Sterile Neutrinos at Future Lepton Colliders

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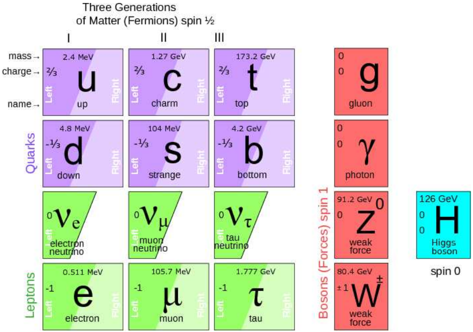
Motivation for Sterile Neutrinos

- Neutrino oscillations require *at least* two massive light/active SM neutrinos.
- This demands an extension of the SM.
- We consider the addition of right-handed fermion singlets – "sterile neutrinos" (N_i).
- Interesting scenario: Symmetry protected seesaw.



Courtesy Marco Drewes

Sterile Neutrinos

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... and others.
Appologies to those who are not on the list!

Outline: How to Test Sterile Neutrinos

- Sterile Neutrino Mass (M) $>$ Electroweak scale ($\Lambda_{\rm EW}$):
  - Indirect effect on precision observables.
  - Electroweak Precision Observables (EWPO) of particular interest.
- $M \sim \Lambda_{\rm EW}$:
  - Indirect effect on low energy precision observables and attenuated effect on EWPO.
  - N decays at the Z pole
  - N decay in 4 lepton final states at and beyond the W threshold
  - Higgs boson branching ratios
- This talk:
  - (ia) Non-unitarity in leptonic mixing, masses $\gg \Lambda_{\rm EW}$.
  - (ib) Two Sterile neutrinos with masses $\sim \Lambda_{\rm EW}$/ TeV scale.
  - (ii) Present bounds from precision data and direct searches.
  - (iii) Sensitivities of the CEPC (ILC and FCC-ee in the Backup).

Part I:
$M \gg \Lambda_{\mathrm{EW}}$ (electroweak scale)

Non-Unitarity of the Leptonic Mixing Matrix

Presence of massive sterile neutrinos (N_i):

$$\mathscr{L}_{\text{Theory}} = \mathscr{L}_{\text{SM}} + \mathscr{L}_N, \qquad \text{e.g. } \mathscr{L}_N \supset y_\alpha \overline{N_R}^I \widetilde{\phi}^\dagger L^\alpha$$

Leads to mixing of the neutral states (ν_{L_α}, N_i), $\alpha = e, \mu, \tau$:

$$\mathcal{U} = \begin{pmatrix} \begin{pmatrix} & \mathcal{N} & \end{pmatrix} & \cdots \\ \vdots & \ddots \end{pmatrix} \qquad \text{with} \qquad \mathcal{U}^\dagger \mathcal{U} = \mathbb{1}$$

- $\mathcal{N}$ is the leptonic mixing matrix
  $\sim$ Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix
- PMNS as submatrix in general **not** unitary ($\mathcal{N}\mathcal{N}^\dagger \neq \mathbb{1}$).

Minimal Unitarity Violation (MUV) Scheme

- For the formalism, see Backup I.
- Effective field theory treatment for energies below M.
- Modification of the weak currents with neutrinos:

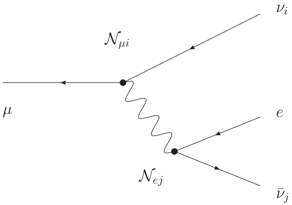
$$\left(J^{\mu\,,\pm}\right)_{\alpha i} = \ell_\alpha \gamma^\mu \nu_i \, \mathcal{N}_{\alpha i} \,, \qquad \left(J^{\mu,\,0}\right)_{ij} = \nu_i \gamma^\mu \nu_j \, \left(\mathcal{N}^\dagger \mathcal{N}\right)_{ij}$$

- Corresponding observables are $\propto \mathcal{N}\mathcal{N}^\dagger \sim \mathcal{N}^\dagger\mathcal{N}$
- Parametrisation: $(\mathcal{N}\mathcal{N}^\dagger)_{\alpha\beta} = \mathbb{1}_{\alpha\beta} + \varepsilon_{\alpha\beta}$

$\Rightarrow$ We have to keep track of the non-unitary PMNS matrix!

Theory Prediction for the EWPO

- Highest precision: M_Z, $\alpha(M_Z)$, G_F.



- Muon decay $\propto \left(\mathcal{N}\mathcal{N}^\dagger\right)_{ee} \left(\mathcal{N}\mathcal{N}^\dagger\right)_{\mu\mu}$
- In the SM: Fermi constant G_F = muon decay constant G_μ.
- MUV: $G_F = \frac{G_\mu}{\sqrt{(\mathcal{N}\mathcal{N}^\dagger)_{ee}(\mathcal{N}\mathcal{N}^\dagger)_{\mu\mu}}} = \frac{\alpha\pi}{\sqrt{2}s_W^2 c_W^2 m_Z^2}$
- Analogous: other observables involving weak decays.

Some examples

Prediction in MUV	SM Prediction	Experiment
$[R_{inv}]_{\mathrm{SM}}\,(1+0.75(\varepsilon_{ee}+\varepsilon_{\mu\mu})+0.67\varepsilon_{\tau})$	5.9723(10)	5.942(16)
$[M_W]_{\mathrm{SM}}(1-0.11(\varepsilon_{ee}+\varepsilon_{\mu\mu}))/\mathrm{GeV}$	80.359(11)	80.385(15)
$[\Gamma_{\mathrm{lept}}]_{\mathrm{SM}}(1-0.59(\varepsilon_{ee}+\varepsilon_{\mu\mu}))/\mathrm{MeV}$	83.966(12)	83.984(86)
$[(s_{W,\mathrm{eff}}^{\ell,\mathrm{lep}})^2]_{\mathrm{SM}}(1+0.71(\varepsilon_{ee}+\varepsilon_{\mu\mu}))$	0.23150(1)	0.23113(21)
$[(s_{W,\mathrm{eff}}^{\ell,\mathrm{had}})^2]_{\mathrm{SM}}(1+0.71(\varepsilon_{ee}+\varepsilon_{\mu\mu}))$	0.23150(1)	0.23222(27)

Global Fit to Precision Data

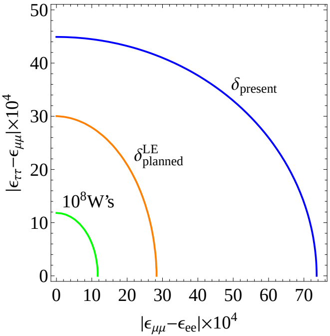
- MUV theory prediction for 34 precision observables: EWPO, lepton universality, rare charged lepton flavour violation, CKM unitarity
- See Backup II, III, IV, V for a complete list.
- MCMC fit of six parameters $\varepsilon_{\alpha\beta}$, including correlations.
- Highest posterior density intervals at 90% Bayesian C.L.*:

$$\boxed{\begin{array}{rcccr@{\qquad}lcl}
-0.0021 & \leq & \varepsilon_{ee} & \leq & -0.0002 & |\varepsilon_{e\mu}| & < & 1.0 \times 10^{-5} \\
-0.0004 & \leq & \varepsilon_{\mu\mu} & \leq & 0 & |\varepsilon_{e\tau}| & < & 2.1 \times 10^{-3} \\
-0.0053 & \leq & \varepsilon_{\tau\tau} & \leq & 0 & |\varepsilon_{\mu\tau}| & < & 8.0 \times 10^{-4}
\end{array}}$$

$\Rightarrow$ Inconclusive, used as constraints.

* This talk: 90% C.L. if not stated otherwise.

Sensitivity to Non-Unitarity from Lepton Universality Tests



Antusch, Fischer (2014)

- Assumption: SM is true ($\varepsilon \equiv 0$ & $O^{exp} = O^{SM}$).
- Blue line: experimental constrains (present).
- Orange line: experimental sensitivity (planned).
  MOLLER, TRIUMF, PSI, NA62, Tau/Charm factories
- Green line: Statistical precision from 10^8 W bosons.

Improvements of the Electroweak Precision Observables

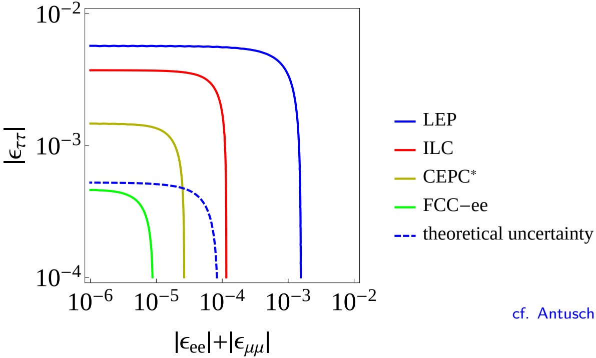
Observable	ILC	FCC-ee	CEPC	CEPC[*]
R_ℓ	0.004	0.001	0.01	0.003[*]
R_{inv}	0.01	0.002	0.012	0.006[*]
R_b	0.0002	0.00002	0.00017	0.00007[*]
M_W [MeV]	2.5	0.5	0.5	0.5
$s^{2,\ell}_{eff}$	1.3×10^{-5}	1×10^{-6}	2.3×10^{-5}	3.3×10^{-6}[*]
σ^0_h [nb]	0.025	0.0025	n.a.	0.008[*]
Γ_ℓ [MeV]	0.042	0.0042	n.a.	0.014[*]
Reference	1310.6708	1308.6176	Ruan (2014)[†]	scaled[*]

† Private communication.

∗ Assumption: CEPC produces 10^{11} Z bosons, compared to the 10^{12} Z bosons @FCC-ee.

⇒ Uncertainties scaled: $\delta_{\mathrm{CEPC}} = \delta_{\mathrm{FCC-ee}} \times \sqrt{10}$.

Improved sensitivity from Future Colliders



cf. Antusch, Fischer (2014)

- Indirect non-unitarity constraints from the EWPO only.
- Connection between ε and Yukawa couplings:

$$\varepsilon_{\alpha\beta} = -y^*_\alpha y_\beta v^2_{EW}/(2\,M^2)$$

$\Rightarrow$ For $y_\alpha = \mathcal{O}(1)$ CEPC tests M up to $\sim$ 40 TeV.

$\Rightarrow$ Improvements in the theoretical uncertainties needed.

Part II:
$M \sim \Lambda_{\mathrm{EW}}$ (electroweak scale)

Low Scale Seesaw Scenario

with two sterile neutrinos N_i and protective symmetry

$$\mathscr{L}_N = -\frac{1}{2}\overline{N_R}^I M^N_{IJ} N^{cJ}_R - y_\alpha \overline{N_R}^I \widetilde{\phi}^\dagger L^\alpha + \mathrm{H.c.}$$

- The leptonic mixing matrix to leading order in the active-sterile mixing parameters:

$$\mathcal{U} = \begin{pmatrix} \mathcal{N}_{1e} & \mathcal{N}_{1\mu} & \mathcal{N}_{1\tau} & -\mathrm{i}\,\frac{\theta_e}{\sqrt{2}} & \frac{\theta_e}{\sqrt{2}} \\ \mathcal{N}_{2e} & \mathcal{N}_{2\mu} & \mathcal{N}_{2\tau} & -\mathrm{i}\,\frac{\theta_\mu}{\sqrt{2}} & \frac{\theta_\mu}{\sqrt{2}} \\ \mathcal{N}_{3e} & \mathcal{N}_{3\mu} & \mathcal{N}_{3\tau} & -\mathrm{i}\,\frac{\theta_\tau}{\sqrt{2}} & \frac{\theta_\tau}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{\mathrm{i}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\theta_e & -\theta_\mu & -\theta_\tau & -\mathrm{i}\frac{1-\theta^2}{\sqrt{2}} & \frac{1-\theta^2}{\sqrt{2}} \end{pmatrix}.$$

- Active-sterile neutrino mixing parameters:

$$\theta_\alpha = \frac{y_\alpha}{\sqrt{2}}\frac{v_{\mathrm{EW}}}{M}, \qquad \alpha = e, \mu, \tau$$

Interactions between Heavy Neutrinos and the SM

- **Charged current** (CC):

$$j_\mu^\pm = \frac{g}{2}\,\theta_\alpha\,\bar{\ell}_\alpha\,\gamma_\mu\,(-\mathrm{i}N_1 + N_2)$$

- **Neutral current** (NC):

$$j_\mu^0 = \frac{g}{2\,c_W}\left[\theta^2\,\bar{N}_2\gamma_\mu\,N_2\; + (\bar{\nu}_i\,\gamma_\mu\,\xi_{\alpha 1}\,N_1 + \bar{\nu}_i\,\gamma_\mu\,\xi_{\alpha 2}\,N_2\; + \;\mathrm{H.c})\right]$$

- Higgs boson **Yukawa** interaction:

$$\mathscr{L}_{\mathrm{Yukawa}} = \sum_{i=1}^{3}\xi_{\alpha 2}\frac{\sqrt{2}\,M}{v_{\mathrm{EW}}}\nu_i\phi^0\left(\overline{N}_1 + \overline{N}_2\right)$$

- With the mixing parameters:

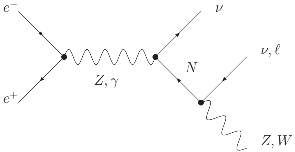
$$\xi_{\alpha 1} = \sum_{\beta=e,\mu,\tau}(-\mathrm{i})\,\mathcal{N}^*_{\alpha\beta}\frac{\theta_\beta}{\sqrt{2}}\,, \qquad \text{and} \qquad \xi_{\alpha 2} = \sum_{\beta=e,\mu,\tau}\mathcal{N}^*_{i\beta}\frac{\theta_\beta}{\sqrt{2}}$$

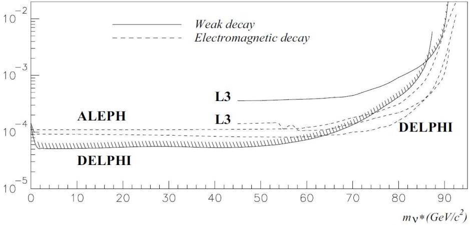
Decays involving Heavy Neutrinos

$$\begin{aligned}
\Gamma_{W\to N\ell} &= \frac{|\theta_\alpha|^2}{2}\frac{G_F\, m_W^3}{6\sqrt{2}\pi}\Pi^W_{(1+1)} & \Gamma_{N\to W\,\ell} &= \frac{|\theta_\alpha|^2}{2}\frac{G_F\, M^3}{4\sqrt{2}\pi}\Pi^W_{(1+1)} \\
\Gamma_{Z\to\nu N} &= |\xi_{ij}|^2\frac{G_F\, m_Z^3}{6\sqrt{2}\pi}\Pi^Z_{(1+1)} & \Gamma_{N\to Z\,\nu} &= |\xi_{ij}|^2\frac{G_F\, M^3}{4\sqrt{2}\pi}\Pi^Z_{(1+1)} \\
\Gamma_{Z\to N\,N} &= |\xi_{55}|^2\frac{G_F\, m_Z^3}{6\sqrt{2}\pi}\Pi^Z_{(2)} & \Gamma_{N\to h\,\nu} &= |\xi_{ij}|^2\frac{M}{16\,\pi}\left(1-\frac{m_h^2}{M^2}\right)^2 \\
\Gamma_{h\to\nu N} &= \frac{m_h\,\theta^2\,M^2}{8\,\pi\,v_{\mathrm{EW}}^2}\left(1-\frac{M^2}{m_h^2}\right)^2
\end{aligned}$$

Π^W, Π^Z : Phase space factors.

Sterile Neutrino searches @ the Z pole I





DELPHI collaboration, Abreu **et al.** (1997)

- Search for $Z \to \nu\, N$ in Z–pole data from Delphi at LEP.
- Analysis yields null result $\Rightarrow$ Upper limit on sterile-active neutrino mixing.

Sterile Neutrino searches @ the Z pole II

- Exclusion contour at 95% confidence level:

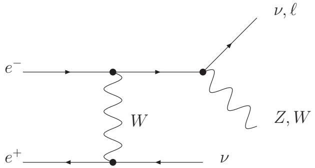
$$\theta^2 = \sum_{\alpha=e,\mu,\tau} |\theta_\alpha|^2 \leq \frac{2.1 \times 10^{-5}}{\Pi^Z_{(1+1)}}$$

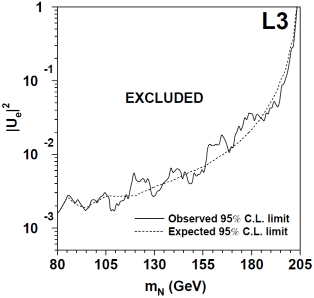
- Complementary: Search for displaced vertices. Investigated for the FCC-ee (TLEP) by Blondel, Graverini, Serra, Shaposhnikov (2014).
- In general: the sensitivity scales with the luminosity.

⇒ Limit on Yukawa couplings:

$$\theta_\alpha = \frac{y_\alpha}{\sqrt{2}} \frac{v_{\mathrm{EW}}}{M}, \qquad \Rightarrow \sum_{\alpha=e,\mu,\tau} |y_\alpha|^2 \leq \frac{M}{v_{\mathrm{EW}}} \frac{3 \times 10^{-5}}{\Pi^Z_{(1+1)}}$$

Searches in 4ℓ Final States for $\sqrt{s} \geq 2\,m_W$





L3 collaboration, Achard **et al.** (2001)

- N decay also contributes to 4ℓ final states.
- We use the experimental uncertainty from the Opal measurement of the WW production cross section.

$$\delta_{Opal} = \frac{\delta n_{WW}^{Opal}}{n_{WW}^{\mathrm{SM}}} = 1.002 \pm 0.011_{stat} \pm 0.007_{syst} \pm 0.005_{theory}$$

OPAL collaboration, Abbiendi **et al.** (2007)

Present Indirect Constraints from Precision Data

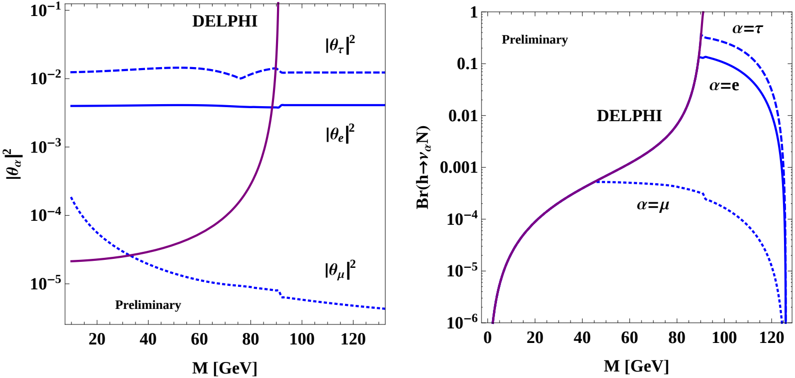
The following sets of precision observables are used:

- Electroweak Precision Observables (mainly LEP).
- Non-universality observables at low Energy (decays of μ, τ, π, K).
- Rare charged lepton flavour violating decays.
- CKM unitarity tests.
- Low energy measurements of the weak mixing angle.

Note:

$$R_{inv} = [R_{inv}]_{\mathrm{SM}} \left(1 - \frac{2}{3} \sum_{\alpha} |\theta_\alpha|^2 \left(1 - \Pi^{Z}_{(1+1)}\right) - 0.09(|\theta_e|^2 + |\theta_\mu|^2\right)$$

Bounds on Mixing Parameters and Higgs Branching Ratios



Antusch, Fischer (2015) *to appear*

- Reminder: Delphi constraints 95% confidence level.
- Large branchings of Higgs to sterile neutrinos are possible.
- Investigated also by Dev, Franceschini, Mohapatra (2012) and Cely, Ibarra, Molinaro, Petcov (2013).

Constraints from Higgs Branching Ratios

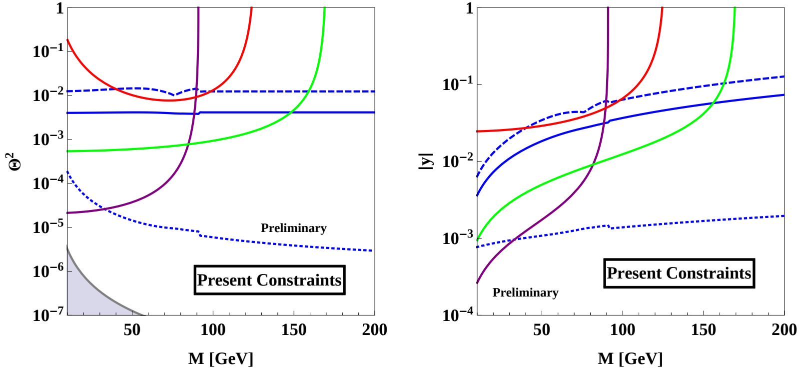
- Focus on the currently best measured branching ratios $h \to ZZ, WW, \gamma\gamma$.
- Assumptions: N decays within the detector and decay products are misidentified as vector boson events.
- Branching ratios become modified by heavy neutrinos:

$$Br_{h\to XX} = r\, Br_{h\to XX,\,\mathrm{SM}} + c_X\, Br_{h\to\nu N}$$

$$r = \frac{\Gamma_{h,\,\mathrm{SM}}}{\Gamma_{h,\,\mathrm{SM}} + \Gamma_{h\to\nu N}}, \qquad c_X = \begin{cases} \frac{1}{2}, & X = Z, W \\ 0, & X = \gamma,\, f \end{cases}$$

- CMS+Atlas: $Br_{h\to\gamma\gamma} = 1.15(27)$

Combination of Present Bounds



Direct searches

- 🟪 Delphi (N decays) @2σ: $|y|=\sqrt{\sum_\alpha |y_\alpha|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- 🟥 LHC (Higgs decays*) @1σ: $|y|=\sqrt{\sum_\alpha |y_\alpha|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- 🟩 Opal ($e^+e^- \to 4$ leptons) @1σ: $|y|=|y_e|$, $\Theta^2=|\theta_e|^2$

Other (global fit)

- ▬ $|y|= |y_e|$, $\Theta^2=|\theta_e|^2$
- ┅ $|y|= |y_\mu|$, $\Theta^2=|\theta_\mu|^2$
- ╍ $|y|= |y_\tau|$, $\Theta^2=|\theta_\tau|^2$

Antusch, Fischer (2015) *to appear*

∗ Currently dominated by $h \to \gamma\gamma$.

Improvements in Precision at Future Lepton Colliders

- Higgs measurements (per year of data taking for one detector):

Branching ratio	ILC [%]	CEPC [%]	FCC-ee [%]
$\delta Br_{h\to WW}$	6.4	1.3	0.9
$\delta Br_{h\to ZZ}$	19	5.1	3.1
$\delta Br_{h\to \gamma\gamma}$	35	8	3.0
Reference	1310.6708	1411.5606	1308.6176

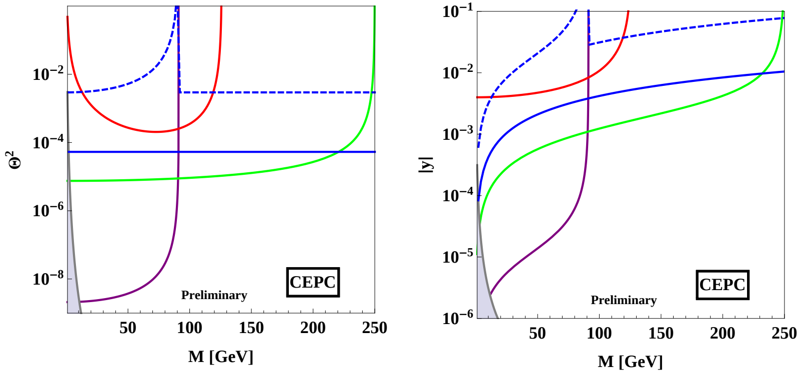
- Analysis uses 10 years of data taking with two detectors.

- Expected W boson yield:

	Opal	ILC	CEPC	FCC-ee
$\#W$'s prod.	10^4	10^7	10^8	2×10^8
$\delta_{\text{stat.}}$	0.011	3×10^{-4}	10^{-4}	7×10^{-4}
$\delta_{\text{syst.}}$	0.007	n.a.	n.a.	n.a.

- We consider the statistical uncertainty, but for realistic results $\delta_{\text{syst.}}$ is required.

Prospects of Sensitivity at the CEPC



Direct searches

- Z pole search @2σ: $|y|=\sqrt{\sum_\alpha |y_\alpha|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- Higgs → WW @1σ: $|y|=\sqrt{\sum_\alpha |y_\alpha|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- $e^+e^- \to 4$ leptons* @1σ: $|y|=|y_e|$, $\Theta^2=|\theta_e|^2$

Other

- Precision constraints: $|y|=\sqrt{|y_e|^2+|y_\mu|^2}$, $\Theta^2=|\theta_e|^2+|\theta_e|^2$
- Precision constraints: $|y|=|y_\tau|$, $\Theta^2=|\theta_\tau|^2$

Antusch, Fischer (2015) *to appear*

∗ Preliminary estimate using statistical uncertainty only.

Summary and Conclusions

- Sterile neutrinos are well motivated extensions of the SM.
- Symmetry protected scenarios allow for electroweak scale sterile neutrino masses and $\mathcal{O}(1)$ active-sterile mixings.
- Present constraints are dominated by Delphi for $M < m_Z$ and from indirect precision data for $M > m_Z$.
- LHC starts to constrain Higgs branching ratios to sterile neutrinos.
- Searches in Z, W, h decay data @ CEPC are very sensitive probes of sterile neutrino extensions of the SM.
- **Performance benchmarks**:
  (If) we want to compare apples with apples.
- At this point, **communication** is very important.
- There is **a lot** to do.

Thank you for your attention.

Backup I - Minimal Unitarity Violation (MUV) Formalism

- Lepton number violating mass operator:

$$\delta\mathscr{L}^{d=5} = \frac{1}{2}\, c^{d=5}_{\alpha\beta}\, (\overline{L^c}_\alpha \tilde{\phi}^*)(\tilde{\phi}^\dagger\, L_\beta)$$

- Lepton number conserving "kinetic" operator:

$$\delta\mathscr{L}^{d=6} = c^{d=6}_{\alpha\beta}\, (\overline{L}_\alpha \tilde{\phi}) i\partial\!\!\!/ (\tilde{\phi}^\dagger L_\beta)$$

- Mass-mixing & kinetic terms ⇒ MUV ≠ SM.
- Theory prediction for observable O: separating tree- and loop-level:

$$\begin{aligned} O_{\mathrm{MUV}} &= O^{\mathrm{tree}}_{\mathrm{MUV}} + \delta O^{\mathrm{loop}}_{\mathrm{MUV}} \\ &= O^{\mathrm{tree}}_{\mathrm{SM}}(1+\delta^{\mathrm{tree}}_{\mathrm{MUV}}) + \delta O^{\mathrm{loop}}_{\mathrm{SM}}(1+\delta^{\mathrm{loop}}_{\mathrm{MUV}})\,, \\ &= O_{\mathrm{SM}} + O^{\mathrm{tree}}_{\mathrm{SM}}\,\delta^{\mathrm{tree}}_{\mathrm{MUV}} + \delta O^{\mathrm{loop}}_{\mathrm{SM}}\delta^{\mathrm{loop}}_{\mathrm{MUV}} \\ &= O_{\mathrm{SM}} + (O_{\mathrm{SM}} - \delta O^{\mathrm{loop}}_{\mathrm{SM}})\,\delta^{\mathrm{tree}}_{\mathrm{MUV}} + \delta O^{\mathrm{loop}}_{\mathrm{SM}}\,\delta^{\mathrm{loop}}_{\mathrm{MUV}} \\ &= O_{\mathrm{SM}}(1+\delta^{\mathrm{tree}}_{\mathrm{MUV}}) + \ldots\,, \end{aligned}$$

- Theory prediction at leading order in the MUV parameters: $\delta^{tree}_{\mathrm{MUV}}$ is sufficient at the moment.

Backup II - EWPO

Experimental results and SM predictions for the EWPO, and the modification in the MUV scheme, to first order in $\varepsilon_{\alpha\alpha}$.

Prediction in MUV	SM Prediction	Experiment
$[R_\ell]_{\rm SM}\,(1-0.15(\varepsilon_{ee}+\varepsilon_{\mu\mu}))$	20.744(11)	20.767(25)
$[R_b]_{\rm SM}\,(1+0.03(\varepsilon_{ee}+\varepsilon_{\mu\mu}))$	0.21577(4)	0.21629(66)
$[R_c]_{\rm SM}\,(1-0.06(\varepsilon_{ee}+\varepsilon_{\mu\mu}))$	0.17226(6)	0.1721(30)
$\left[\sigma^0_{had}\right]_{\rm SM}\,(1-0.25(\varepsilon_{ee}+\varepsilon_{\mu\mu})-0.27\varepsilon_\tau)/\text{nb}$	41.470(15)	41.541(37)
$[R_{inv}]_{\rm SM}\,(1+0.75(\varepsilon_{ee}+\varepsilon_{\mu\mu})+0.67\varepsilon_\tau)$	5.9723(10)	5.942(16)
$[M_W]_{\rm SM}(1-0.11(\varepsilon_{ee}+\varepsilon_{\mu\mu}))/\text{GeV}$	80.359(11)	80.385(15)
$[\Gamma_{\rm lept}]_{\rm SM}(1-0.59(\varepsilon_{ee}+\varepsilon_{\mu\mu}))/\text{MeV}$	83.966(12)	83.984(86)
$[(s^{\ell,\rm lep}_{W,\rm eff})^2]_{\rm SM}(1+0.71(\varepsilon_{ee}+\varepsilon_{\mu\mu}))$	0.23150(1)	0.23113(21)
$[(s^{\ell,\rm had}_{W,\rm eff})^2]_{\rm SM}(1+0.71(\varepsilon_{ee}+\varepsilon_{\mu\mu}))$	0.23150(1)	0.23222(27)

Backup III - Lepton Universality

MUV prediction:

$$R_{\alpha\beta} = \sqrt{\frac{(NN^\dagger)_{\alpha\alpha}}{(NN^\dagger)_{\beta\beta}}} \simeq 1 + \frac{1}{2}\left(\varepsilon_{\alpha\alpha} - \varepsilon_{\beta\beta}\right) \, .$$

	Process	Bound
$R^{\ell}_{\mu e}$	$\frac{\Gamma(\tau \to \nu_\tau \mu \bar{\nu}_\mu)}{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)}$	1.0018(14)
$R^{\ell}_{\tau\mu}$	$\frac{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)}{\Gamma(\mu \to \nu_\mu e \bar{\nu}_e)}$	1.0006(21)
$R^{W}_{e\mu}$	$\frac{\Gamma(W \to e \bar{\nu}_e)}{\Gamma(W \to \mu \bar{\nu}_\mu)}$	1.0085(93)
$R^{W}_{\tau\mu}$	$\frac{\Gamma(W \to \tau \bar{\nu}_\tau)}{\Gamma(W \to \mu \bar{\nu}_e)}$	1.032(11)

	Process	Bound
$R^{\pi}_{\mu e}$	$\frac{\Gamma(\pi \to \mu \bar{\nu}_\mu)}{\Gamma(\pi \to e \bar{\nu}_e)}$	1.0021(16)
$R^{\pi}_{\tau\mu}$	$\frac{\Gamma(\tau \to \nu_\tau \pi)}{\Gamma(\pi \to \mu \bar{\nu}_\mu)}$	0.9956(31)
$R^{K}_{\tau\mu}$	$\frac{\Gamma(\tau \to K \nu_\tau)}{\Gamma(K \to \mu \bar{\nu}_\mu)}$	0.9852(72)
$R^{K}_{\tau e}$	$\frac{\Gamma(\tau \to K \nu_\tau)}{\Gamma(K \to e \bar{\nu}_e)}$	1.018(42)

Backup IV - CKM Unitarity Constraint

Current world averages: $V_{ud} = 0.97427(15)$, $V_{ub} = 0.00351(15)$

In the MUV scheme:

$|V_{ij}^{th}|^2 = |V_{ij}^{exp}|^2(1 + f^{\text{process}}(\varepsilon_{\alpha\alpha}))$,

$|V_{ud}^{th}|^2 = |V_{ud}^{exp,\beta}|^2(NN^\dagger)_{\mu\mu}$.

For the kaon decay processes we have:

$|V_{us}^{th}|^2 = |V_{us}^{exp,K\to e}|^2(NN^\dagger)_{\mu\mu}$,

$|V_{us}^{th}|^2 = |V_{us}^{exp,K\to\mu}|^2(NN^\dagger)_{ee}$.

Process	$V_{us} f_+(0)$
$K_L \to \pi e \nu$	0.2163(6)
$K_L \to \pi \mu \nu$	0.2166(6)
$K_S \to \pi e \nu$	0.2155(13)
$K^\pm \to \pi e \nu$	0.2160(11)
$K^\pm \to \pi \mu \nu$	0.2158(14)
Average	0.2163(5)

Processes involving tau leptons:

Process	$f^{\text{process}}(\varepsilon)$	$\lvert V_{us}\rvert$
$\frac{B(\tau\to K\nu)}{B(\tau\to\pi\nu)}$	$\varepsilon_{\mu\mu}$	0.2262(13)
$\tau \to K\nu$	$\varepsilon_{ee} + \varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}$	0.2214(22)
$\tau \to \ell,\ \tau \to s$	$0.2\varepsilon_{ee} - 0.9\varepsilon_{\mu\mu} - 0.2\varepsilon_{\tau\tau}$	0.2173(22)

Backup V - Lepton Flavour Violation

- Present experimental limits at 90% C.L.:

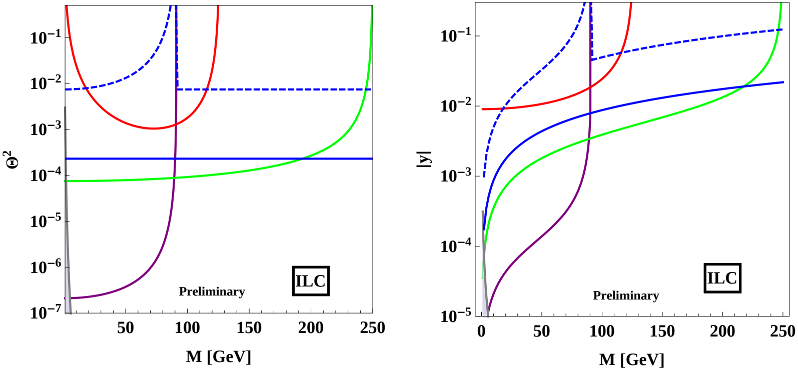
Process	MUV Prediction	Bound	Constraint on $\lvert\varepsilon_{\alpha\beta}\rvert$
$\mu \to e\gamma$	$2.4 \times 10^{-3} \lvert\varepsilon_{\mu e}\rvert^2$	5.7×10^{-13}	$\varepsilon_{\mu e} < 1.5 \times 10^{-5}$
$\tau \to e\gamma$	$4.3 \times 10^{-4} \lvert\varepsilon_{\tau e}\rvert^2$	1.5×10^{-8}	$\varepsilon_{\tau e} < 5.9 \times 10^{-3}$
$\tau \to \mu\gamma$	$4.1 \times 10^{-4} \lvert\varepsilon_{\tau \mu}\rvert^2$	1.8×10^{-8}	$\varepsilon_{\tau \mu} < 6.6 \times 10^{-3}$

- Estimated sensitivities of planned experiments at 90% C.L.:

Process	MUV Prediction	Bound	Sensitivity
$Br_{\tau e}$	$4.3 \times 10^{-4} \lvert\varepsilon_{\tau e}\rvert^2$	10^{-9}	$\varepsilon_{\tau e} \geq 1.5 \times 10^{-3}$
$Br_{\tau \mu}$	$4.1 \times 10^{-4} \lvert\varepsilon_{\tau \mu}\rvert^2$	10^{-9}	$\varepsilon_{\tau \mu} \geq 1.6 \times 10^{-3}$
$Br_{\mu eee}$	$1.8 \times 10^{-5} \lvert\varepsilon_{\mu e}\rvert^2$	10^{-16}	$\varepsilon_{\mu e} \geq 2.4 \times 10^{-6}$
$R_{\mu e}^{Ti}$	$1.5 \times 10^{-5} \lvert\varepsilon_{\mu e}\rvert^2$	2×10^{-18}	$\varepsilon_{\mu e} \geq 3.6 \times 10^{-7}$

$\Rightarrow$ $R_{\mu e}^{Ti}$ yields a sensitivity to m_{ν_R} up to 0.3 PeV.

Backup VI - ILC Summary Plot



Direct searches

- Z pole search @2σ: $|y|=\sqrt{\sum_\alpha |y_\alpha|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- Higgs → WW @1σ: $|y|=\sqrt{\sum_\alpha |y_\alpha|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- $e^+e^- \to$ 4 leptons* @1σ: $|y|=|y_e|$, $\Theta^2=|\theta_e|^2$

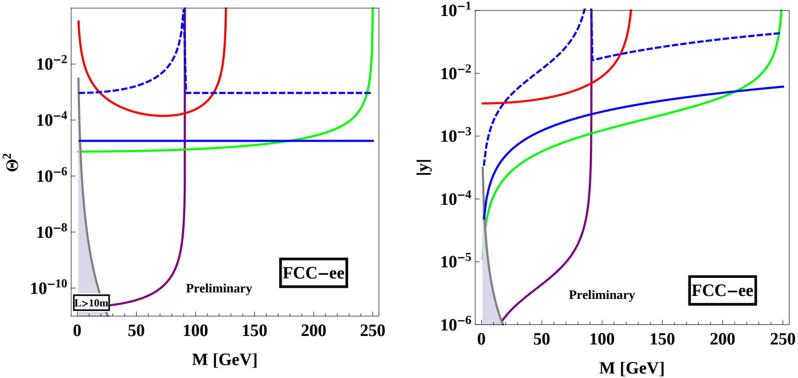
Other

- Precision constraints: $|y|=\sqrt{|y_e|^2+|y_\mu|^2}$, $\Theta^2=|\theta_e|^2+|\theta_e|^2$
- Precision constraints: $|y|=|y_\tau|$, $\Theta^2=|\theta_\tau|^2$

Antusch, Fischer (2015) *to appear*

∗ Preliminary estimate using statistical uncertainty only.

Backup VII - FCC-ee (TLEP) Summary Plot



Direct searches

- ▬ Z pole search @2σ: $|y|=\sqrt{\sum_\alpha |y_\alpha|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- ▬ Higgs → WW @1σ: $|y|=\sqrt{\sum_\alpha |y_\alpha|^2}$, $\Theta^2=\sum_\alpha|\theta_\alpha|^2$
- ▬ $e^+e^-\to$ 4 leptons* @1σ: $|y|=|y_e|$, $\Theta^2=|\theta_e|^2$

Other

- ▬ Precision constraints: $|y|=\sqrt{|y_e|^2+|y_\mu|^2}$, $\Theta^2=|\theta_e|^2+|\theta_e|^2$
- ╍ Precision constraints: $|y|=|y_\tau|$, $\Theta^2=|\theta_\tau|^2$

Antusch, Fischer (2015) *to appear*

$*$ Preliminary estimate using statistical uncertainty only.