A Conservative Perspective on

Physics BSM

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Philosophical :-

Paul Feyerabend : Against Method

— *the only principle that does not inhibit progress is anything goes*

- don't censor scientific endeavour
- *however*, individuals should have personal perspectives
  discussions/debates necessary

Educated guess **Vs** *Shooting in the dark / Blindly following*

- everyone is qualified to speak up

⋆ intend to provoke discussions

Going beyond the SM — model-*building* :-

not (complementary) model independent constraint analyses

• has to be bottom-up (the energy scale)

down to earth/the colliders (experimental reach)

ALL THEORIES ARE EFFECTIVE THEORIES

• not much about m_ν, CPV, dark matter

$\Longleftarrow$ sources as easy bi-product

$\Longleftarrow$ $m_\nu \leftrightarrow$ CPV : leptogenesis $\rightarrow$ baryogenesis

$\Longleftarrow$ little use as guide

• against unification

— beautiful idea : too high scale

— ? missing pieces; ?? QFT (-like) framework

SIMPLICITY and BEAUTY

Physics Beyond the Standard Model

WHY ?

WHAT ?

WHERE ?

- **Theoretical** — hierarchy problem, couplings *
- **Experimental** — m_ν, Δa_μ, B physics, Higgs (?)

  $\star$ **Flavor Problems**

Standard Model :-

- gauge symmetry
- anomaly free chiral fermion spectrum
- a Higgs multiplet for EW symmetry breaking
- accidental global symmetries

The Gauge **Symmetry :-**

- dictates interactions, gauge bosons
- may dictate all fields (massless)
  + supersymmetry, may dictate all matter content
- scalar field — problematic/arbitrary

⋆ Flavor Problem(s) *Why 3 families ? ! ·*

⋆ Gauge Symmetry fixes spin 1 sector

⋆ The Story of Fermions ⋯

— **3 families** of **15** spin $\frac{1}{2}$ quantum fields (**Weyl 2-spinors**)
under $SU(3)_C \times SU(2)_L \times U(1)_Y$

not 4 (Dirac) particles

- $(3, 2, 1)$: u u u d d d
- $(\bar{3}, 1, -4)$: $\bar{u}$ $\bar{u}$ $\bar{u}$
- $(\bar{3}, 1, 2)$: $\bar{d}$ $\bar{d}$ $\bar{d}$
- $(1, 2, -3)$: ν e^-
- $(1, 1, 6)$: e^+

— **minimal *chiral* set free from all gauge anomalies**

completely nontrivial cancellation (Vs vectorlike pairing)

SM fermion field spectrum for one family :-

minimal chiral set with completely nontrivial anomaly cancellation

Geng & Marshak (89)

— less than appreciated well enough

• taking $SU(3)_C \times SU(2)_L \times U(1)_Y$

• assuming a $(3, 2, 1)$ multiplet

— $SU(3)$ requires $(\bar{3}, 1, a)$ and $(\bar{3}, 1, b)$

— $SU(2)$ requires an extra $(1, 2, c)$

— $U(1)$ anomalies have no solution

$\longrightarrow$ adding a $(1, 1, k)$ give *the* unique solution

$\star$ idea extended to derive the 3-family spectrum O.K. MPLA11, PRD55 (97)

? alternative solution :

- $(3,2,0)$: q^+ q^+ q^+ q^- q^- q^-
- $(\bar{3},1,-1)$: $\bar{q}^-$ $\bar{q}^-$ $\bar{q}^-$
- $(\bar{3},1,1)$: $\bar{q}^+$ $\bar{q}^+$ $\bar{q}^+$
- $(1,2,0)$: **missing/needed BUT** ***not chiral***
- $(1,1,0)$: **missing/no need**

— some lectures on EW (from famous theorist)
"What would be the simplest fully chiral representation of ...... give an anomaly free theory? ...
... one generation of the Standard Model,... fails by little ...
The simplest representation is in fact a charge 1/2 quark, ..."

- **Witten's anomaly requires at least $(1,2,0)$, ...**

Principle of Gauge-Chiral Fields

Why there is what there is — why the list ?

- gauge symmetry / canceled anomaly $\Longrightarrow$ full Lagrangian
- massless (before symmetry breaking)

— if massive, at model cut-off scale / decoupled

Georgi : survival hypothesis (79)

— no (non-chiral) scalars (SUSY $\Rightarrow$ chiral scalar)

- 'chiral matter' + gauge bosons (DICTATED)

—all fields massless by gauge symmetry

- **SM — two problems**

— needs EWSB : dynamical symmetry breaking; & SUSY (?)

— the most fundamental mystery : *Why Three Families ?*

- against vectorlike pair – Georgi's survival hypothesis

  invariant mass at cutoff scale

- SM → BSM — hierarchy/fine-tuning problem

  scalar field is somewhat sick

- scalar field content — only part arbitrary (*cf.* gauge symmetry)

- SUSY — **technically** natural hierarchy

  scalar as (part of) **chiral** superfield (constrained as fermions)

  Vs

BUT μ-problem — vectorlike pair of Higgs superfields

- **SNJL models solve our problem**

— and avoid fine-tuning of "four-quark" coupling(s)

Energy scale issue :-

- **cutoff is 'natural'; Vs conformal theories**

— **other small scales should be generated**

- **need SSB – w/o put-in scale**

— **NJL gives best/simplest DSB framework**

— **SNJL : bi-superfield condensate**

HSNJL model— viable version for MSSM

O.K. *et.al.* PRD81 (10), JHEP01 (12), PRD87 (13)

— **fully chiral background (cf. HSNJL)**

Supersymmetric SM :-

• SUSY is a beautiful symmetry

• one compelling model for TeV scale

R-parity Violation (bottom-up !) —

• early days — faith in baryon and lepton number conservation

• (global) discrete symmetry — *theorists' pretence of elegance in discarding arbitrary admissible terms in an Lagrangian ?*

• given SUSY — simplest, natural, way to have neutrino masses

⋆ flavor problems ⟶ more complicated

• μ-problem — **vector-like Higgs superfields !**
— NJL as a solution (interesting viable SUSY version)

O.K. *et.al.* PRD81 (10), JHEP01 (12), PRD87 (13)

SUSY (w/o R parity) Flavor Physics:-

- one Higgs from L_0 an extra 'lepton' flavor O.K. *et. al.* PLB430 (98)

— clarifying the flavor basis issue $\Longrightarrow$ better formulation

phenomenology from all RPV combinations : O.K. *et. al.* (98-13)

e.g. fermion EDM at 1-loop, $h \rightarrow \mu^{\pm}\tau^{\mp}$

Keum & O.K. PRL86 (01), Choi *et. al.* PRD63 (00)
Arhrib, Cheng, & O.K. PRD87 (13), EPL101 (13)

- for λ_{ijk} and λ'_{ijk} being key source of m_ν

$\Longrightarrow$ no hierarchy or anti-hierarchy down the families

O.K. MPLA14 (99)

- ? flavor structure among soft SUSY breaking terms

Extending SM (vertical) Gauge Symmetry :-

• adding quarks and leptons w/o extending symmetry
— essentially a *no go*

• extending EW symmetry
— interesting flavor physics, flavor $\neq$ family

• little Higgs as models of extended EW symmetries
— ? interesting option of TeV scale effective field theory

⋆ ? 'flavor symmetry' when flavor $\neq$ family

Anomaly Free Gauged $SU(N)_L \times U(1)_X$ **Models :-**

— family non-universal SM embeddings : cancellation among them

- some in the literature

— Frampton PRL69 (92), Singer *et.al.* PRD22, Pisano & Pleitez PRD46, Foot *et.al.* PRD50

- infinite number exist under simple construction rules !

O.K. IJMPA20 (05)

Compatible with little Higgs ?

— works for simplest Higgs little type of models

Kaplan & Schmaltz JHEP10 (03)

- *one* for $N = 3$, *more* for $N = 4$

O.K. PRD70 (04), IJMPA20 (05)

The Construction Rules :-

- (t, b) containing Q^a as $(3, N, X_Q)$
- other quark doublets in $(3, \bar{N}, X_{Q'})$
- because $N_f = N_c \longrightarrow SU(N)_L$ anomaly cancels

by 3 N's - 6 $\bar{N}$'s + *3* family universal leptonic $(1, N, X_L)$'s

- $[SU(N)_L]^2 U(1)_X$ anomaly with correct doublet embeddings

$N_c X_Q + 2\, N_c X_{Q'} + N_f X_L = 0$

e.g. (with $N = 4$*)*

$$\mathcal{Q} = \frac{1}{2}\lambda^3 + \frac{A}{3}\lambda^8 + \frac{B}{6}\lambda^{15} + X$$

$\longrightarrow$ condition : $A + B + X_Q = \frac{1}{6}$ etc.

- add singlets to keep QCD & QED spectra vectorlike

331 little Higgs Model :-

— there is a solution (existence not *a priori* clear)

	Gauge anomalies					$U(1)_Y$ states
	tX	LLL	LLX	CCX	X^3	
$(\mathbf{3_C}, \mathbf{3_L}, \frac{\mathbf{1}}{\mathbf{3}})$	3	3	1	1	$\frac{1}{3}$	$\frac{\mathbf{1}}{\mathbf{6}}[Q]$ $\frac{\mathbf{2}}{\mathbf{3}}(T)$
$2\ (\mathbf{3_C}, \mathbf{\bar{3}_L}, \mathbf{0})$	0	-6	0	0	0	$2\ \frac{\mathbf{1}}{\mathbf{6}}[Q]$ $2\ \frac{\mathbf{-1}}{\mathbf{3}}(D, S)$
$3\ (\mathbf{1_C}, \mathbf{3_L}, \frac{\mathbf{-1}}{\mathbf{3}})$	-3	3	-1		$\frac{-1}{3}$	$3\ \frac{\mathbf{-1}}{\mathbf{2}}[L]$ $3\ \mathbf{0}(N)$
$4\ (\mathbf{\bar{3}_C}, \mathbf{1_L}, \frac{\mathbf{-2}}{\mathbf{3}})$	-8			$\frac{-8}{3}$	$\frac{-32}{9}$	$4\ \frac{\mathbf{-2}}{\mathbf{3}}\ (\bar{u}, \bar{c}, \bar{t}, \bar{T})$
$5\ (\mathbf{\bar{3}_C}, \mathbf{1_L}, \frac{\mathbf{1}}{\mathbf{3}})$	5			$\frac{5}{3}$	$\frac{5}{9}$	$5\ \frac{\mathbf{1}}{\mathbf{3}}\ (\bar{d}, \bar{s}, \bar{b}, \bar{D}, \bar{S})$
$3\ (\mathbf{1_C}, \mathbf{1_L}, \mathbf{1})$	3				3	$3\ \mathbf{1}\ (e^+, \mu^+, \tau^+)$
Total	0	0	0	0	0	

⋆ realistic little Higgs models $\Longrightarrow$ *new picture* on flavor physics

(also neutrino physics)

• may need to go to $SU(4)_L \times U(1)_X$ model (*also fine*)

$SU(4)_L \times U(1)_X$ **little Higgs Model** — 2nd example

	$U(1)_Y$-states		
$(\mathbf{3_C}, \mathbf{4_L}, \frac{\mathbf{1}}{\mathbf{6}})$	$\frac{\mathbf{1}}{\mathbf{6}}[Q]$	$\frac{\mathbf{2}}{\mathbf{3}}(T)$	$\frac{\mathbf{-1}}{\mathbf{3}}(B)$
$2\ (\mathbf{3_C}, \bar{\mathbf{4}}_\mathbf{L}, \frac{\mathbf{1}}{\mathbf{6}})$	$2\ \frac{\mathbf{1}}{\mathbf{6}}[2\ Q]$	$2\ \frac{\mathbf{-1}}{\mathbf{3}}(D, S)$	$2\ \frac{\mathbf{2}}{\mathbf{3}}(U, C)$
$3\ (\mathbf{1_C}, \mathbf{4_L}, \frac{\mathbf{-1}}{\mathbf{2}})$	$3\ \frac{\mathbf{-1}}{\mathbf{2}}[3\ L]$	$3\ \mathbf{0}(3\ N)$	$3\ \mathbf{-1}(3\ E^-)$
$6\ (\bar{\mathbf{3}}_\mathbf{C}, \mathbf{1_L}, \frac{\mathbf{-2}}{\mathbf{3}})$	$4\ \frac{\mathbf{-2}}{\mathbf{3}}\ (\bar{u}, \bar{c}, \bar{t}, \bar{T})$		$2\ \frac{\mathbf{-2}}{\mathbf{3}}\ (\bar{U}, \bar{C})$
$6\ (\bar{\mathbf{3}}_\mathbf{C}, \mathbf{1_L}, \frac{\mathbf{1}}{\mathbf{3}})$	$5\ \frac{\mathbf{1}}{\mathbf{3}}\ (\bar{d}, \bar{s}, \bar{b}, \bar{D}, \bar{S})$		$\frac{\mathbf{1}}{\mathbf{3}}\ (\bar{B})$
$6\ (\mathbf{1_C}, \mathbf{1_L}, \mathbf{1})$	$3\ \mathbf{1}\ (e^+, \mu^+, \tau^+)$		$3\ \mathbf{1}(3\ E^+)$

Flavor Structure of the 331-little Higgs Model(s) :-

- top Yukawa — $y_1\,\bar{t}'_a\,\Phi_1\,Q^a + y_2\,\bar{T}'_a\,\Phi_2\,Q^a \longrightarrow y_t\,\bar{t}\,h\begin{pmatrix} t \\ b \end{pmatrix}$
- bottom Yukawa — no $\bar{b}\,\Phi_i^\dagger\,Q^a$ but $\bar{b}\,\Phi_i^\dagger\,\Phi_j^\dagger\,Q^a$
- extra S and D may be relevant to B (b) physics

⋆ others — from the gauge symmetry only

— u and c Yukawa $\rightarrow 1_L\,\Phi_i\,\Phi_j\,\bar{3}_L$

— d and s Yukawa $\rightarrow 1_L\,\Phi_i^\dagger\,\bar{3}_L$

— *family universal* leptonic Yukawa $\rightarrow \ell^+\,\Phi_i^\dagger\Phi_j^\dagger\,L$

- also extra singlet neutrinos $\cdots\cdots$

quark masses — admit generic mass matrices

note : L-handed quark mixings

$$\mathcal{L}_{top} = \lambda_1^t\, \bar{t'}_a\, \Phi_1\, Q^a + \lambda_2^t\, \bar{T'}_a\, \Phi_2\, Q^a$$

$$= f\,(\lambda_1^t\, \bar{t'} + \lambda_2^t\, \bar{T'})\, T + \frac{i}{\sqrt{2}}\,(\lambda_1^t\, \bar{t'} - \lambda_2^t\, \bar{T'})\, h \begin{pmatrix} t \\ b \end{pmatrix} + \cdots$$

$$\mathcal{L}_{Q'} = \frac{1}{M}\lambda^u_{\alpha j}\, \bar{u'}_\alpha\, \Phi_1\, \Phi_2\, Q'_j = \frac{-i\sqrt{2}\, f}{M}\lambda^u_{\alpha j}\, \bar{u'}_\alpha\, h \begin{pmatrix} u_j \\ d_j \end{pmatrix} + \cdots$$

$$\mathcal{L}_{down} = \lambda^{d1}_{\beta j}\, \bar{d'}_\beta\, \Phi_1^\dagger\, Q'_j + \lambda^{d2}_{\beta j}\, \bar{d'}_\beta\, \Phi_2^\dagger\, Q'_j + \frac{1}{M}\lambda^b_\beta\, \bar{d'}_\beta\, \Phi_1^\dagger\, \Phi_2^\dagger\, Q$$

$$= f\,(\lambda^{d1}_{\beta j}\, \bar{d'}_\beta + \lambda^{d2}_{\beta j}\, \bar{d'}_\beta)\, D_j - \frac{i}{\sqrt{2}}\,(\lambda^{d1}_{\beta j}\, \bar{d'}_\beta - \lambda^{d2}_{\beta j}\, \bar{d'}_\beta)\, h^\dagger \begin{pmatrix} u_j \\ d_j \end{pmatrix}$$

$$+ \frac{i\sqrt{2}\, f}{M}\lambda^b_\beta\, \bar{d'}_\beta\, h^\dagger \begin{pmatrix} t \\ b \end{pmatrix} + \cdots$$

$$\mathcal{M}^q = \begin{pmatrix} m^q & 0 \\ m^{Qq} & m^Q \end{pmatrix} \quad — \begin{cases} 3+1 & \mathcal{M}^u \\ 3+2 & \mathcal{M}^d \end{cases}$$

SMALL PART OF THE STORY :-

• Z^0 boson couplings — $T^3_f - \mathcal{Q}_f \sin^2\theta_W$

— mixings of states with different T^3_f values

— CKM unitarity violation, FCNC, ...

$$\Longrightarrow \quad U^f_L = \begin{pmatrix} K^f & R^f \\ S^f & T^f \end{pmatrix} \qquad e.g. \quad S^c \simeq \frac{-1}{m_T}\, m^{Tc}$$

$$g_L(c) = \tfrac{1}{2}\left[1 - |S_c|^2\right] - \tfrac{2}{3}\sin^2\theta_W \qquad g_L(\bar{u}c) = \tfrac{1}{2}\left[-S^*_u S_c\right]$$

— hadronic Z-width gives < 0.014

3-family Models (with gauge-chiral fields ?) :-

- construction of minimal (?) chiral (fermion) spectrum
  with extended (gauge) symmetry

- require consistent SM embedding
  —— 1 fully chiral spectrum —— + **SSB**
  $\Longrightarrow$ 3 SM families + vectorlike SM fermions

- note: extending embedding to kill all anomaly always possible

— spectrum may be huge (*esthetic !*)

— yielding new chiral SM fermion is phenomenologically fatal

- beyond $3N1$ stories, $N321$ models

O.K. MPLA430, PRD55 (97)

$e.g.\ SU(4)_A \times SU(3)_C \times SU(2)_L \times U(1)_X$

multiplets	X	Gauge anomalies					$U(1)_Y$ states	
		t-1	441	331	221	1^3		
$(\mathbf{4},\mathbf{3},\mathbf{2})$	**1**	24	6	8	12	24	3 $\mathbf{1}(Q)$	$\mathbf{-5}(Q^{'})$
$(\bar{\mathbf{4}},\bar{\mathbf{3}},\mathbf{1})$	**5**	60	15	20		1500	3 $\mathbf{-4}(\bar{u})$	$\mathbf{2}(\bar{d})$
$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	**3**	24	6		12	216	3 $\mathbf{-3}(L)$	$\mathbf{3}(\bar{L})$
$(\bar{\mathbf{4}},\mathbf{1},\mathbf{1})$	**9**	36	9			2916	3 $\mathbf{-6}(\bar{E})$	$\mathbf{0}(N)$
$(\mathbf{6},\mathbf{1},\mathbf{1})$	**-18**	-108	-36			-34992	3 $\mathbf{6}(E)$	3 $\mathbf{12}(S)$
$(\mathbf{1},\bar{\mathbf{3}},\mathbf{2})$	**-10**	-60		-20	-30	-6000	$\mathbf{5}(\bar{Q}^{'})$	
$(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$	**-4**	-12		-4		-192	$\mathbf{2}(\bar{d})$	
$(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})$	**-4**	-12		-4		-192	$\mathbf{2}(\bar{d})$	
$(\mathbf{1},\mathbf{1},\mathbf{2})$	**6**	12			6	432	$\mathbf{-3}(L)$	
3 $(\mathbf{1},\mathbf{1},\mathbf{1})$	**24**	72				41472	3 $\mathbf{-12}(\bar{S})$	
3 $(\mathbf{1},\mathbf{1},\mathbf{1})$	**-12**	-36				-5184	3 $\mathbf{6}(E)$	
Total		0	0	0	0	0		

Back to Horizontal Symmetry

— $SU(3)_H \times SU(3)_C \times SU(2)_L \times U(1)_Y$

	Scheme I $U(1)_Y$-states	*Scheme II* $U(1)_Y$-states
$(\mathbf{3}, \mathbf{3}, \mathbf{2})$	$3\ \mathbf{1}(Q)$	$3\ \mathbf{1}(Q)$
$(\bar{\mathbf{3}}, \bar{\mathbf{3}}, \mathbf{1})$	$3\ \mathbf{2}(\bar{d})$	$3\ \mathbf{-4}(\bar{u})$
$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{2},)$	$3\ \mathbf{-3}(L)$	$3\ \mathbf{-3}(L)$
$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1})$	$3\ \mathbf{-6}(\bar{E})$	$3\ \mathbf{-12}(\bar{S}^{''})$
$3\ (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$	$3\ \mathbf{-4}(\bar{u})$	$3\ \mathbf{2}(\bar{d})$
$3\ (\mathbf{1}, \mathbf{1}, \mathbf{1})$	$3\ \mathbf{6}(E)$	$3\ \mathbf{6}(E)$
$3\ (\mathbf{1}, \mathbf{1}, \mathbf{1})$	$3\ \mathbf{6}(E)$	$3\ \mathbf{12}(S^{''})$

- simple gauge version of horizontal(/family) symmetry
- 3 SM families in one minimal chiral fermion spectrum

Summary of Basic Models :-

• NJL (1961)

$$\mathcal{L}_\psi = i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$$

• SNJL (1982) — dim 6 four-superfield interaction

$$\mathcal{L}_\psi = \int d^4\theta \left(\Phi_+^\dagger\Phi_+ + \Phi_-^\dagger\Phi_-\right)(1-\tilde{m}^2\theta^2\bar{\theta}^2) + \int d^4\theta\; g^2\Phi_+^\dagger\Phi_-^\dagger\Phi_+\Phi_-(1-\tilde{m}_c^2\theta^2\bar{\theta}^2)$$

• HSNJL (2010) — dim 5 four-superfield interaction

$$\mathcal{L}_\psi = \int d^4\theta \left(\Phi_+^\dagger\Phi_+ + \Phi_-^\dagger\Phi_-\right)(1-\tilde{m}^2\theta^2\bar{\theta}^2) - \int d^2\theta\; \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-(1+B\theta^2)$$

Concluding Remarks :-

- **BSM flavor $\neq$ family; Vs horizontal/family symmetry**

— ***how*** **is quite model dependent**

- **Principle of gauge-chiral fields may work**

— **need SUSY and/or DSB (? NJL)**

needs soft SUSY breaking (?)

- **(fermion) mass pattern may also be dictated**

— **higher-D operators give suppressed mass**

- **need more imaginative ideas and deep thinking**

— **also experimental data (new particles)**

Application to EW Symmetry Breaking :-

- top mode SM   Miransky, W. Bardeen, · · · '89/'90 (Nambu)
  — infared (quasi-)fixed point (IQFP)   (Pendleton-Ross), Hill, Marciano, · · ·
  prediction : top mass > 200 GeV ; VEV – top condensate

- supersymmetric NJL (formal – '82, SSM – '90)
  — $m_t = y_t \cdot v$, $m_b = y_b \cdot v'$; NJL predicts y not m ; $y_b < y_t$
  — other not very nice features as MSSM
  — lighter top fine, *but* . . .   (172.1 GeV top, $\tan\beta < 1.5$)

- our **holomorphic SNJL** (alternative supersymmetrization)
  — non-chiral symmetric 4-superfield interaction with t and b
  — superfield condensate : both scalar and fermion condensate
  — $y_t < y_b$;   nice , experimentally viable (LHC)

Nambu–Jona-Lasinio Model :-

- dynamical symmetry breaking
- four-fermion interaction

$$\mathcal{L}_\psi = i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$$

$$\longrightarrow \mathcal{L}_\psi - (\mu\phi^\dagger + g\psi_+\psi_-)(\mu\phi + g\bar{\psi}_+\bar{\psi}_-)$$

$$= i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- - \mu^2\phi^\dagger\phi - \mu g(\phi^\dagger\bar{\psi}_+\bar{\psi}_- + \phi\psi_+\psi_-)$$

- auxiliary scalar field ϕ (no kinetic term)
- EL-eq for $\phi^\dagger$ gives ϕ as composite

$$\phi = -g/\mu\,\bar{\psi}_+\bar{\psi}_-$$

- $\langle\phi\rangle \neq 0 \Longrightarrow$ symmetry breaking and fermion mass

Supersymmetrizing the NJL Model (Naively):-

- $i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ \quad \longrightarrow \quad \int d^4\theta \;\; \Phi_+^\dagger\Phi_+$
- $g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_- \quad \longrightarrow \quad \int d^4\theta \;\; g^2\Phi_+^\dagger\Phi_-^\dagger\Phi_+\Phi_-$
- $-\mu g\,\phi\psi_+\psi_- \quad \longrightarrow \quad \int d^2\theta \;\; \mu g\,\Phi\Phi_+\Phi_-$
- $-\mu^2\phi^*\phi \quad \longrightarrow \quad \int d^2\theta \;\; \frac{\mu}{2}\Phi\Phi$

BUT :-

- $\phi = -g/\mu\,\bar{\psi}_+\bar{\psi}_-$ implies

$$\mu^2\phi^*\phi = -\mu g\,\phi\psi_+\psi_- = g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_- \qquad \text{(no SUSY !)}$$

- no nontrivial vacuum without SUSY breaking

The Supersymmetric NJL Model :-

Buchmüller & Love 82

- $i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ \longrightarrow \int d^4\theta\ \Phi_+^\dagger\Phi_+\ (1-\tilde{m}^2\theta^2\bar{\theta}^2)$
- $g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_- \longrightarrow \int d^4\theta\ g^2\Phi_+^\dagger\Phi_-^\dagger\Phi_+\Phi_- \dashrightarrow \int d^4\theta\ \Phi_1^\dagger\Phi_1$
- $-\mu g\,\phi\psi_+\psi_- \longrightarrow \int d^2\theta\ \mu g\,\Phi_2\Phi_+\Phi_-$
- $-\mu^2\phi^*\phi \longrightarrow \int d^2\theta\ \mu\,\Phi_1\Phi_2$

BUT :-

- EL-eq for Φ_2 gives $\Phi_1 = -g\,\Phi_+\Phi_-$ implies

$$\int d^4\theta\ \bar{\Phi}_1\Phi_1 = \int d^4\theta\ g^2\Phi_+^\dagger\Phi_-^\dagger\Phi_+\Phi_-$$

- Φ_2 not the composite Φ_1 plays the Higgs superfield $\langle\Phi_1\rangle = 0$

An Alternative Supersymmetrization ?

Jung, O.K., Lee 2010

- $i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ \quad \longrightarrow \quad \int d^4\theta \;\; \Phi_+^\dagger\Phi_+ \;\; (1-\tilde{m}^2\theta^2\bar{\theta}^2)$
- $-\mu g\,\phi\psi_+\psi_- \quad \longrightarrow \quad \int d^2\theta \;\; \mu g\,\Phi_0\Phi_+\Phi_-$
- $-\mu^2\phi^*\phi \quad \longrightarrow \quad \int d^2\theta \;\; \frac{\mu}{2}\Phi_0\Phi_0$

$$\Longrightarrow \qquad \mathcal{L} = \int d^4\theta \left[(\Phi_+^\dagger\Phi_+ + \Phi_-^\dagger\Phi_-)(1-\tilde{m}^2\theta^2\bar{\theta}^2)\right] + \int d^2\theta \left[\frac{\mu}{2}\Phi_0^2 + \sqrt{\mu G}\Phi_0\Phi_+\Phi_-\right] + h.c.$$

- consider superpotential $\quad W = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$

$$\longrightarrow W - \frac{1}{2}(\sqrt{\mu}\Phi_0 + \sqrt{G}\Phi_+\Phi_-)(\sqrt{\mu}\Phi_0 + \sqrt{G}\Phi_+\Phi_-)$$

With Holomorphic Four-Chiral Superfield Interaction :-

- $W = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$ contains no $g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$
- EL-eq for auxiliary superfield Φ_0 gives $\Phi_0 = -\sqrt{G/\mu}\,\Phi_+\Phi_-$

  implies $\quad \frac{\mu}{2}\Phi_0^2 = -\frac{\sqrt{\mu G}}{2}\Phi_0\Phi_+\Phi_- = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$
- $\langle\Phi_0\rangle \implies \frac{G}{2}\langle\Phi_+\Phi_-\rangle\ \Phi_+\Phi_-$ Dirac mass for Φ_+–Φ_-
- kinetic term for Φ_0 from wave-function renormalization

  through Φ_+–Φ_- loop with Yukawa vertices

Towards the MSSM :-

- consider $W = G\,\varepsilon_{\alpha\beta}\,\hat{Q}^{\alpha}\hat{U}^{c}\hat{Q}'^{\beta}\hat{D}^{c}\,(1 + B\theta^2)$

$$
\begin{aligned}
W \;\longrightarrow\; & W - \mu\,(\hat{H}_d - \lambda_u \hat{Q}\hat{U}^c)(\hat{H}_u - \lambda_d \hat{Q}'\hat{D}^c)(1 + B\theta^2) \\
& = (-\mu \hat{H}_d \hat{H}_u + y_u \hat{Q}\hat{H}_u \hat{U}^c + y_d \hat{H}_d \hat{Q}'\hat{D}^c)(1 + B\theta^2)
\end{aligned}
$$

- two composites — $\hat{H}_u = \frac{y_d}{\mu}\,\hat{Q}'\hat{D}^c$ and $\hat{H}_d = \frac{y_u}{\mu}\,\hat{Q}\hat{U}^c$

- low energy effective theory looks like MSSM ($A = B$)
- symmetric role for $\hat{H}_u$ and $\hat{H}_d$ $\left(\mathrm{also:}\;\; \mu\lambda_\mathrm{u}\lambda_\mathrm{d} = \frac{\mathrm{y_u y_d}}{\mu} = \mathrm{G}\right)$

— numerical lifted through non-universal soft masses

— expect $\langle h_u \rangle \gtrsim \langle h_d \rangle$ (Vs UBB in D-flat)

Non-perturbative Analysis of DSB :-

- Dirac mass parameter ($\sim$ Higgs VEV) with SUSY breaking

e.g. Miller 83

$$\mathcal{M} = m - \theta^2 \eta$$

- superfield propagator with (soft) SUSY breaking

Scholl 84, Helayel-Neto 84

- superfield generating functional with SUSY breaking

$$\Gamma = \int \frac{d^4 p}{2\pi^4} \int d^2\theta \, \Phi_+(-p,\theta) \, \Gamma^{(2)}_{+-}(p,\theta^2) \, \Phi_-(p,\theta) + h.c. + \cdots$$

$\Longrightarrow$ gap equation : $\quad -\mathcal{M} = \Sigma^{(loop)}_{+-}(p,\theta^2)\Big|_{\text{on-shell}}$

$(from\ supergraphs)$

New Gap Equation Results (with nontrivial solutions) :-

- SNJL model

$$m = 2mg^2 I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)$$
$$\eta = -\eta\, g^2 \tilde{m}_C^2\, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)$$

— solution known considering only m, as $\eta = 0$, or $\tilde{m}_C^2 = 0$

Büchmuller & Ellwanger 84

— interesting general solution

- HSNJL model

$$m = \frac{\bar{\eta} G}{2}\, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)$$
$$\eta = \bar{m} G\, I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) + \frac{\bar{\eta} G B}{2}\, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)\ .$$

— tightly coupled, cannot see solution when neglecting η part

Majorana Vs Dirac :-

- beyond Dirac mass generation

— HSNJL model has Majorana mass option

- $\frac{G}{2}\langle\Phi_+\Phi_-\rangle\ \Phi_+\Phi_-$ Dirac mass for $\Phi_+\Phi_-$
- $\frac{G}{2}\langle\Phi_+\Phi_+\rangle\ \Phi_-\Phi_-$ Majorana mass for $\Phi_-\Phi_-$

  and $\frac{G}{2}\langle\Phi_-\Phi_-\rangle\ \Phi_+\Phi_+$ for $\Phi_+\Phi_+$ mass

- $\mathcal{L} = \int d^4\theta \left[\Phi_+^\dagger\Phi_+(1-\tilde{m}_+^2\,\theta^2\bar{\theta}^2) + \Phi_-^\dagger\Phi_-(1-\tilde{m}_-^2\,\theta^2\bar{\theta}^2)\right]$

  $- \int d^2\theta \big[\frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_- - \mu(H_+ + \lambda_-\Phi_-\Phi_-)(H_- + \lambda_-\Phi_+\Phi_+)\big] + h.c.$

Gap Equations for Majorana Mass Analysis :-

$$m_+ = \frac{\bar{\eta}_- G}{2}\, I_2(|m_-|^2, \tilde{m}_-^2, |\eta_-|, \Lambda^2)$$

$$\eta_+ = \bar{m}_- G\, I_1(|m_-|^2, \tilde{m}_-^2, |\eta_-|, \Lambda^2) + \frac{\bar{\eta}_- G B}{2}\, I_2(|m_-|^2, \tilde{m}_-^2, |\eta_-|, \Lambda^2)$$

$$m_- = \frac{\bar{\eta}_+ G}{2}\, I_2(|m_+|^2, \tilde{m}_+^2, |\eta_+|, \Lambda^2)$$

$$\eta_- = \bar{m}_+ G\, I_1(|m_+|^2, \tilde{m}_+^2, |\eta_+|, \Lambda^2) + \frac{\bar{\eta}_+ G B}{2}\, I_2(|m_+|^2, \tilde{m}_+^2, |\eta_+|, \Lambda^2)$$

- $\tilde{m}_+^2 = \tilde{m}_-^2 \Longrightarrow m_+ = m_-\ , \eta_+ = \eta_-$ same equations as Dirac case

★ completing symmetry breaking/mass generation scenarios

- for $\tilde{m}_-^2 = 0$, no Majorana mass solution $(B = 0)$

Concluding Remarks :-

- **our HSNJL works** $\longrightarrow$

  dynamical symmetry breaking, mass generation

- **may provide more interesting version of MSSM**

- **key to analysis**

— generating functional with SUSY breaking part

— maybe used for spontaneous SUSY breaking

- **completing Majorana Vs Dirac**

— splitted soft masses favor Dirac

— important for MSSM, also needed for m_t **Vs** m_b

THANK YOU !