

Beam-Beam Effects and Luminosity Optimization for e⁺e⁻ Colliders at High Energies

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Introduction

We will discuss the features of beam-beam interaction and luminosity optimization for high energy e^+e^- colliders. The considered energy range: from 45.5 to 175 GeV.

These studies were initiated by development of the FCC-ee project at CERN. Similar problems arise in the CEPC project in China.

The main points of interest are: 45.5 GeV (Z), 80 GeV (W), 120 GeV (H), 175 GeV (tt).

The peculiarity of these new generation of e^+e^- colliders is *high energy* and *high luminosity*. Consequently, a significant role starts to play the effect which has never been recorded in circular colliders, however, were considered in the projects of linear colliders (e.g. ILC): *beamstrahlung*.

The beamstrahlung seriously affects the luminosity optimization at both “high” (120, 175 GeV) and “low” (45.4, 80 GeV) energies. In this presentation we will discuss the most important issues caused by this effect.

Luminosity

For flat beams (both head-on and crossing angle collision):

$$L = \frac{\gamma}{2er_e} \cdot \frac{I_{tot} \xi_y}{\beta_y^*} \cdot R_H \propto \frac{\xi_y}{\beta_y^*}$$

I_{tot} – total beam current (defined by SR power, e.g. 50 MW)

ξ_y – vertical betatron tune shift, its limit depends on the collision scheme

R_H – hour-glass factor: $R_H \approx [0.86, 0.71, 0.60]$ for $L_i/\beta_y^* = [1, 2, 3]$

L_i – length of the interaction area:

$$L_i = \frac{\sigma_z}{\sqrt{1 + \phi^2}} \quad \phi = \frac{\sigma_z}{\sigma_x} \operatorname{tg}\left(\frac{\theta}{2}\right) \quad \text{– Piwinski angle}$$

β_y^* should be minimized, but there are restrictions:

- Beta-function at the final quads raises as $1/\beta_y^*$, that affects dynamic aperture and creates problems with chromaticity corrections.
- L_i should be squeezed to $L_i \sim \beta_y^*$. On the other hand, too short L_i may cause problems with beamstrahlung lifetime.

When performing optimizations, we do not care about the bunch population N_p and the number of bunches N_b . Namely, N_p is adjusted according to beam-beam limit, and it defines N_b (since the total beam current I_{tot} is fixed).

Beamstrahlung

At very high energies, the luminosity is limited by the beamstrahlung lifetime:

$$\tau_{bs} \propto \exp\left(\frac{2\alpha\eta\rho}{3r_e\gamma^2}\right) \cdot \frac{\rho\sqrt{\eta\rho}}{L_i\gamma^2}$$

α – fine structure constant

η – energy acceptance

ρ – average bending radius of particle's trajectory at the IP

Obviously, the major tool for reducing the negative effect of beamstrahlung is making ρ larger. For flat beams, ρ is inversely proportional to the surface charge density:

$$\frac{1}{\rho} \propto \frac{N_p}{\gamma\sigma_x\sigma_z} \propto \frac{\xi_y}{L_i} \sqrt{\frac{\epsilon_y}{\beta_y^*}} \propto L \cdot \sqrt{\frac{\epsilon_y}{\beta_y^*}}$$

The last transformation is based on assumption that $L_i \sim \beta_y^*$. We want to increase the luminosity L while keeping the lifetime (and therefore ρ) large enough. It follows that:

- The vertical emittance (i.e. both the betatron coupling and the horizontal emittance) should be minimized.
- β_y^* (together with L_i) should be *maximized*! What does it mean? Increase of β_y^* by a factor k may result in luminosity gain by $k^{1/2}$ (with ρ unchanged), but ξ_y will grow by $k^{3/2}$.
We can do this until ξ_y remains below the beam-beam limit.

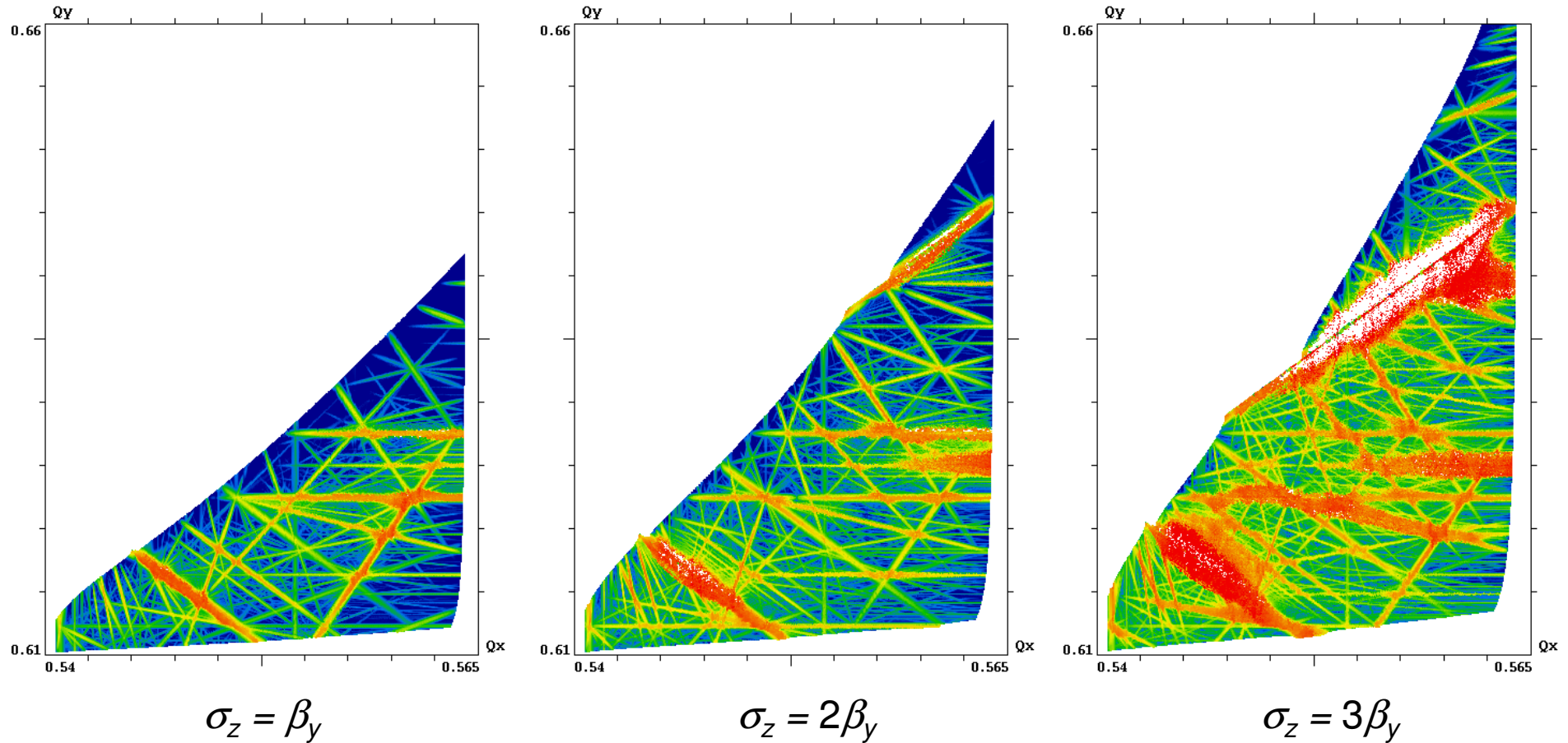
What is the optimum β_y at “high” energies ?

- The general rule for optimization: *if there are multiple limiting factors, maximum performance happens when all limits are reached simultaneously.*
- In our case it means that β_y^* (together with L_i) should be adjusted in such a way that both τ_{bs} and ξ_y achieve their limits.
- It follows that shifting the balance towards “limit by the beamstrahlung lifetime” (e.g. decrease of η , increase of γ , ε_y) will require increase of L_i (together with β_y^*), and vice versa.
Example: $E = 175 \text{ GeV}$, $\eta = 0.02$, $\varepsilon_y = 2.6 \text{ pm} \Rightarrow \beta_y^* \approx 2 \text{ mm}$
- Such adjustment of L_i can be easily done in head-on collision by means of the bunch length.
- In collision with crossing angle L_i is smaller and it is more difficult to control, as it depends on σ_x rather than σ_z . This creates some disadvantage of crossing angle at 175 GeV.

“Low” energies: head-on or crossing angle ?

- At “low” energies (45.5, 80 GeV) τ_{bs} allows to squeeze β_y^* below 1 mm. Then we reach another limitation: lattice of IR, chromaticity correction and DA require $\beta_y^* \geq 1$ mm.
- We want to have large $\xi_y \geq 0.1$ with smaller $\beta_y^* \sim 1$ mm. In this case beamstrahlung again is strong, but it manifests itself in the bunch lengthening, and not in the lifetime ($\eta \sim 0.01$ is enough).
- It is *impossible* to have $\sigma_z \sim 1$ mm and $\xi_y \geq 0.1$ simultaneously: beamstrahlung will make $\sigma_z \sim 2 \div 3$ mm (depends on ξ_y).
- If we increase β_y^* to $2 \div 3$ mm, we lose the luminosity. If we have $\sigma_z / \beta_y^* \sim 2 \div 3$, the hour-glass effect will be very strong in head-on collision, and its negative impact is enhanced by weak damping at low energies.
- **We need crossing angle** to make $\beta_y^* \ll \sigma_z$, if we want to keep hour-glass small.

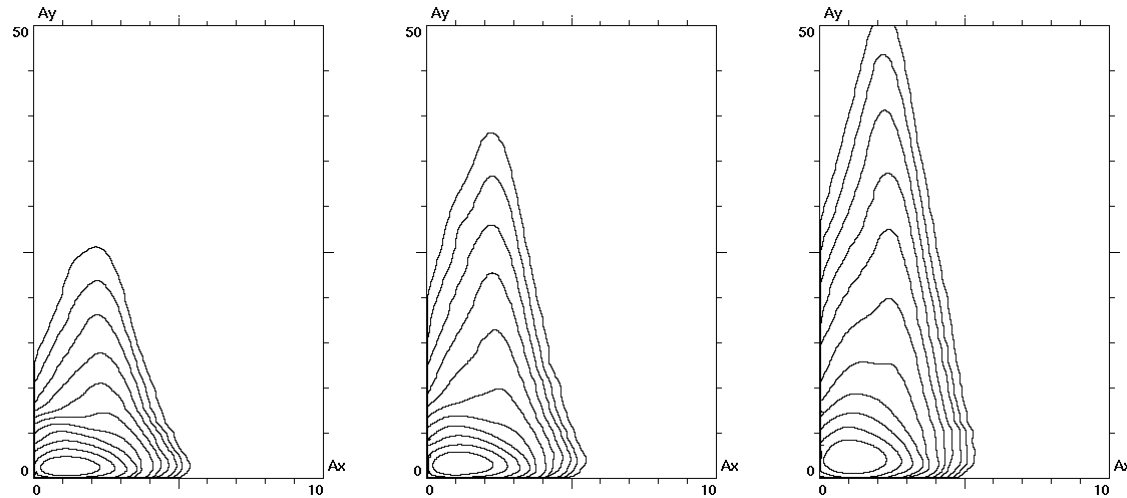
Impact of hour-glass: tune shift



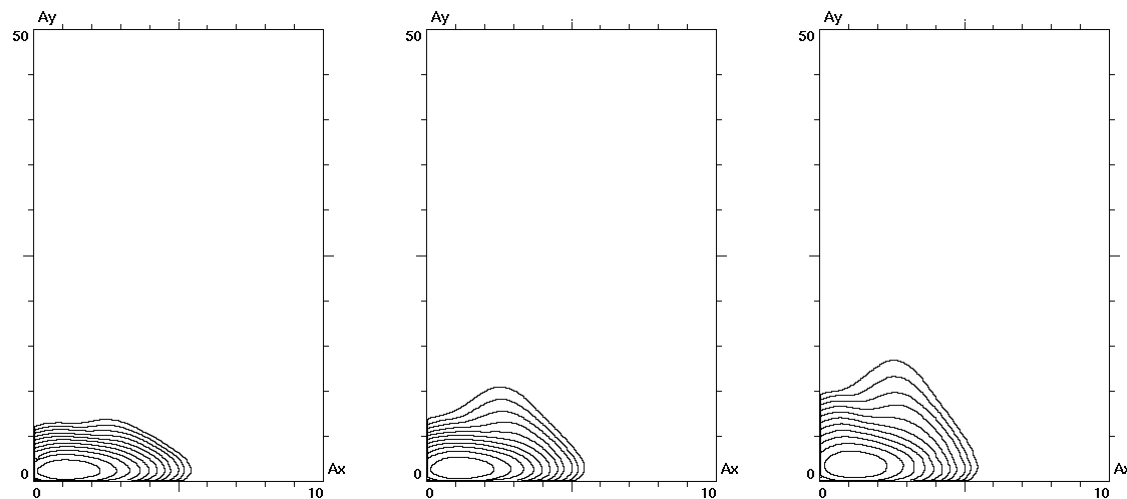
FMA footprints in the plane of betatron tunes, synchrotron amplitude: $A_s = 1$ sigma.

Parameters as for TLEP Z from FCC-ACC-SPS-0004, $\xi_x \approx \xi_y \approx 0.03$ (nominal).

Impact of hour-glass vs. damping



Damping as for 45.5 GeV



Damping as for 120 GeV
(20 times stronger)

$$\sigma_z = \beta_y, R_H \approx 0.86$$

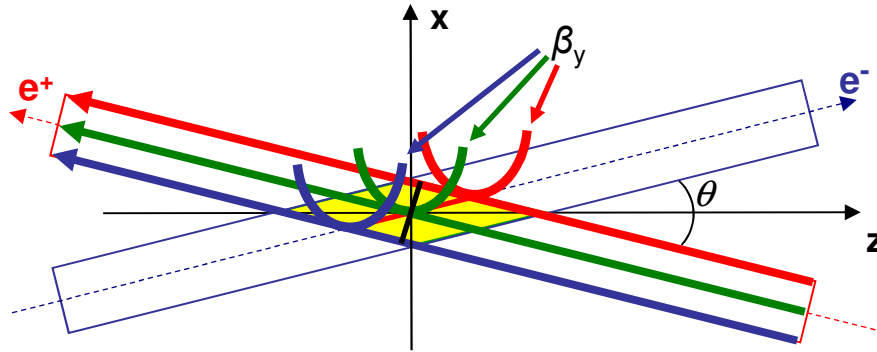
$$\sigma_z = 2\beta_y, R_H \approx 0.71$$

$$\sigma_z = 3\beta_y, R_H \approx 0.60$$

Contour plots of equilibrium distribution in the space of normalized betatron amplitudes. Density between successive contour lines decreases by a factor of e .

Crab Waist scheme

P. Raimondi, 2006



$$\phi = \frac{\sigma_z}{\sigma_x} \operatorname{tg}\left(\frac{\theta}{2}\right) - \text{Piwinski angle}$$

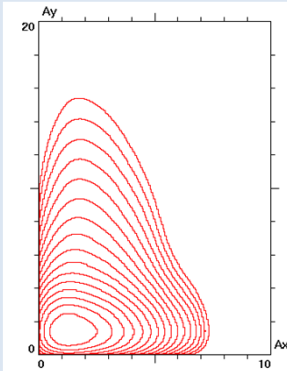
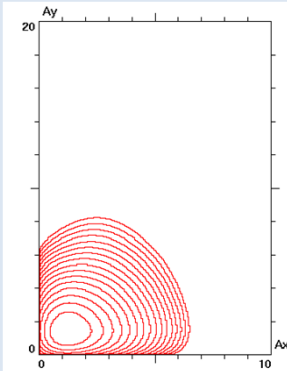
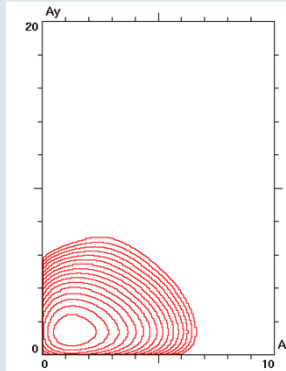
- 1) Large Piwinski angle: $\phi \gg 1$
- 2) β_y approx. equals to overlapping area: $\beta_y \approx \sigma_z / \phi$
- 3) Crab Waist: minimum of β_y along the axis of the opposite beam

Advantages:

- ✓ Impact of hour-glass is small and does not depend on bunch lengthening
- ✓ Suppression of betatron coupling resonances allows to achieve $\xi_y \sim 0.2$

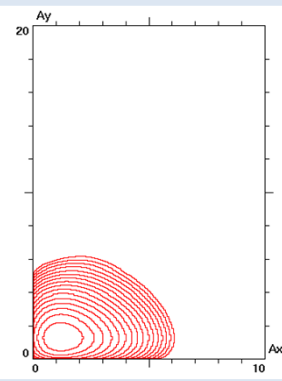
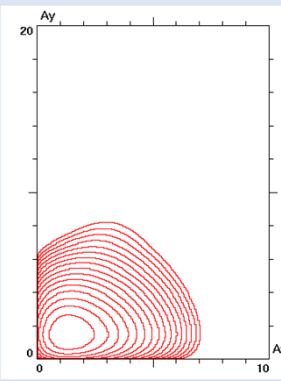
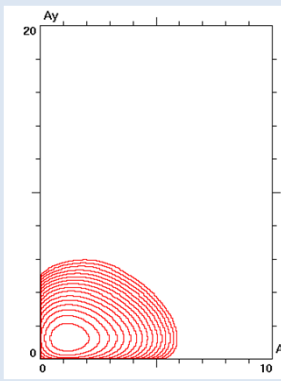
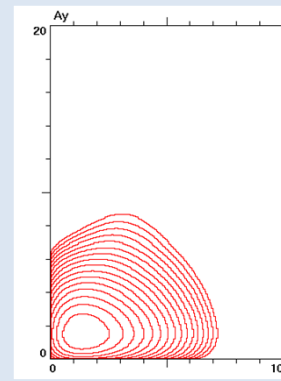
As a result, luminosity can be significantly increased!

FCC-ee @175 GeV, different collision schemes

Collision scheme	Head-on	30 mrad	Crab Waist
N_p	$1.8 \cdot 10^{11}$	$2.1 \cdot 10^{11}$	$2.2 \cdot 10^{11}$
N_b	66	57	54
σ_z / σ_{zbs} [mm]	2.41 / 2.80	2.41 / 2.87	2.41 / 2.89
v_x / v_y	0.56 / 0.61	0.54 / 0.57	0.54 / 0.57
$\Delta v_x / \Delta v_y$	0.126 / 0.141	0.056 / 0.084	0.057 / 0.092
L [$\text{cm}^{-2}\text{s}^{-1}$]	$1.35 \cdot 10^{34}$	$1.06 \cdot 10^{34}$	$1.23 \cdot 10^{34}$
τ_{bs} [min]	30	30	30
Density contour plots $10\sigma_x \times 20\sigma_y$			

Flip-flop @ 175 GeV (30 mrad, crab waist)

triggered by asymmetry in the bunch currents and
the bunch lengthening due to beamstrahlung

Asymmetry	10 %		15 %	
Bunch	“strong”	“weak”	“strong”	“weak”
σ_{zbs} [mm]	2.68	3.06	2.62	3.11
L [cm ⁻² s ⁻¹]	1.09·10 ³⁴		1.02·10 ³⁴	
τ_{bs} [min]	~900	5	> 3000	3
Density contour plots $10\sigma_x \times 20\sigma_y$				

To work at the maximum luminosity, the bunch currents asymmetry must be < 10%.

Flip-flop @ “low” energies

Asymmetry in the bunch currents leads to asymmetry in the bunch lengths (due to beamstrahlung).



Asymmetry in the bunch lengths leads to the “weak” ε_x growth (depends on crossing angle and ξ_x).



To suppress flip-flop, we need to reduce the horizontal emittance growth. This can be achieved by decrease of β_x (and ξ_x decreases in the same proportion).



Due to betatron coupling, vertical emittance of the “weak” bunch also increases.



In simulations, this effect depends on the model: how ε_y is affected by ε_x .



The vertical emittance blowup enhances beamstrahlung for the “weak” bunch, and its lengthening.



Particles which experience the maximum beamstrahlung: $|y| > 2\sigma_y$ (sigma of the opposite beam).



Summary

- Crab Waist collision scheme provides higher luminosity than head-on at “low” energies (45.5, 80 GeV).
- Beamstrahlung is one of the most important factors that affect the collider performance. At “high” energies (120, 175 GeV) it manifests itself mainly in limiting the beam lifetime; at “low” energies (45.5, 80 GeV) – in the bunch lengthening.
- The general recipe to reduce beamstrahlung: decrease the bunch population (while keeping ξ_y large!), therefore increase the number of bunches and decrease emittances.
- Flip-flop instability, which is enhanced by the bunch lengthening due to beamstrahlung, also may limit the luminosity. To avoid flip-flop, β_x^* (which is proportional to ξ_x when $\phi \gg 1$) should be not large.
- The whole optimization process is rather complicated and should be performed with the help of beam-beam simulations in strong-strong (or quasi-strong-strong) model.