagonal non-unitarity parameters are always ≤ 0:

$$\varepsilon_{\alpha\alpha} \leq 0 .$$

Constraints on Leptonic Non-Unitarity: Most relevant processes

- Electroweak precision observables (EWPOs)
- Universalitiy tests (at low E and via W decays)
- Charged LFV decays

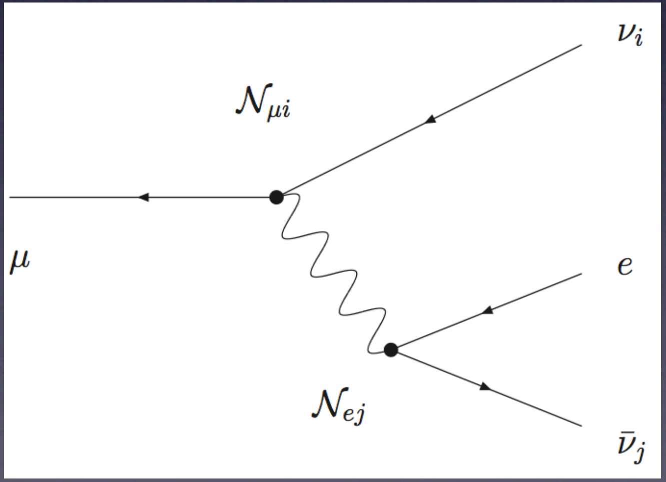
Electroweak precision observables

- Remark: As usual, one expresses the predictions for the EWPOs in terms of the very well measured quantities

$$
\begin{aligned}
\alpha(m_z)^{-1} &= 127.944(14)\ , \\
G_F &= 1.1663787(6)\ \times 10^{-5}\mathrm{GeV}^{-2}\ , \\
m_Z &= 91.1875(21)\ ,
\end{aligned}
$$

- However: G_F is measured in muon decays ... and thus gets modified due to the non-unitary mixing N:

$$G_\mu^2 = G_F^2 (NN^\dagger)_{\mu\mu}(NN^\dagger)_{ee}$$



$\longrightarrow$ affects many observables

Summary of modified EWPO predictions

Present constraints

Prediction in MUV	Prediction in the SM	Experiment
$[R_\ell]_{\rm SM}\,(1 - 0.15(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	20.744(11)	20.767(25)
$[R_b]_{\rm SM}\,(1 + 0.03(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	0.21577(4)	0.21629(66)
$[R_c]_{\rm SM}\,(1 - 0.06(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	0.17226(6)	0.1721(30)
$[\sigma^0_{had}]_{\rm SM}\,(1 - 0.25(\varepsilon_{ee} + \varepsilon_{\mu\mu}) - 0.27\,\varepsilon_{\tau\tau})$	41.470(15) nb	41.541(37) nb
$[R_{inv}]_{\rm SM}\,(1 + 0.75(\varepsilon_{ee} + \varepsilon_{\mu\mu}) + 0.67\,\varepsilon_{\tau\tau})$	5.9721(10)	5.942(16)
$[M_W]_{\rm SM}(1 - 0.11(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	80.359(11) GeV	80.385(15) GeV
$[\Gamma_{\rm lept}]_{\rm SM}(1 - 0.59(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	83.966(12) MeV	83.984(86) MeV
$[(s^{\ell,{\rm lep}}_{W,{\rm eff}})^2]_{\rm SM}(1 + 0.71(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	0.23150(1)	0.23113(21)
$[(s^{\ell,{\rm had}}_{W,{\rm eff}})^2]_{\rm SM}(1 + 0.71(\varepsilon_{ee} + \varepsilon_{\mu\mu}))$	0.23150(1)	0.23222(27)

S.A., O. Fischer (1407.6607)

Sensitivity via G_F!

Other sensitive probes: Universality tests

Consider ratios of observables of the form:

$$R_{\alpha\beta} = \sqrt{\frac{(NN^\dagger)_{\alpha\alpha}}{(NN^\dagger)_{\beta\beta}}} \simeq 1 + \frac{1}{2}(\varepsilon_{\alpha\alpha} - \varepsilon_{\beta\beta})$$

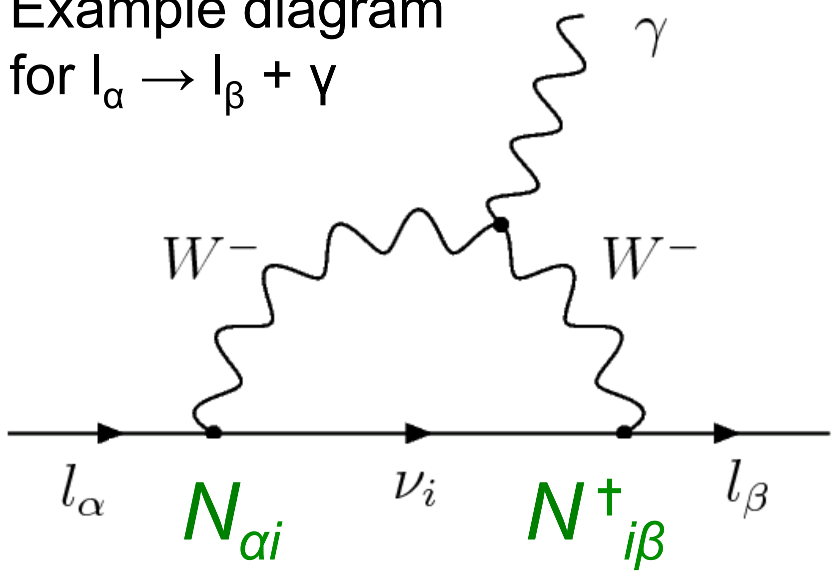
	Process	Bound
$R^{\ell}_{\mu e}$	$\frac{\Gamma(\tau \to \nu_\tau \mu \bar{\nu}_\mu)}{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)}$	$1.0018(14)$
$R^{\ell}_{\tau\mu}$	$\frac{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)}{\Gamma(\mu \to \nu_\mu e \bar{\nu}_e)}$	$1.0006(21)$
$R^{W}_{e\mu}$	$\frac{\Gamma(W \to e \bar{\nu}_e)}{\Gamma(W \to \mu \bar{\nu}_\mu)}$	$1.0085(93)$
$R^{W}_{\tau\mu}$	$\frac{\Gamma(W \to \tau \bar{\nu}_\tau)}{\Gamma(W \to \mu \bar{\nu}_e)}$	$1.032(11)$

	Process	Bound
$R^{\pi}_{\mu e}$	$\frac{\Gamma(\pi \to \mu \bar{\nu}_\mu)}{\Gamma(\pi \to e \bar{\nu}_e)}$	$1.0021(16)$
$R^{\pi}_{\tau\mu}$	$\frac{\Gamma(\tau \to \nu_\tau \pi)}{\Gamma(\pi \to \mu \bar{\nu}_\mu)}$	$0.9956(31)$
$R^{K}_{\tau\mu}$	$\frac{\Gamma(\tau \to K \nu_\tau)}{\Gamma(K \to \mu \bar{\nu}_\mu)}$	$0.9852(72)$
$R^{K}_{\tau e}$	$\frac{\Gamma(\tau \to K \nu_\tau)}{\Gamma(K \to e \bar{\nu}_e)}$	$1.032(11)$

Constraints from LFV

- Bounds on LFV μ and τ decays $l_i \to l_j\, \gamma$ (and on $\mu \to 3e$ and μ → e conversion in nuclei) lead to constraints on the $|\varepsilon_{\alpha\beta}|$:

Example diagram for $l_\alpha \to l_\beta + \gamma$



$$\frac{\Gamma(\ell_\alpha \to \ell_\beta \gamma)}{\Gamma(\ell_\alpha \to \nu_\alpha \ell_\beta \overline{\nu}_\beta)} = \frac{3\alpha}{32\pi} \frac{|\sum_k N_{\alpha k} N^\dagger_{k\beta} F(x_k)|^2}{(NN^\dagger)_{\alpha\alpha}(NN^\dagger)_{\beta\beta}}$$

irrelevant for unitary mixing matrix, but can lead to sizable Br's for non-unitary N!

$$F(x) \equiv \frac{\boxed{10} - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \ln x}{3(x-1)^4}$$

where:

$$x_k \equiv m_k^2/M_W^2$$

m_k: light neutrinos' masses

Present constraints

S.A., O. Fischer (1407.6607)

- We performed a global MCMC fit of the six MUV parameters

$$\boxed{\varepsilon_{ee},\ \varepsilon_{\mu\mu},\ \varepsilon_{\tau\tau},\ |\varepsilon_{e\mu}|,\ |\varepsilon_{e\tau}|,\ |\varepsilon_{\mu\tau}|}$$

to the most relevant 34 observables (including also tests of CKM matrix unitarity, ...).

Results of a global fit to the present data

S.A., O. Fischer (1407.6607)

- Highest posterior density intervals at 90% Bayesian C.L.:

$-0.0021 \leq \varepsilon_{ee} \leq -0.0002$	$\lvert\varepsilon_{e\mu}\rvert < 1.0 \times 10^{-5}$
$-0.0004 \leq \varepsilon_{\mu\mu} \leq 0$	$\lvert\varepsilon_{e\tau}\rvert < 2.1 \times 10^{-3}$
$-0.0053 \leq \varepsilon_{\tau\tau} \leq 0$	$\lvert\varepsilon_{\mu\tau}\rvert < 8.0 \times 10^{-4}$

$\rightarrow$ treated as constraints

constraints from charged LFV

Results of a global fit to the present data

S.A., O. Fischer (1407.6607)

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-0.0053	$\leq \varepsilon_{\tau\tau}$	0	$\lvert\varepsilon_{\mu\tau}\rvert$	$<$	8.0×10^{-4}

$\rightarrow$ treated as cons

constraints from charged LFV

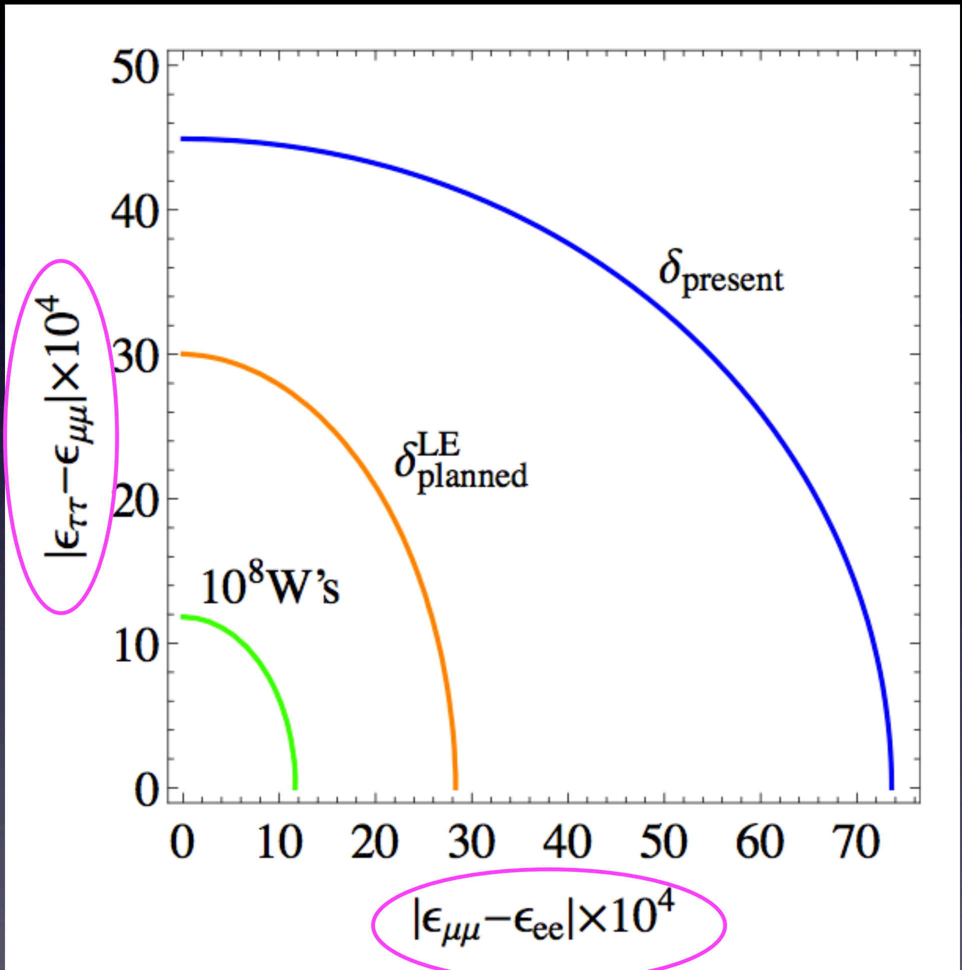
Interesting: Non-zero best-fit point for ε_{ee}

$\Rightarrow$ ε_{ee} = - 0.0012

(but of course only a mild indication of non-zero ε_{ee}, at the level of ~ 2σ)

How much can the CEPC improve the sensitivity to leptonic non-unitarity?

CEPC: Very sensitive tests of universality with W decays



S.A., O. Fischer (1407.6607)

- Blue line: Present constraints
- Orange line: Planned exprimental sensitivity of universality tests at low E [MOLLER, TRIUMF, PSI, NA62, Tau/Charm factories]
- Green line: Sensitivity (statistical uncertainties only) with 10^8 W bosons [as possible at the CEPC]

universality tests probe the differences between the $\varepsilon_{\alpha\alpha}$

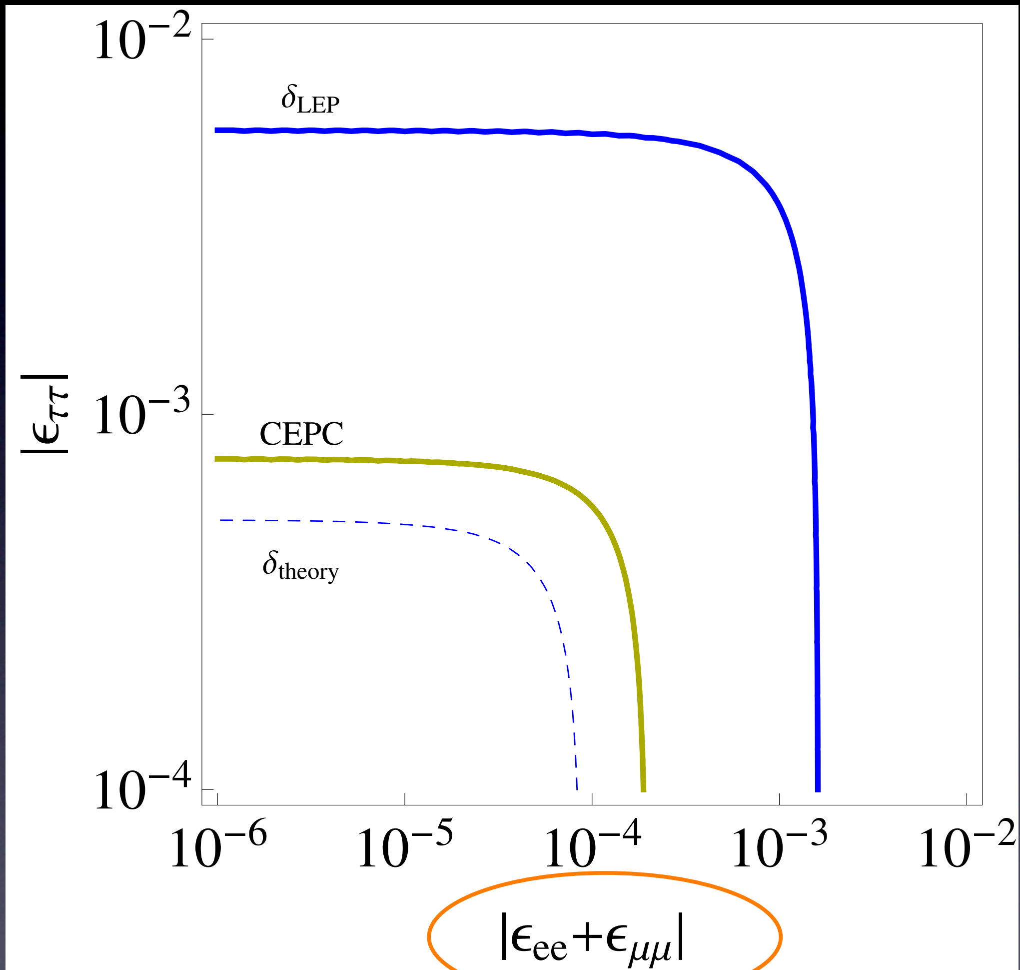
CEPC: Greatly improved precision for EWPOs

With 10^{11} Z bosons produced at CEPC

Observable	LEP precision	CEPC CDR
M_W [MeV]	33	3
$\sin^2\theta_W^{\rm eff}$	0.07%	0.01%
R_b	0.3%	0.08%
R_c	0.3%	0.07%
R_{inv}	0.27%	8.9×10^{-4}
R_ℓ	0.1%	0.1%
Γ_ℓ	0.1%	0.1%
σ_h^0 [nb]	8.9×10^{-4}	1×10^{-4}

- Values taken from the CEPC pre-CDR where available (otherwise estimates were used)

Sensitivities of future EWPO measurements



- For Yukawa couplings y_α = O(1), e.g., the CEPC can probe sterile neutrinos with masses up to M ~ 12 TeV!

[Using connection to sterile neutrino parameters in minimal symmetry protected seesaw scenarios (SPSS):

$$\varepsilon_{\alpha\beta} = - y_\alpha^* \, y_\beta \, v_{EW}^2/(2 M^2)$$]

cf. S.A., O. Fischer (1407.6607)

The EWPOs are very sensitive to the sum $|\varepsilon_{ee} + \varepsilon_{\mu\mu}|$

Summary

- Non-unitarity of the leptonic mixing matrix is a generic (and rather model-independent) consequence of new physics in the neutrino sector to generate the observed neutrino masses.

- Effective field theory: MUV (“Minimal Unitarity Violation”)
  ➔ represents a large class of models

- EWPOs and universality tests at the CEPC (with pre-CDR parameters) can probe leptonic non-unitarity with very high sensitivity (i.e. sums and differences of $\varepsilon_{\alpha\alpha}$ up to $O(10^{-4})$).

… as announced, I have a few more slides ➔

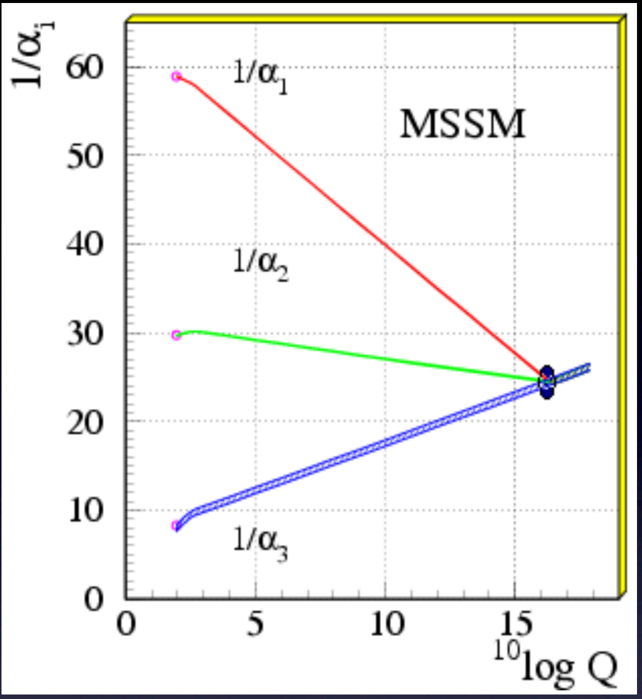
A different topic (in 2 slides):

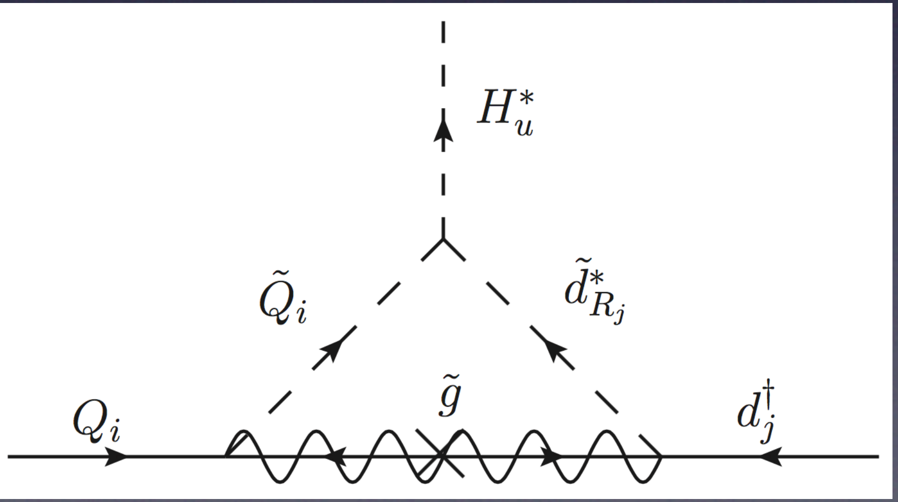
Comment on the motivation for SUSY at the SppC

Predicting the SUSY spectrum from GUTs

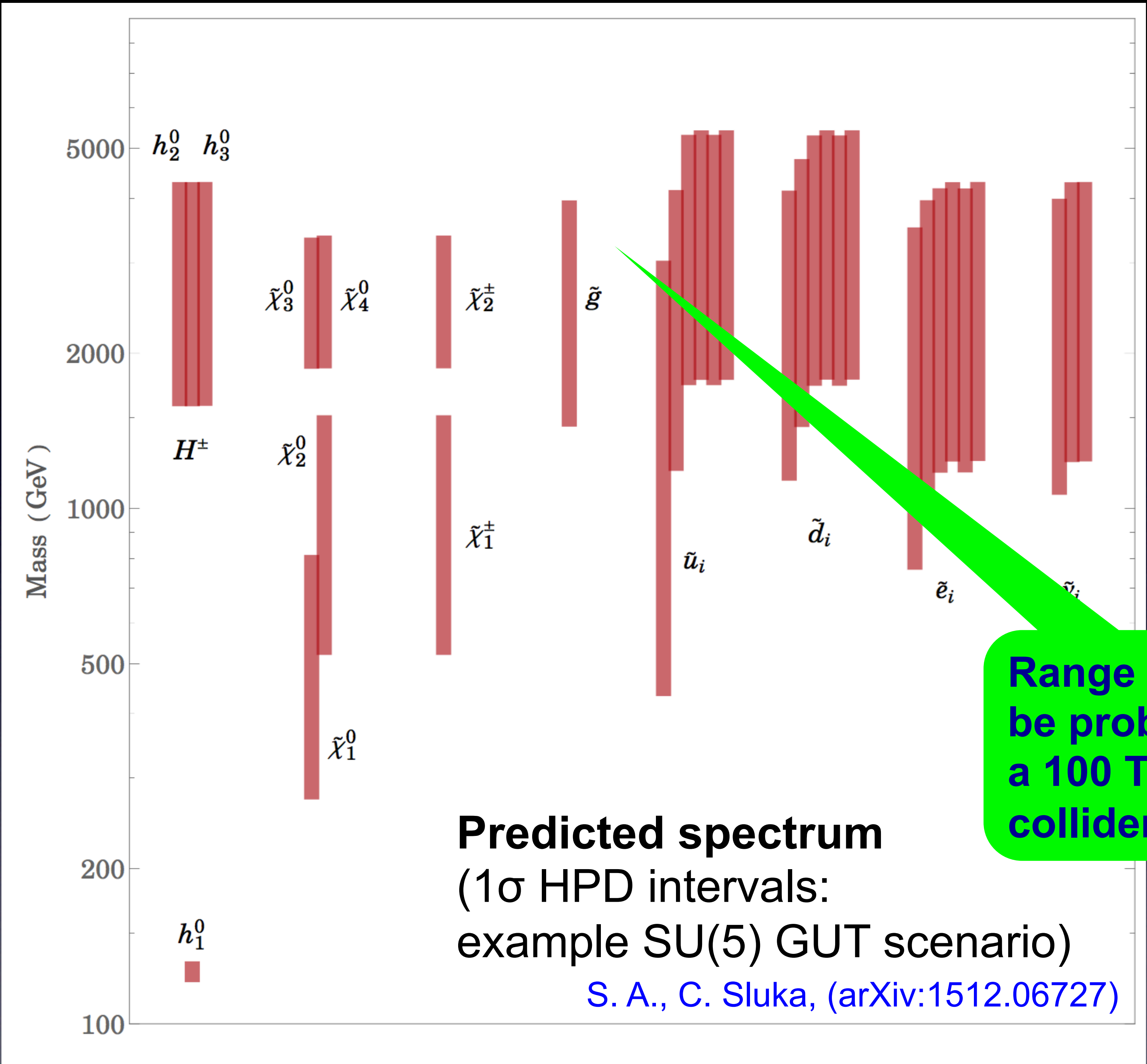
- GUTs not only predict gauge coupling unification, but also unification of quarks and leptons in joint multiplets at high energy
  ➔ Predictions for the Yukawa coupling ratios, e.g. y_τ/y_b,= 1 or 3/2, y_μ/y_s = 3, 9/2 or 6, y_e/y_d = ½ or ⅓

See e.g.: Georgi, Jarlskog ('79), S.A., Spinrath ('09)
S.A., King, Spinrath ('13)





- Performing the RG running to low energies and including the SUSY threshold corrections, the predictions can be compared to the experimental data on quark and lepton masses
  ➔ **Link to the sparticle spectrum**



Note: No contraints from present SUSY searches applied. Sparticles out of present LHC reach is a result!

Range can be probed at a 100 TeV pp collider

Predicted spectrum
(1σ HPD intervals: example SU(5) GUT scenario)

S. A., C. Sluka, (arXiv:1512.06727)

New tool: **SusyTC**

Thanks for your attention!

Extra slides

Future sensitivities to the $\varepsilon_{\alpha\beta}$ from LFV

- Estimated sensitivities of planned experiments at 90% C.L.:

Process	MUV Prediction	Bound	Sensitivity
$Br_{\tau e}$	$4.3 \times 10^{-4} \lvert\varepsilon_{\tau e}\rvert^2$	10^{-9}	$\varepsilon_{\tau e} \geq 1.5 \times 10^{-3}$
$Br_{\tau \mu}$	$4.1 \times 10^{-4} \lvert\varepsilon_{\tau \mu}\rvert^2$	10^{-9}	$\varepsilon_{\tau \mu} \geq 1.6 \times 10^{-3}$
$Br_{\mu eee}$	$1.8 \times 10^{-5} \lvert\varepsilon_{\mu e}\rvert^2$	10^{-16}	$\varepsilon_{\mu e} \geq 2.4 \times 10^{-6}$
$R_{\mu e}^{Ti}$	$1.5 \times 10^{-5} \lvert\varepsilon_{\mu e}\rvert^2$	2×10^{-18}	$\varepsilon_{\mu e} \geq 3.6 \times 10^{-7}$

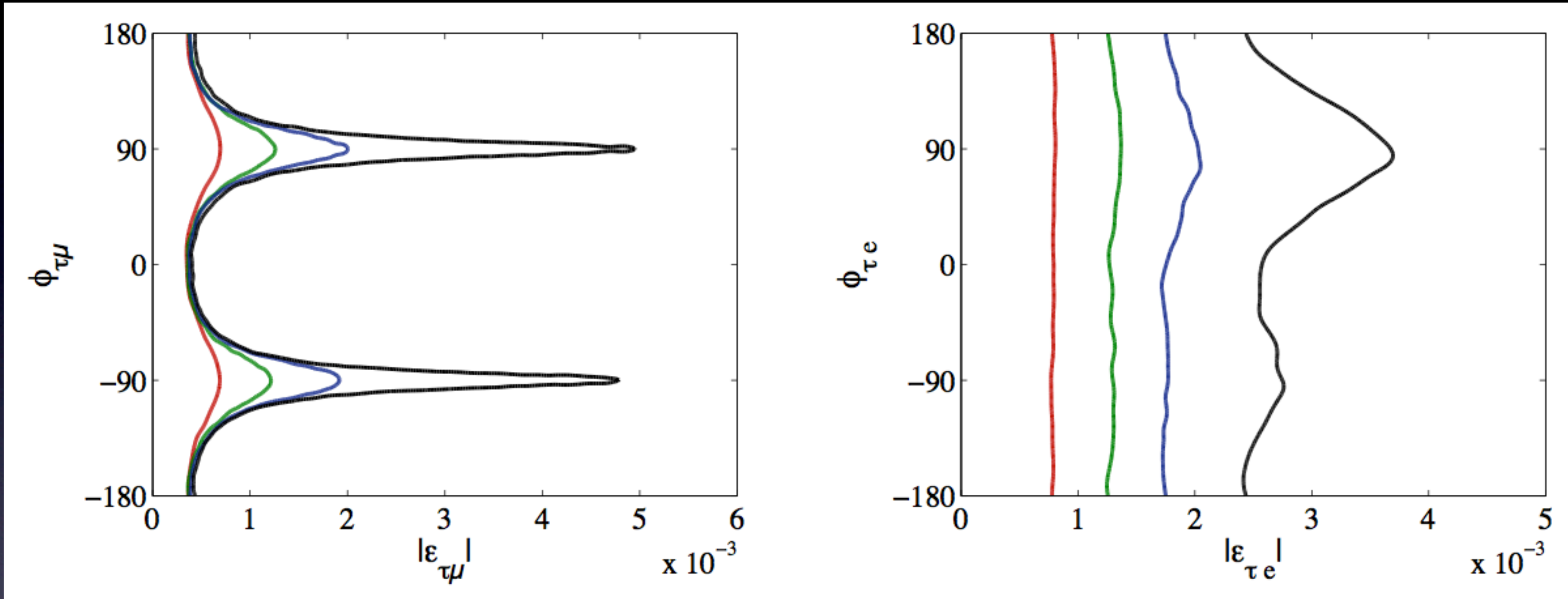
Note: A measurement of LFV would guarantee non-zero $\varepsilon_{\alpha\alpha}$

S.A., O. Fischer (1407.6607)

- Also very sensitive at the CEPC: cLFV Z decays Z → l_1 l_2

cf. Abada, Dr Romeri, Monteil, Orloff, Teixeira (1412.6322)

Possible sensitivity also to the phases of $\varepsilon_{\alpha\beta}$ at a neutrino factory



- Interplay of (tau-sensitive) near and far detectors could provide information on the phase of the MUV parameters $\varepsilon_{\tau\mu}$ and $\varepsilon_{\tau e}$ (i.e. on the phases of - $\theta_\tau^*\theta_\mu$ and - $\theta_\tau^*\theta_e$)

S.A., M. Blennow, E. Fernandez-Martinez, J. Lopez-Pavon (arXiv:0903.3986)

Example: Precision vs. direct tests in the “Symmetry Protected Seesaw Scenario (SSMS)”

cf.: S.A., O. Fischer (arXiv:1502.05915)

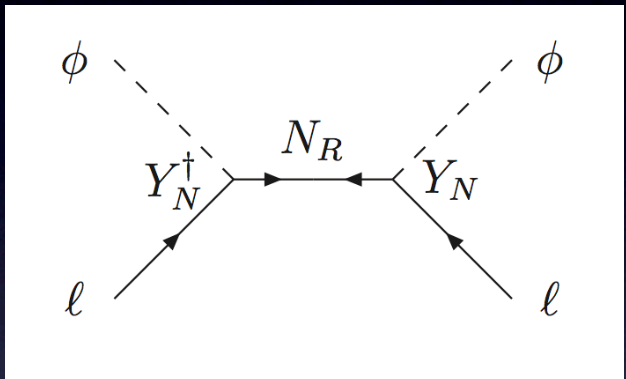
Sterile neutrinos with masses ~ EW scale

Similar: “inverse” seesaw, “linear” seesaw

Symmetry Protected low scale Seesaw Scenario (SSPS)

In the symmetry limit:

$$\mathscr{L}_N = - \overline{N_R}^1 M \, N_R^{c\,2} - y_\alpha \overline{N_R}^1 \tilde{\phi}^\dagger L^\alpha + \mathrm{H.c.}$$



Parameters:
M, y_α, (α=1,2,3)

+ possible additional “decoupled” sterile neutrinos, which do not mix with the active neutrinos in the symmetry limit

E.g.:

$$M_N = \begin{pmatrix} 0 & M \\ M & \mu \end{pmatrix}$$

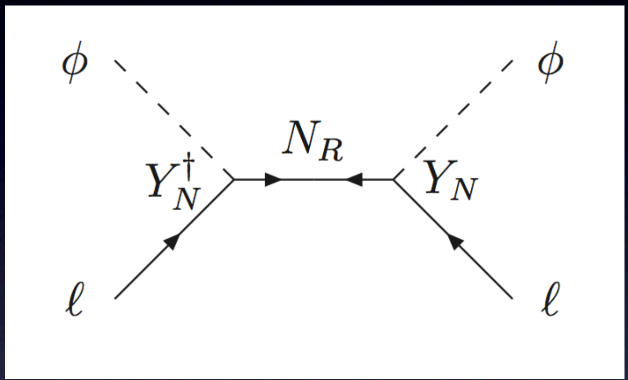
Minimal: Two sterile neutrinos forming a pseudo-Dirac pair

→ Assumption: The two sterile neutrinos N_R^1 and N_R^2 dominate phenomenology

Sterile neutrinos with masses ~ EW scale

Similar: “inverse” seesaw, “linear” seesaw

Symmetry Protected low scale Seesaw Scenario (SSPS)



E.g.:

$$M_N = \begin{pmatrix} 0 & M \\ M & \mu \end{pmatrix}$$

Parameters:
M, y_α, (α=1,2,3)
or equivalently
M, θ_α, (α=1,2,3)

In the symmetry limit:

$$\boxed{\mathscr{L}_N = - \overline{N_R}^{1} M \; N_R^{c\,2} - y_\alpha \overline{N_R}^{1} \widetilde{\phi}^\dagger L^\alpha + \text{H.c.}}$$

- The leptonic mixing matrix to leading order in the active-sterile mixing parameters:

$$U = \begin{pmatrix} \mathcal{N}_{1e} & \mathcal{N}_{1\mu} & \mathcal{N}_{1\tau} & -\frac{\mathrm{i}}{\sqrt{2}}\theta_e & \frac{1}{\sqrt{2}}\theta_e \\ \mathcal{N}_{2e} & \mathcal{N}_{2\mu} & \mathcal{N}_{2\tau} & -\frac{\mathrm{i}}{\sqrt{2}}\theta_\mu & \frac{1}{\sqrt{2}}\theta_\mu \\ \mathcal{N}_{3e} & \mathcal{N}_{3\mu} & \mathcal{N}_{3\tau} & -\frac{\mathrm{i}}{\sqrt{2}}\theta_\tau & \frac{1}{\sqrt{2}}\theta_\tau \\ 0 & 0 & 0 & \frac{\mathrm{i}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\theta_e^* & -\theta_\mu^* & -\theta_\tau^* & \frac{-\mathrm{i}}{\sqrt{2}}(1-\frac{1}{2}\theta^2) & \frac{1}{\sqrt{2}}(1-\frac{1}{2}\theta^2) \end{pmatrix}$$

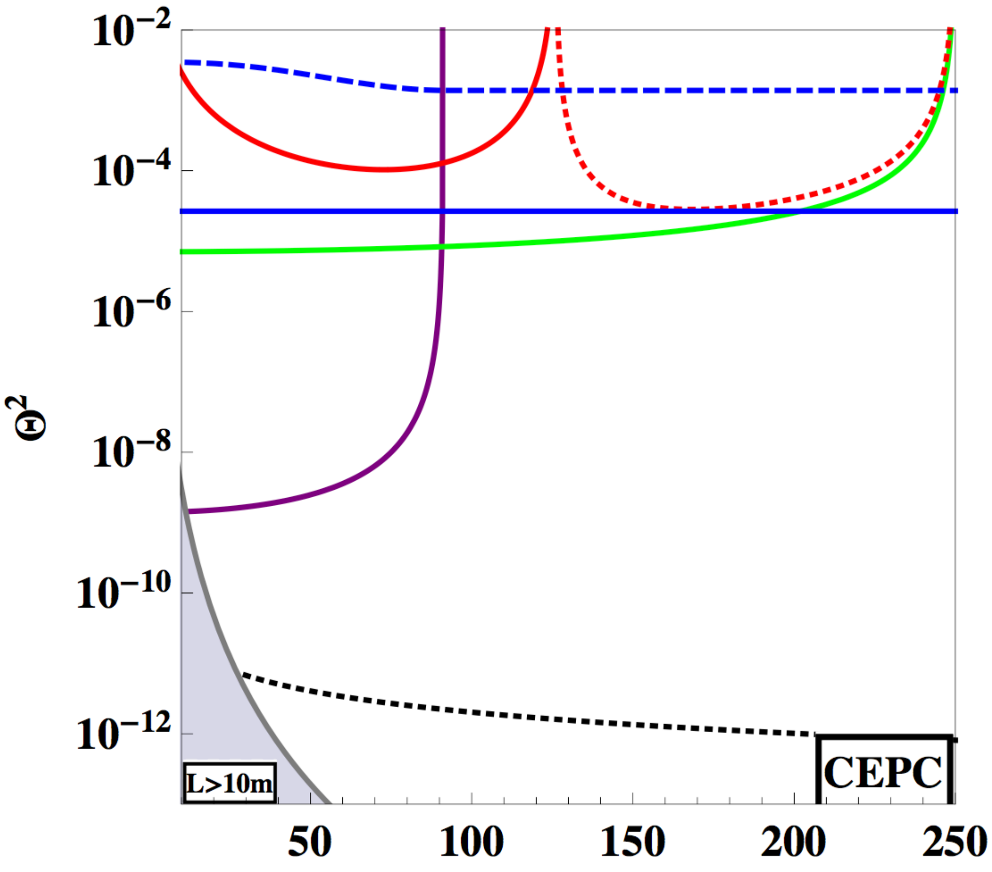
- Active-sterile neutrino mixing parameters:

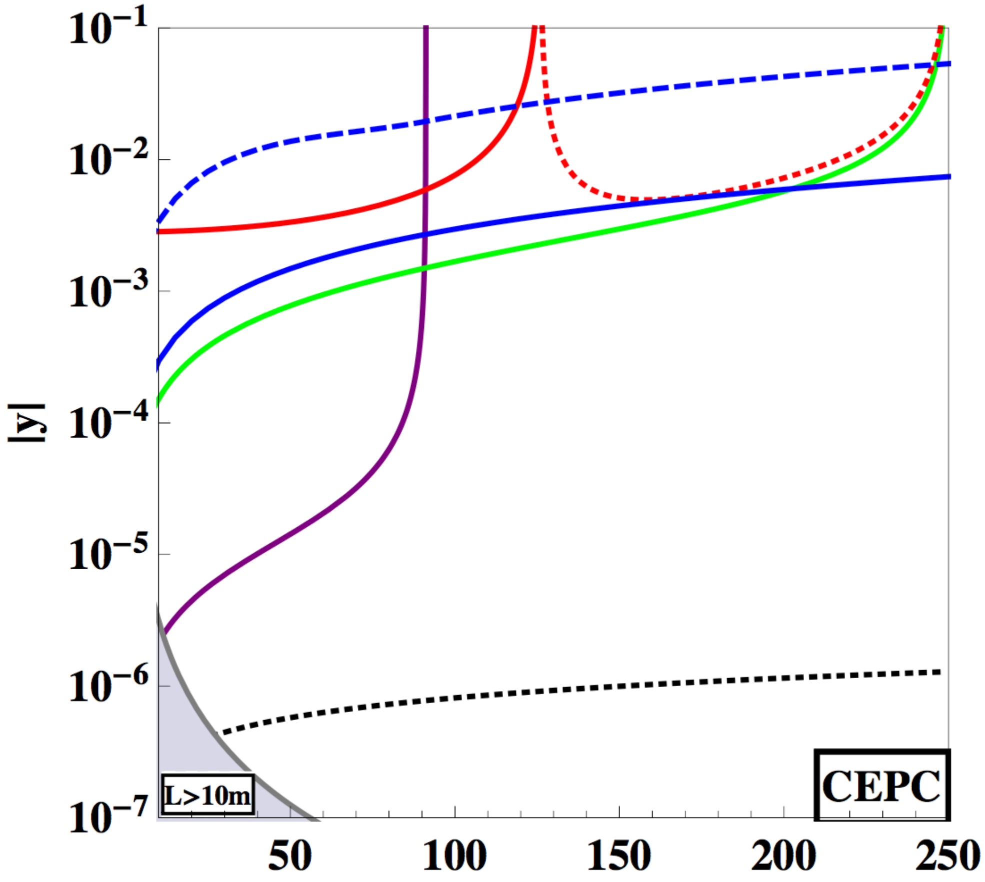
$$\theta_\alpha = \frac{y_\alpha^*}{\sqrt{2}}\frac{v_{\rm EW}}{M}, \qquad \alpha = e, \mu, \tau$$

At low energy: Relation to parameters of the effective theory (MUV)

	y_{ν_α}	θ_α	$\varepsilon_{\alpha\beta}$
$y_{\nu_\alpha} =$	$-$	$\frac{\sqrt{2}M}{v_{\mathrm{EW}}}\theta_\alpha^*$	$-\frac{\sqrt{2}M}{v_{\mathrm{EW}}}\varepsilon_{\beta\alpha}/\sqrt{-\varepsilon_{\beta\beta}}$
$\theta_\alpha =$	$\frac{v_{\mathrm{EW}}}{\sqrt{2}M}y_{\nu_\alpha}^*$	$-$	$-\varepsilon_{\beta\alpha}/\sqrt{-\varepsilon_{\beta\beta}}$
$\varepsilon_{\alpha\beta} =$	$-\frac{v_{\mathrm{EW}}^2 y_{\nu_\alpha}^* y_{\nu_\beta}}{2M^2}$	$-\theta_\alpha^*\theta_\beta$	$-$

Comparison: Non-unitarity (precision) vs direct probes





S.A., O. Fischer (arXiv:1502.05915)

Direct searches

- Z pole search @2σ: $|y|=\sqrt{\sum_\alpha |y_{\nu_\alpha}|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- Higgs $\to$ WW @1σ: $|y|=\sqrt{\sum_\alpha |y_{\nu_\alpha}|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- $e^+e^- \to h + ME_{(T)}$ @1σ: $|y|=|y_{\nu_e}|$, $\Theta^2=|\theta_e|^2$
- $e^+e^- \to l\nu l\nu^*$ @1σ: $|y|=|y_{\nu_e}|$, $\Theta^2=|\theta_e|^2$

Other

- Precision constraints: $|y|=\sqrt{|y_{\nu_e}|^2+|y_{\nu_\mu}|^2}$, $\Theta^2=|\theta_e|^2+|\theta_\mu|^2$
- Precision constraints: $|y|=|y_{\nu_\tau}|$, $\Theta^2=|\theta_\tau|^2$
- "Unprotected" type−I seesaw