Charged Particle Optics in Circular Higgs Factory

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Luminosity

• Bunch luminosity

\[ L_b = f_{\text{rev}} \frac{N_b^2}{4\pi\sigma_x\sigma_y} R_h \]

where \( R_h \) is a geometrical reduction from the hourglass effect and is written as

\[ R_h = \sqrt{\frac{2}{\pi}} ae^{a^2} K_0(a^2), a = \frac{\beta_y^*}{\sqrt{2\sigma_z}} \]

• Total luminosity

\[ L = n_b L_b \]
Beam-Beam Limit

- Given the beam-beam parameter
  \[ \xi_y = \frac{r_e N_b \beta_y^*}{2\pi \gamma \sigma_y (\sigma_x + \sigma_y)} \]

  The luminosity can be re-written as
  \[ L = \frac{cI\gamma \xi_y}{2r_e^2 I_A \beta_y^*} R_h \]

  where \( I_A = 17045 \) A. Smaller \( \beta_y^* \) is absolutely necessary. For example, in this design we have \( I = 14.4 \) mA, \( E_0 = 120 \) GeV, \( \xi_y = 0.07 \), \( R_h = 0.76 \), \( \beta_y^* = 2 \) mm, gives \( 1 \times 10^{34} \) cm\(^{-2}\)s\(^{-1}\) in luminosity. So what is the beam-beam limit for Higgs factory?
Figure 1: Measured $\xi_y$ at 94.5 GeV versus bunch current. The data is fitted with ("Model fit") and without ("Linear fit") beam-beam beam limitation.

R. Assmann and K. Cornelis, Proceeding of EPAC 2000, Vienna, Austria
Power Limitation

• Synchrotron radiation

\[ U_0 = \frac{4\pi r_e m c^2}{3 \rho} \gamma^4 \]

• Beam power given by RF

\[ P_b = \frac{U_0 I}{e} \]

• Limits the total beam current I

For example, \( E_0 = 120 \text{ GeV}, \rho = 5.2 \text{ km}, U_0 = 3.6 \text{ GeV}, \]
\( I = 14.4 \text{ mA}, \) lead to \( P_b = 50 \text{ MW} \) in this design.
Scaling of Luminosity

• If there is a beam-beam limit as suggested by the simulation and beam power is also limited, the luminosity can be re-written as

\[ L = \frac{3c}{8\pi r_e^3} \frac{\xi_y \rho}{\gamma^3 \beta_y^*} \frac{P_b}{P_A} R_h \]

where \( P_A = mc^2 l_A/e = 8.7 \text{ GW} \). This scaling was first given by B. Richter, Nucl. Instr. Meth. 136 (1976) 47-60.
Beamstrahlung Effects

• Beam lifetime due to large single photon emission (for 30 minutes, Telnov, 2012)

\[
\frac{N_b}{\sigma_x \sigma_z} < 0.1\eta \frac{\alpha}{3\sqrt{r_e}^2}
\]

• Large RF-buckets and large momentum aperture \( \eta \)

• Large \( \sigma_z \) and \( \sigma_x \). Favors longer and larger horizontal beam size.

• Limits bunch population \( N_b \)

Are there any reasonable solutions?
Analysis of Design Constraints

• To achieve the beam-beam parameter and assuming $\beta_y = \kappa \beta \beta_x$ and $\varepsilon_y = \kappa \varepsilon \varepsilon_x$ we have

$$\frac{N_b}{\varepsilon_x} = \frac{2\pi \gamma \xi_y}{\varepsilon_x} \sqrt{\frac{\kappa_{\varepsilon}}{\kappa_{\beta}}}$$

• To have adequate beam lifetime (due to beamstrahlung)

$$\frac{N_b}{\sqrt{\varepsilon_x}} < 0.1 \eta \frac{\alpha \sigma_z}{3 \gamma r_e^2} \sqrt{\frac{\beta_{y}^*}{\kappa_{\beta}}}$$

• Clearly, smaller coupling $\kappa_{\varepsilon}$ is better and larger momentum acceptance $\eta$ is better but they have their own limits.
Solution of the Constraints

• Given a momentum acceptance $\eta$, we solve

$$\varepsilon_x < \left( \frac{0.1\eta\alpha\sigma_z}{6\pi\gamma^2 \xi_y r_e} \right)^2 \frac{\beta^*_y}{\kappa_e}$$

Note that it does not depends on $\kappa_\beta$. We can use $\kappa_\beta$ to adjust the bunch population $N_b$ or the number of bunches $n_b$. Clearly, there are many possible solutions. But is there any self-consistent one?

• The requirement of accommodating beamstrahlung is translated to design a low emittance lattice.

• Normally, the emittance scales as $\gamma^2$. This relation requires a scaling of $\gamma^{-4}$, indicating a difficulty to design a machine with much higher energy than 120 GeV.
## Design Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LEP2</th>
<th>CHF2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Energy [GeV]</td>
<td>104.5</td>
<td>120.0</td>
</tr>
<tr>
<td>Circumference [km]</td>
<td>26.7</td>
<td>47.5</td>
</tr>
<tr>
<td>Beam current [mA]</td>
<td>4</td>
<td>14.4</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>Bunch population [10^{10}]</td>
<td>57.5</td>
<td>32.0</td>
</tr>
<tr>
<td>Horizontal emittance [nm]</td>
<td>48</td>
<td>1.7</td>
</tr>
<tr>
<td>Vertical emittance [nm]</td>
<td>0.25</td>
<td>0.0043</td>
</tr>
<tr>
<td>Momentum compaction factor</td>
<td>18.5\times10^{-5}</td>
<td>1.43\times10^{-5}</td>
</tr>
<tr>
<td>$\beta_x^*$ [mm]</td>
<td>1500</td>
<td>200</td>
</tr>
<tr>
<td>$\beta_y^*$ [mm]</td>
<td>50</td>
<td>2</td>
</tr>
<tr>
<td>Hourglass factor</td>
<td>0.98</td>
<td>0.76</td>
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<tr>
<td>SR power [MW]</td>
<td>11</td>
<td>50</td>
</tr>
<tr>
<td>Bunch length [mm]</td>
<td>16.1</td>
<td>1.5</td>
</tr>
<tr>
<td>Beam-beam parameter</td>
<td>0.07</td>
<td>0.07</td>
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<tr>
<td>Luminosity [10^{34} cm^{-2}s^{-1}]</td>
<td><strong>0.0125</strong></td>
<td><strong>1.01</strong></td>
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</table>
Main Challenges in Lattice Design

Compared with LEP2, we want a factor of 100 increase of luminosity at beam energy 120 GeV with affordable cost

- Low emittance lattice at high energy
- High packing factor of magnets
- Strong final focusing
- Large momentum acceptance
- Short bunches
Emittance of Lattice

For an electron ring, the horizontal emittance is given by

$$\varepsilon_0 = F_c \frac{C_q \gamma^2}{J_x} \theta^3$$

$\theta$ is the bending angle of the dipoles.

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc},$$

- The quantum constant $C_q = 3.8319 \times 10^{-13}$ m for electron
- $\gamma$ is the Lorentz factor (energy)

Lower emittance implies
- More cells, more dipoles and smaller dispersion
- Stronger focusing, more quadrupoles
- Stronger$^2$ of sextupoles reduces dynamic aperture
60°/60° Arc Cell

6 cells make an achromat (unit transformation).
Arc Design

60°/60° FODO Lattice

Dynamic/Momentum Aperture

Natural emittance: $\varepsilon_x=1.7$ nm and cell length is 28.0 m

$\eta>2\%$

1000 $\sigma_y$

100 $\sigma_x$
Characteristics of Phase Space

Stable region with largest amplitudes

$v_x = 0.23$
Transformation to Normalized Coordinates

Transformation is approximated by a 10\textsuperscript{th} order Taylor map

Physical coordinates \rightarrow Normalized coordinates
Footprint in Tune Space

Frequency analysis
Tracking & FFT

Normal form analysis
Taylor map & Lie form
Presentations of Magnetic Elements

### Lie factors
- Dragt-Finn

### Taylor map
- TPSA

### Symplectic Integrator

\[
\prod_{i=1}^{n} e^{\frac{H_0}{2} \Delta s} e^{-H_1 \Delta s} e^{\frac{H_0}{2} \Delta s}
\]

- engine in MARYLIE (A. Dragt)
  - violates symplecticity when evaluates
- engine in TRANSPORT, MAD, COSY (K. Brown and M. Berz), simple R-matrix
  - but high-order one violates

- engine in PTC, SAD, TRACY-II, LEGO
  - preserves symplecticity
  - simple and based on several known solutions
  - emphasis on numerical process

1/21/2015 Yunhai Cai, SLAC
Cancellation of All Geometric 3rd and 4th Resonances Driven by Sextupoles (Arcs) except $2\nu_x-2\nu_y$

There are still three tune shift terms.

K.L. Brown & R.V. Servranckx

Yunhai Cai
Final Focus System

at IP:

\( \beta_x^* = 200 \text{ mm} \)

\( \beta_y^* = 2 \text{ mm} \)

\( L^* = 2 \text{ m} \)

chromaticity:

\( \xi_y \sim L^*/\beta_y^* \)
Local Chromatic Correction

Use two pairs of sextupole reaching residual of 1.0%.

$\delta_y/p_x = 0.$

Table name = TWISS
Local Chromatic Compensation in Single-Lie Form

\[
\begin{bmatrix}
0, 2, 0, 0, 1
\end{bmatrix}
\]

Horizontal

\[
\begin{bmatrix}
0, 0, 0, 2, 1
\end{bmatrix}
\]

Vertical

\( \delta \)
5th-Order Geometric-Chromatic Aberrations

Aberrations are largest at the IP angle phase:

Identified first by Oide. Can be corrected by asymmetric dispersion at positions of the pair.

The largest aberration up to 5th order. There are more smaller terms.
Lattice of Collider Ring

at IP:
\[ \beta_x = 200 \text{ mm} \]
\[ \beta_y = 2 \text{ mm} \]
\[ L^* = 2 \text{ m} \]
## Tune as Function of Betatron Amplitudes and Momentum Deviation

<table>
<thead>
<tr>
<th>$\nu_x$</th>
<th>$\nu_y$</th>
<th>$J_x$</th>
<th>$J_y$</th>
<th>$\delta$</th>
<th>unit</th>
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<tr>
<td>0.23</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
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<tr>
<td>$2.94 \times 10^5$</td>
<td>$9.91 \times 10^5$</td>
<td>1</td>
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<td>0</td>
<td>$m^{-1}$</td>
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<td>$1.07 \times 10^5$</td>
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<td>1</td>
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<td>$m^{-1}$</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>$m^{-2}$</td>
</tr>
<tr>
<td>$2.02 \times 10^{12}$</td>
<td>$1.99 \times 10^{12}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$m^{-2}$</td>
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<tr>
<td>$9.96 \times 10^{11}$</td>
<td>$5.11 \times 10^{11}$</td>
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<td>$1.11 \times 10^9$</td>
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<td>1</td>
<td>$m^{-1}$</td>
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<tr>
<td>$1.75 \times 10^9$</td>
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<td>$m^{-1}$</td>
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<td>$1.36 \times 10^5$</td>
<td>$1.27 \times 10^6$</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>-</td>
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</tbody>
</table>

- Geometric
- Geometric & Chromatic
- Chromatic
Dynamic Aperture

\[ 144 \sigma_y \]

\[ 16 \sigma_x \]
Design Issues

1. Tune shifts vs. amplitudes are very large due to interlaced sextupoles in the arcs.

2. “Second-order dispersion” in the interaction region leads out to the arcs.

3. Second-order chromatic optics is not yet well matched between the arcs and the interaction region.

4. Huge geometric-chromatic aberration seen in 5th-order Lie operators in the interaction region. They may well be the bottle neck of the lattice.
Betatron Tunes vs Momentum

Two families

Eight families
Sextupole Strengths in Arcs

<table>
<thead>
<tr>
<th>All ARCS</th>
<th>ARCS NEXT TO IR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.684199818638</td>
<td>0.773642*2.0</td>
</tr>
<tr>
<td>-1.055036557526</td>
<td>-1.066286*2.0</td>
</tr>
<tr>
<td>0.684199818638</td>
<td>0.656123*2.0</td>
</tr>
<tr>
<td>-1.055036557526</td>
<td>-0.977044*2.0</td>
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<tr>
<td>0.684199818638</td>
<td>ARCS AWAY FROM IR</td>
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<tr>
<td>-1.055036557526</td>
<td>-0.077861*2.0</td>
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<tr>
<td>0.684199818638</td>
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<tr>
<td>0.684199818638</td>
<td>-0.093970*2.0</td>
</tr>
</tbody>
</table>

\[ K_2 = \frac{B'''}{B \rho} \]  
In unit of m^{-3}
$5^{th}$-Order Geometric-Chromatic Aberrations and Correction

A pair of decapoles are inserted.
Dynamic Aperture

with 8 families of sextupoles in arcs and octupoles and decapoles in FFS
Summary

- Impact on lattice design due to beamstrahlung is analyzed. We found a formula of minimum natural emittance that is necessary for beam lifetime. A systematic design procedure is outlined.

- Lie method is used to analyze aberrations in the final focusing system. In particular, accumulation of aberration as a function of distance from the interaction point provides us insights. Most importantly, it helps us to develop schemes of compensation.

- We have almost achieved 2% momentum bandwidth in a lattice with an ultra-low beta interaction region. More families of sextupole in arcs seem necessary. Some pair of octupoles and decapoles in the final focusing system are helpful to correct high order chromatic and geometric aberrations.