How to Determine the Tunnel Circumference
or
Scaling Behaviour of Circular Colliders Dominated by Synchrotron Radiation

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Abstract

- The natural next future circular collider is a circular e+e- Higgs Factory and, after that, a post-LHC p,p collider in the same tunnel.
- The main Higgs factory cost-driving parameter choices include: tunnel circumference $\mathcal{C}$, whether there is to be one ring or two, what is the installed power, and what is the "Physics" for which the luminosity deserves to be maximized.
- Of these, the tunnel circumference is the first parameter to be chosen, and therefore the most important at this time.
- My white paper discusses some of the trade-offs among these choices, but with special emphasis on the simultaneous optimization of both an e,p Higgs factory and a next generation p,p collider in the same tunnel.
- It attempts to show that the optimization goals for the Higgs factory and the later p,p collider are entirely compatible.

3 Outline

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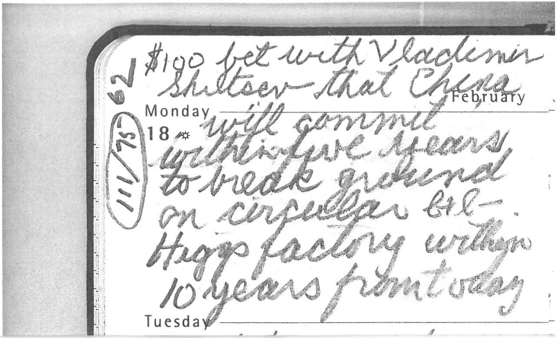
4 Why We Must Do It, Where, When, and How?

- The full white paper, "Scaling Behavior of Circular Colliders Dominated by Synchrotron Radiation", is available on the conference site.
- It was the basis for two "tutorial" lectures last week, in which I developed the results to be used in this lecture.

In the second lecture I described a high stakes wager I made with Vladimir Shiltsev, almost two years ago (February 18, 2013) at a time when I knew very little about "future circular colliders":


62
February
Monday
18
$100 bet with Vladimir Shiltsev that China will commit within five years to break ground on circular e+e- Higgs factory within 10 years from today
Tuesday

My reasons for bringing this up now are:

- to disclose my financial conflict of interest,
- to show that building a Higgs Factory followed by a next generation p,p collider has been a “no brainer” from the start, and still is, at least as far as I am concerned. (Vladimir can speak for himself.)
- to acknowledge that I am “rooting” for the CEPC/SPPS project or, in the jargon, “this is where I am coming from”.
- Even when I am critical of the design so far, I admire the effort. As it happens, I am also one of the many authors of the CEPC pre-CDR report—an entirely unjustified honor. So when I am critical, I am criticizing myself.
- In any case, I criticize only details, not overall design. There will be plenty of time to iron out the details.
- Most important, the time scale is set. Ground has to be broken before 2024.

- I leave it to particle theorists and experimentalists to explain what motivation we should have for proceeding to a next generation of particle colliders, and how best to persuade the government to fund the effort.
- For myself, I hold some truths to be self-evident. One is that the world should proceed with all deliberate speed to start work on a next generation of colliders.
- I have no doubt that at least the Higgs Factory phase can be completed, and will be successful.
- I am dubious about some of the luminosity claims of both CEPC and FCC-ep, but I propose changes to make them more credible.

8 The Theme for Three Lectures

- A theme of all three lectures has been how to perform the seemingly impossible task of optimizing both an e,p collider and a p,p collider in the same tunnel.
- It seems obvious that "bigger is better" for the p,p collider (to maximize the beam energy for an achievable magnetic field).
- The task therefore is to make the case that "bigger is better" also for the e,p ring.
- In lectures I and II I promised no computer simulations. Today I also promise only to show as few formulas as I can.

9 The Theme for Three Lectures (cont)

- In lecture I, I introduced the "radius times RF power invariant product" and used it to show that "bigger is **as good** as smaller", (because increasing R and decreasing P_{RF} proportionally leaves the luminosity constant.)
- In lecture II, I showed that "bigger is **better than** smaller" (because the ratio of dynamic aperture to beam size increases with increasing R;—how helpful this will be depends on the intersection region optical design, which depends only weakly on the ring circumference.)
- I also promised that today's lecture III would show that "bigger is **both better and cheaper** than smaller". This is more of a stretch—more a hope than a promise. But I will marshall arguments as to why it could be true, at least in principle.
- Certainly every effort should be made to make the ring as big as possible, but **without jeopardizing the prospects for approval of the project.**

10 Radius x Power Scaling Law Invariant. Why Bigger is Better

- Dominating everything is the synchrotron radiation formula

$$\Delta E \propto \frac{E^4}{R}, \tag{1}$$

  relating energy loss per turn ΔE, beam energy E and bend radius R.
- The formulas in the white paper represent vanilla electron collider design. They were formulated mainly with the e+e- Higgs factory in mind.
- I have more recently become persuaded that 100 TeV is such a high energy that synchrotron radiation will "dominate" p,p design, just as it has always dominated e+e- design. This accounts for the phrase "Dominated by Synchrotron Radiation" in the title.
- But the dominance of synchrotron radiation in a proton ring, with its low temperature magnets, is less direct than in an electron ring. The synchrotron radiation power in a 100 TeV would be unimportant, except to the extent it is dissipated at low temperature, vastly magnifying its cost.
- One could imagine such a perfect beam screen that none of the radiation is absorbed at low temperature. But the difficulty of doing this increases rapidly with increasing beam energy, as the photon spectrum changes from UV to x-rays.

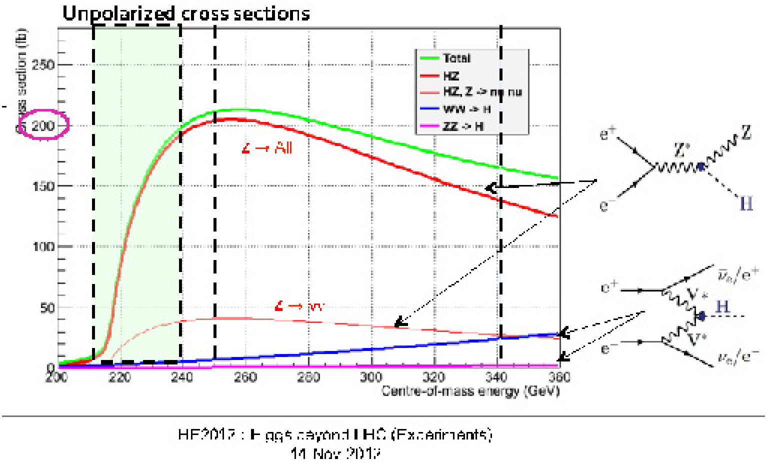


Figure: Higgs particle cross sections up to $\sqrt{s} = 0.3\,\mathrm{TeV}$ (copied from Patrick Janot); $\mathcal{L} \geq 2 \times 10^{34}\ /\mathrm{cm}^2/\mathrm{s}$, will produce 400 Higgs per day in this range.

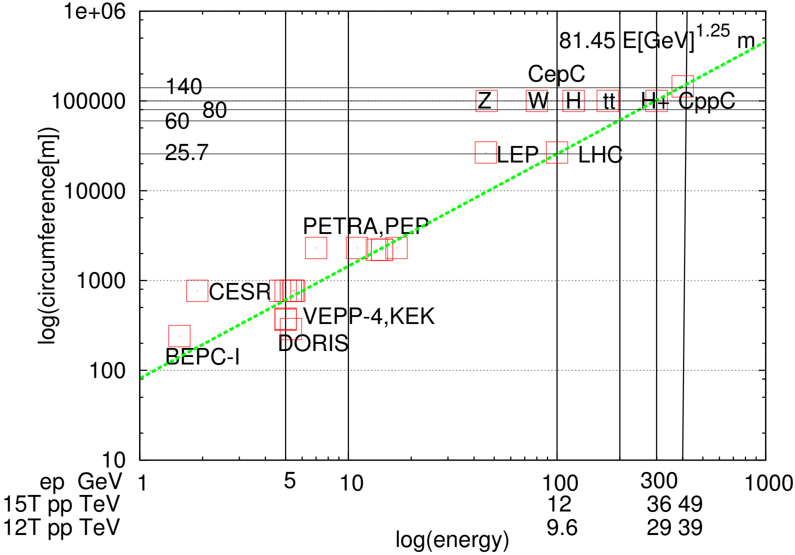


Figure: Dependence of circumference on beam energy, both for GeV-scale electron colliders, and for TeV-scale proton colliders of magnetic field 12T or 15T.

Parameter	Symbol	Proportionality	Scaling
phase advance per cell	μ		1
collider cell length	L_c		$R^{1/2}$
bend angle per cell	ϕ	$= L_c/R$	$R^{-1/2}$
quad strength $(1/f)$	q	$1/L_c$	$R^{-1/2}$
dispersion	D	ϕL_c	1
beta	β	L_c	$R^{1/2}$
tunes	Q_x, Q_y	R/β	$R^{1/2}$
Sands's "curly H"	$\mathcal{H}$	$= D^2/\beta$	$R^{-1/2}$
partition numbers	$J_x/J_y/J_\epsilon$	$= 1/1/2$	1
horizontal emittance	ϵ_x	$\mathcal{H}/(J_x R)$	$R^{-3/2}$
fract. momentum spread	σ_δ	$\sqrt{B}$	$R^{-1/2}$
arc beam width-betatron	$\sigma_{x,\beta}$	$\sqrt{\beta\epsilon_x}$	$R^{-1/2}$
-synchrotron	$\sigma_{x,synch.}$	$D\sigma_\delta$	$R^{-1/2}$
sextupole strength	S	q/D	$R^{-1/2}$
dynamic aperture	$x^{\max}$	q/S	1
relative dyn. aperture	$x^{\max}/\sigma_x$		$R^{1/2}$
pretzel amplitude	x_p	σ_x	$R^{-1/2}$

Table: *Constant dispersion* scaling is the result of choosing cell length $L \propto R^{1/2}$. The entry "1" in the last column of the shaded "dispersion" row, indicates that the dispersion is independent of R when the cell length L_c varies proportional to $\sqrt{R}$ with the phase advance per cell μ held constant.

14 Ring Parameters Scaled for 50 and 100 km Circumference

Parameter	Symbol	Value	Unit	Energy-scaled	Radius-	scaled
bend radius	R	**3026**	m	**3026**	**5675**	**11350**
	$R/3026$			1	1.875	3.751
Beam Energy	E	**45.6**/91.5	GeV	**120**	**120**	**120**
Circumference	$\mathcal{C}$	26.66	km	26.66	50	100
Cell length	L_c		m	79	108	153
Momentum compaction	α_c	1.85e-4		1.85e-4	0.99e-4	0.49e-4
Tunes	Q_x	90.26		90.26	123.26	174.26
	Q_y	76.19		76.19	104.19	147.19
Partition numbers	$J_x/J_y/J_\epsilon$	1/1/2		1/1.6/1.4 !	1/1/2	1/1/2
Main bend field	B_0	0.05/0.101	T	0.1316	0.0702	0.0351
Energy loss per turn	U_0	0.134/2.05	GeV	6.49	3.46	1.73
Radial damping time	τ_x	0.06/0.005	s	0.0033	0.0061	0.0124
	τ_x/T_0	679/56	turns	37	69	139
Fractional energy spread	σ_δ	0.946e-3/1.72e-3		0.0025	0.0018	0.0013
Emittances (no BB), x	ϵ_x	22.5/30	nm	21.1	8.2	2.9
y	ϵ_y	0.29/0.26	nm	1.0	0.4	0.14
Max. arc beta functs	$\beta_x^{\max}$	125	m	125	171	242
Max. arc dispersion	$D^{\max}$	0.5	m	0.5	0.5	0.5
Beta functions at IP	β_x^*, β_y^*	2.0,0.05	m	1.25/0.04	N/Sc.	N/Sc.
Beam sizes at IP	σ_x^*, σ_y^*	211, 3.8	μm	178/11	N/Sc.	N/Sc.
Beam-beam parameters	ξ_x, ξ_y	0.037,0.042		0.06/0.083	N/Sc.	N/Sc.
Number of bunches	N_b	8		4	N/Sc.	N/Sc.
Luminosity	$\mathcal{L}$	2e31	$\mathrm{cm}^{-2}\mathrm{s}^{-1}$	1.0e32	N/Sc.	N/Sc.
Peak RF voltage	V_{RF}	380	MV	3500	N/Sc.	N/Sc.
Synchrotron tune	Q_s	0.085/0.107		0.15	N/Sc.	N/Sc.
Low curr. bunch length	σ_z	0.88	cm	$\frac{\alpha_c R \sigma_e}{Q_s E}$	N/Sc.	N/Sc.

15 The Radius x Power Scaling Law Invariant.

- Conclusions in this paper are based on scaling laws, with respect to beam energy E or bending radius R.
- Higgs production was just barely beyond the reach of LEP's top energy, by the ratio $125\,\mathrm{GeV}/105\,\mathrm{GeV} = 1.19$. In such an extrapolation it is increased radius more than increased beam energy that is mainly required.
- One can note that, for a ring three times the size of LEP, the ratio of E^4/R (synchrotron energy loss per turn) is $1.19^4/3 = 0.67$—i.e. less than final LEP operation.
- For a given RF power P_{rf}, the maximum total number of stored particles is proportional to R^2—doubling the ring radius cuts in half the energy loss per turn and doubles the time interval over which the loss occurs. Expressed as a scaling law

$$n_1 = \text{number of stored electrons per MW} \ \propto R^2. \tag{2}$$

- This favors large circumference for both electron and (radiation-dominated) proton colliders.

- There are three distinct upper limit constraints on the luminosity. Maximum luminosity results when the ring parameters have been optimized so the three constraints yield the same upper limit for the luminosity.
- For now we concentrate on just the simplest luminosity constraint $\mathcal{L}^{\rm RF}_{\rm pow}$, the maximum luminosity for given RF power $P_{\rm rf}$. With n_1 being the number of stored particles per MW; f the revolution frequency; N_b the number of bunches, which is proportional to R; σ_y^* the beam height at the collision point; and aspect ratio σ_x^*/σ_y^* fixed (at a large value such as 15);

$$\mathcal{L}^{\rm RF}_{\rm pow} \propto \frac{f}{N_b}\left(\frac{n_1 P_{\rm rf}[\rm MW]}{\sigma_y^*}\right)^2. \tag{3}$$

- Variations for which

$$P_{\rm rf} \propto \frac{1}{R}. \tag{4}$$

leave $\mathcal{L}^{\rm RF}_{\rm pow}$ invariant.

- The dependencies on R are, $N_b \propto R$, $f \propto 1/R$, and $n_1 \propto R^2$. With the $P_{\rm rf} \propto 1/R$ scaling of $\mathcal{L}$ is independent of R. In other words, the luminosity depends on R and $P_{\rm rf}$ only through their product $RP_{\rm rf}$.
- This scaling law will be used in the form

$$\boxed{\mathcal{L}(R, P_{\rm rf}) = f(RP_{\rm rf}),} \tag{5}$$

  the luminosity depends on R and $P_{\rm rf}$ as a function $f(RP_{\rm rf})$ of only their product.
- This radius/power scaling formula can be checked numerically by comparing tables to be shown in the white paper.

Staged Optimization

For best chance of initial approval and best eventual p,p performance, the cost of the first step has to be minimized and the tunnel circumference maximized. Surprisingly, these requirements are consistent. Consider optimization principles for three collider stages:

- **Stage I, e+e-:** Starting configuration. Minimize cost at "respectable" luminosity, e.g. 10^{34}. Constrain the number of rings to 1, and the number of IP's to $N^* = 2$.
- **Stage II, e+e-:** Maximize luminosity/cost for production Higgs (etc.) running. Upgrade the luminosity by some combination of: $P_{\rm rf} \rightarrow 2P_{\rm rf}$ or $4P_{\rm rf}$, one ring $\rightarrow$ two rings, increasing N^* from 2 to 4, or decreasing β_y^*.
- **Stage III, pp:** Maximize the ultimate physics reach, i.e. center of mass energy, i.e. maximize tunnel circumference.

19 Cost Optimization

Treating the cost of the 2 detectors as fixed, and letting C be the cost exclusive of detectors, the cost can be expressed the sum of a term proportional to size and a term proportional to power;

$$C = C_R + C_P \equiv c_R R + c_P P_{\rm rf} \tag{6}$$

where c_R and c_P are unit cost coefficients. The radius x power scaling law gives

$$P_{\rm rf} = \frac{\mathcal{L}}{k_1 R}. \tag{7}$$

Minimizing C at fixed $\mathcal{L}$ leads to

$$\boxed{R_{\rm opt} = \sqrt{\frac{1}{k_1}\,\frac{c_P}{c_R}\,\mathcal{L}}.} \tag{8}$$

Conventional thinking has it that c_P is universal world wide but, at the moment, c_R is thought to be somewhat cheaper in China than elsewhere. If so, the optimal radius should be somewhat greater in China than elsewhere.

20 Exploiting $P_{\mathrm{rf}} \propto \mathcal{L}/R$, some estimated costs (in arbitrary cost units) and luminosities for Stages 1 and 2 are given in the table.

	R	P_{rf}	C_{tun}	C_{acc}	Phase-I cost	$\mathcal{L}^{I}$ (Higgs)	$\mathcal{L}^{I}$ (Z_0)	$\mathcal{L}^{II}$ (Higgs)
	km	MW	arb.	arb.	arb.	10^{34}	10^{34}	10^{34}
1	5	50	0.5	2.5	3.0	1.2	2.6	2
	10	25	1.0	2.5*	**3.5**	1.2	5.2	**5**
	10	50	1.0	4.0	5.0	2.3	10.4	5
2	5	50	0.5	4.5	5.0	1.2	21	2
	10	25	1.0	4.5*	5.5	1.2	21	5
	10	50	1.0	7.0	8.0	2.3	42	5

Table: Estimated costs, one ring in the upper table, two in the lower. C_{tun} is the tunnel cost, C_{acc} is the cost of the rest of the accelerator complex. *A crude LEP spread sheet shows that doubling the radius and halving the power leaves the accelerator cost not very much changed. Magnet costs are to be discussed later. The cost ratios are crudely extracted from the LEP "Pink Book".

- As an aside particle experimentalists should note that, to be consistent with this scheme, they have to resist the temptation to be too greedy in the first Higgs factory phase.

21 Cost Increases Implied by Doubling R

- My consistent bias is to increase the bending radius R and reduce the power P, both for electrons and protons.
- In doubling R one must acquire a prejudice as to the effect on the vacuum chamber bore diameter.
- Formulas in this paper suggest that leaving the bore unchanged is sensible for a first iteration.
- By reducing RF power, three important costs have been reduced. The RF power infrastructure, the matching cooling infrastructure, and the long term power costs.
- But increasing R has increased other costs.
- Probably the most serious is the vacuum chamber cost which will be more or less proportional to R. Nothing can be done about this.
- One might reflexively accept that doubling R will double the magnet cost. This will be addressed next.

Holding Magnet Cost Down

- Mentioned in passing was the fact that my optimized cell lengths L_c were more than twice as long as assumed in the CEPC and FCC-ee designs.
- Either I am making a mistake or they are. Let us tentatively say they are.
- Only half as many magnets suggests "cheaper".
- Immediate protest. The magnets are already shorter than the cell length. Same number of magnets. Same cost. I will consider this next.

23 Some Magnet Considerations

- Iron electromagnet costs are sometimes expressed as dollars/energy where "energy" is the magnetic energy.
- From this (completely misleading) point of view, the magnet cost **falls** proportional to R because $B \propto 1/R$ and we are holding the transverse magnet area fixed.
- Some say "the costs is all in the magnet ends". Others say, "the cost is all in transporting and installation". Others: "the cost is all in the pedestals".
- To hold down magnet costs, my inescapable conclusion is that the magnets have to be built *in situ*, in their final positions in the Higgs factory tunnel. This is the only way to prevent the magnet cost from scaling proportional to the tunnel circumference, or worse.
- (The same may be true also for superconducting magnets in the later p,p phase of the project.)
- Built in place, the magnets can be almost arbitrarily long.

24 More Magnet Considerations

- It is not at all challenging to build the Higgs factory collider magnets in place. With top-off injection these magnets do not have to ramp up in field. As a result they have no eddy currents and therefore do not need to be laminated.
- Regrettably the same is not true for the injector magnet, which will be more challenging, and may be more expensive, than the collider magnet.

25 Accelerator Ring as Power Transmission Line

- A quixotic argument for building the magnet in place starts by comparing the arcs of the collider to high voltage electrical power lines, which carry vast amounts of power over vast distances.
- A 10^6 V power line, carrying 10^3 A, carries 10^9 W of power over a distance of 100 Km, with fractional energy loss of 1%.
- The arcs of the Higgs factory will carry 10^{11} V at 10^{-2} A over a distance of 100 Km with fractional energy loss of 1%.
- Same power, same loss.
- One would not even think of building overland power lines in a factory before transporting them to where they are needed.
- The same should be true for accelerator magnets.

26 How Would Bob Wilson Do It?

- As a disciple of Robert Wilson, one cannot avoid contemplating how he would have approached the design challenge of establishing Higgs factory parameters when so little is known about what to expect.
- I remind you of (some of) Bob Wilson's credentials.
- Two of the most important developments in high energy physics have been the establishment of Fermilab and the development of superconducting magnets for the Tevatron. Bob was the most important figure in each case.
- Certainly Bob Wilson would have endorsed the curious formulation of Nima Arkani-Hamed according to which the bigness of a 100 TeV collider is a reason for building it, rather than the contrary.

- "How would Bob do it?" suggests unconventional design approaches. At the early design stage, based on good, but limited, understanding of the task, one of his traits was to insist on round numbers, "It is important for the parameter choices to be easy to remember". He could only remember round numbers, preferably inches for small things, meters for big things.
- Bob would certainly have liked the round numbers in a statement such as "We need a ring with 100 km circumference to obtain 100 TeV collisions," especially because of (or, possibly, in spite of) the fact that the CERN FCC group favors just these values.
- These are just style. How about policy?

- Bob would never think of asking an engineer how much it will cost to produce something.
- He would specify the cost and instruct the engineer to produce the design for that cost.
- Later when the engineer returns, saying "it cannot be done for this cost", Bob would tell him how to do it and send him back to try again.
- It was Bob's attitude that, if a competent physicist (where he had himself in mind) could conceptualize an elegant solution to a mechanical design problem, consistent with the laws of physics, then a competent engineer (where he again had himself in mind) could certainly do it.
- In other words, ask not what it will cost, ask the engineer to figure out how to do it for the assigned dollars. If he or she says it cannot be done, explain/suggest how it can be.

29 Luminosity Detector Length Trade-Off

The paper obtains the dependence of luminosity on free space length L^*, which is the half-length of the drift space into which the detector must fit;

$$\mathcal{L} = \frac{4 \times 10^{31}\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{m}}{\beta_y^*} \tag{9}$$

or, using the relation between beta function β_Y* at the IP, and maximum beta function nearby, β_y^{max},

$$\mathcal{L} = 1.6 \times 10^{31}\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{m} \times \frac{\beta_y^{\mathrm{max}}}{L^{*2}}. \tag{10}$$

- *The constant of proportionality in these equation is not determined by the scaling formula.* It has been chosen to match a preliminary CEPC luminosity estimate.
- Since the next scaling law sets an upper limit on β_y^{max}, this formula imposes a serious constraint on the detectors.

$$L * \times \mathcal{L} \quad \text{is fixed.} \tag{11}$$

- It has always been known that to get higher luminosity requires reducing β_y^*.
- This is easier said than done, since reducing β_y^* increases $\beta_y^{\max}$, which inevitably makes the collider more erratic, often unacceptably so.
- Though it is the IR optics that causes β to be unacceptably large near the IP, the IR elements themselves are typically not the source of the problem.
- A high energy collider ring is "high strung" and high beta anywhere makes it more "skittish".
- Even single beam operation is hampered by high β anywhere in the ring.
- Sensitivity to beam-beam effects is also greatly magnified by large β anywhere in the ring.

Maximum β_Y Phenomenology, Based on Transverse Sensitivity

- A "transverse sensitivity length" $DL_C/\beta^{\max}$ can be used to compare different rings, either proton or electron, independent of their beam energies.
- (Inverse) transverse sensitivity lengths are listed for various accelerators are given in the table.

β_y^* m	Ring		D m	L_c m	DL_c m^2	$\beta_y^{\max}$ m	$\frac{\beta_y^{\max}}{DL_c}$ 1/m
0.015	CESR	exp.	1.1	17	18.7	95	5.1
0.08	PETRA	exp.	0.32	14.4	4.6	225	49
	HERA	exp.	1.5	48	72	2025	28
0.05	LEP	exp.	0.8	79	63	441	7.0
0.007	KEKB	exp.	0.5	20	10	290	29
	LHC	exp.	1.6	79	126	4500	36
0.01	CepC_1	des.	0.31	47	14.6	1225	84
0.01	CepC_2	des.	1.03	153	158	1225	8.8
0.001	CEPC	des.	0.31	47	14.6	6000	410
0.001	FCC-ee	des.	0.10	50	5.0	9025	1805

32 Notes on Transverse Sensitivity Comparison Table

- Lattice parameters and inverse transverse sensitivity lengths $\beta_y^{\max}/DL_c$ for various e+e- colliders.
- The upper rows contain experimentally measured values, the lower rows contain design values.
- CepC_1 copies the L_c and D values from CEPC, while CepC_2 copies them from Table **??**.
- The IR design is assumed identical for CepC_1 and CepC_2, with $\beta_y^* = 10\,\mathrm{mm}$.
- In principle nothing in the table depends directly on β_y^*. But, indirectly, large $\beta_y^{\max}$ values are correlated with small β_y^* values.

33 More Comments on the Transverse Sensitivity Figure of Merit

- Electron rings and proton rings are like apples and oranges.
- Compared in this way the transverse tolerances of KEKB and LHC are close in value, even though, as storage rings, they could scarcely be more disimilar; KEKB is a "small" electron collider, LHC is a large proton collider.
- The near agreement between a modern electron ring KEKB and a modern proton beam LHC, lends some confidence in this sensitivity measure for comparing them.
- When β_y^{max} is large, it is always because β_y^* is small.
- But the value of β_y^*, in itself, does not influence the dynamic aperture. Nevertheless β_y^* values are given in the table.
- The pessimistic behavior of LEP can be blamed on the absence of top-off injection, which led to the tortuous ramping and beta squeeze operations. This limited the β_y^* to be not less than 5 cm.

34 Extra Material

35 Beam Tune Shift Limit—Good News for Both e,p and p,p

Table: Parameters of some circular, flat beam, e+e- colliding rings, and the saturation tune shift values predicted (with no free parameters) by the simulation.

Ring IP's	Q_x/IP	Q_y/IP	Q_s/IP	σ_z	β_y^*	$10^4\delta_y$	$\xi_{\rm th.}$	$\Delta Q_{y,\rm exp.}$	th/exp
VEPP4 1	8.55	9.57	0.024	0.06	0.12	1.68	0.028	0.046	0.61
PEP-1IP 1	21.296	18.205	0.024	0.021	0.05	6.86	0.076	0.049	1.55
PEP-2IP 2	5.303	9.1065	0.0175	0.020	0.14	4.08	0.050	0.054	0.93
CESR-4.7 2	4.697	4.682	0.049	0.020	0.03	0.38	0.037	0.018	2.06
CESR-5.0 2	4.697	4.682	0.049	0.021	0.03	0.46	0.034	0.022	1.55
CESR-5.3 2	4.697	4.682	0.049	0.023	0.03	0.55	0.029	0.025	1.16
CESR-5.5 2	4.697	4.682	0.049	0.024	0.03	0.61	0.027	0.027	1.00
CESR-2000 1	10.52	9.57	0.055	0.019	0.02	1.113	0.028	0.043	0.65
KEK-1IP 1	10.13	10.27	0.037	0.014	0.03	2.84	0.046	0.047	0.98
KEK-2IP 2	4.565	4.60	0.021	0.015	0.03	1.42	0.048	0.027	1.78
PEP-LER 1	38.65	36.58	0.027	0.0123	0.0125	1.17	0.044	0.044	1.00
KEK-LER 1	45.518	44.096	0.021	0.0057	0.007	2.34	0.042	0.032	1.31
BEPC 1	5.80	6.70	0.020	0.05	0.05	0.16	0.068	0.039	1.74

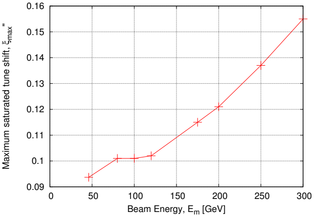


Figure: Plot of maximum tune shift $\xi_{\rm max}$ as a function of maximum beam energy for rings such that $E \propto R^{5/4}$. The non-smoothness has to be blamed on statistical fluctuations in the Monte Carlo program calculation. The maximum achieved tune shift parameter 0.09 at 100 GeV at LEP was less than shown, but their torturous injection and energy ramping seriously constrained their operations.

- “Final focus” (like “funeral”) is a place where electrons in a linear collider go to die.
- The “advantage” a circular collider has over a linear collider is that every particle has millions of chances to collide with a particle in the other beam.
- The term “intersection region” or “IR” is appropriate for a section of a storage ring in which the particles survive.
- Applying the term “final focus” to the IR of a circular collider is a crime against language.
- This is not just pedanttry. It is the source of the common mistake of assuming the linear collider final focus optics can simply be inserted into a storage ring.
- The “disadvantage” of a circular collider is that a particle has to survive millions of passages through the other beam. This makes the storage ring IR optics far more difficult.

37 Two Rings or One

name	E GeV	ϵ_x nm	β_y^* mm	ϵ_y pm	$\xi_{\rm sat}$	$N_{\rm tot}$	σ_y μm	σ_x μm	u_c^* GeV	$n_{\gamma,1}^*$	$\mathcal{L}^{\rm RF}$ 10^{34}	$\mathcal{L}_{\rm trans}^{\rm bs}$ 10^{34}	$\mathcal{L}^{\rm bb}$ 10^{34}	N_b	β_x^* m	$P_{\rm rf}$ MW
Z	46	0.916	2	61.1	0.094	7.3e+14	0.35	5.24	0.000	1.97	52.5	96.8	52.513	33795	0.03	50
W	80	0.323	2	21.6	0.101	7.6e+13	0.208	3.12	0.001	2.06	9.66	16.2	9.661	5696	0.03	50
LEP	100	0.215	2	14.3	0.101	3.1e+13	0.169	2.54	0.002	2.10	4.95	8	4.947	2814	0.03	50
H	120	0.153	2	10.2	0.102	1.5e+13	0.143	2.15	0.003	2.13	2.86	4.48	2.863	1581	0.03	50
tt	175	0.077	2	5.12	0.118	3.3e+12	0.101	1.52	0.006	2.19	0.923	1.35	0.923	478	0.03	50
Z	46	16.5	5	1100	0.094	7.3e+14	2.35	35.21	0.001	2.12	21	33.2	21.005	1872	0.075	50
W	80	5.88	5	392	0.101	7.6e+13	1.4	20.99	0.003	2.22	3.86	5.52	3.864	313	0.075	50
LEP	100	3.91	5	261	0.101	3.1e+13	1.14	17.12	0.005	2.26	1.98	2.71	1.979	154	0.075	50
H	120	2.80	5	187	0.102	1.5e+13	0.966	14.50	0.007	2.30	1.15	1.52	1.145	86	0.075	50
tt	175	1.41	5	94	0.118	3.3e+12	0.686	10.28	0.016	2.38	0.369	0.455	0.369	26	0.075	50
Z	46	149	10	9900	0.094	7.3e+14	9.95	149.28	0.002	2.24	10.5	14.7	10.503	208	0.15	50
W	80	53.1	10	3540	0.101	7.6e+13	5.95	89.26	0.007	2.36	1.93	2.42	1.932	34	0.15	50
LEP	100	35.4	10	2360	0.101	3.1e+13	4.86	72.88	0.011	2.41	0.989	1.19	0.989	17	0.15	50
H	120	25.4	10	1700	0.102	1.5e+13	4.12	61.78	0.016	2.45	0.573	0.663	0.573	9.5	0.15	50
tt	175	12.9	10	857	0.118	3.3e+12	2.93	43.92	0.035	2.54	0.185	0.198	0.185	2.9	0.15	50

Table: Luminosity influencing parameters and luminosities with unlimited number of bunches N_b, assuming 50 km circumference ring and 50MW per beam RF power.

E GeV	β_y^* m	$\xi_{\rm sat}$	$\mathcal{L}_{\rm actual}$ 10^{34}	$N_{\rm b,actual}$	$P_{\rm rf}$ MW
46	0.002	0.094	0.174	112	50
80	0.002	0.1	0.190	112	50
100	0.002	0.1	0.197	112	50
120	0.002	0.1	0.203	112	50
175	0.002	0.12	0.216	112	50
46	0.005	0.094	1.256	112	50
80	0.005	0.1	1.380	112	50
100	0.005	0.1	1.434	112	50
120	0.005	0.1	1.145	86.6	50
175	0.005	0.12	0.369	26.1	50
46	0.010	0.094	5.644	112.0	50
80	0.010	0.1	1.932	34.7	50
100	0.010	0.1	0.989	17.1	50
120	0.010	0.1	0.573	9.5	50
175	0.010	0.12	0.185	2.9	50

Table: Luminosity influencing parameters and luminosities with the number of bunches limited to $N_b = 112$, assuming 50 km circumference ring and 50MW per beam RF power.

name	E	ϵ_x	β_y^*	ϵ_y	$\xi_{\rm sat}$	$N_{\rm tot}$	σ_y	σ_x	u_c^*	$n_{\gamma,1}^*$	$\mathcal{L}^{\rm RF}$	$\mathcal{L}_{\rm trans}^{\rm bs}$	$\mathcal{L}^{\rm bb}$	N_b	β_x^*	$P_{\rm rf}$
	GeV	nm	mm	pm		10^{12}	μm	μm	GeV		10^{34}	10^{34}	10^{34}		m	MW
Z	46	0.949	2	63.3	0.094	1500	0.356	5.34	0.000	2.01	52.5	103	52.5	65243	0.03	25
W	80	0.336	2	22.4	0.101	150	0.212	3.17	0.001	2.10	9.66	17.2	9.6	10980	0.03	25
LEP	100	0.223	2	14.9	0.101	62	0.172	2.59	0.002	2.13	4.95	8.46	4.94	5421	0.03	25
H	120	0.159	2	10.6	0.102	30	0.146	2.19	0.003	2.17	2.86	4.74	2.86	3044	0.03	25
tt	175	0.078	2	5.33	0.118	6.6	0.103	1.55	0.006	2.24	0.923	1.43	0.92	920	0.03	25
Z	46	17.2	5	1140	0.094	1500	2.39	35.89	0.001	2.16	21	35.1	21.	3605	0.075	25
W	80	6.11	5	408	0.101	150	1.43	21.42	0.003	2.26	3.86	5.83	3.86	602	0.075	25
LEP	100	4.07	5	271	0.101	62	1.16	17.47	0.005	2.31	1.98	2.86	1.97	296	0.075	25
H	120	2.92	5	195	0.102	30	0.987	14.80	0.008	2.35	1.15	1.6	1.14	166	0.075	25
tt	175	1.47	5	98.1	0.118	6.6	0.7	10.51	0.017	2.43	0.369	0.479	0.37	49	0.075	25
Z	46	155	10	10300	0.094	1500	10.2	152.3	0.002	2.29	10.5	15.5	10.5	400	0.15	25
W	80	55.4	10	3690	0.101	150	6.08	91.17	0.007	2.41	1.93	2.55	1.93	66	0.15	25
LEP	100	37.0	10	2470	0.101	62	4.97	74.48	0.011	2.46	0.989	1.25	0.99	32	0.15	25
H	120	26.6	10	1770	0.102	30	4.21	63.15	0.016	2.50	0.573	0.696	0.57	18.3	0.15	25
tt	175	13.5	10	898	0.118	6.6	3.0	44.94	0.036	2.60	0.185	0.207	0.19	5.5	0.15	25

Table: The major factors influencing luminosity, assuming 100 km circumference and 25 MW/beam RF power. The predicted luminosity is the smallest of the three luminosities, $\mathcal{L}^{\rm RF}$, $\mathcal{L}_{\rm trans}^{\rm bs}$, and $\mathcal{L}^{\rm bb}$. All entries in this table apply to either one ring or two rings, except where the number of bunches N_b is too great for a single ring.

- With one ring, the maximum number of bunches is limited to approximately ≤ 200.
- For $N_b > 200$ the luminosity $\mathcal{L}$ has to be de-rated accordingly; $\mathcal{L} \rightarrow \mathcal{L}_{\mathrm{actual}} = \mathcal{L} \times N_b/200$. This correction has been applied in Table 7 (showed earlier).
- When the optimal number of bunches is less than (roughly) 200, single ring operation is satisfactory, and hence favored.
- When the optimal number of bunches is much greater than 200, for example at the Z_0 energy, two rings are better.
- Note though, that the Z_0 single ring luminosities are still very healthy. In fact, with β_y^*=10 mm, which is a more conservative estimate than most others in this paper and in other FCC reports, the Z_0 single ring penalty is substantially less.

E GeV	β_y^* m	ξ_{sat}	$\mathcal{L}_{\mathrm{actual}}$ 10^{34}	N_{actual}	P_{rf} MW/beam
46	0.002	0.094	0.161	200	25
80	0.002	0.1	0.176	200	25
100	0.002	0.1	0.182	200	25
120	0.002	0.1	0.188	200	25
175	0.002	0.12	0.200	200	25
46	0.005	0.094	1.165	200	25
80	0.005	0.1	1.282	200	25
100	0.005	0.1	1.334	200	25
120	0.005	0.1	1.145	166	25
175	0.005	0.12	0.369	50	25
46	0.010	0.094	5.247	200	25
80	0.010	0.1	1.932	66.5	25
100	0.010	0.1	0.989	32.7	25
120	0.010	0.1	0.573	18.3	25
175	0.010	0.12	0.185	5.5	25

Table: Luminosites achievable with a single ring with number of bunches N_b limited to 200, 100 km circumference and 25 MW/beam RF power. The luminosity entries in (earlier) Table 2 were obtained from this table.