Physics at a Higgs Factory

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Based in part on:
arXiv:1411.1054 and 1412.3107
(JiJi Fan, MR, and Lian-Tao Wang)
CEPC pre-CDR

Future e^+e^- Physics

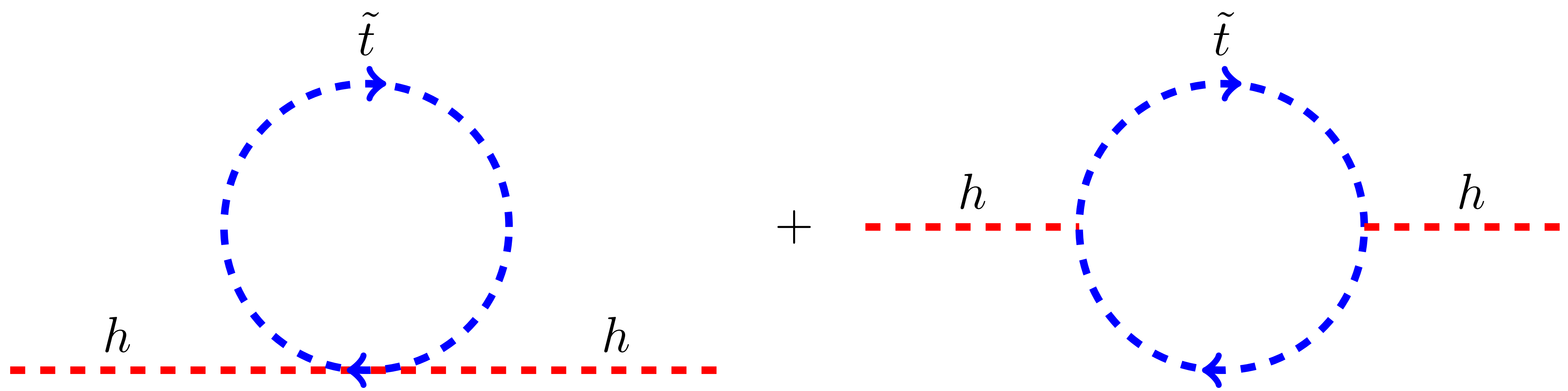
- ILC, FCC-ee, or CEPC will be a precision measurement machine! As a Higgs factory, **measuring Higgs couplings** precisely is a major goal.

- Aside from the “Higgs factory” run, these machines potentially also do *Z*-pole physics and top threshold physics. Part of this talk: **what’s the relative importance of these?**

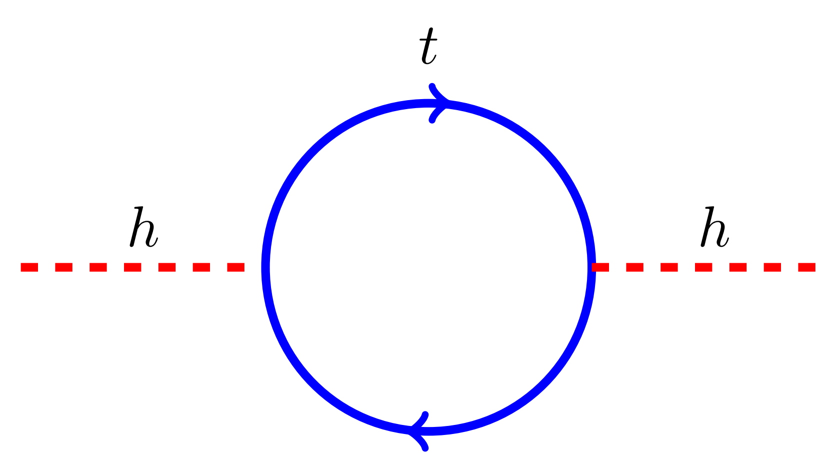
- I also want to give **context**: what could the measurements tell us about what lies beyond the Standard Model?

Stops

To contextualize the results, I'll begin by focusing on one illustrative case for what the new physics could be: stops.

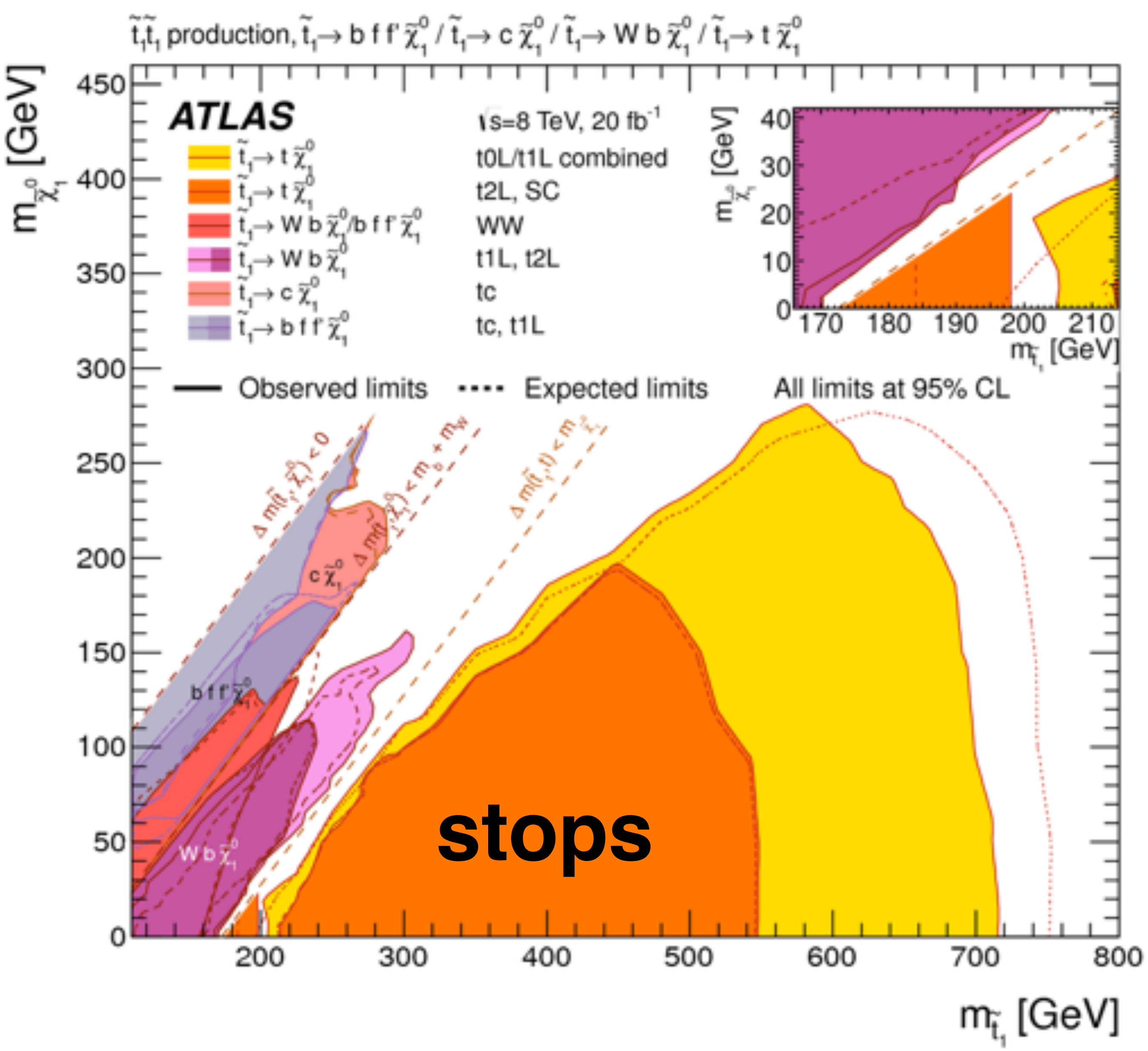


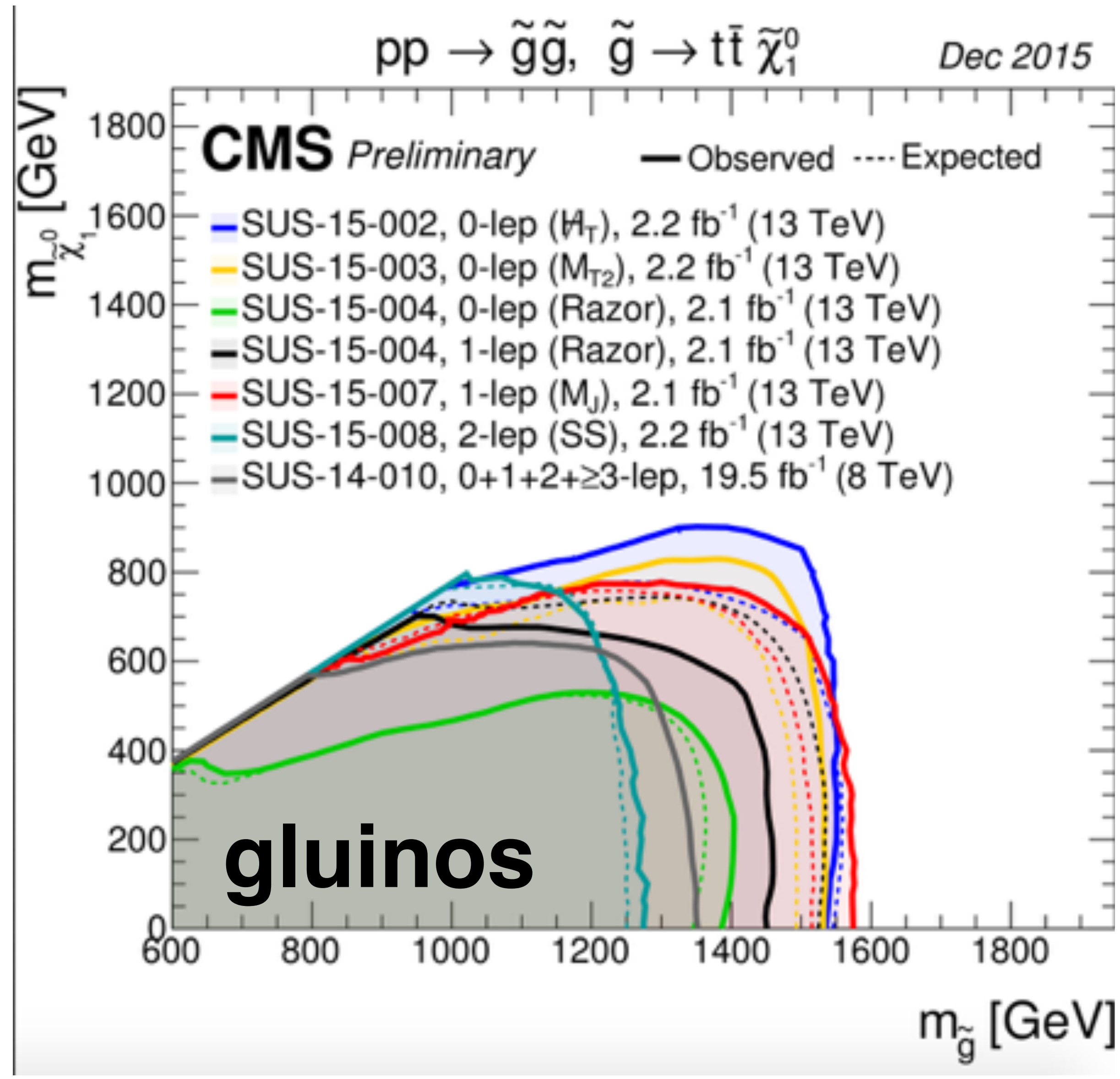
Different-spin pieces combine to cancel large corrections.



"Stop" or "scalar top":
cancels the biggest correction.
~10% tuned if mass ~ 700 GeV.

LHC: Towards Fine-Tuning?





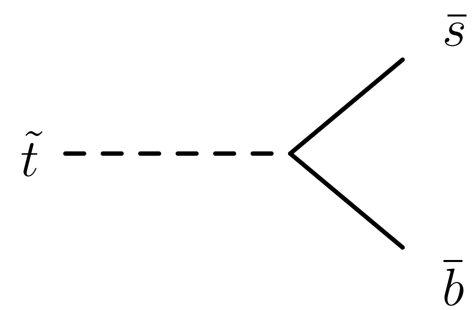
Direct searches for the superpartners are so far coming up empty. But lots of still-uncharted stop territory.

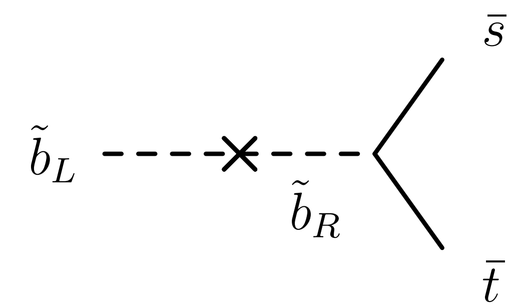
LHC Stop Prospects

Exhausting all possibilities at the LHC requires a systematic search of many different channels and kinds of physics, e.g.:

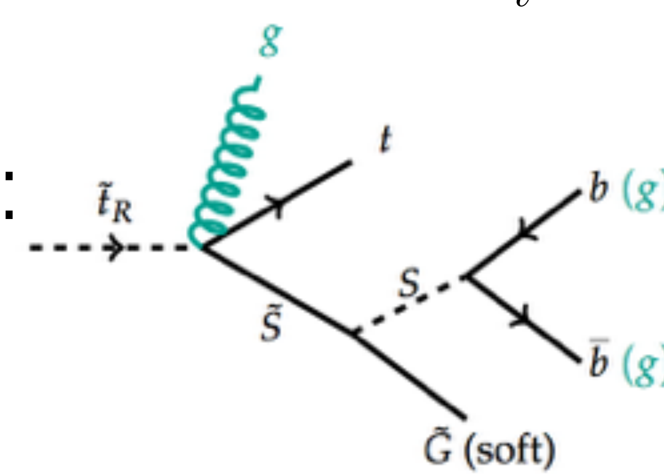
Compressed stops (see e.g. Kilic/Tweedie; An/Wang; Macaluso/Park/Shih/Tweedie)

R-parity violating stops:





Decay to hidden sector (e.g. stealth SUSY):
(Fan/Krall/Pinner/Reece/Ruderman)



Despite our best efforts, gaps can and likely will remain in LHC direct search coverage.

Indirect Observables

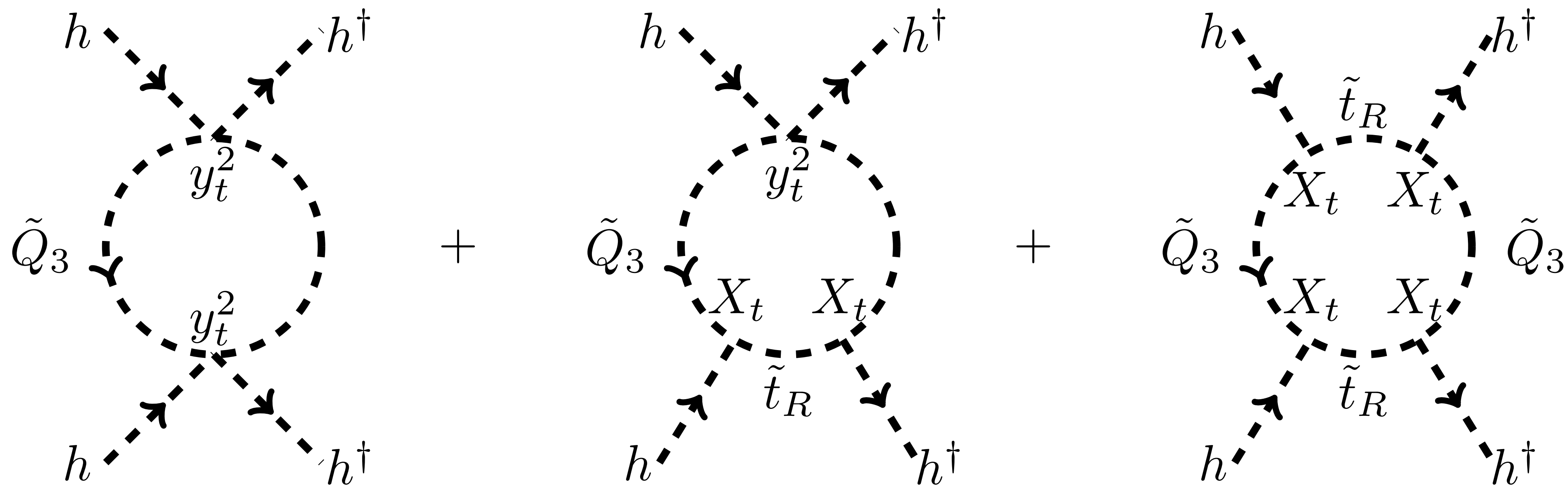
The same physics that is relevant for naturalness—couplings to the Higgs boson—can enter in loops to produce modifications of Standard Model electroweak observables.

S parameter: $S\left(\frac{\alpha}{4 s_W c_W v^2}\right) h^\dagger \sigma^i h W^i_{\mu\nu} B^{\mu\nu}$

T parameter: $-T\left(\frac{2\alpha}{v^2}\right)\left|h^\dagger D_\mu h\right|^2$

Higgs decays: $c_{hgg} h^\dagger h G^a_{\mu\nu} G^{a\mu\nu} + c_{h\gamma\gamma} h^\dagger h F_{\mu\nu} F^{\mu\nu}$

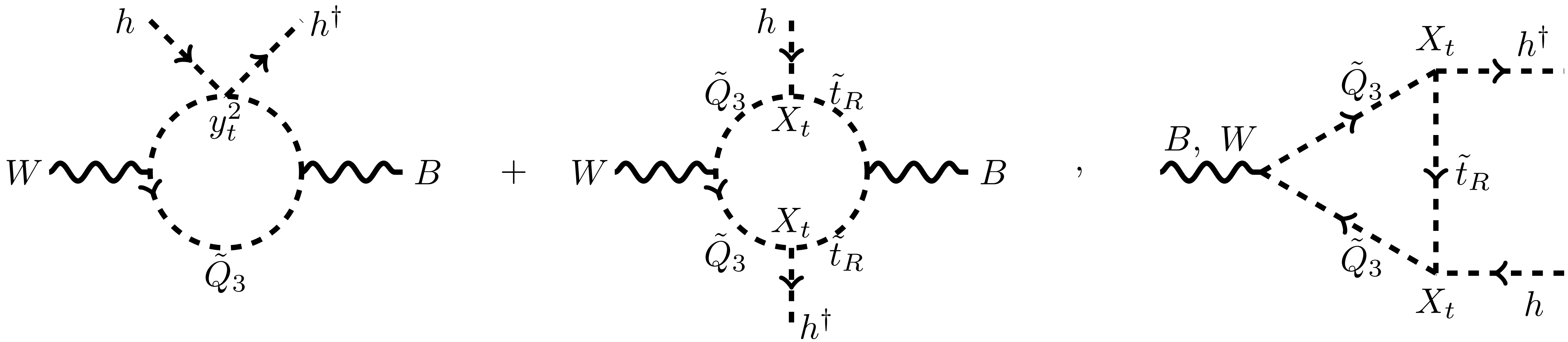
Stops: *T* Parameter



$$T \approx \frac{m_t^4}{16\pi \sin^2\theta_W m_W^2 m_{\tilde{Q}_3}^2} + \mathcal{O}\left(\frac{m_t^2 X_t^2}{4\pi m_{\tilde{Q}_3}^2 m_{\tilde{u}_3}^2}\right).$$

A Higgs quartic coupling! These are the same diagrams that lift the Higgs mass in the MSSM, *except* that we are reading off subleading momentum dependence: $D_\mu^2/m_{\rm stop}^2 \sim m_Z^2/m_{\rm stop}^2$.

The S Parameter



The diagram on the right, at first glance, doesn't seem to generate the right operator. In fact, it generates

$$i\partial^\nu B_{\mu\nu} h^\dagger \overleftrightarrow{D}^\mu h$$

But if we work with a minimal basis of operators, equations of motion turn this into a linear combination including the S parameter.

Why Focus on *S*, *T* ?

Any SU(2)$_L$-charged particles, coupling to the Higgs or not, contribute at one loop to two other dimension-6 operators:

$$c_{WWW}\, g\epsilon_{ijk} W^i_{\mu\nu} W^{j\nu}_{\rho} W^{k\rho\mu}$$

"TGC"

$\lambda_\gamma = \lambda_Z$

$$c_{WWW} = \frac{g^2}{2880\pi^2} \sum_{\text{rep } R,\ \text{mass } M} (-1)^F \frac{T(R)}{M^2},$$

$$c_{JJ} D^\mu W^i_{\mu\nu} D_\lambda W^{i\lambda\nu}$$

"*W* parameter"

$$c_{JJ} = -\frac{g^2}{960\pi^2} \sum_{\text{rep } R,\ \text{mass } M} a_F \frac{T(R)}{M^2},$$

where $a_F = 4$ for Weyl fermions and 1 for complex scalars.

Unfortunately, their perturbative coefficients are very small. (Could be lucky to have many new degrees of freedom?)

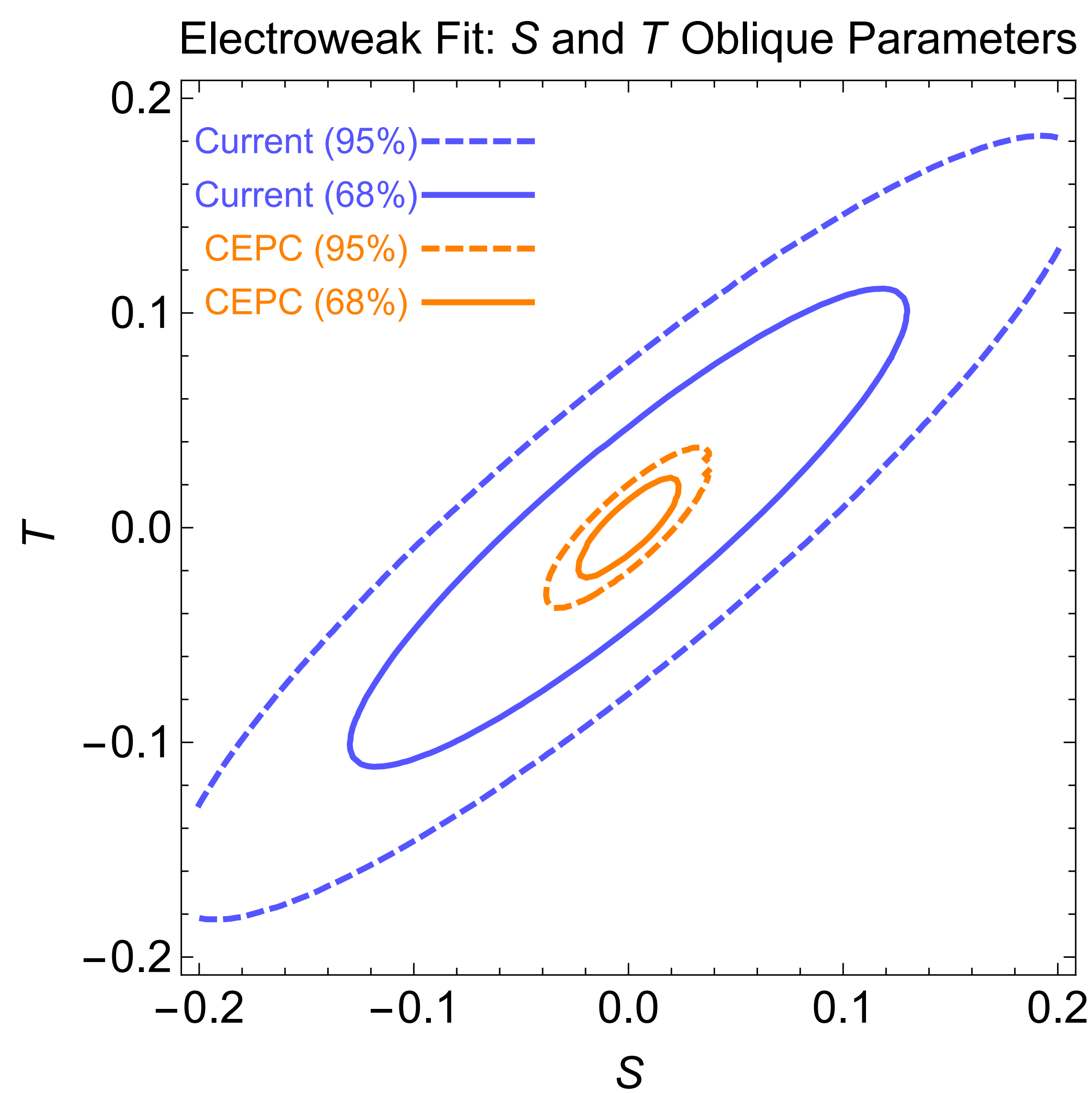
The *U* parameter is dimension 8: $c_U \left(h^\dagger \sigma^i h W^{i\mu\nu}\right)^2$

Electroweak Fit

	Present data	CEPC fit
$\alpha_s(M_Z^2)$	0.1185 ± 0.0006 [23]	$\pm 1.0 \times 10^{-4}$ [24]
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$	$(276.5 \pm 0.8) \times 10^{-4}$ [25]	$\pm 4.7 \times 10^{-5}$ [26]
m_Z [GeV]	91.1875 ± 0.0021 [27]	$\mathbf{\pm 0.0005}$
m_t [GeV] (pole)	$173.34 \pm 0.76_{\text{exp}}$ [28] $\pm 0.5_{\text{th}}$ [26]	$\pm 0.2_{\text{exp}} \pm 0.5_{\text{th}}$ [29, 30]
m_h [GeV]	125.14 ± 0.24 [26]	$< \pm 0.1$ [26]
m_W [GeV]	$80.385 \pm 0.015_{\text{exp}}$ [23] $\pm 0.004_{\text{th}}$ [31]	$(\pm \mathbf{3}_{\text{exp}} \pm 1_{\text{th}}) \times 10^{-3}$ [31]
$\sin^2 \theta_{\text{eff}}^{\ell}$	$(23153 \pm 16) \times 10^{-5}$ [27]	$(\pm \mathbf{2.3}_{\text{exp}} \pm 1.5_{\text{th}}) \times 10^{-5}$ [32]
Γ_Z [GeV]	2.4952 ± 0.0023 [27]	$(\pm \mathbf{5}_{\text{exp}} \pm 0.8_{\text{th}}) \times 10^{-4}$ [33]
$R_b \equiv \Gamma_b/\Gamma_{\text{had}}$	0.21629 ± 0.00066 [27]	$\pm \mathbf{1.7} \times 10^{-4}$
$R_\ell \equiv \Gamma_{\text{had}}/\Gamma_\ell$	20.767 ± 0.025 [27]	$\mathbf{\pm 0.007}$

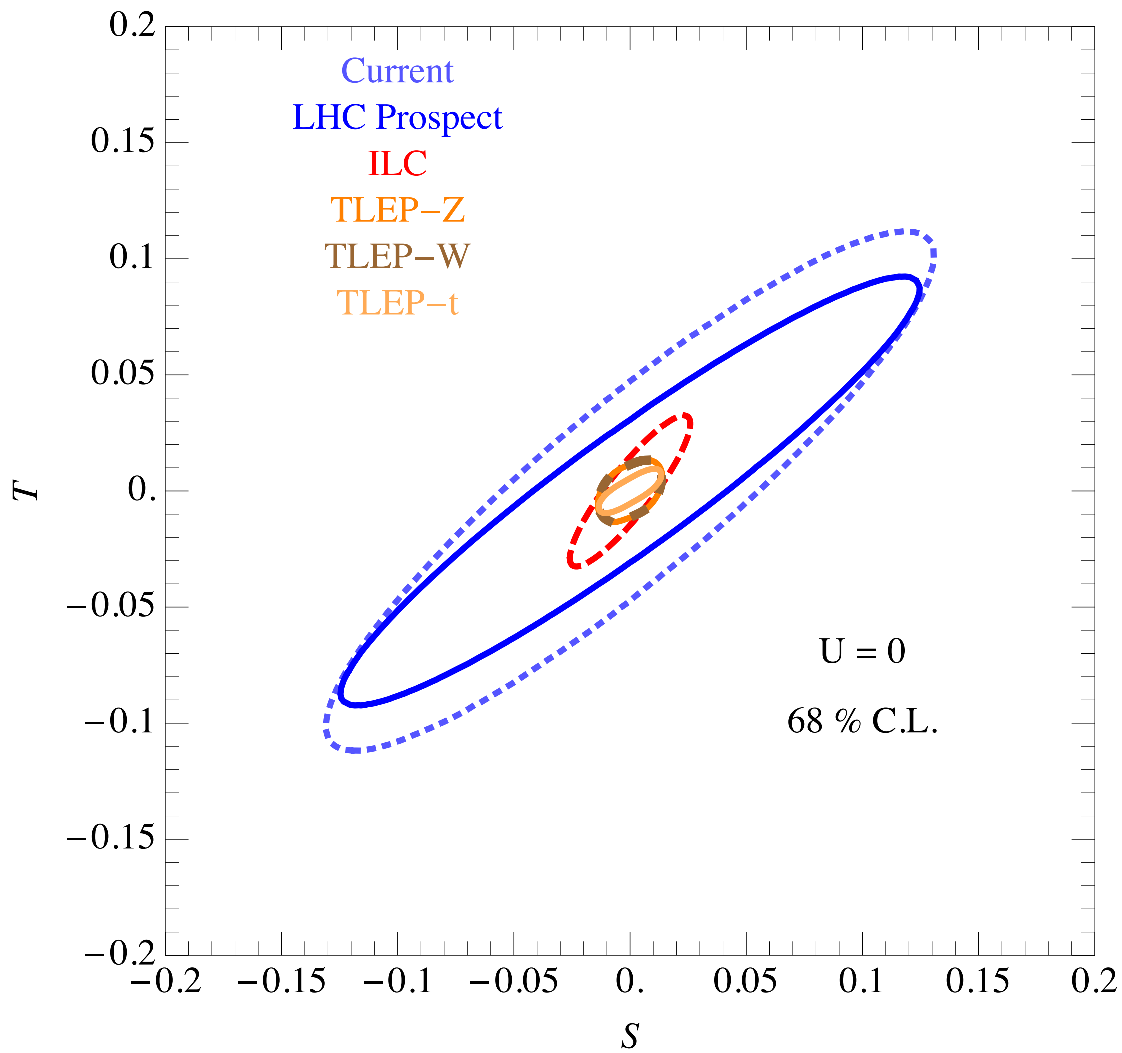
Numbers in **boldface**: major CEPC inputs to the electroweak precision fit.

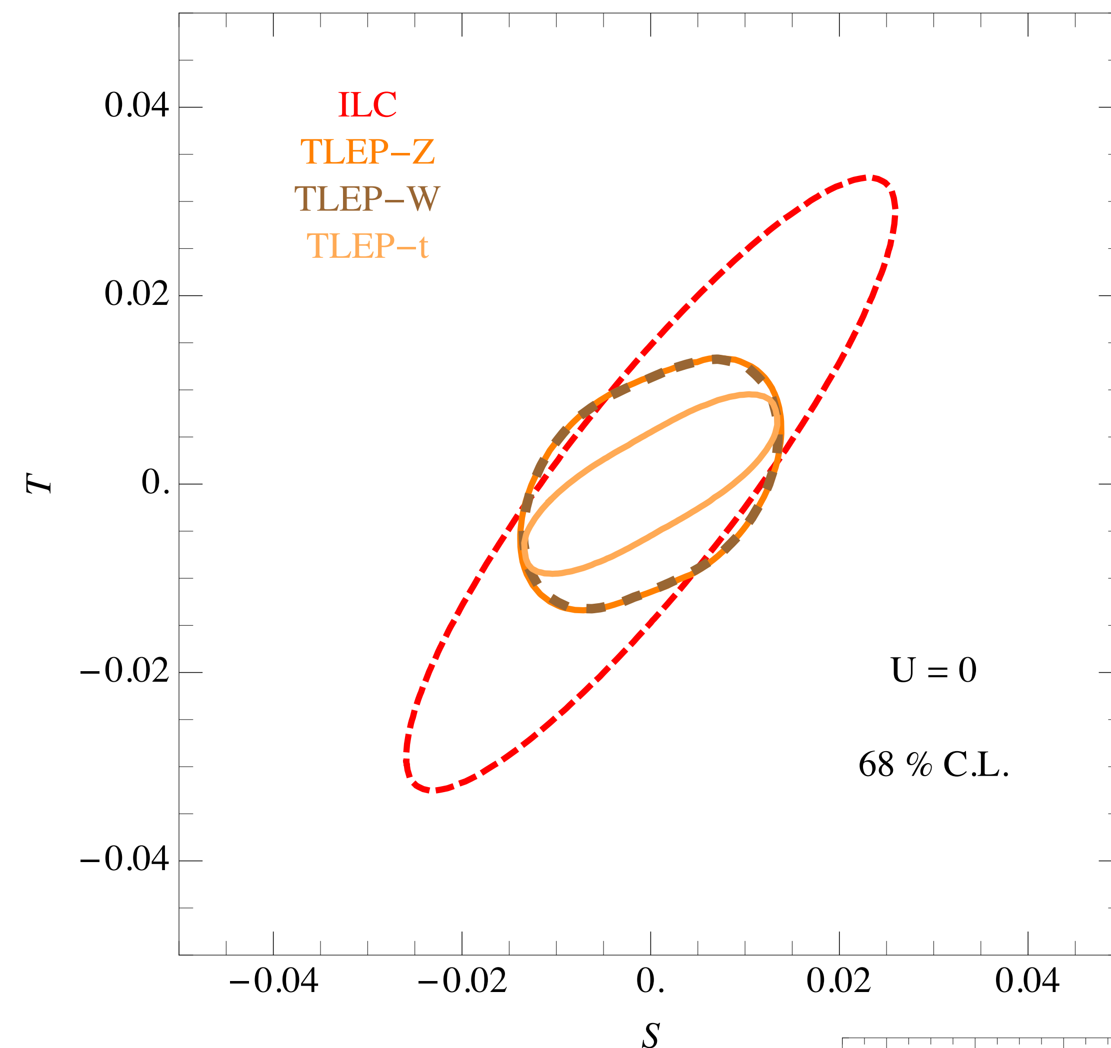
Baseline Fit



Even with conservative estimates, CEPC will provide a substantial improvement over existing data.

ILC and FCC-ee

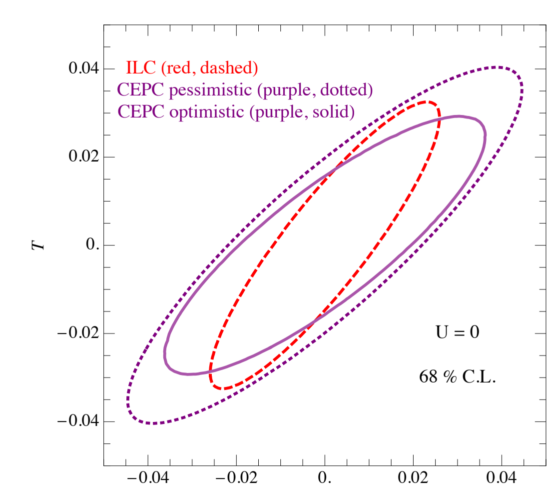




W mass: 5 MeV at ILC, 1.2 MeV at FCC-ee

$\sin^2\theta_W$: 1.3×10^{-5} at ILC, 0.3×10^{-5} at FCC-ee

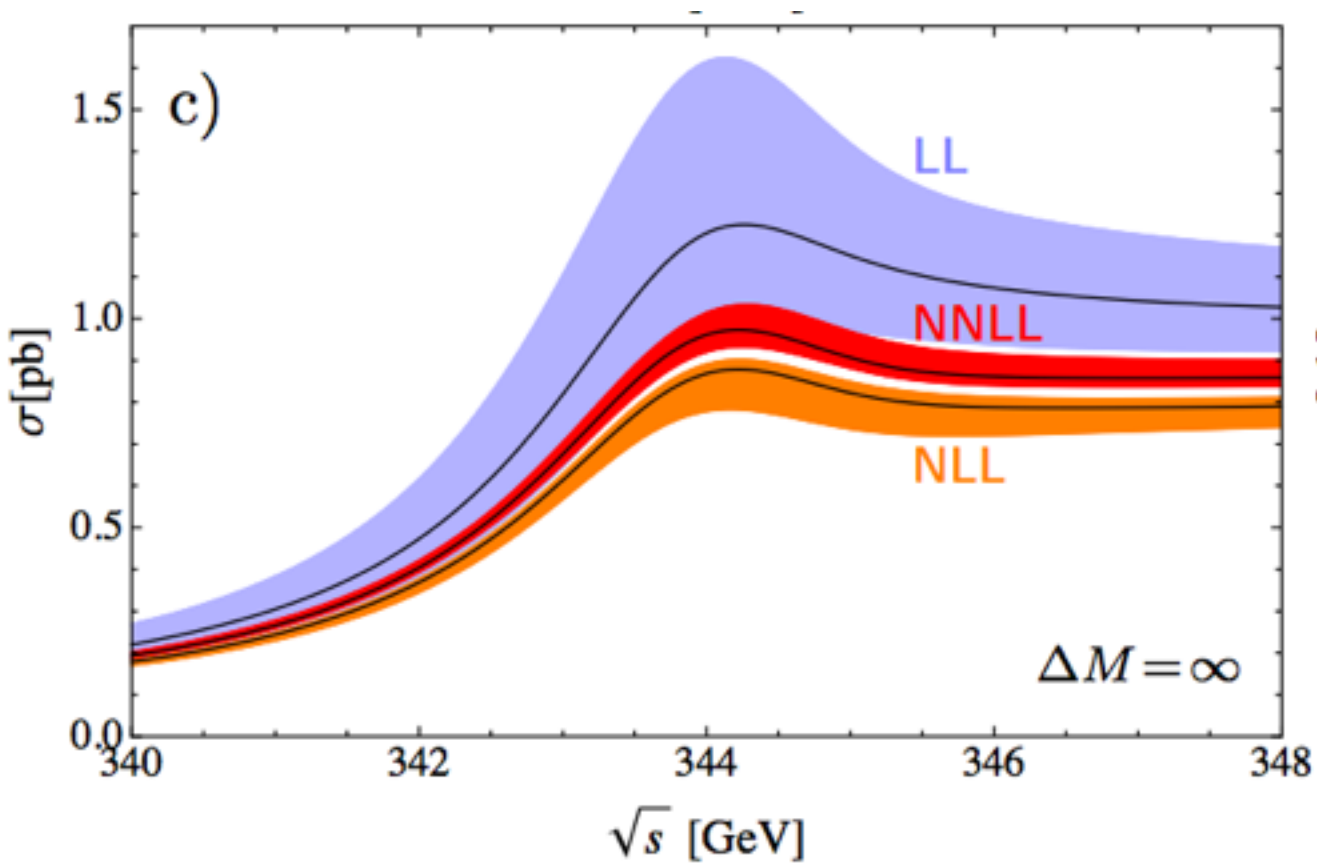
top mass: 30 MeV at ILC, 20 MeV at FCC-ee
(theory-dominated: 100 MeV)



Refining the t Mass

Much easier to do at the ILC than CEPC, due to synchrotron losses.

Top mass measurements at the LHC are subject to significant theoretical uncertainties due to hadron physics.



Hoang and Stahlhofen, 1309.6323

The threshold scan can determine the top mass and width in the 1S scheme, which is less subject to large corrections than the pole mass measured kinematically.

Prospect for better than 100 MeV accuracy.

Refining the Z Mass

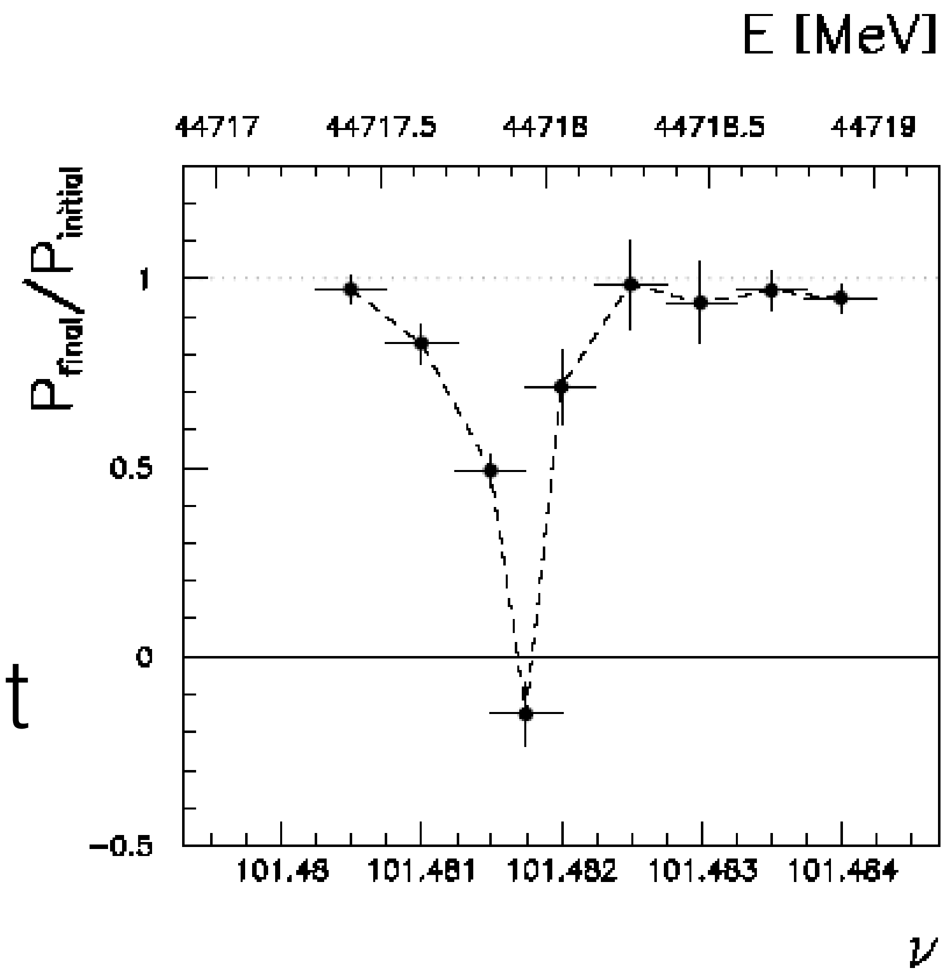
Energy calibration at FCC-ee is better than ILC.

Error bar on m_Z can be reduced: 2 MeV to < 0.5 MeV.

At a **circular** collider, the electron beams can be *polarized* (have their spins aligned rather than randomized.)

The spins *precess* in the bending field.

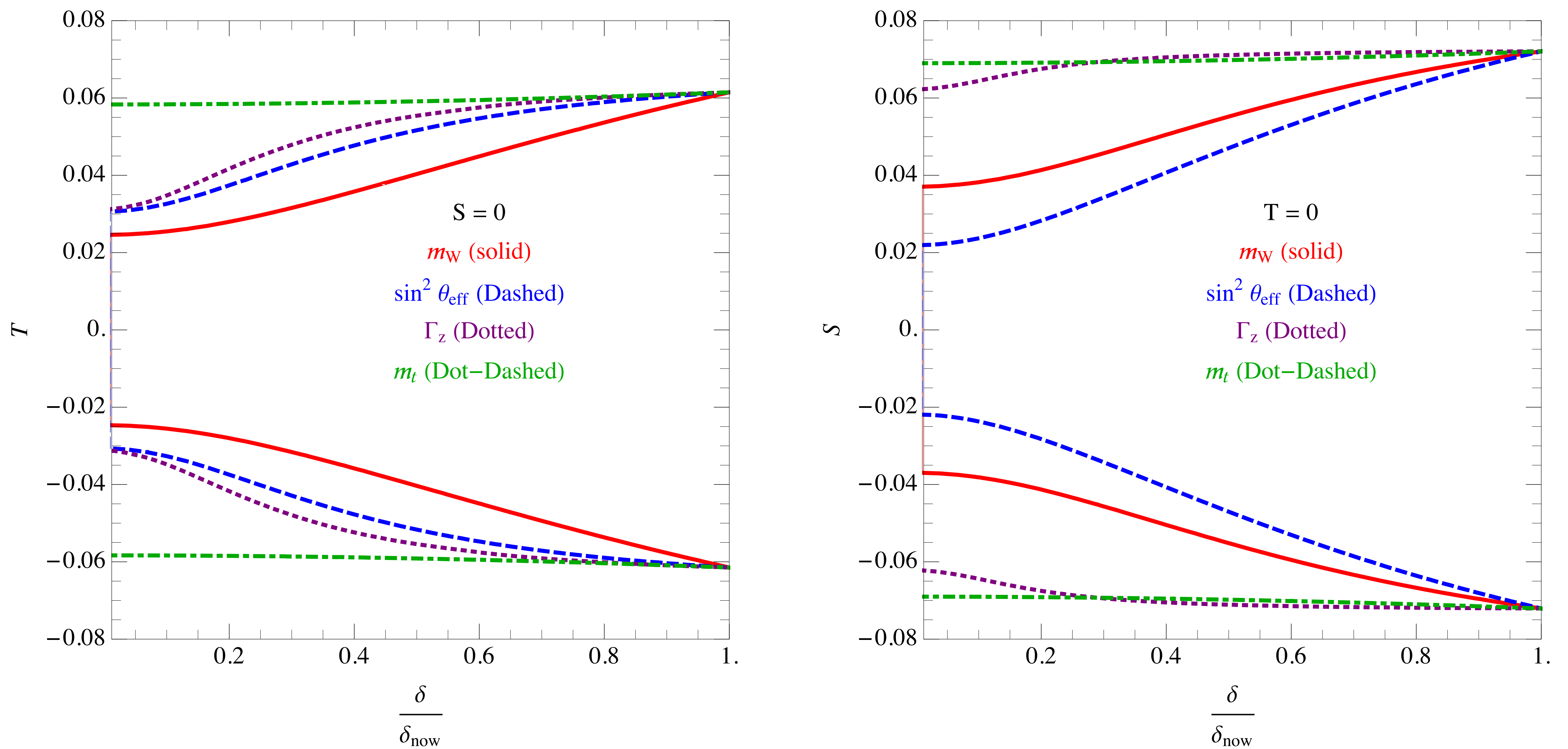
Applying an orthogonal field at the right frequency can *depolarize* the electrons.



Like NMR: carefully measuring the **resonant depolarization** transition rate can calibrate the energy of the beam!

Limiting Measurements

If we only improved one input to fit at a time, hit limits:



W mass is priority for measuring *T*.

$\sin^2\theta_W$ is priority for measuring *S*

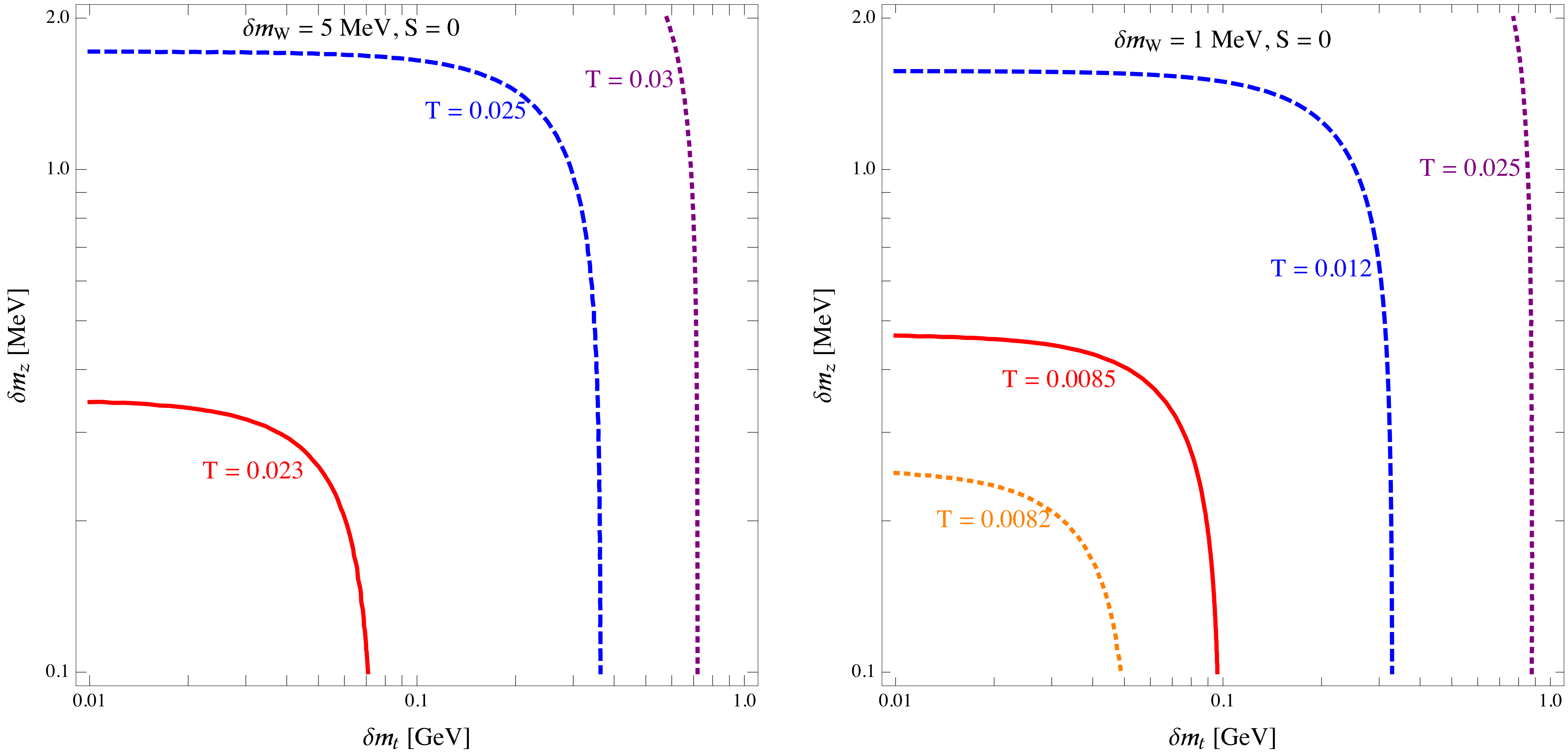
Comment

The most high-priority measurement that requires *Z* pole running is $\sin^2\theta_W$. The *W* mass will be measured well at 240 GeV.

$\sin^2\theta_W$ benefits from the large cross section on the *Z* peak. It seems unlikely that it could be inferred as precisely from any observables at 240 GeV.

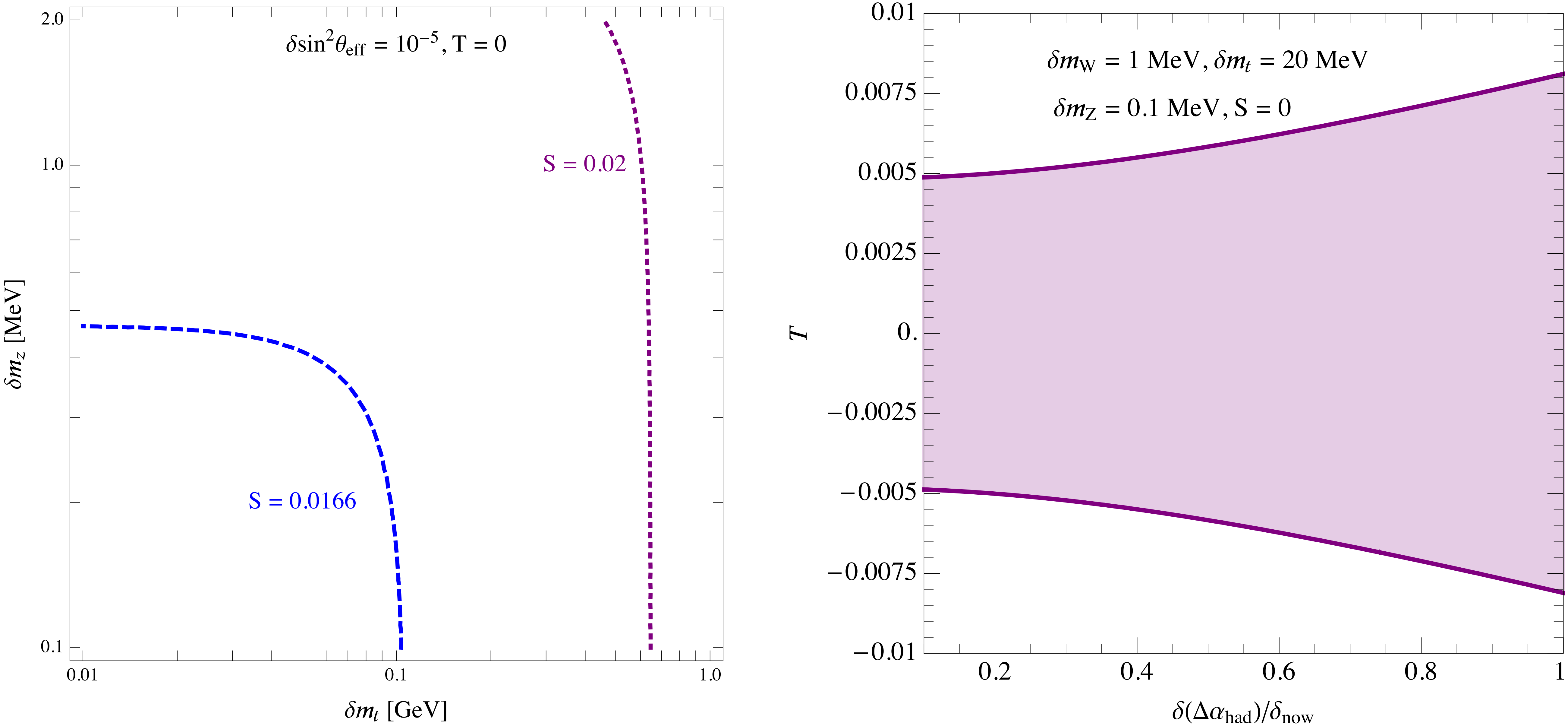
(However, I don't know of detailed studies of how much can be done at 240.)

Role of Top and Z Mass for T Parameter



At left: 5 MeV error on W mass. At right: 1 MeV error. Top/Z masses play much larger role once W error is very small. If error stuck at 5 MeV, limited improvement.

Role of Top/Z mass for S Parameter, $\Delta\alpha_{\text{had}}$ for T Parameter



Again, all the ingredients help, but first must achieve sufficient precision on crucial numbers like m_W and $\sin^2\theta_W$.

A Wish List

Of course, we want the best measurements possible of many quantities. But here are reasonable goals to probe loops of ~TeV particles. **CEPC will deliver what's in bold.**

- **Measure m_W to better than 5 MeV (now 15 MeV) and $\sin^2\theta_W$ to better than 2×10^{-5} (now 16×10^{-5})**

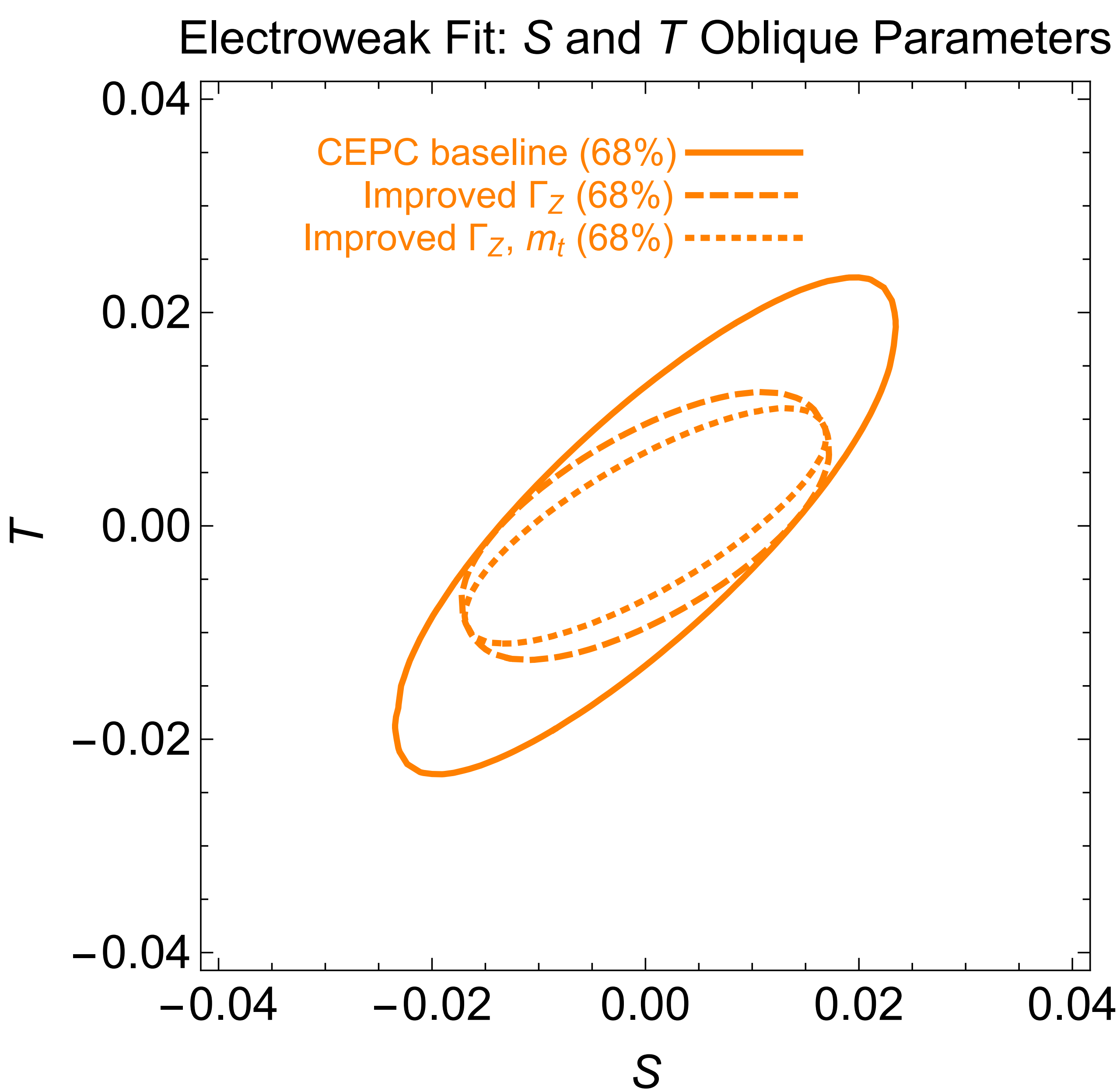
- **Measure m_Z to 500 keV precision (now 2 MeV)**

- Measure m_t to 100 MeV precision (now ~0.8 GeV*)

- Have precise enough theory to make use of these results: *at least 3-loop* calculations.

Improving on the Baseline?

CEPC	$\Gamma_Z(m_Z)$ [GeV]	m_t [GeV]
Improved Error	$(\pm 1_{\rm exp} \pm 0.8_{\rm th}) \times 10^{-4}$ (± 0.0001)	$\pm 0.03_{\rm exp} \pm 0.1_{\rm th}$



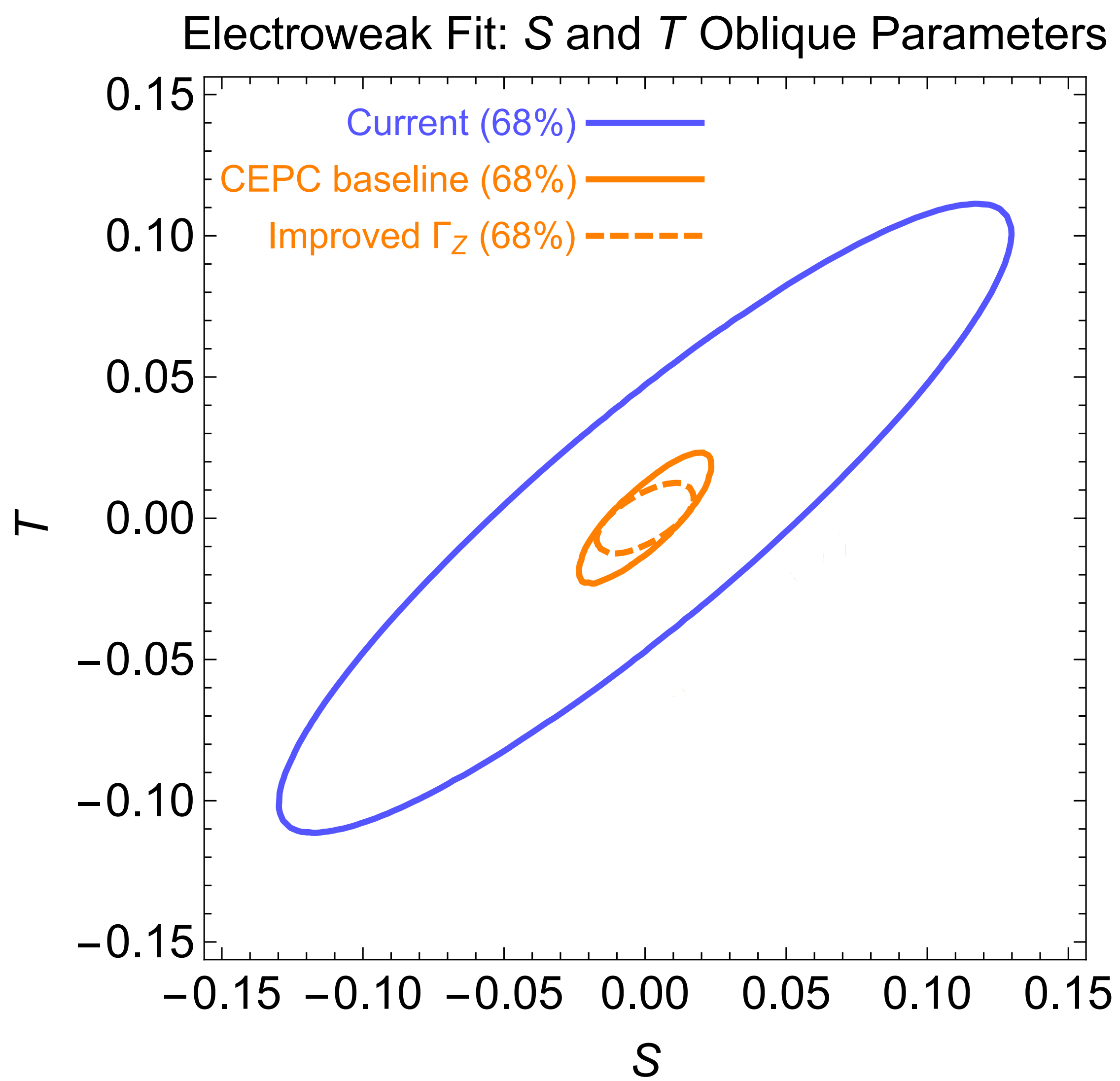
Improving the Z width measurement requires a better energy calibration. Improving the top mass measurement requires an e+e- collider threshold scan. (Beyond CEPC energy plans, but planned at ILC and FCC-ee.)

Summary: CEPC Fit

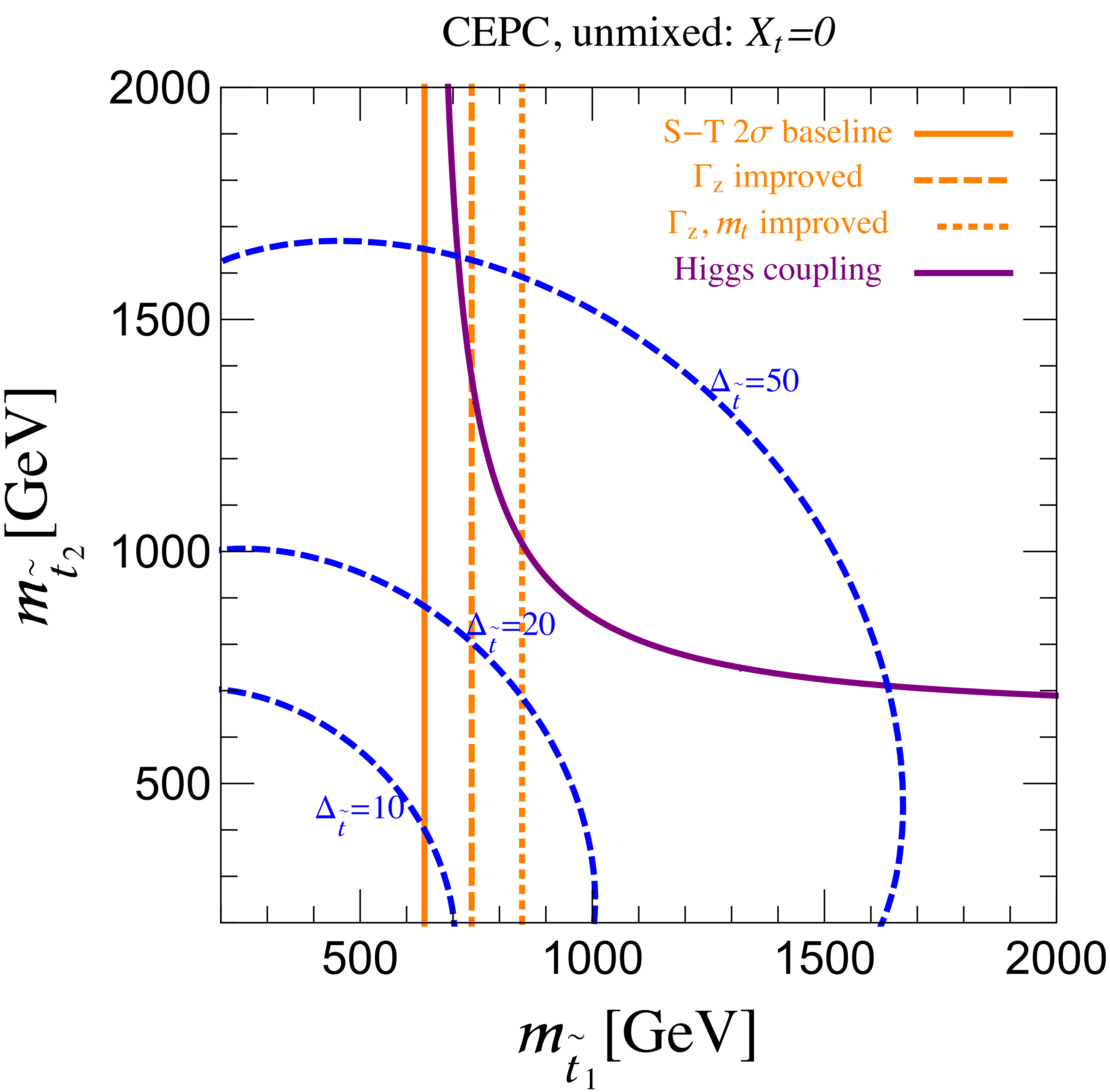
Parameter	Current	CEPC baseline	Improved Γ_Z (and m_Z)	Also improved m_t
S	3.6×10^{-2}	9.3×10^{-3}	9.3×10^{-3}	7.1×10^{-3}
T	3.1×10^{-2}	9.0×10^{-3}	6.7×10^{-3}	4.6×10^{-3}

Results at $\Delta\chi^2 = 1$

The CEPC would provide order-of-magnitude improvement over the current results from LEP, Tevatron, and LHC.



CEPC and Stops

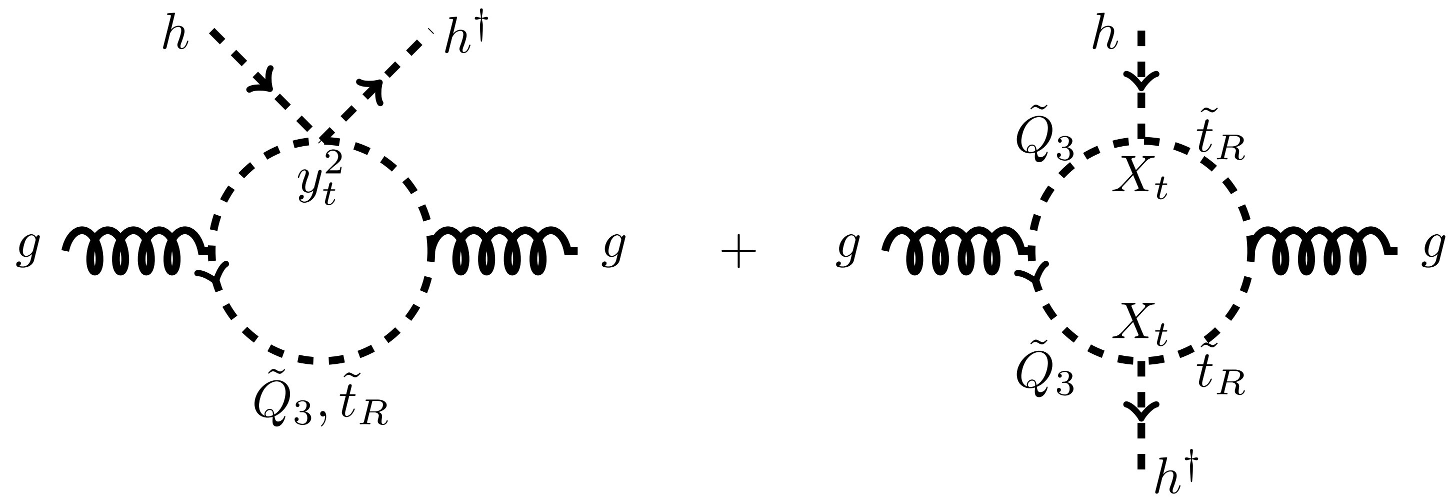


No mixing:

Similar mass reach via *T*-parameter and Higgs couplings. Pushes tuning to the few % level.

Definitively close LHC loopholes (hidden, stealthy, compressed stops).

Higgs Couplings

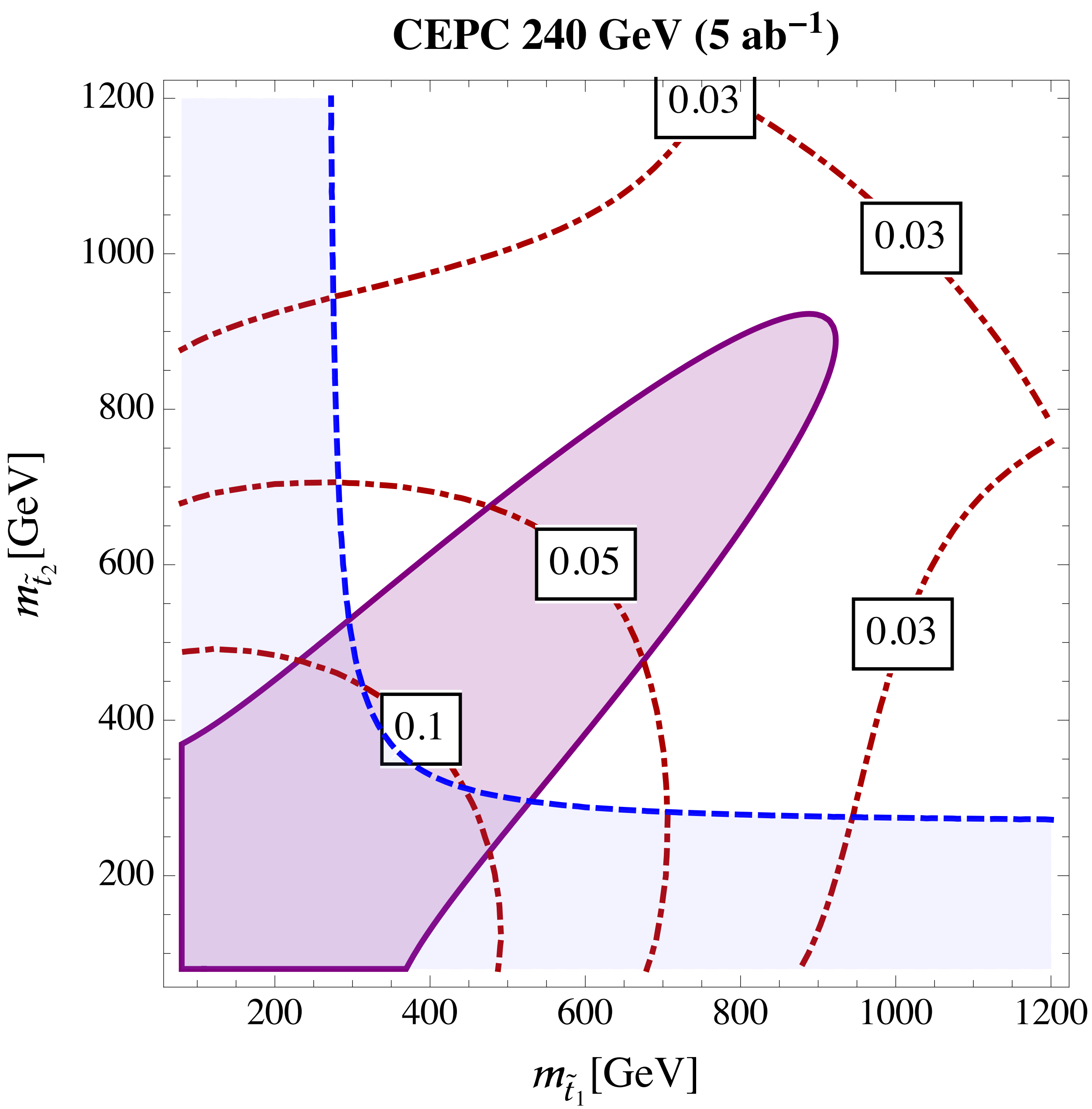


$$r_G^{\tilde{t}} \equiv \frac{c_{hgg}^{\tilde{t}}}{c_{hgg}^{\mathrm{SM}}} \approx \frac{1}{4}\left(\frac{m_t^2}{m_{\tilde{t}_1}^2} + \frac{m_t^2}{m_{\tilde{t}_2}^2} - \frac{m_t^2 X_t^2}{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}\right)$$

Familiar low-energy theorem: beta function coefficients times $\sum \frac{\partial \log M}{\partial \log v}$

Similar result for photons (except SM contribution dominated by *W* loop)

Higgs Couplings and Stops

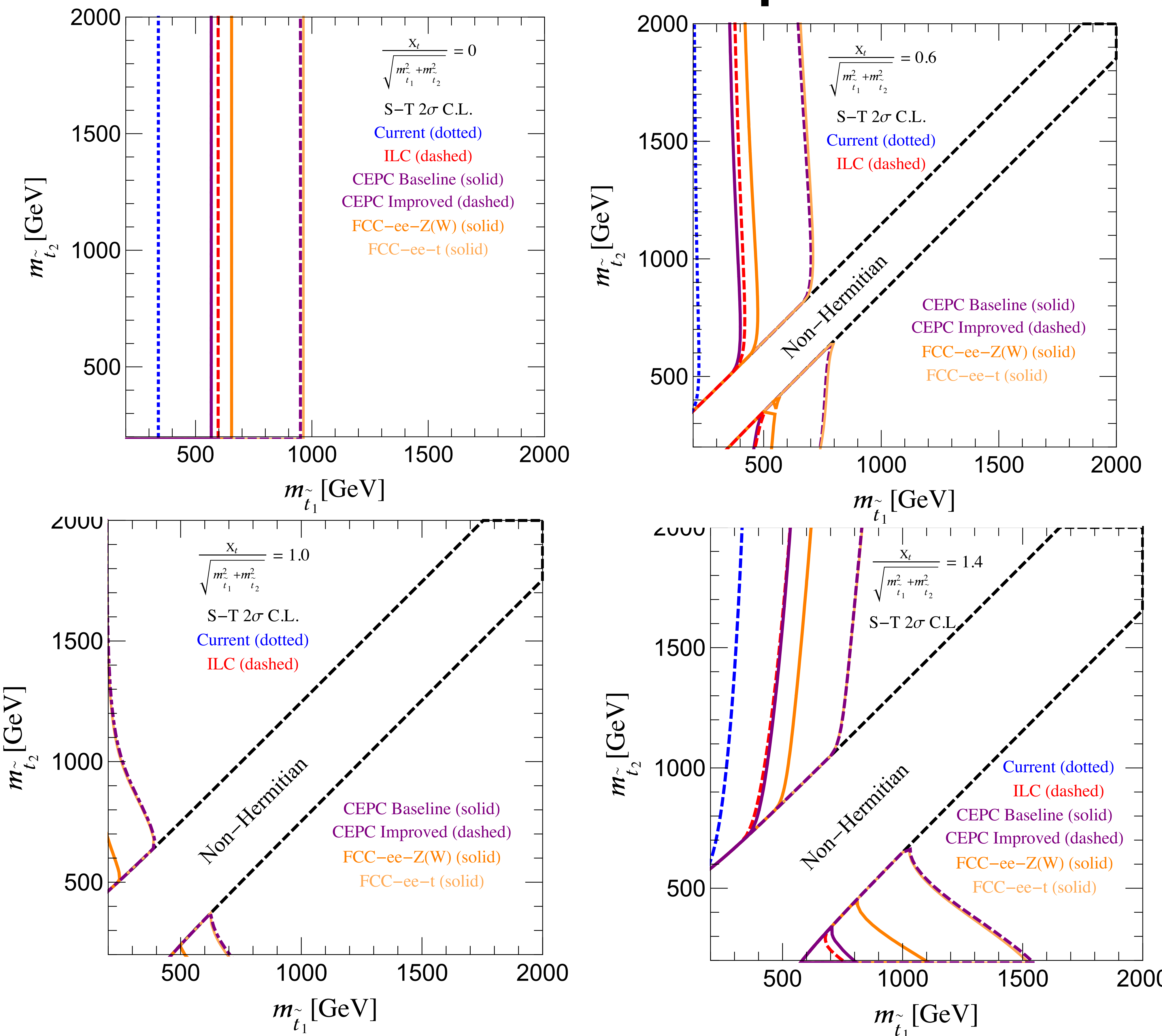


The purple region can be excluded for *any* mixing angle. (Because large mixing forces the mass eigenvalues away from the diagonal.)

Blue region is excluded unless mixing angle is tuned by a factor of 10.

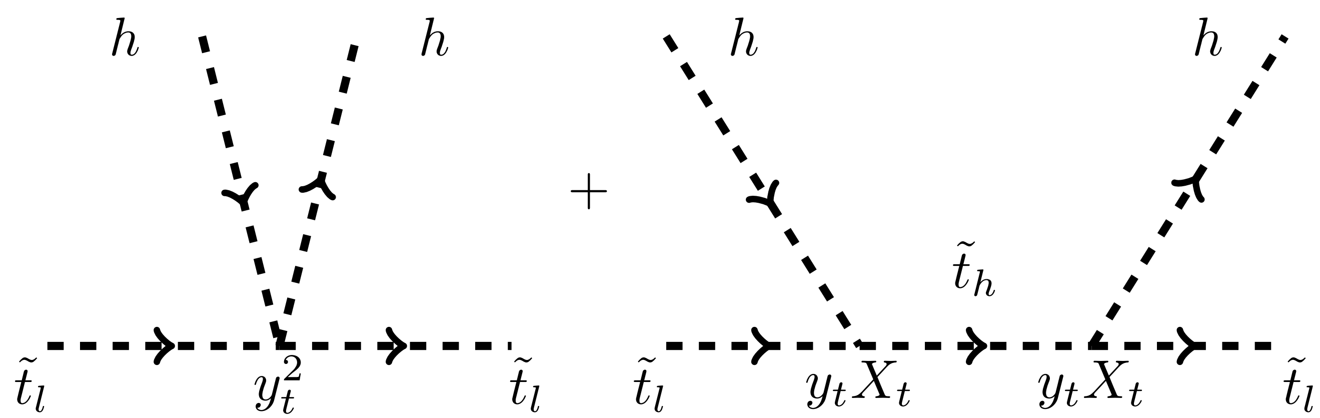
(also see J. Fan, MR arXiv:1401.7671)

EWPT and Stop Mixing



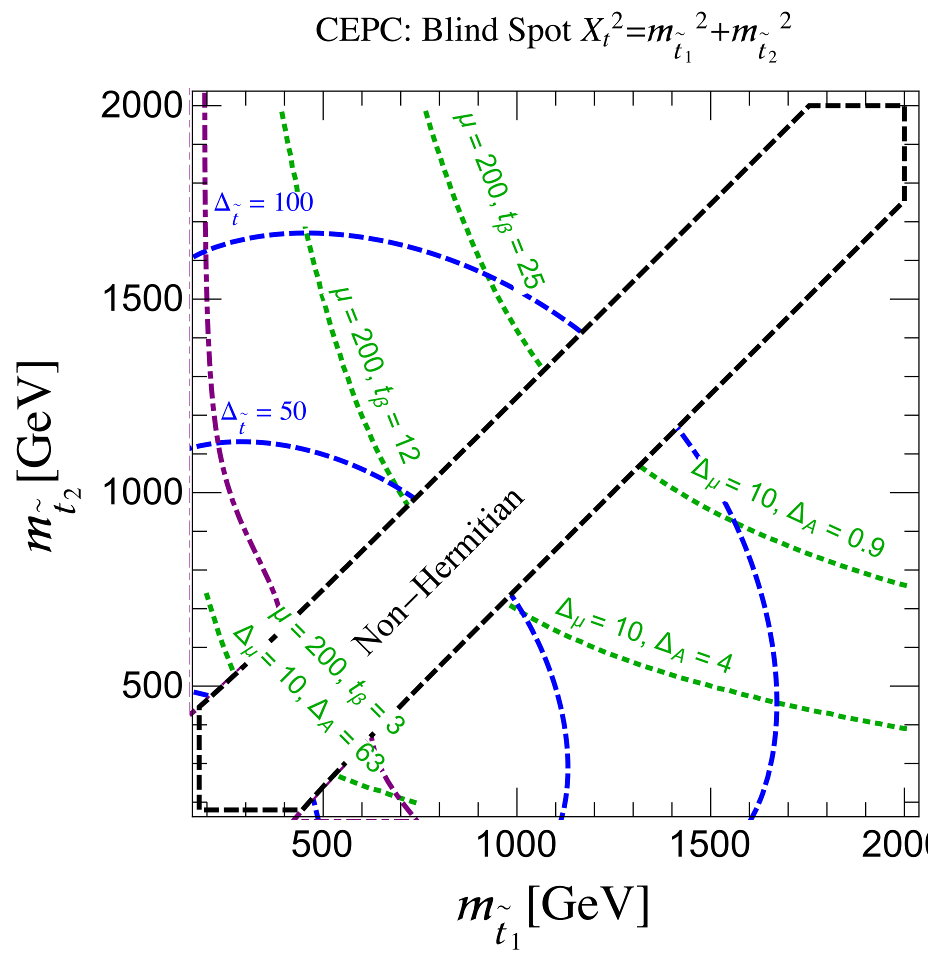
“Blind Spot” for Stops

The light stop mass eigenstate may be decoupled from the Higgs at tree level, at a certain critical mixing angle:



$$X_t^* = \left(m_{\tilde{t}_h}^2 - m_{\tilde{t}_l}^2\right)^{1/2}.$$

If the light stop is decoupled from the Higgs, it’s irrelevant for naturalness! Then it’s the heavy stop that matters.



Purple: CEPC EWPT
Green: $b \to s\gamma$

(also see Craig, Farina, McCullough, Perelstein 1411.0676)

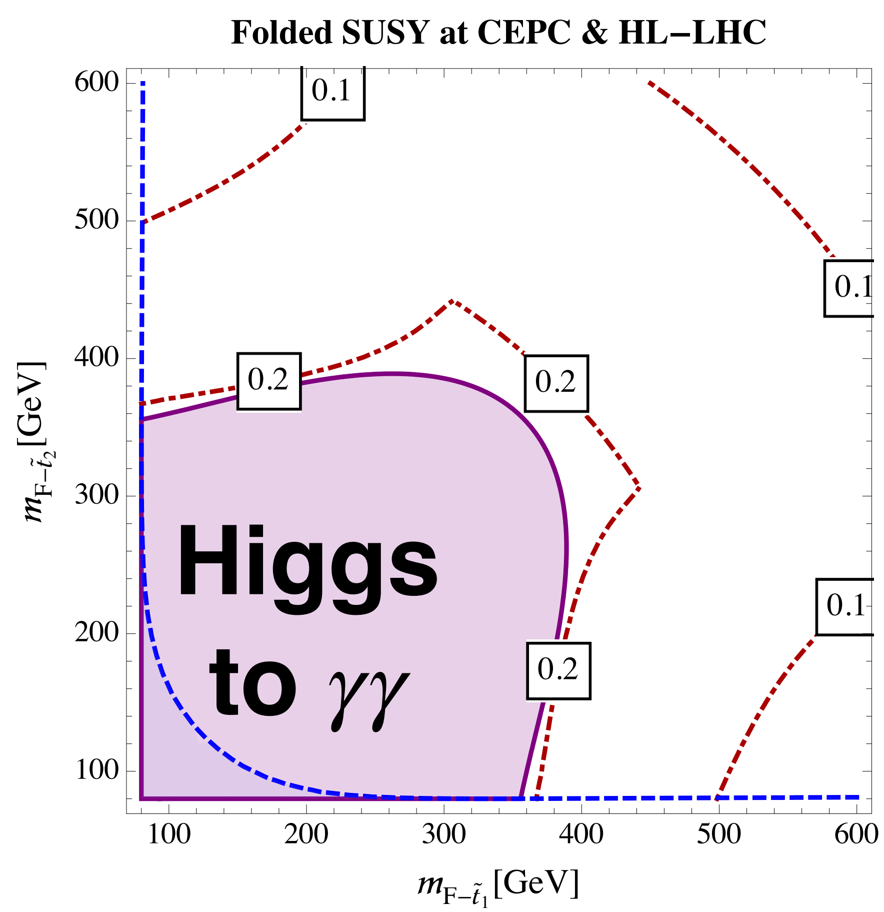
Folded SUSY

In folded SUSY, stops have **no QCD color** (makes life difficult at LHC). But still have electroweak interactions.

Measuring Higgs decays to photons and the *T* parameter can help constrain folded SUSY stops.

The *T*-parameter bounds previously shown for stops are *exactly* the same for folded stops!

Higgs factories have exciting potential for uncolored naturalness!



Other Precision Z Physics

Rare *Z* decays: Standard Model predicts

$$\begin{aligned} \mathrm{Br}(Z^0 \to J/\psi\,\gamma) &\approx 8 \times 10^{-8} \\ \mathrm{Br}(Z^0 \to \Upsilon(nS)\,\gamma) &\approx 1.0 \times 10^{-7}. \end{aligned}$$

(Grossman, Koenig, Neubert 1501.06569; see also Huang, Petriello 1411.5924)

No current collider has had a large enough *Z* sample to see them. The CEPC or FCC-ee, with > 10^{10} Z bosons, can explore a **new frontier of the SM**.

CEPC could also search for *flavor-violating* processes like Z to muon + tau. Probe of high-scale flavor violation beyond the Standard Model!

Higgs Coupling Measurements (CEPC pre-CDR)

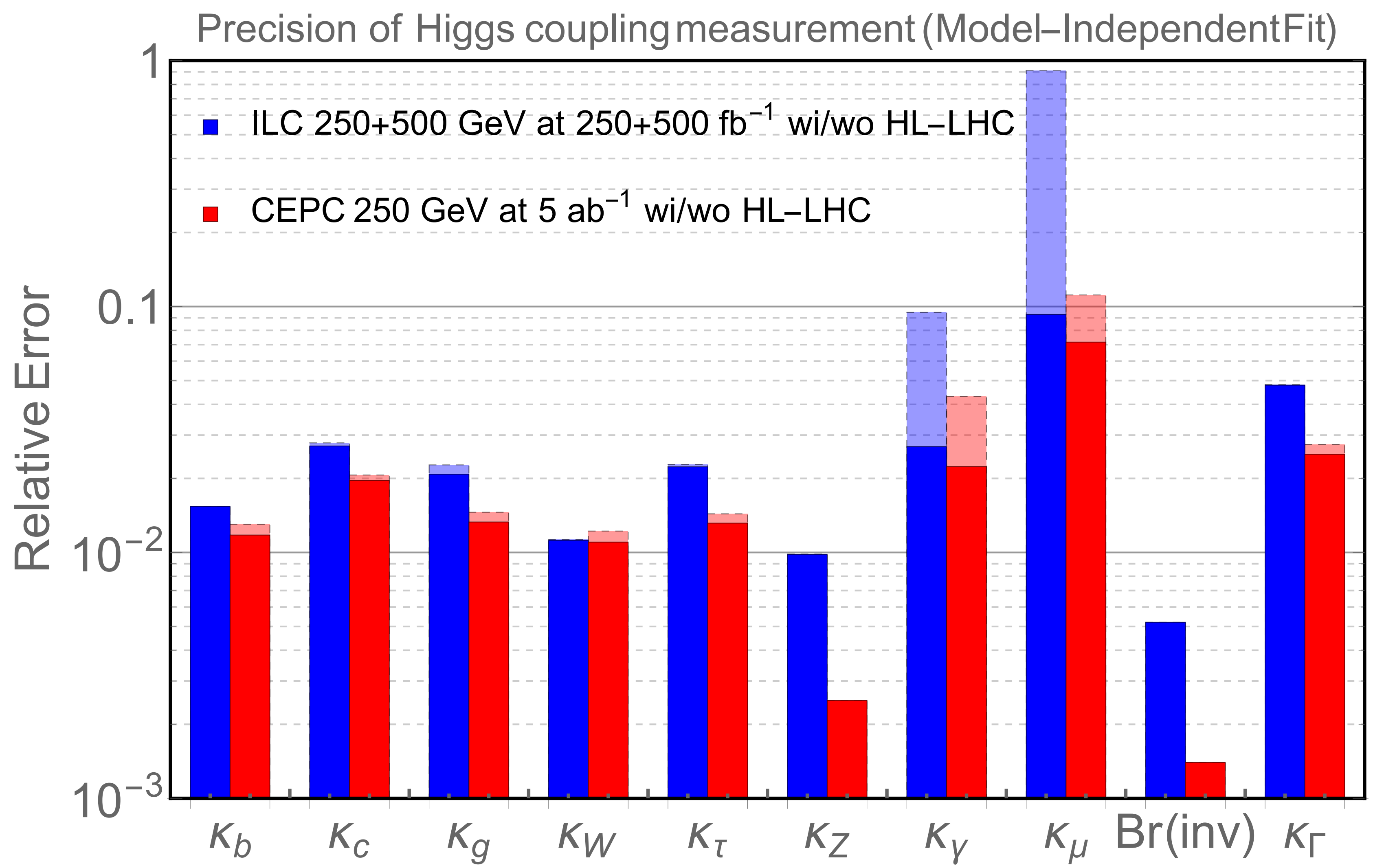
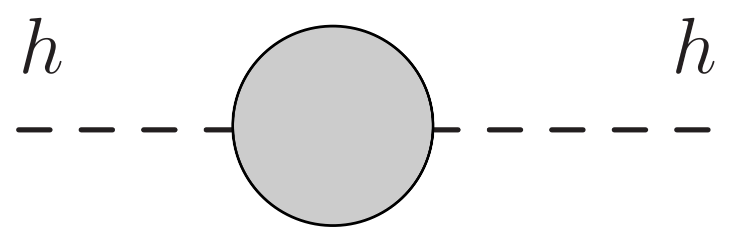


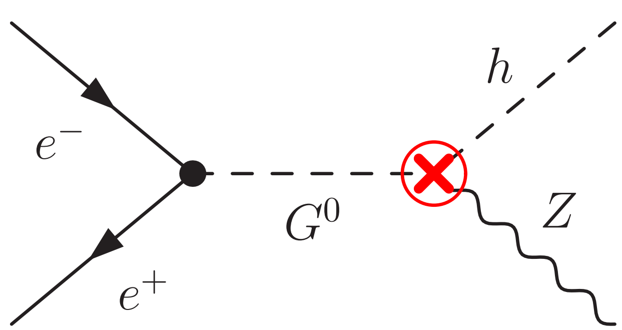
Figure 3.20 The 10 parameter fit result and comparison with the ILC. The CEPC at 250 GeV with 5 ab^{-1} integrated luminosity and the ILC 250+500 GeV at 250+500 fb^{-1} are shown. The CEPC and ILC result without combination with HL-LHC input as shown in dashed edges.

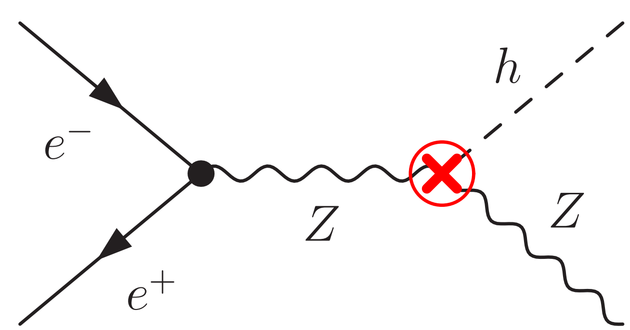
Higgs Wavefunction Renormalization

Craig, Englert, McCullough 1305.5251

$$\delta Z_h, \delta m_h^2 \sim$$ 

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{c_H}{m_\phi^2}\left(\frac{1}{2}\partial_\mu |H|^2 \partial^\mu |H|^2\right) + \ldots$$





Probes *Any* Natural Physics

E.g. toy model:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_i \left(|\partial_\mu \phi_i|^2 - m_i^2 |\phi_i|^2 - \lambda_i |H|^2 |\phi_i|^2 \right)$$

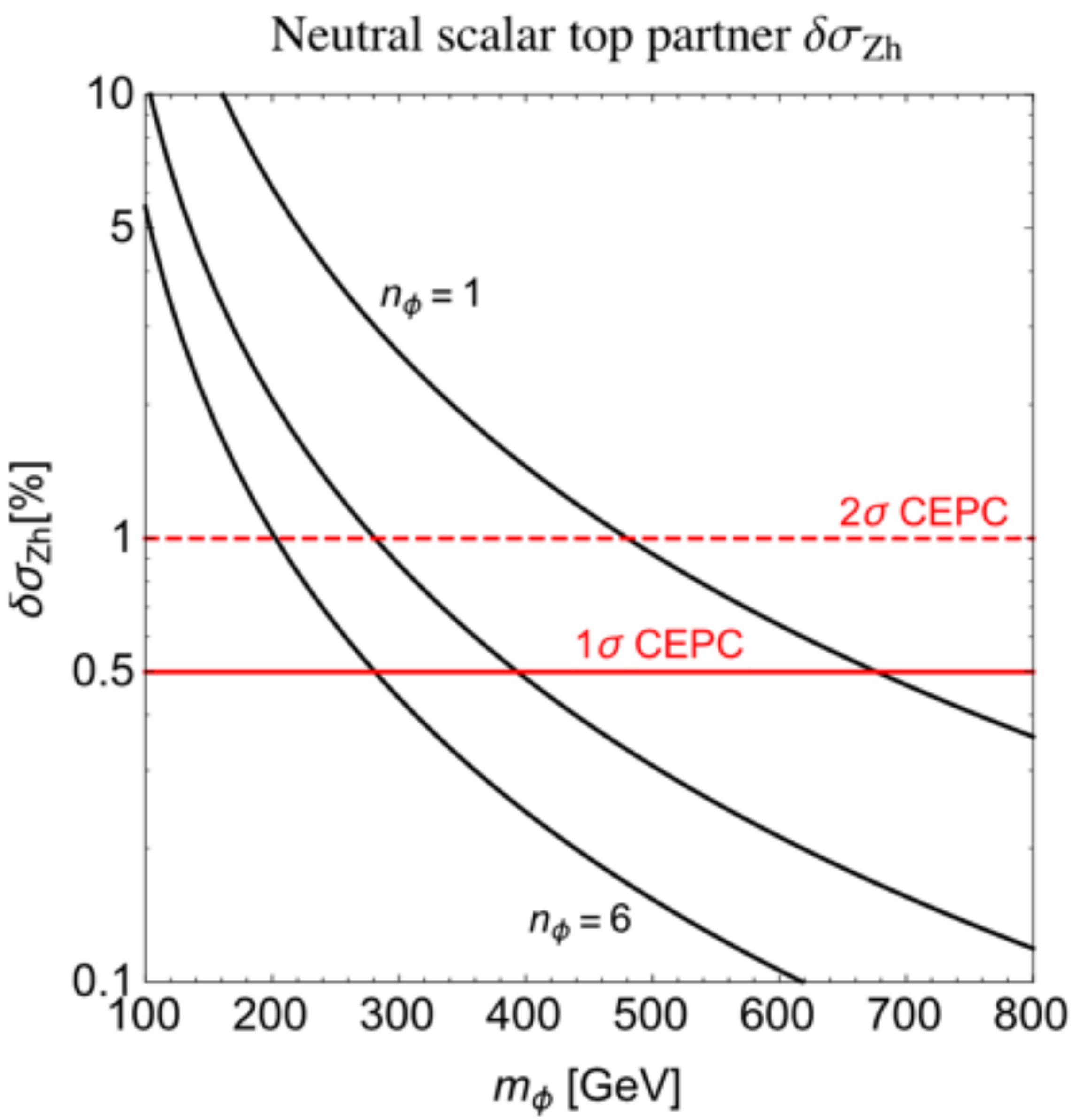
New singlets; undetectable; cancel divergences if:

$$\sum_i \lambda_i = 6\lambda_t^2$$

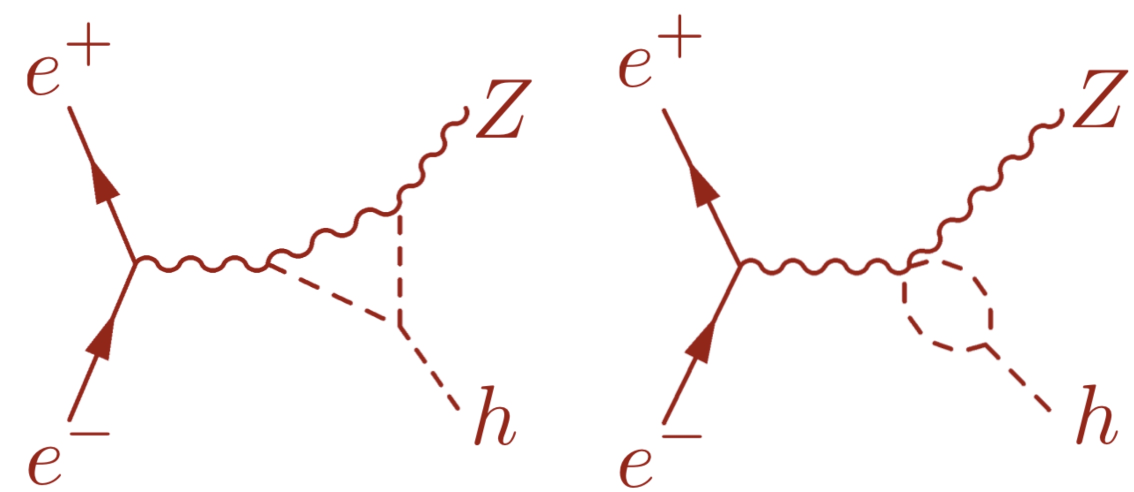
Less "toy" analogues include Twin Higgs or Folded Supersymmetry: cancel top loops with partner particles that do *not* have QCD color and so are hard to make directly.

Reach for new physics

Craig, Englert, McCullough; CEPC pre-CDR



Also probe Higgs self-coupling through loop effect (McCullough 1312.3322)



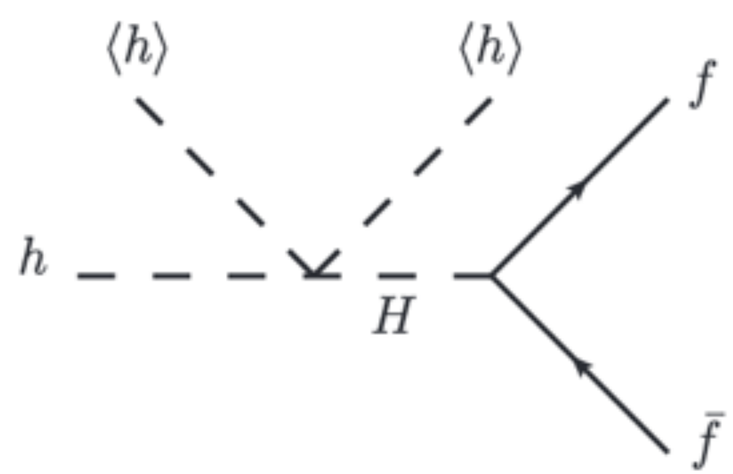
$$\delta_\sigma^{240} = 100\,(2\delta_Z + 0.014\delta_h)\,\%$$

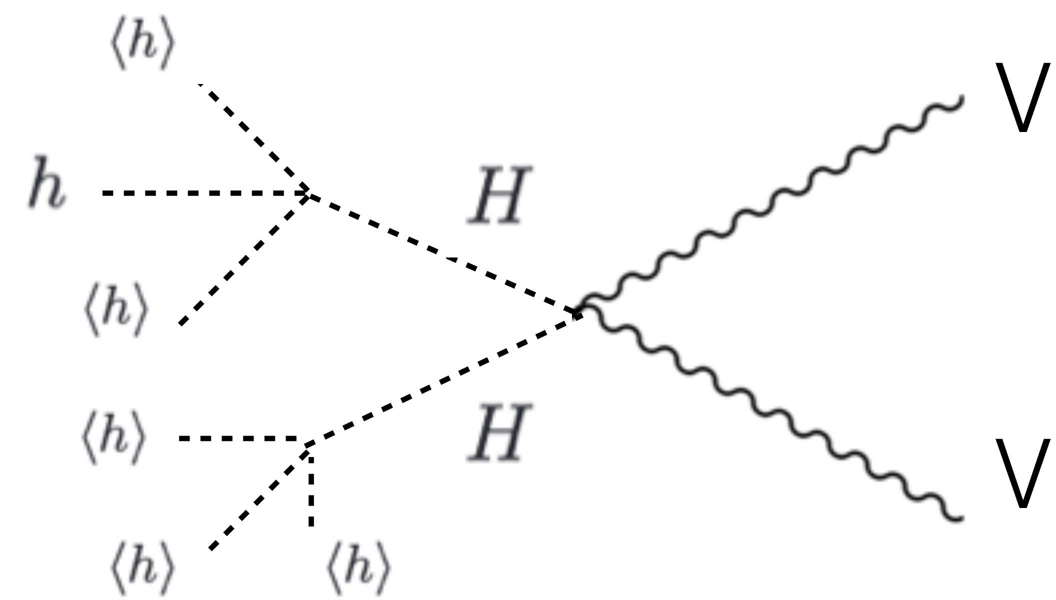
Would see effect if order-one deviation from SM!

(also useful to probe EW baryogenesis: e.g. Katz, Perelstein 1401.1827)

Two Higgs Doublets

Size of corrections: work in basis of **doublet *h* with VEV** and ***H* with no VEV**. Mixing hH and quartic h^3H terms exist and are related by absence of H tadpole.





Fermion couplings deviate at ~v^2/m_H^2; gauge boson couplings at ~v^4/m_H^4.

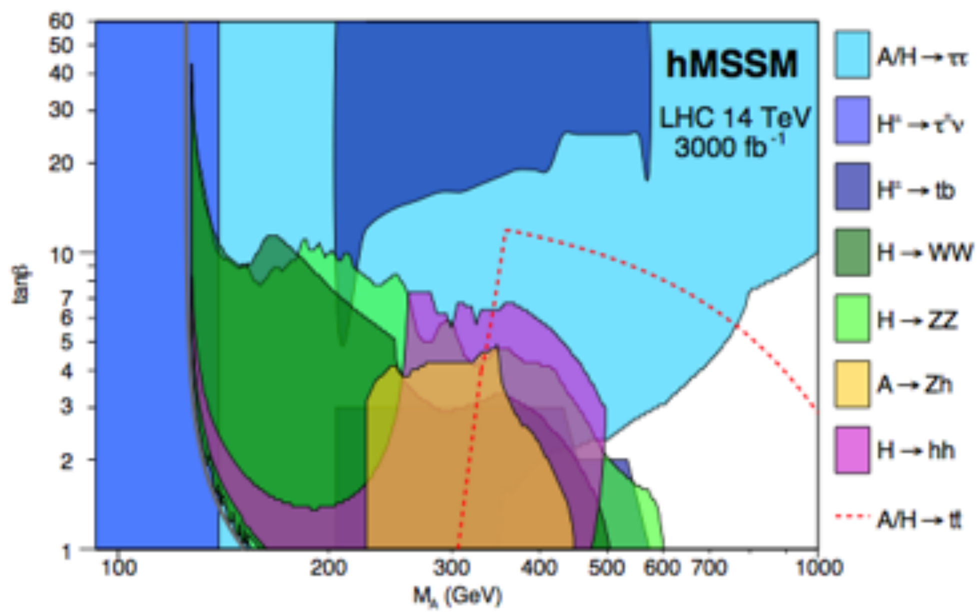
Leading effect: shift in $\Gamma(h\rightarrow b\bar{b})$, all branching ratios change.

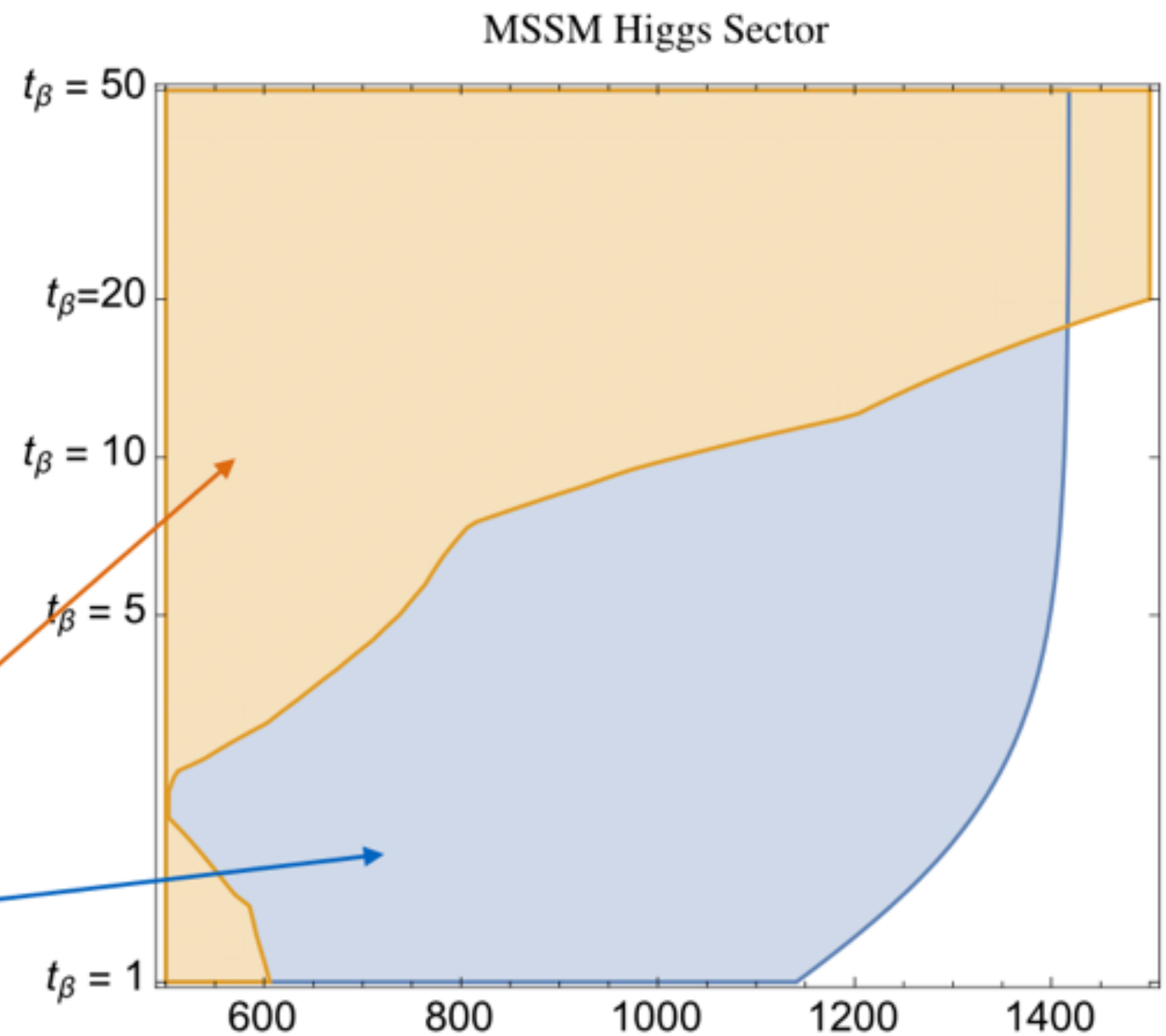
(see e.g. 1212.5240 by Gupta, Montull, Riva for a clear exposition)

N. Craig's slide from August CEPC meeting:

HL-LHC direct reach vs. CEPC coupling reach

Djouadi, Maiani, Polosa, Quevillon, Riquer [1502.05653]





HL-LHC reach (direct)

CEPC reach (indirect)

Cover the "wedge" region to 1.2 TeV

Composite Higgs

(see Contino 1005.4269 for a review)

Tuning in Higgs VEV for a light Higgs. Specifically: for Higgs as a pseudo-Goldstone, expect a potential something like

$$V(h) \sim \frac{a\lambda^2}{16\pi^2}\cos(h/f) + \frac{b\lambda^2}{16\pi^2}\sin^2(h/f)$$

This has $v \sim f$ unless:

$$-2\cos(h/f) - (1+\epsilon)\sin^2(h/f) \Rightarrow \langle h \rangle^2 \approx 2\epsilon f^2$$

We tune $v << f$ by making $\epsilon << 1$.

(Exception: “little Higgs” with extended symmetry structure. Pay a big price in complexity.)

Composite Higgs

Constraints: *S*-parameter $S \approx \frac{4\pi v^2}{m_\rho^2}, \quad m_\rho^{(NDA)} \sim \frac{4\pi f}{\sqrt{N}}$

Higgs couplings: $a = \frac{g_{VVH}}{g_{VVh}^{\mathrm{SM}}} = \sqrt{1 - \frac{v^2}{f^2}}$

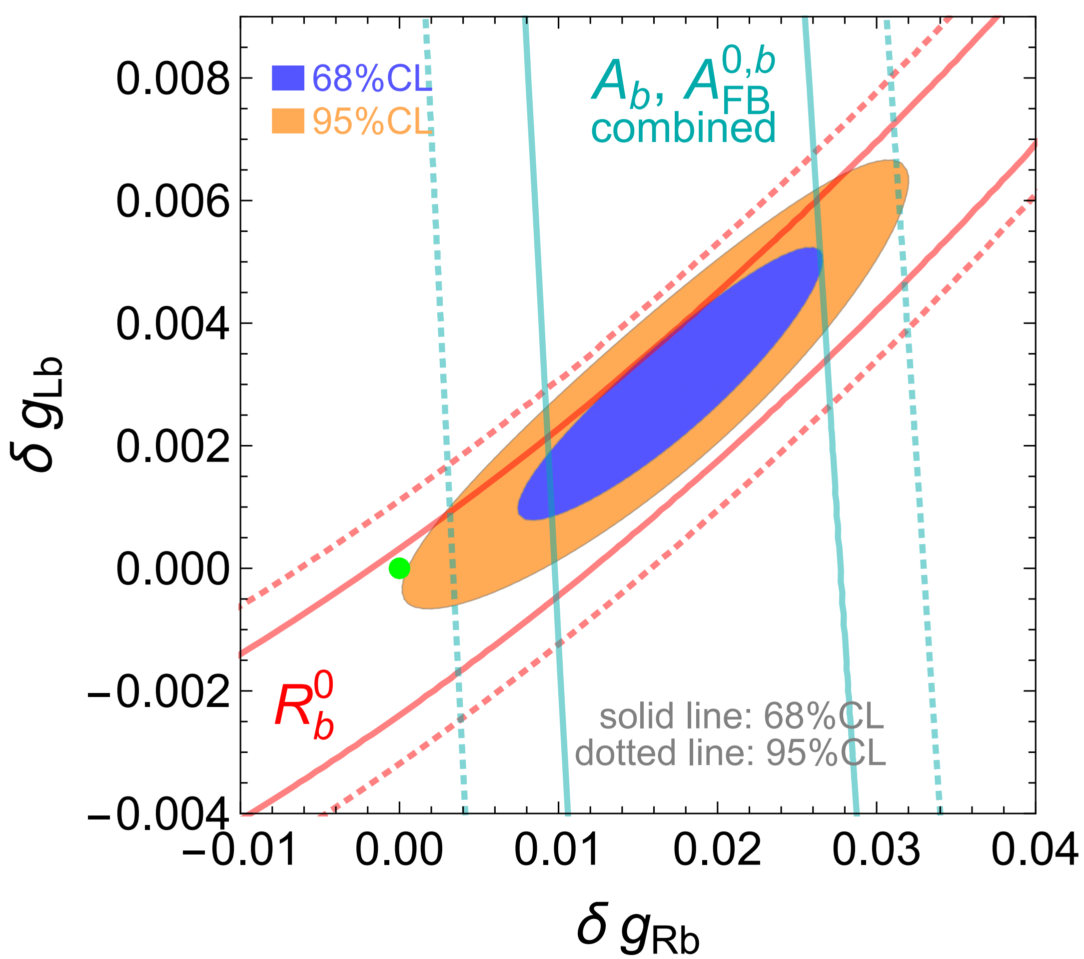
Currently bounds from *S* and Higgs couplings translate to roughly

$$m_\rho \gtrsim 3 \text{ TeV}, \quad f \gtrsim \max(\sqrt{\frac{N}{3}} \times 400 \text{ GeV}, 550 \text{ GeV})$$

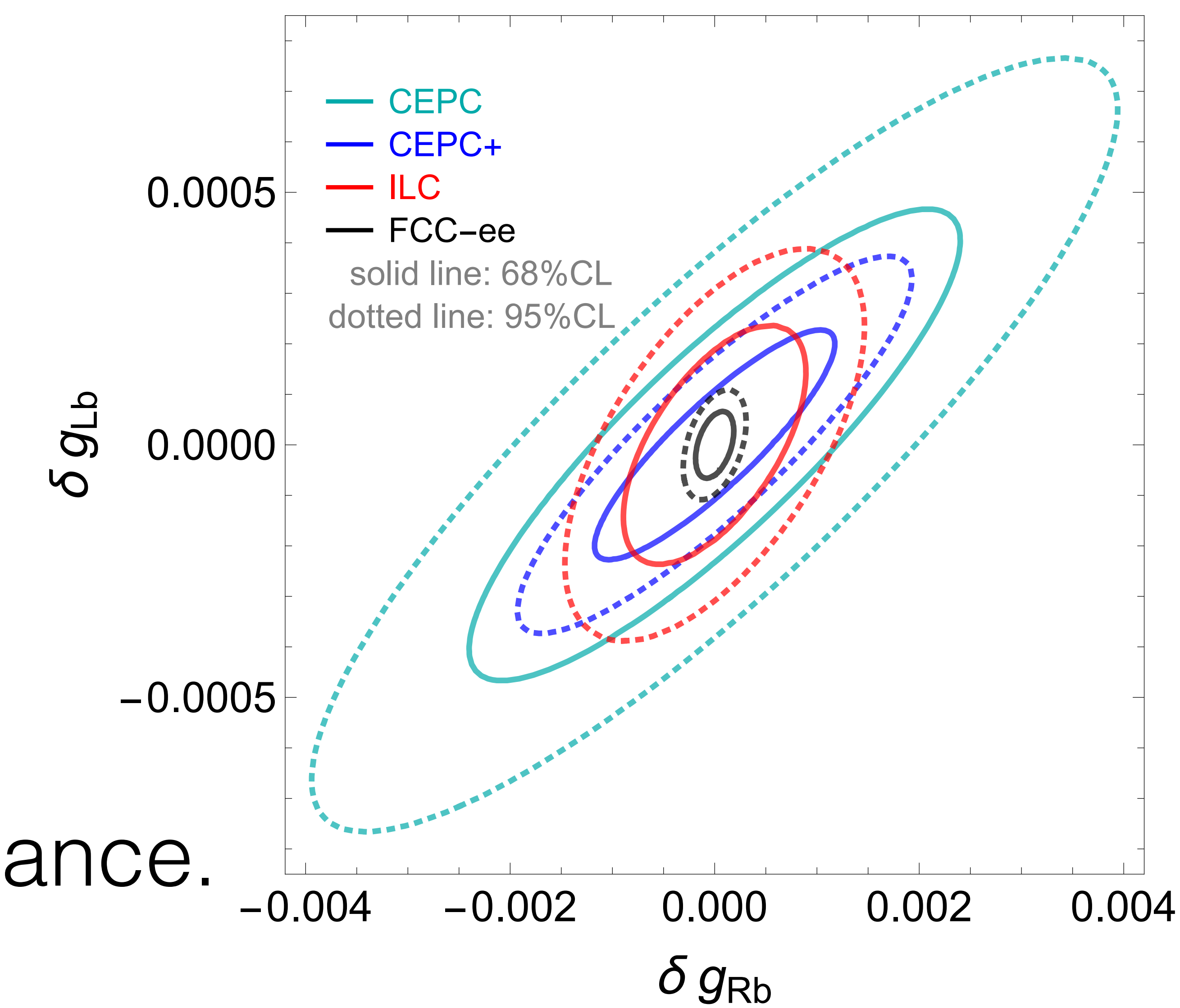
FCC-ee would bring the *ZZh* coupling measurement to the 0.1% level, probing *f* ~ 6 TeV and achieving **a factor of ~ 1000 tuning in the Higgs VEV**

Bottom quark couplings

S. Gori, J. Gu, L.-T. Wang 1508.07010



existing data: some tension with Standard Model predictions.



Future reach: if deviations are present, detect at high significance.

Higgs vs. EWPT

Whether the (S, T) fit or Higgs coupling measurements are more sensitive to new physics depends on the model. Two well-motivated examples:

Composite Higgs: probe scale *f* via ***ZH***, *S*-parameter
Left-handed stops: probe mass via *Hgg*, ***T*-parameter**

Experiment	κ_Z (68%)	f (GeV)	κ_g (68%)	$m_{\tilde{t}_L}$ (GeV)
HL-LHC	3%	1.0 TeV	4%	430 GeV
ILC500	0.3%	3.1 TeV	1.6%	690 GeV
ILC500-up	0.2%	3.9 TeV	0.9%	910 GeV
CEPC	0.2%	3.9 TeV	0.9%	910 GeV
TLEP	0.1%	5.5 TeV	0.6%	1.1 GeV

Experiment	S (68%)	f (GeV)	T (68%)	$m_{\tilde{t}_L}$ (GeV)
ILC	0.012	1.1 TeV	0.015	890 GeV
CEPC (opt.)	0.02	880 GeV	0.016	870 GeV
CEPC (imp.)	0.014	1.0 TeV	0.011	1.1 GeV
TLEP-Z	0.013	1.1 TeV	0.012	1.0 TeV
TLEP-t	0.009	1.3 TeV	0.006	1.5 TeV

(from 1411.1054 Fan, MR, Wang)

Higgs-Z Interplay

I've shown you results from fits of Higgs properties, and results from Z-pole (and near-Z-pole) physics. But these are not really independent. For instance, the S parameter operator

$$h^\dagger \sigma^i h W^i_{\mu\nu} B^{\mu\nu}$$

will affect the Higgs decay rate to two neutral gauge bosons (photons or Z bosons)—though other operators do too.

In the end, we should perform a global fit all the data together, including all the electroweak operators. Use all the information. For instance, angular observables in Higgs properties can also enhance the physics reach (Craig et al.1512.06877).

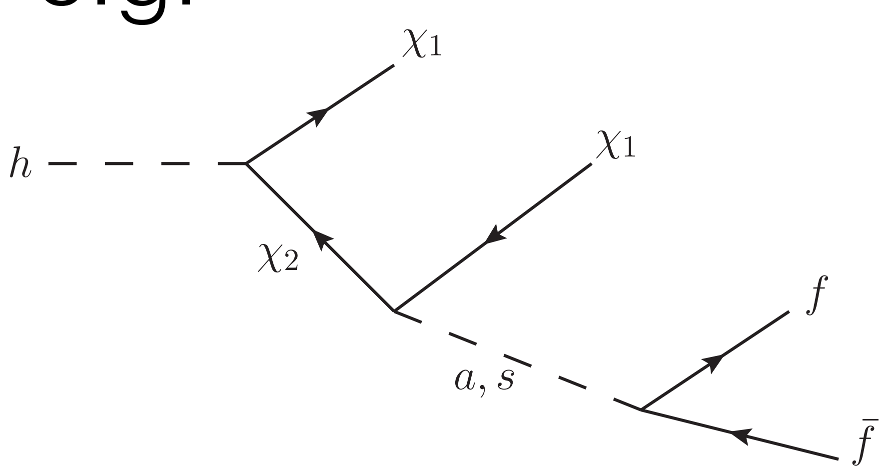
Exotic Higgs Decays

Because the Higgs coupling to *b*-quarks is so small, there is ample room for small couplings to new physics to lead to significant decay rates beyond the Standard Model.

Pseudoscalars, dark photons, dark matter, hidden valleys, …

e.g.

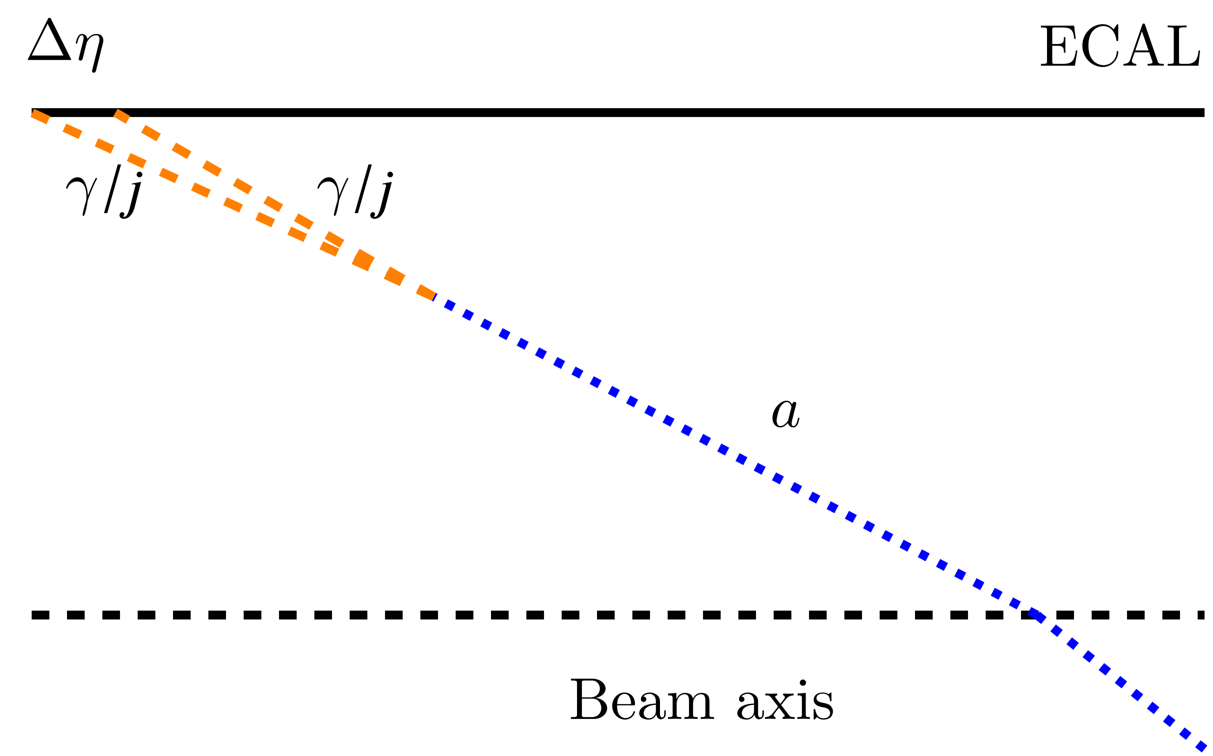
review **for LHC**: Curtin et al., 1312.4992



Little study so far for Higgs factories! **The leading "direct discovery" possibility for the circular machines.**

Exotic Higgs Decays

Could involve particles that propagate macroscopic distances in the detector before decaying!



We care about possible discoveries and not just Standard Model measurements. Important to be careful when designing detectors that opportunities to see exotic physics aren't unnecessarily closed off!

Linear vs Circular

Complementary: in an ideal world, do both.

- Linear: go to higher energy. **Higher direct discovery potential *if* new electroweak states exist.**
-
- Linear: easy to reach top threshold, **improve top mass as input to electroweak fits**.

- Circular: **resonant spin depolarization** gives precise energy calibration and **Z mass & width measurement**

- Circular: future as **high energy hadron machine**. *Important to build a large enough tunnel that foreseeable magnet technology can reach desired energies!*

Conclusions

The LHC has great potential to study colored particles, but it can miss light uncolored particles or even colored particles that decay in ways that mimic backgrounds.

Higgs factories can exhaustively probe particles that interact with Higgs bosons, whether or not the LHC can see them. **EWPT and Higgs measurements** contribute. Example: the T-parameter could be the strongest constraint on folded stops.

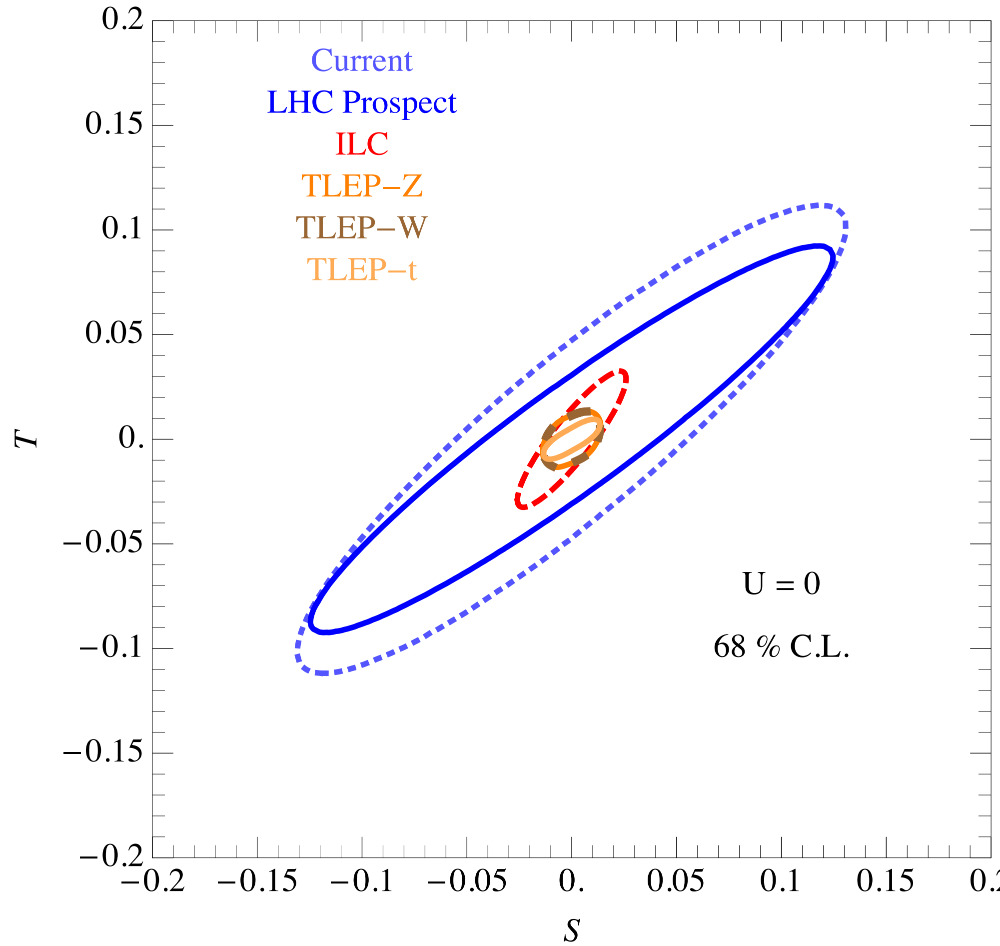
Linear and circular machines have different strengths. We must take all the options seriously.

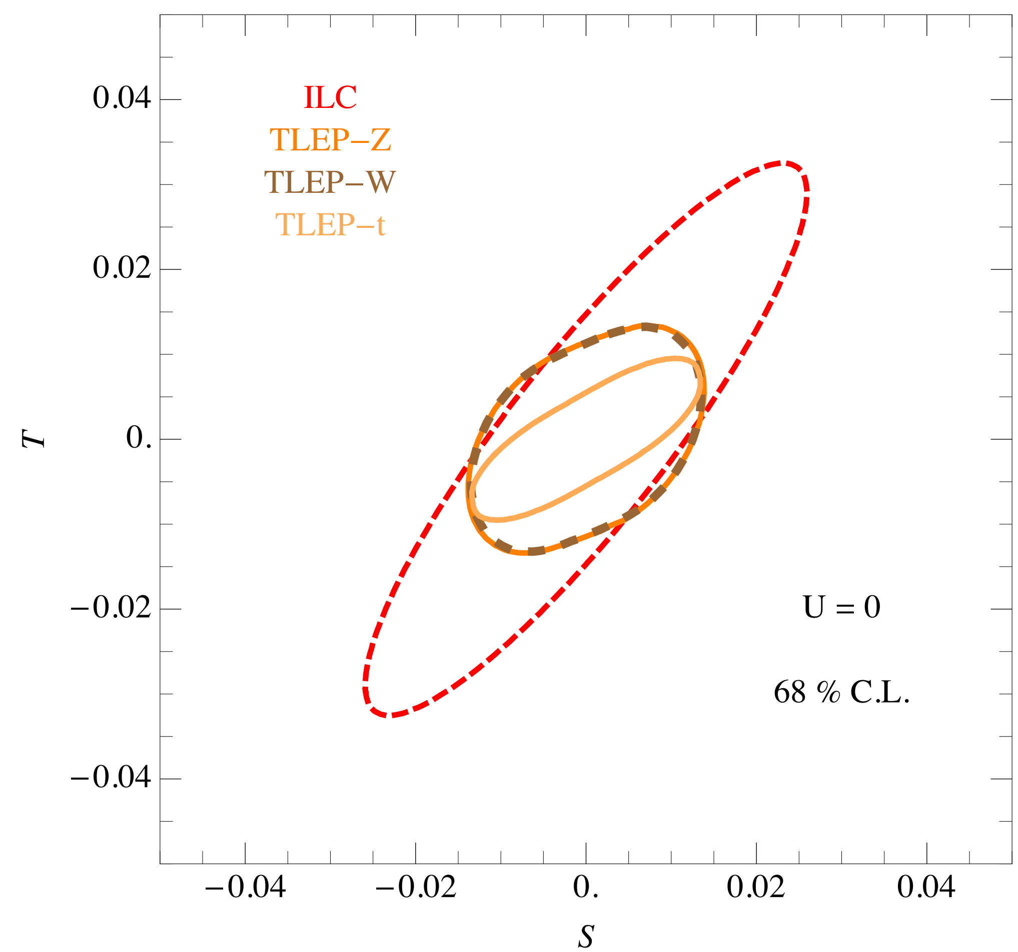
Backup

Other Colliders

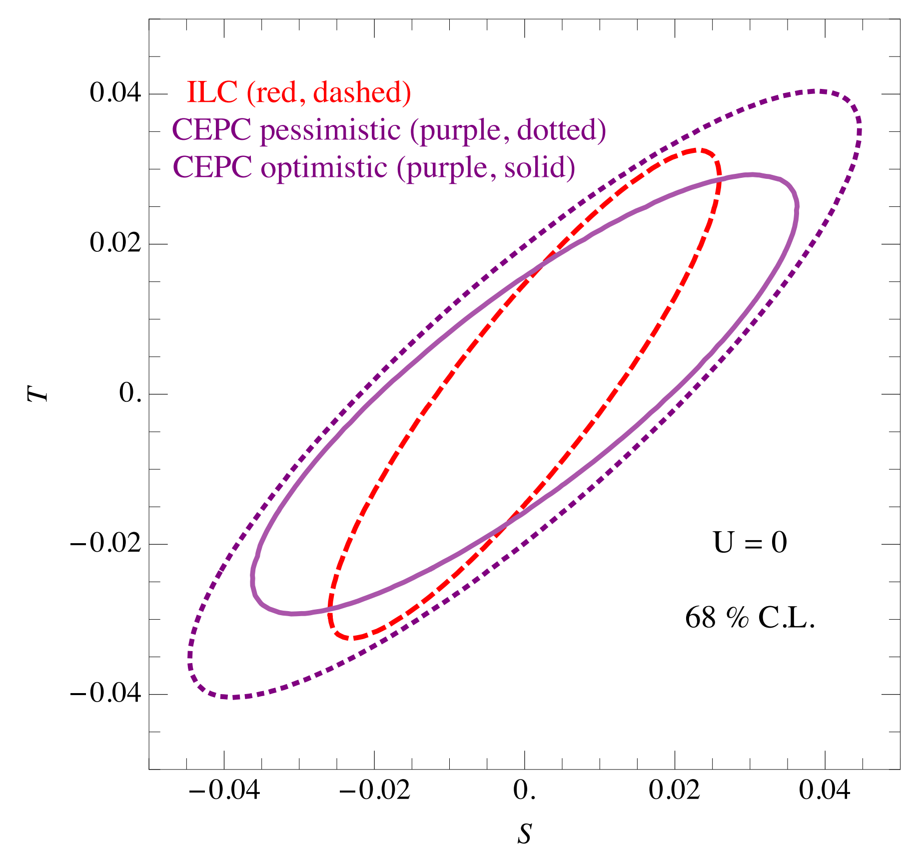
	Present data	LHC14	ILC/GigaZ
$\alpha_s(M_Z^2)$	0.1185 ± 0.0006 [34]	± 0.0006	$\pm 1.0 \times 10^{-4}$ [35]
$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$	$(276.5 \pm 0.8) \times 10^{-4}$ [36]	$\pm 4.7 \times 10^{-5}$ [23]	$\pm 4.7 \times 10^{-5}$ [23]
m_Z [GeV]	91.1875 ± 0.0021 [27]	± 0.0021 [23]	± 0.0021 [23]
m_t [GeV] (pole)	$173.34 \pm 0.76_{\rm exp}$ [37] $\pm 0.5_{\rm th}$ [23]	$\pm 0.6_{\rm exp} \pm 0.25_{\rm th}$ [23]	$\pm 0.03_{\rm exp} \pm 0.1_{\rm th}$ [23]
m_h [GeV]	125.14 ± 0.24 [23]	$< \pm 0.1$ [23]	$< \pm 0.1$ [23]
m_W [GeV]	$80.385 \pm 0.015_{\rm exp}$ [34]$\pm 0.004_{\rm th}$ [24]	$(\pm 8_{\rm exp} \pm 4_{\rm th}) \times 10^{-3}$ [23, 24]	$(\pm 5_{\rm exp} \pm 1_{\rm th}) \times 10^{-3}$ [23, 38]
$\sin^2\theta_{\rm eff}^{\ell}$	$(23153 \pm 16) \times 10^{-5}$ [27]	$\pm 16 \times 10^{-5}$	$(\pm 1.3_{\rm exp} \pm 1.5_{\rm th}) \times 10^{-5}$ [20, 38]
Γ_Z [GeV]	2.4952 ± 0.0023 [27]	± 0.0023	± 0.001 [39]

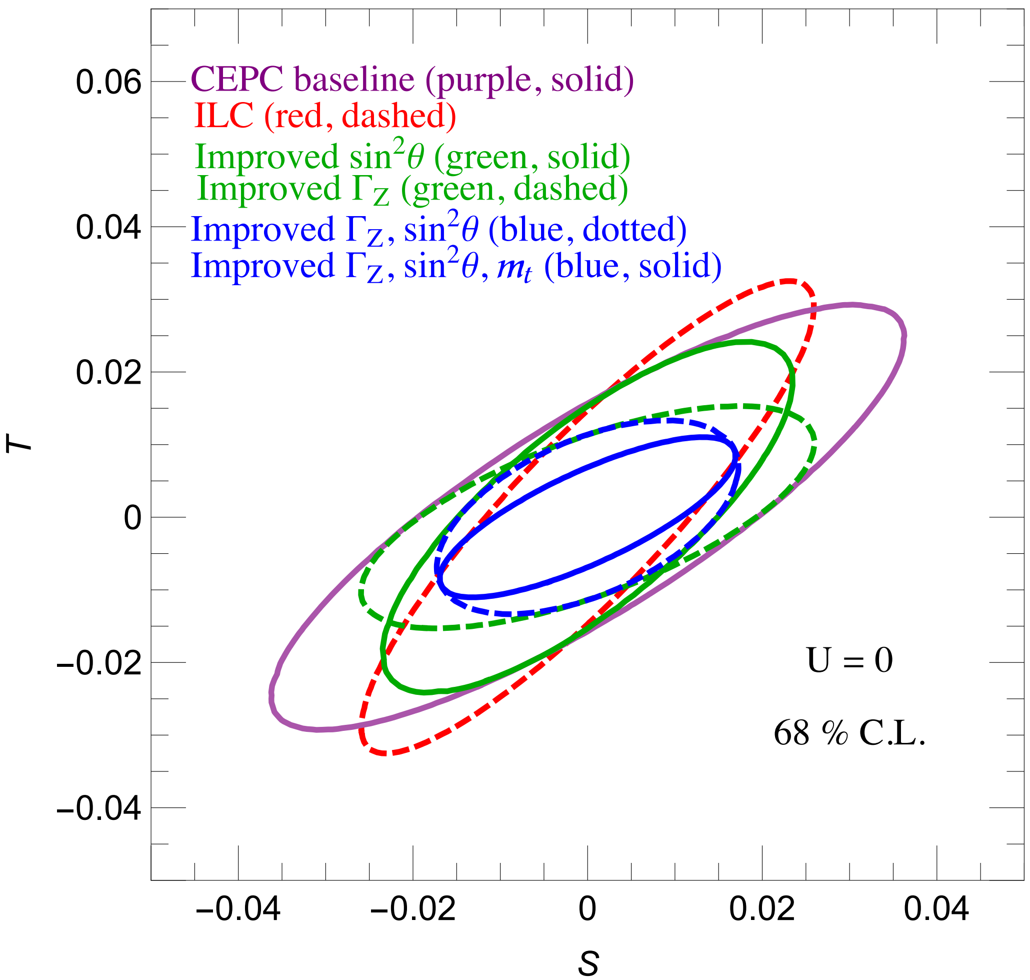
	TLEP-Z	TLEP-W	TLEP-t
$\alpha_s(M_Z^2)$	$\pm 1.0 \times 10^{-4}$ [35]	$\pm 1.0 \times 10^{-4}$ [35]	$\pm 1.0 \times 10^{-4}$ [35]
$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$	$\pm 4.7 \times 10^{-5}$	$\pm 4.7 \times 10^{-5}$	$\pm 4.7 \times 10^{-5}$
m_Z [GeV]	$\pm 0.0001_{\rm exp}$ [2]	$\pm 0.0001_{\rm exp}$ [2]	$\pm 0.0001_{\rm exp}$ [2]
m_t [GeV] (pole)	$\pm 0.6_{\rm exp} \pm 0.25_{\rm th}$ [23]	$\pm 0.6_{\rm exp} \pm 0.25_{\rm th}$ [23]	$\pm 0.02_{\rm exp} \pm 0.1_{\rm th}$ [2, 23]
m_h [GeV]	$< \pm 0.1$	$< \pm 0.1$	$< \pm 0.1$
m_W [GeV]	$(\pm 8_{\rm exp} \pm 1_{\rm th}) \times 10^{-3}$ [23, 38]	$(\pm 1.2_{\rm exp} \pm 1_{\rm th}) \times 10^{-3}$ [20, 38]	$(\pm 1.2_{\rm exp} \pm 1_{\rm th}) \times 10^{-3}$ [20, 38]
$\sin^2\theta_{\rm eff}^{\ell}$	$(\pm 0.3_{\rm exp} \pm 1.5_{\rm th}) \times 10^{-5}$ [20, 38]	$(\pm 0.3_{\rm exp} \pm 1.5_{\rm th}) \times 10^{-5}$ [20, 38]	$(\pm 0.3_{\rm exp} \pm 1.5_{\rm th}) \times 10^{-5}$ [20, 38]
Γ_Z [GeV]	$(\pm 1_{\rm exp} \pm 0.8_{\rm th}) \times 10^{-4}$ [2, 26]	$(\pm 1_{\rm exp} \pm 0.8_{\rm th}) \times 10^{-4}$ [2, 26]	$(\pm 1_{\rm exp} \pm 0.8_{\rm th}) \times 10^{-4}$ [2, 26]





	CEPC
$\alpha_s(M_Z^2)$	$\pm 1.0 \times 10^{-4}$ [35]
$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$	$\pm 4.7 \times 10^{-5}$
m_Z [GeV]	$\pm(0.0005 - 0.001)$ [41]
m_t [GeV] (pole)	$\pm 0.6_{\rm exp} \pm 0.25_{\rm th}$ [23]
m_h [GeV]	$< \pm 0.1$
m_W [GeV]	$(\pm(3-5)_{\rm exp} \pm 1_{\rm th}) \times 10^{-3}$ [24, 38, 41]
$\sin^2\theta_{\rm eff}^{\ell}$	$(\pm(4.6-5.1)_{\rm exp} \pm 1.5_{\rm th}) \times 10^{-5}$ [25, 38, 41]
Γ_Z [GeV]	$(\pm(5-10)_{\rm exp} \pm 0.8_{\rm th}) \times 10^{-4}$ [26, 41]





Current	m_t	m_Z	m_h	α_s	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$
δm_W [MeV]	4.6	2.6	0.1	0.4	1.5
$\delta\sin^2\theta_{\rm eff}^{\ell}(10^{-5})$	2.4	1.5	0.1	0.2	2.8
$\delta\Gamma_Z$ [MeV]	0.2	0.2	0.004	0.30	0.08

ILC	m_t	m_Z	m_h	α_s	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$
δm_W [MeV]	0.2	2.6	0.05	0.06	0.9
$\delta\sin^2\theta_{\rm eff}^{\ell}(10^{-5})$	0.09	1.5	0.04	0.03	1.6
$\delta\Gamma_Z$ [MeV]	0.007	0.2	0.002	0.05	0.04

TLEP-$Z(W)$	m_t	m_Z	m_h	α_s	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$
δm_W [MeV]	3.6	0.1	0.05	0.06	0.9
$\delta\sin^2\theta_{\rm eff}^{\ell}(10^{-5})$	1.9	0.07	0.04	0.03	1.6
$\delta\Gamma_Z$ [MeV]	0.1	0.01	0.002	0.05	0.04

TLEP-t	m_t	m_Z	m_h	α_s	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$
δm_W [MeV]	0.1	0.1	0.05	0.06	0.9
$\delta\sin^2\theta_{\rm eff}^{\ell}(10^{-5})$	0.06	0.07	0.04	0.03	1.6
$\delta\Gamma_Z$ [MeV]	0.004	0.01	0.002	0.05	0.04

CEPC	m_t	m_Z	m_h	α_s	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$
δm_W [MeV]	3.6	0.6-1.3	0.05	0.06	0.9
$\delta\sin^2\theta_{\rm eff}^{\ell}(10^{-5})$	1.9	0.4-0.7	0.04	0.03	1.6
$\delta\Gamma_Z$ [MeV]	0.1	0.05-0.1	0.002	0.05	0.04

Table 5. Parametric errors from each free parameter in the fit for current, ILC, TLEP-Z (TLEP-W), TLEP-t and CEPC scenarios.