Chromaticity correction of the interaction region

A. Bogomyagkov

Budker Institute of Nuclear Physics
Novosibirsk

IAS Program on High Energy Physics, Hong Kong
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Introduction

Momentum acceptance is needed for

- synchrotron radiation sources to provide long beam life time
  - limited by nonlinear chromaticity of the rigid low emittance cell,
- colliders with extremely small IP beta
  - limited by nonlinear chromaticity of the interaction region.

Two examples

Energy acceptance optimization with the help of the beta function chromaticity.

- FCC-ee with dedicated local chromaticity correction sections,
- imaginary collider with distributed chromaticity correction.

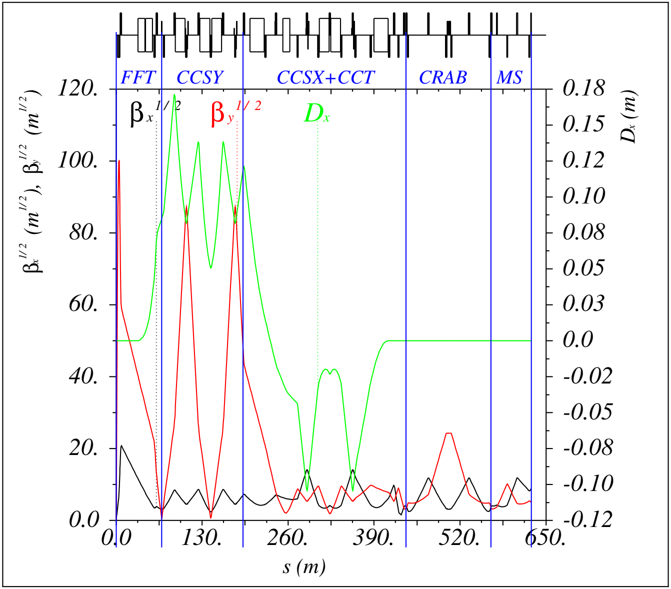
Example 1: FCC-ee parameters for crab waist

	Z	W	H	tt
Energy [GeV]	45	80	120	175
Perimeter [km]	100			
Crossing angle [mrad]	30			
Particles per bunch [10^{11}]	1	4	4.7	4
Number of bunches	29791	739	127	33
Energy spread [10^{-3}]	1.1	2.1	2.4	2.6
Emittance hor. [nm]	0.14	0.44	1	2.1
Emittance ver. [pm]	1	2	2	4.3
β_x^*/β_y^* [m]	0.5 / 0.001			
Luminosity / IP [$10^{34}\ cm^{-2}s^{-1}$]	212	36	9	1.3
Energy loss / turn [GeV]	0.03	0.3	1.7	7.7

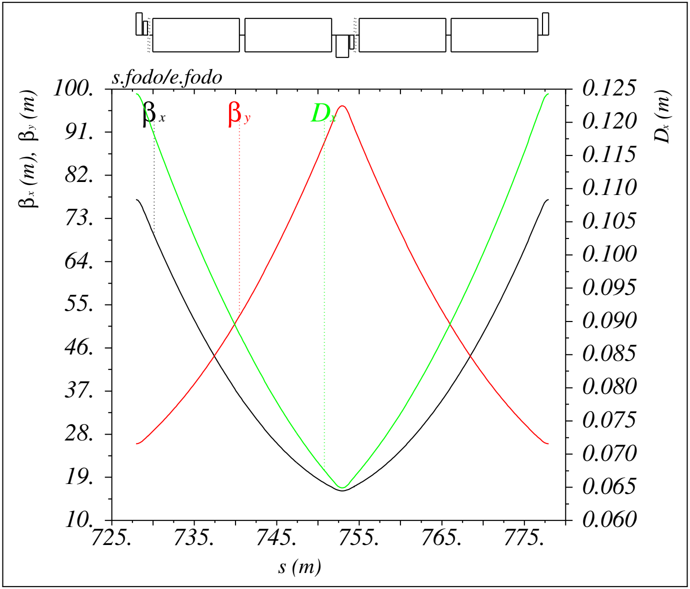
Parameters of one quarter of the ring

	tt
Energy [GeV]	175
Perimeter [m]	24655.9
Momentum compaction	$5.7 \cdot 10^{-6}$
Emittance hor. [nm]	1.3
Energy spread [10^{-3}]	1.6
β_x^*/β_y^* [m]	0.5 / 0.001
Q_x/Q_y	124.54 / 84.57
Q'_x/Q'_y	-152 / -816
Energy loss / turn [GeV]	2.12

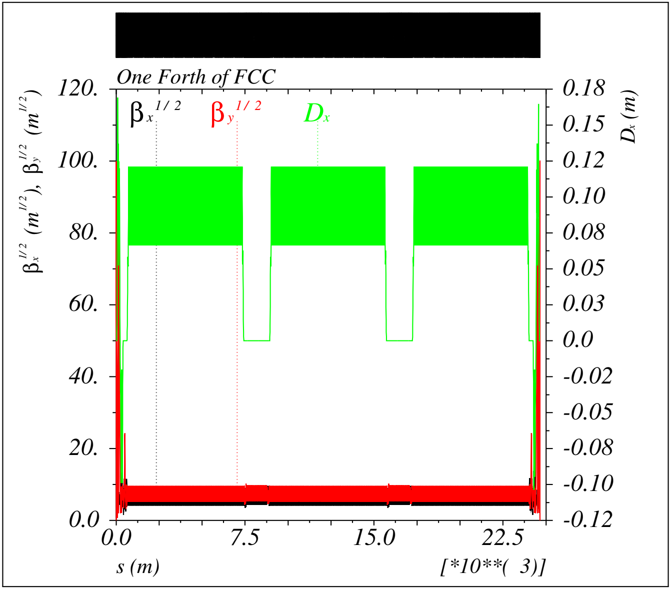
Interaction Region optical functions



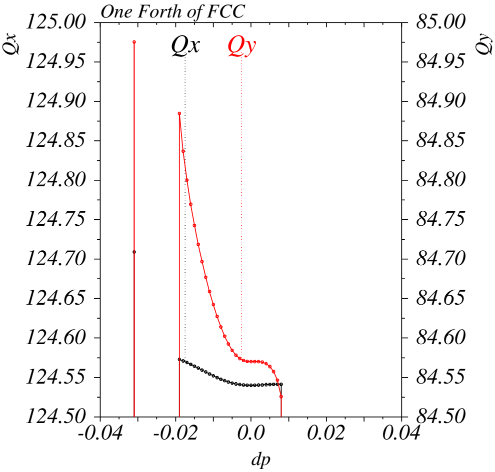
Arc cell optical functions: $\mu_x = \pi/2$, $\mu_y = \pi/3$



One quarter ring optical functions

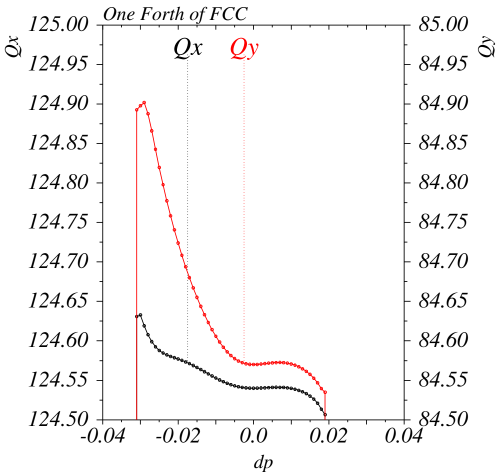


Energy acceptance I: [-1.9%;+0.8%]



	Value	$\Delta Q(2\%)$
Q_x	124.54	
Q'_x	0	0
Q''_x	170	0.034
Q'''_x	$-4.5 \cdot 10^4$	-0.059
Q''''_x	$-5.3 \cdot 10^6$	-0.035
Q_y	84.57	
Q'_y	0	0
Q''_y	387	0.077
Q'''_y	$-5.3 \cdot 10^5$	-0.7
Q''''_y	$-4.3 \cdot 10^6$	-0.029

Energy acceptance II: [-3.1%;+1.9%]



	Value	$\Delta Q(2\%)$
Q_x	124.54	
Q'_x	0	0
Q''_x	170	0.034
Q'''_x	$-5.1 \cdot 10^4$	-0.068
Q''''_x	$-4.8 \cdot 10^6$	-0.032
Q_y	84.57	
Q'_y	0	0
Q''_y	387	0.077
Q'''_y	$-1.4 \cdot 10^5$	-0.182
Q''''_y	$1.9 \cdot 10^6$	-0.013

How does it work (chromaticity estimations)?

Montague functions first and second order

$$b_{y,1} = \frac{1}{\beta_y}\frac{\partial \beta_y}{\partial \delta}\,, \qquad b_{y,2} = \frac{1}{\beta_y}\frac{\partial^2 \beta_y}{\partial \delta^2}\,, \qquad \frac{d^2 b_i}{d\phi^2} + 4b_i = f_i\,,$$

$$a_{y,1} = \frac{\partial \alpha_y}{\partial \delta} - \frac{\alpha_y}{\beta_y}\frac{\partial \beta_y}{\partial \delta}\,, \quad a_{y,2} = \frac{\partial^2 \alpha_y}{\partial \delta^2} - \frac{\alpha_y}{\beta_y}\frac{\partial^2 \beta_y}{\partial \delta^2}\,, \quad \frac{d^2 a_i}{d\phi^2} + 4a_i = g_i\,.$$

Dispersion

$$x = \eta_0\delta + \eta_1\delta^2 + \eta_2\delta^3\,,$$

$$\eta = \eta_0 + \eta_1\delta + \eta_2\delta^2\,.$$

How does it work (chromaticity estimations)?

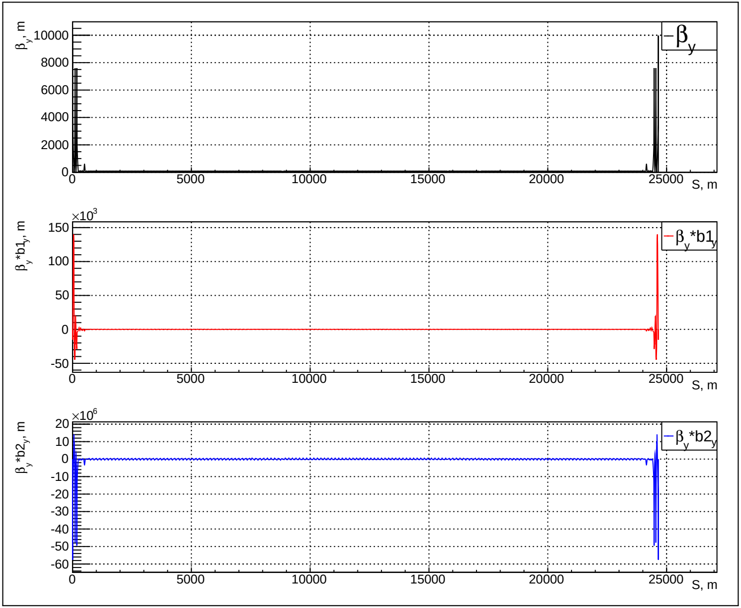
Chromaticity

$$\frac{\partial \varphi_y}{\partial \delta} = \frac{1}{2}\int_0^{\Pi} \beta_y (K_1 - K_2\eta_0) ds$$

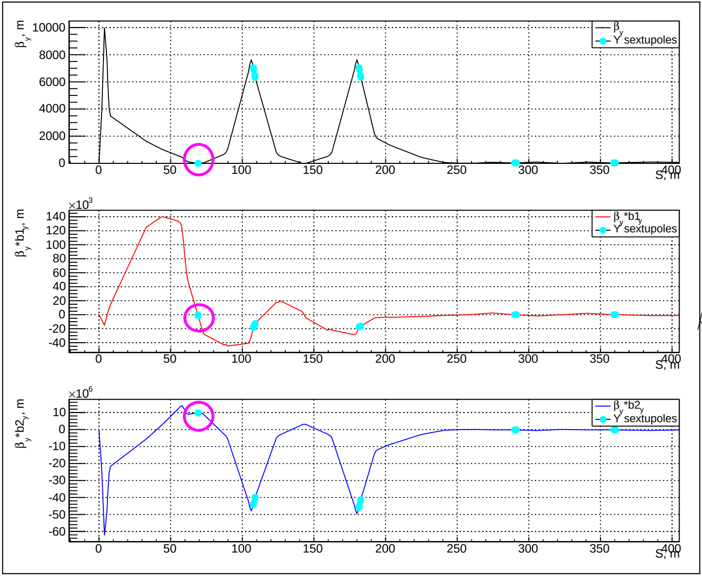
$$\frac{\partial^2 \varphi_y}{\partial \delta^2} = -2\frac{\partial \varphi_y}{\partial \delta} - \int_0^{\Pi} \beta_y (K_2\eta_1 + K_3\frac{\eta_0^2}{2}) ds + \frac{1}{2}\int_0^{\Pi} \beta_y b_{y,1} (K_1 - K_2\eta_0) ds$$

$$\frac{\partial^3 \varphi_y}{\partial \delta^3} = 6\frac{\partial \varphi_y}{\partial \delta} - 3\int_0^{\Pi} \beta_y (a_{y,1}^2 + b_{y,1}^2)(K_1 - K_2\eta_0) ds + 3\int_0^{\Pi} \beta_y \left(K_3 \left(\frac{\eta_0^2}{2} - \eta_0\eta_1 \right) + K_2(\eta_1 - \eta_2) \right) ds + \frac{3}{2}\int_0^{\Pi} \beta_y b_{y,2} (K_1 - K_2\eta_0) ds$$

Beta chromaticity



Beta chromaticity



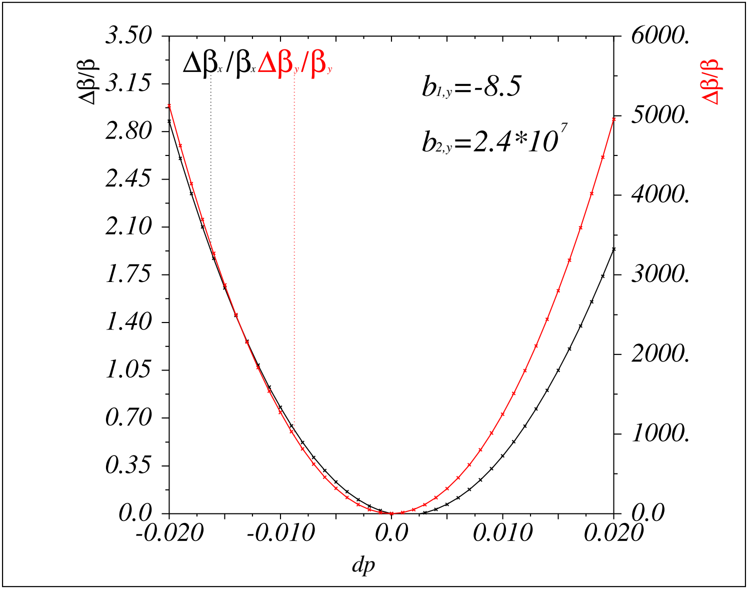
$$\eta_0 = 0.085\text{m}$$

$$K_2 L = -2\text{m}^{-2}$$

$$\beta_y b_{y,2} = 10^7\text{m}$$

$$\Delta Q''' = 4 \cdot 10^5$$

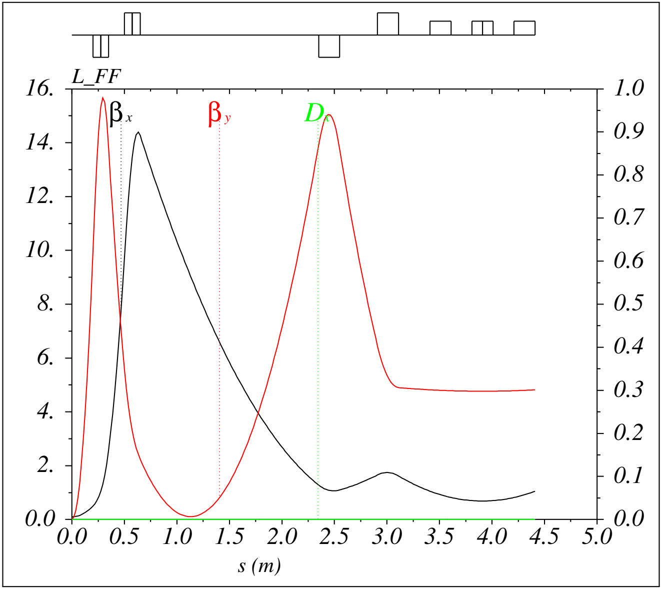
Final Focus Telescope: beta chromaticity



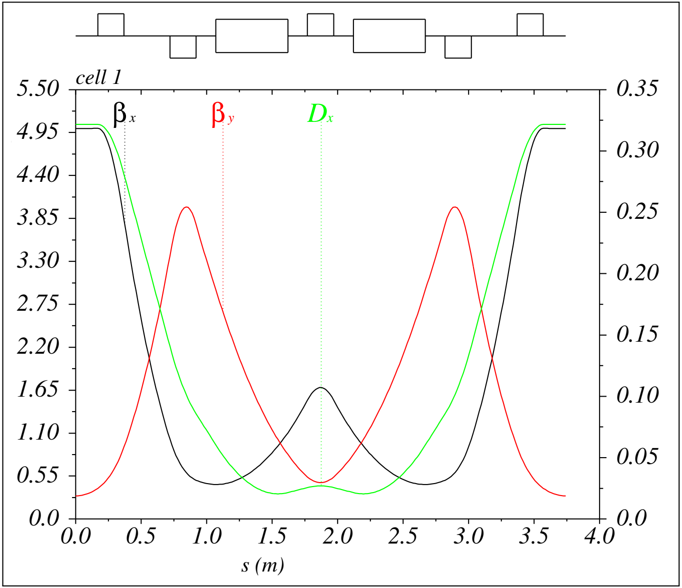
Example 2: 80 m collider

Energy [GeV]	1
Perimeter [m]	83
Crossing angle [mrad]	100
Current [A]	2.3
Number of bunches	80
Particles per bunch [10^{10}]	4.97
Emittance hor. [nm]	9(IBS)/4.35
Emittance ver. [pm]	90
$\beta_x^*[m]/\beta_y^*[m]$	0.1/0.004
Luminosity [$10^{34} cm^{-2} s^{-1}$]	1.5
Momentum compaction	2.3×10^{-3}
Q_x/Q_y	12.543 / 12.574
Q'_x/Q'_y	-24 / -35.6

Interaction Region optical functions



Arc cell optical functions: $\mu_x = 5\pi/4$, $\mu_y = 5\pi/4$

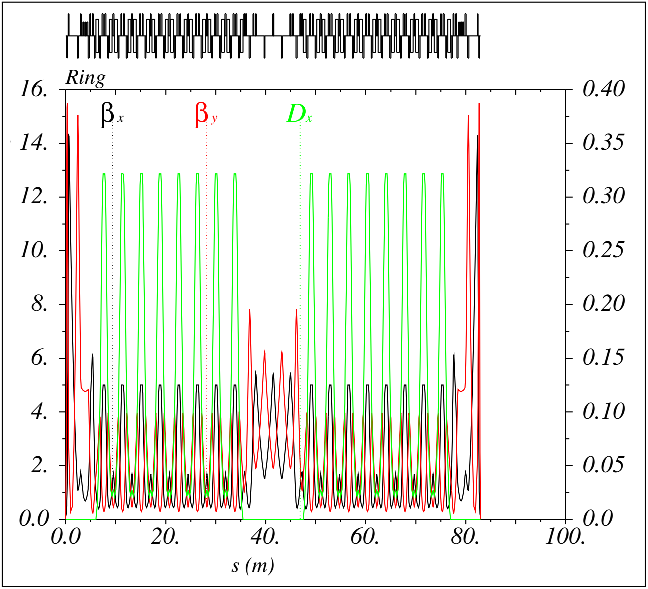


A.Bogomyagkov, E.Levichev, P.Piminov, S.Sinyatkin

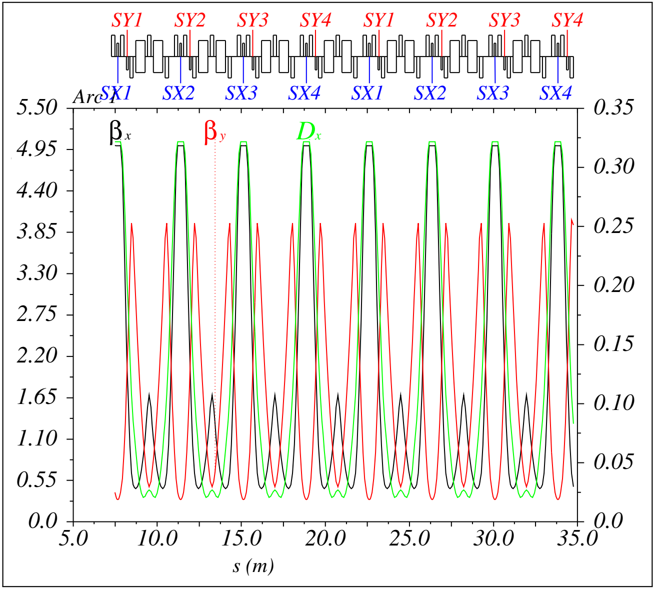
"Ultimate synchrotron radiation source with horizontal field wigglers"

Low Emittance Rings 2014 Workshop

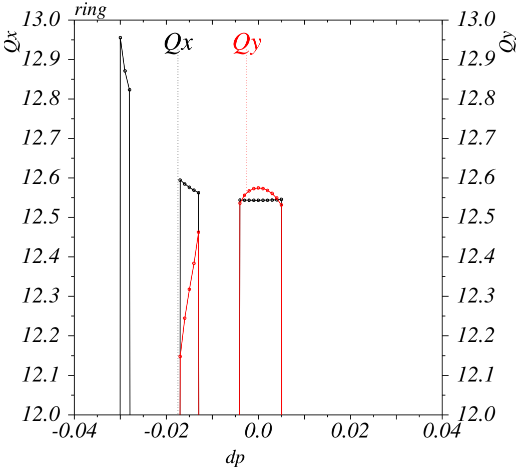
Ring optical functions



Four -I families of sextupoles

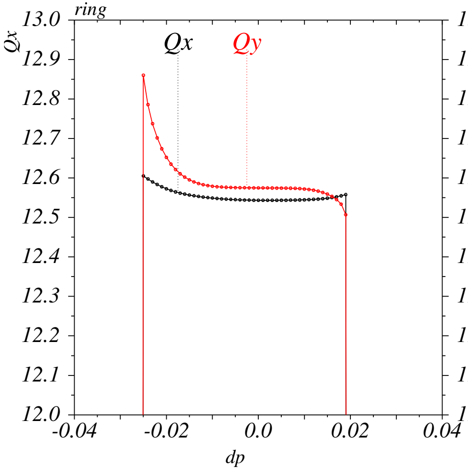


Energy acceptance I: two families [-0.4%;+0.5%]



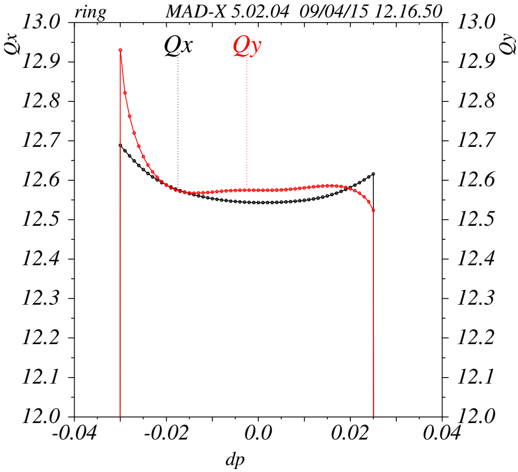
	Value	$\Delta Q(2\%)$
Q_x	12.543	
Q'_x	0	0
Q''_x	25	0.005
Q'''_x	$5.3 \cdot 10^4$	0.07
Q''''_x	$2 \cdot 10^7$	0.13
Q_y	12.575	
Q'_y	0	0
Q''_y	$-3.4 \cdot 10^3$	-0.67
Q'''_y	$4.2 \cdot 10^5$	0.57
Q''''_y	$-2.8 \cdot 10^8$	-1.9

Energy acceptance II: four families [-2.5%;+1.9%]



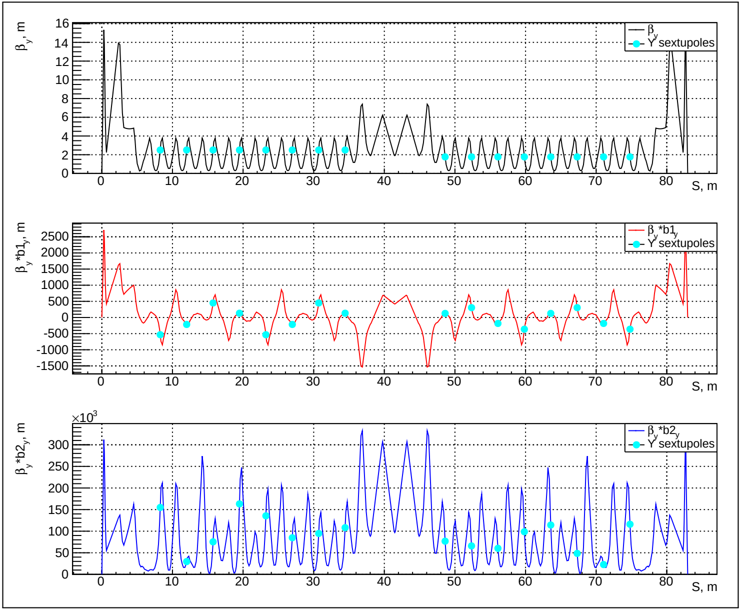
	Value	$\Delta Q(2\%)$
Q_x	12.543	
Q'_x	0	0
Q''_x	4.5	0.001
Q'''_x	$-1.9 \cdot 10^3$	-0.003
Q''''_x	$-3 \cdot 10^5$	-0.002
Q_y	12.575	
Q'_y	0	0
Q''_y	0	0
Q'''_y	0	0
Q''''_y	$-5 \cdot 10^5$	-0.003

Energy acceptance III: four families [-3%;+2.5%]

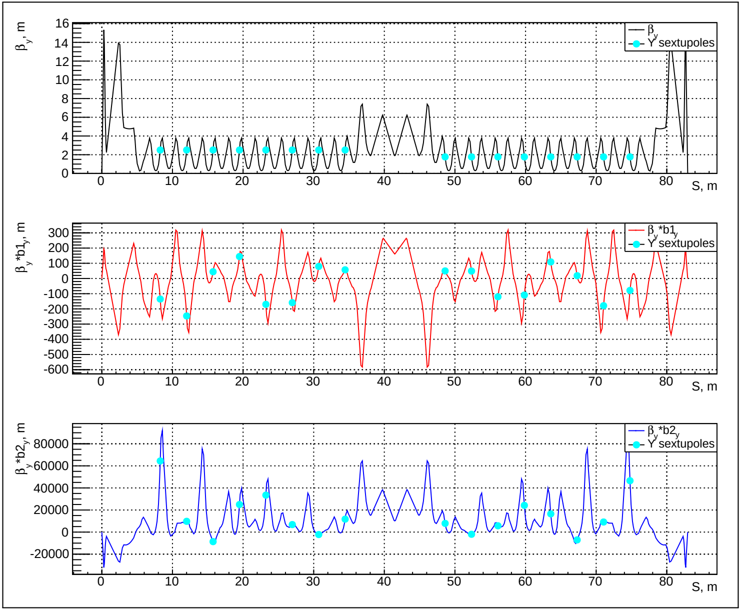


	Value	$\Delta Q(2\%)$
Q_x	12.543	
Q'_x	0	0
Q''_x	90	0.018
Q'''_x	$-4.9 \cdot 10^3$	-0.006
Q''''_x	$2.8 \cdot 10^5$	0.002
Q_y	12.575	
Q'_y	0	0
Q''_y	0	0
Q'''_y	$6 \cdot 10^4$	0.08
Q''''_y	$6.6 \cdot 10^4$	0.0004

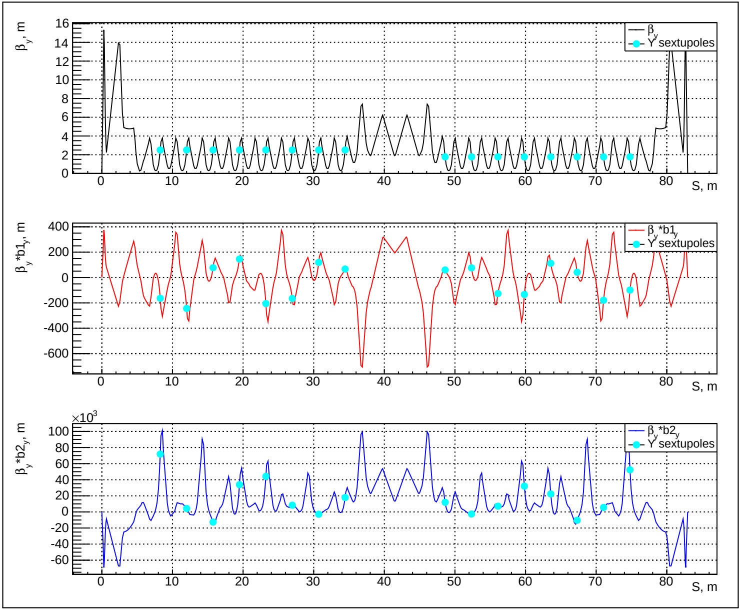
Beta chromaticity I: two families [-0.4%;+0.5%]



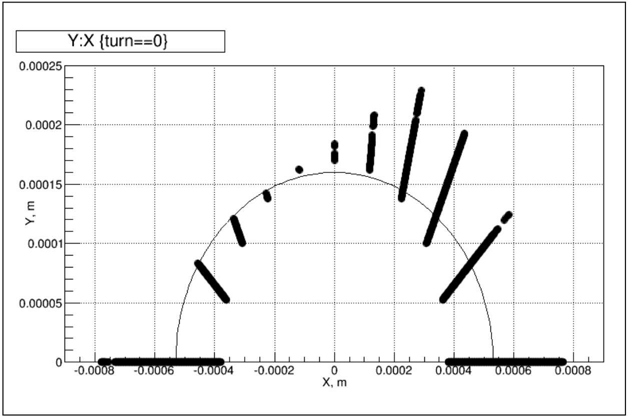
Beta chromaticity II: four families [-2.5%;+1.9%]



Beta chromaticity III: four families [-3%;+2.5%]



Dynamic aperture: on momentum, PTC



$\beta_x = 0.1$ m
$\beta_y = 0.004$ m
$R_x = 5.3 \cdot 10^{-4}$ m
$R_y = 1.6 \cdot 10^{-4}$ m
$\sigma_x = 3 \cdot 10^{-5}$ m
$\sigma_y = 6 \cdot 10^{-7}$ m

$ksy1 = -330$ T/m^2
$ksx1 = 495$ T/m^2
$ksy2 = -2695$ T/m^2
$ksx2 = 1161$ T/m^2
$ksy3 = -2886$ T/m^2
$ksx3 = 1414$ T/m^2
$ksy4 = -3754$ T/m^2
$ksx4 = 916$ T/m^2
$Ls = 0.1$ m

Conclusion

- Presented two examples of colliders, where nonlinear chromaticity up to fourth order is corrected by sextupoles.
- In the first example correction is performed by using local chromaticity section and additional sextupole with second order beta chromaticity.
- In the second example correction is performed by using four -I pairs of interleaved of sextupoles with phase advance between sextupoles of $5\pi/4$.

Reference

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- A. Bogomyagkov, “Lattice Momentum Acceptance Optimization”, 1st Workshop on Low Emittance Lattice Design 23-24 April 2015, Barcelona, Spain. https://indico.cern.ch/event/370770/
- A.V. Bogomyagkov, E.B. Levichev, “Interaction region for crab waist scheme of the Future Electron-Positron Collider (CERN)”, in Proc. 6th International Particle Accelerator Conference, Richmond, VA, USA, paper TUPTY018, pp. 2034-2037, ISBN: 978-3-95450-168-7, http://jacow.org/IPAC2015/papers/tupty018.pdf, 2015.