Amplitudes beyond four-dimensions

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Why? why?

- QFT exists in $D \neq 4$
- Questions rephrased in terms of physical observables

  Allow for application of consistency conditions:
- Structures based on Unitarity + Locality, should be universal.

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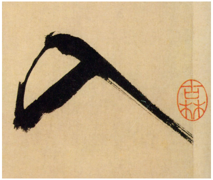
Where should we look?

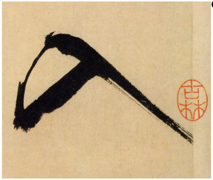
Spinor magic:



Where should we look?

$\alpha \leftarrow \mathrm{SL}(2, \mathrm{C})$

$\mathrm{D} = 4:$



Little Group $\mathrm{U}(1)$

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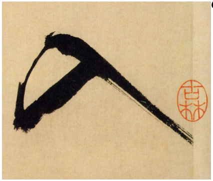


$\alpha \leftarrow \mathrm{SL}(2, \mathrm{C})$

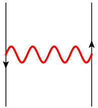
Little Group $\mathrm{U}(1)$

$\mathrm{D} = 3: \quad \lambda^{\alpha} \quad \alpha \rightarrow \mathrm{SL}(2, \mathrm{R})$, $\mathrm{D} = 6: \quad \lambda^{\mathrm{A}}_{\mathrm{a}} \quad \mathrm{A} \rightarrow \mathrm{SU}(4), \; \mathrm{a} \rightarrow \mathrm{SU}(2)$

Complicated questions rephrased (D=3)

Is the $\mathcal{N}=8$ interacting CFT unique?

Four-point amplitude: ($\mathcal{N}=8$ SCS + factorization)



$$\mathcal{A}_4 = \frac{\delta^3(P)\delta^8(Q)}{\langle 12\rangle\langle 23\rangle\langle 13\rangle}$$

It is completely anti-symmetric in $(1,2,3,4) \rightarrow$ must be dressed with $f^{[1234]}$ (SU(2)$\times$SU(2))

The theory is unique if it has a perturbative S-matrix

Complicated questions rephrased (D=6)

Can self-dual tensors interact?

- Difficult to non-abelianize

$$\delta B_{\mu\nu} = \partial_\mu \Lambda_\nu + \ ????$$

- Difficult to construct self-dual cations $H_{\mu\nu\rho} = \partial_\mu B_{\neq\rho} + \cdots = (H^{\sigma\rho\tau})^*$

Degenerate three-point kinematics: $s_{12} = 0 \rightarrow \left(\lambda_{1a}\right)^A \left(\tilde{\lambda}_{2\dot{a}}\right)_A = u_{1a}\tilde{u}_{2\dot{a}}$

Cheung, O' Connell

$$\langle BBB \rangle = 0, \quad \langle BB\mathcal{O} \rangle = 0$$

Three self-dual tensors cannot interact if the S-matrix for the theory exists

Test of structure beyond four-dimensions

Where to search for new structures ?

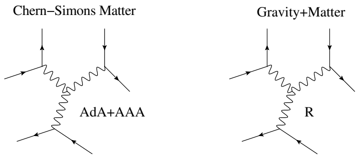
Test of structure beyond four-dimensions

Consider scattering amplitudes in three-dimensions:

- Large class of pure SCFT with different supersymmetry (same physical singularities)

$$\mathcal{L} = \mathcal{L}_{CS} + \mathcal{L}_{\phi,Kin} + \mathcal{L}_{\psi,Kin} + \mathcal{L}_{4\phi^2\psi^2} + \mathcal{L}_{6\phi^6}$$

- Both gauge and gravity are perturbation of topological theories



- Rich UV and IR-physics.

Test of structure beyond four-dimensions

Simpler amplitudes:

- Odd-amplitudes vanishes to all orders.
- Vanishing soft-limits for gravity:

$$\mathcal{M}_{n+1}(1,\cdots,n,s) = \mathcal{S}_G^0\mathcal{M}_n(1,\cdots,n) + \mathcal{S}_G^1\mathcal{M}_n(1,\cdots,n) + \cdots$$

$$\mathcal{S}^0 = \sum_{a=1}^{n}\left(\frac{\langle\mu a\rangle}{\langle\mu s\rangle}[sa]\right)^2\frac{1}{\langle as\rangle[sa]} \xrightarrow{\text{3-d kin}} \sum_{a=1}^{n}\frac{\langle\mu a\rangle^2}{\langle\mu s\rangle^2} = -1$$

  All bosonic states of the theory satisfy a duality symmetry See Wei-Ming Chen's talk
- Leading UV and IR-divergences are absent from Odd-loops.

Test of structure beyond four-dimensions

What structures should we search for?

$\mathcal{N} = 4$ Super Yang-Mills

- The planar theory enjoys SU(2,2|4) DSCI
- The string sigma model enjoys fermionic self T-duality
- The (super)amplitude is dual to a (super)Wilson-loop
- The IR-divergence structure captured by BDS
- The leading singularities is given by residues of $Gr(k,n)$ $\int [dC]_M \delta(C \cdot Z)$
- The amplitude has uniform trancsendentality
- Geometrization of locality and unitarity

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Test of structure beyond four-dimensions

$\mathcal{N} = 6$ Chern-Simons matter theory ABJM:

- The planar theory enjoys SU(2,2|4) DSCI $\rightarrow$ OSp(6|4)
- The leading singularities is given by residues of $Gr(k,n) \rightarrow$ OG(k,2k) S. Lee

$$\int [dC]_M \delta(C^T C)\delta(C \cdot Z)$$

Known unknowns:

1. The amplitudes appears to be uniform transcendental (proof?)

2. Why is the IR-divergence (Dual conformal anomaly equation) the same? Y-t, W. Chen, S. Caron-Huot

$$\mathcal{A}_4^{2-\text{loop}} = \left(\frac{N}{k}\right)^2 \frac{\mathcal{A}_4^{\text{tree}}}{2} \text{BDS}_4$$

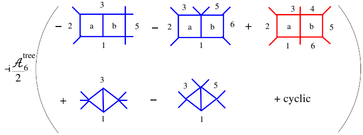
$$\mathcal{A}_6^{2-\text{loop}} = \left(\frac{N}{k}\right)^2 \left\{ \frac{\mathcal{A}_6^{\text{tree}}}{2} \left[\text{BDS}_6 + R_6\right] + \frac{\mathcal{A}_{6,\text{shifted}}^{\text{tree}}}{4i} \left[\ln\frac{u_2}{u_3} \ln\chi_1 + \text{cyclic} \times 2\right] \right\}$$

At four-point to all orders in ϵ M. Bianchi, M. Leoni, S Penati, exponentiation verified at three-loops M. Bianchi, M. Leoni

Known unknowns: 3. Why is the amplitude non-analytic?

$$\mathcal{A}_6^{\text{1-loop}} = \frac{\mathcal{A}_6^{\text{tree}}}{\sqrt{2}}\Big[I_{box}(3,4,5,1) + I_{box}(1,2,3,4) - I_{box}(4,5,6,1) - I_{box}(6,1,2,4)\Big] + \frac{\mathcal{C}_1 + \mathcal{C}_1^*}{2} I_{tri}(1,3,5) + \frac{\mathcal{C}_2 + \mathcal{C}_2^*}{2} I_{tri}(2,4,6)\,.$$

$$\rightarrow \quad \mathcal{A}_6^{\text{1-loop}} = \left(\frac{N}{k}\right)\frac{-\pi}{2}\mathcal{A}_{6,\text{shifted}}^{\text{tree}}\left(\text{sgn}_c\langle 12\rangle \text{sgn}_c\langle 34\rangle \text{sgn}_c\langle 56\rangle + \text{sgn}_c\langle 23\rangle \text{sgn}_c\langle 45\rangle \text{sgn}_c\langle 61\rangle\right).$$



$$\rightarrow \quad \mathcal{A}_6^{\text{2-loop}} = \left(\frac{N}{k}\right)^2 \left\{\frac{\mathcal{A}_6^{\text{tree}}}{2}\Big[BDS_6 + R_6\Big] + \frac{\mathcal{A}_{6,\text{shifted}}^{\text{tree}}}{2} \times \left[\text{sgn}_c(\langle 12\rangle)\text{sgn}_c(\langle 45\rangle)\frac{(\langle 34\rangle\langle 46\rangle + \langle 35\rangle\langle 56\rangle)}{\sqrt{(\langle 34\rangle\langle 46\rangle + \langle 35\rangle\langle 56\rangle)^2}} \log\frac{u_2}{u_3}\arccos(\sqrt{u_1}) + \text{cyclic} \times 2\right]\right\}$$

Let us see how far can we get by understanding the amplitude through Grassmannian

Orthogonal Grasmmannian

Consider *k*-planes in *n*-dimensional space equipped with a symmetric bi-linear Q^{ij}

The orthogonal grassmannian $\equiv Q^{ij}C_{\alpha i}C_{\beta j} = 0$

Consider $n = 2k$ and $Q^{ij} = \eta^{ij}$ signature $(+, +, +, \cdots, +)$

$$
\begin{aligned}
k &= 1, & C_{\alpha i} &= (1, \pm i) \\
k &= 2, & C_{\alpha i} &= \begin{pmatrix} 1 & \pm i\cos z & 0 & -i\sin z \\ 0 & \pm i\sin z & 1 & i\cos z \end{pmatrix}
\end{aligned}
$$

$$
\mathcal{L}_n = \sum_{res} \int \frac{dC}{(1\cdots k)\cdots(k\cdots n-1)} \delta(Q^{ij}C_{\alpha i}C_{\beta j})\delta^{2k}(C\cdot\lambda)\delta^{3k}(C\cdot\eta)
$$

S. Lee, D. Gang, E. Koh, E. Koh, A. Lipstein, Y-t

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S. Lee, D. Gang, E. Koh, E. Koh, A. Lipstein, Y-t

Positive Orthogonal Grasmmannian

Positivity: $(\mathrm{i},\mathrm{i}+1,\cdots,\mathrm{i}+\mathrm{k})>0$

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$$k=1,\qquad \mathrm{C}_{\alpha\mathrm{i}}=(1,\pm\mathrm{i})$$

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Positivity: ordered $(\mathrm{i}, \cdots, \mathrm{j}) > 0$

$\mathrm{Q}^{\mathrm{ij}} = \eta^{\mathrm{ij}}$ signature $(+, -, +, \cdots, -)$

$$k = 1, \qquad \mathrm{C}_{\alpha\mathrm{i}} = (1, 1)$$

$$k = 2, \qquad \mathrm{C}_{\alpha\mathrm{i}} = \begin{pmatrix} 1 & \cos \mathrm{z} & 0 & -\sin \mathrm{z} \\ 0 & \sin \mathrm{z} & 1 & \cos \mathrm{z} \end{pmatrix}$$

Positive for $0 \leq \mathrm{z} \leq \pi/2$

Volume form w. logarithmic singularity at the boundary: $z = \pi/2$, $z = 0$

$$\frac{\mathrm{dz}}{\cos \mathrm{z} \sin \mathrm{z}} = \mathrm{d} \log \tan \mathrm{z}$$

$$\int \mathrm{d} \log \tan \quad \delta^4(\mathrm{C} \cdot \lambda) \delta^6(\mathrm{C} \cdot \eta)$$

This is not the amplitude $\mathcal{A}_4$!

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For $0 \leq z \leq \pi/2$ Positivity: $(i, \cdots, j) > 0$ and $\pm(i, \cdots, 2k) > 0$

$$\mathcal{A}_4 = \int d \log \tan \quad \delta^4(C \cdot \lambda)\delta^6(C \cdot \eta) + (\overline{OG}_{2+})$$

The four-point amplitude is given by the sum of two branches in OG_{2+}

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Why Two Branches of Positive Orthogonal Grasmmannian

$$k = 2, C_{\alpha i} = \begin{pmatrix} 1 & \cos z & 0 & -\sin z \\ 0 & \sin z & 1 & \cos z \end{pmatrix}$$

$$\delta^4(C \cdot \lambda) \to \begin{array}{c} \lambda_1 + \cos z\lambda_2 - \sin z\lambda_4 = 0 \\ \lambda_3 + \sin z\lambda_2 + \cos z\lambda_4 = 0 \end{array} \to \langle 34 \rangle = \langle 12 \rangle$$

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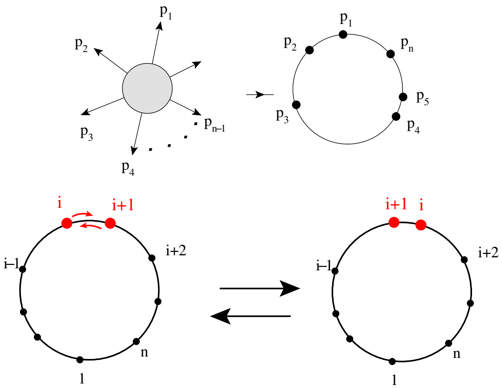
There are two branches in the kinematics as well:

$$\langle 34 \rangle^2 = s_{34} = s_{12} = \langle 12 \rangle^2$$

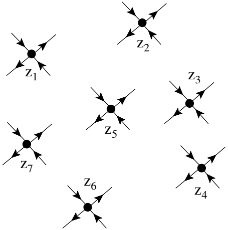
Why Two Branches of Positive Orthogonal Grasmmannian

3D- kinematics is topologically a circle

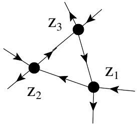
$p_i = (1, \cos\theta_i, \sin\theta_i)$



On-shell diagrams in Orthogonal Grasmmannian

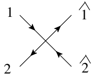


$\rightarrow$

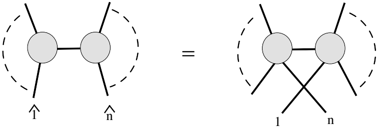


$\rightarrow \quad C_{\alpha i}(z)$

Are these diagrams related to $\mathcal{A}_n$?



$$\delta^4(C \cdot \lambda) \rightarrow \begin{array}{l} \lambda_{\hat{1}} + \sec z\lambda_1 + \tan z\lambda_2 \\ \lambda_{\hat{2}} - \tan z\lambda_1 - \sec z\lambda_2 \end{array}$$



Arkani-Hamed, J. Bourjaily, F. Cachazo, A. Goncharov, A. Postnikov, J. Trnka

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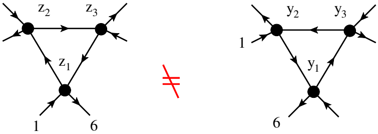
$$\mathcal{A}_6 = \sum_{\text{branch}} \int \mathrm{d}\log\tan_1 \, \mathrm{d}\log\tan_2 \, \mathrm{d}\log\tan_3 \, \delta^{2k}(C\cdot\lambda)\delta^{3k}(C\cdot\eta)$$

No



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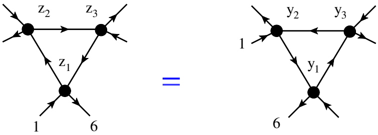
Are these diagrams related to $\mathcal{A}_n$?

$$\mathcal{A}_6 = \sum_{\text{branch}} \int \mathrm{d}\log\tan_1 \mathrm{d}\log\tan_2 \mathrm{d}\log\tan_3 (1+\sin_1\sin_2\sin_3)\delta^{2k}(C\cdot\lambda)\delta^{3k}(C\cdot\eta)$$

Yes



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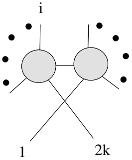
No new singularities $0 \leq z \leq \pi/2$.

On-shell diagrams in Orthogonal Grasmmannian

In general

$$\sum_{i}$$

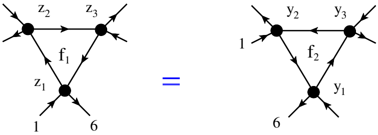


$$\mathcal{A}_n = \sum_{\text{branch}} \sum_{\text{dia}} \int \prod_{i=1}^{k} d \log \tan_i \, \mathcal{J} \delta^{2k}(C \cdot \lambda) \delta^{3k}(C \cdot \eta)$$

How to get $\mathcal{J}$?

On-shell diagrams in Orthogonal Grasmmannian

$$\mathcal{A}_6 = \sum_{\text{branch}} \int \mathrm{d}\log\tan_1 \, \mathrm{d}\log\tan_2 \, \mathrm{d}\log\tan_3 \, (1 + \sin_1 \sin_2 \sin_3)\delta^{2k}(C\cdot\lambda)\delta^{3k}(C\cdot\eta)$$



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$\mathcal{J}$ is naturally associated with faces!

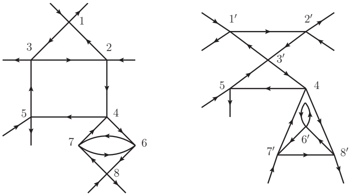
On-shell diagrams in Orthogonal Grasmmannian

$$\mathcal{J} = 1 + \mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3 + \mathcal{J}_{13} + \mathcal{J}_{23}$$

- $\mathcal{J}_1$:

$$\mathcal{J}_1 = \sum_{\text{single}} J_i + \sum_{\text{disjoint pairs}} J_i J_j + \sum_{\text{disjoint triples}} J_i J_j J_k + \dots$$

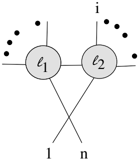
- $\mathcal{J}_2$: Two closed loops sharing a single vertex
- $\mathcal{J}_3$: Two closed loops sharing two vertices without sharing an edge.
- $\mathcal{J}_{13}$ and $\mathcal{J}_{23}$: The effect of the bigger loop from $\mathcal{J}_3$.



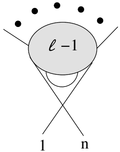
(a) (b)

The loop-level recurssion Arkani-Hamed, J. Bourjaily, F. Cachazo, A. Goncharov, A. Postnikov, J. Trnka

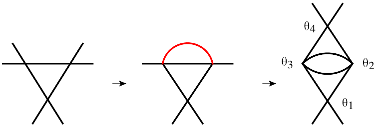
$$\mathcal{A}_n^l = \sum_{l_1+l_2=l} \sum_{i=4}^{n-2}$$



$+$

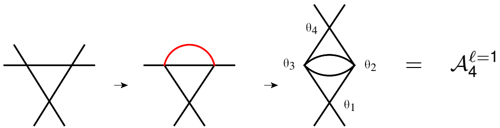


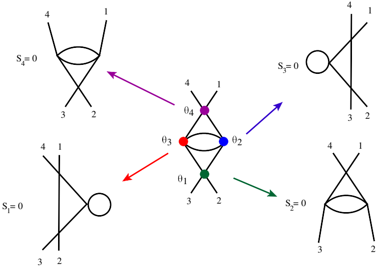
The loop-level recurssion



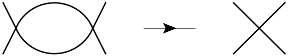
$$= \mathcal{A}_4^{\ell=1}$$

The loop-level recurssion

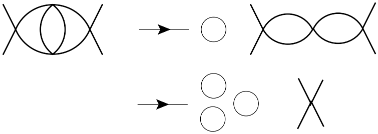




Using reduction:

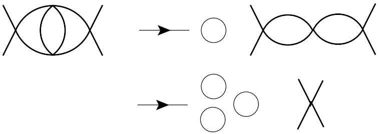


We can separate out the $d\log$ measure



Loop-amplitude and on-shell diagrams in Orthogonal Grasmmannian

Using reduction: We can separate out the *d* log measure



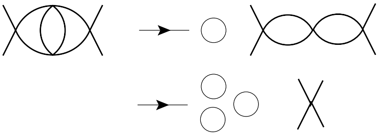
$$I_4 = \int_{X_0^2=0} \frac{\langle X_0 dX_0 dX_0 dX_0 dX_0 \rangle}{X_0^2} \frac{\langle 0,1,2,3,4 \rangle}{(0.1)(0.2)(0.3)(0.4)}, \quad (i.j) \equiv X_i \cdot X_j$$

$$X_0 = a_1 X_1 + a_2 X_2 + a_3 X_3 + a_4 X_4 + a_\epsilon \langle *, 1, 2, 3, 4 \rangle \,.$$

$$I_4 = \int d\log a_2 \, d\log a_3 \, d\log a_4 \,.$$

Loop-amplitude and on-shell diagrams in Orthogonal Grasmmannian

Using reduction: We can separate out the *d* log measure



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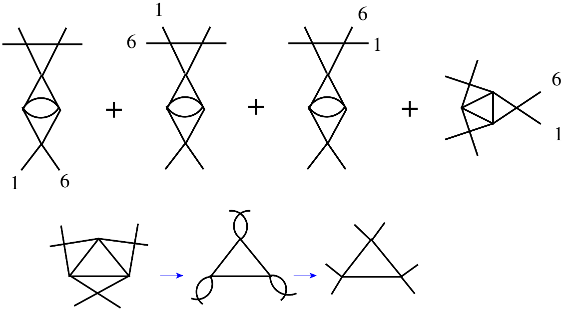
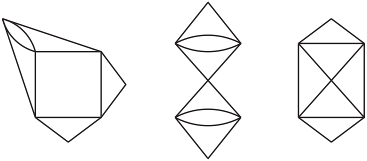
Loop-amplitude and on-shell diagrams in Orthogonal Grasmmannian

$$A_6^{1-\text{loop}} =$$



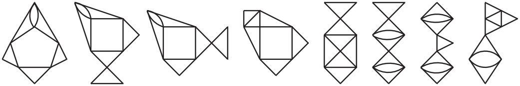
Loop-amplitude and on-shell diagrams in Orthogonal Grasmmannian

$A_4^{2-\text{loop}} =$



$+ \quad (i \to i{+}2)$

$A_6^{2-\text{loop}} =$



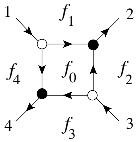
$+ \ (i \to i{+}2)$

Loop-amplitude and on-shell diagrams in Orthogonal Grasmmannian

- The solution to BCFW is manifestly cylic $i \to i+2$
- For each cell, a single chart covers all singularities
- All loop: 4 and 6-point amplitudes is a product of independent $d\log$
- Proved all physical sing present, spurious cancels

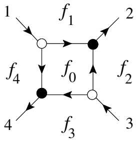
Any hint on the close tie to $\mathcal{N} = 4$ SYM?

Embedding $\mathrm{OG}(k, 2k)$ into $\mathrm{G}(k, 2k)$



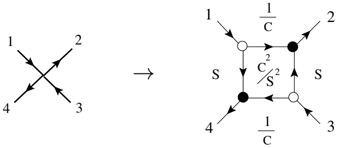
$$C = \begin{pmatrix} 1 & 1/f_1 & 0 & -f_4 \\ 0 & f_2 & 1 & 1/f_3 \end{pmatrix} .$$

Embedding $OG(k, 2k)$ into $G(k, 2k)$



$$C = \begin{pmatrix} 1 & 1/f_1 & 0 & -f_4 \\ 0 & f_2 & 1 & 1/f_3 \end{pmatrix} .$$

$$f_1 = \frac{1}{c},\ f_4 = s,\ f_2 = s,\ f_3 = \frac{1}{c}$$

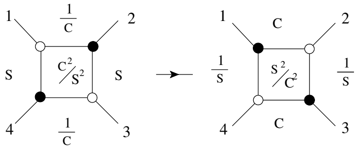


OG_{2+} has an image in $Gr(2, 4)_+$

Embedding $\mathrm{OG}(k, 2k)$ into $\mathrm{G}(k, 2k)$

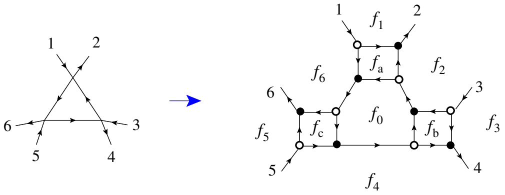
$$C = \begin{pmatrix} 1 & 1/f_1 & 0 & -f_4 \\ 0 & f_2 & 1 & 1/f_3 \end{pmatrix} .$$

Cluster transformation:



$c, s \to \frac{1}{c}, \frac{1}{s}$

Embedding $\mathrm{OG}(k,2k)$ into $\mathrm{G}(k,2k)$

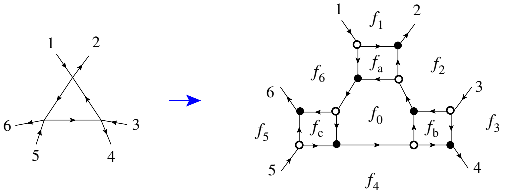


$$(f_a, f_b, f_c) = (c_1^2/s_1^2, c_2^2/s_2^2, c_3^2/s_3^2),\ f_0 = \frac{1}{c_1 c_2 c_3}$$

$$f_1 = \frac{1}{c_1},\ f_2 = s_1 s_2,\ f_3 = \frac{1}{c_3},\ f_4 = s_2 s_3,\ f_5 = \frac{1}{c_3},\ f_6 = s_1 s_3$$

- The variable for the k new faces is simply $f = c^2/s^2$.
- Take a clockwise orientation on each face. The contribution from each vertex is $1/c$ if one first encounters the black vertex, otherwise the contribution is s.

Embedding $\mathrm{OG}(k, 2k)$ into $\mathrm{G}(k, 2k)$



$$(f_a, f_b, f_c) = (c_1^2/s_1^2, c_2^2/s_2^2, c_3^2/s_3^2),\ f_0 = \frac{1}{c_1 c_2 c_3}$$

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In terms of the Grassmannian coordinates, ABJM is at the boundary of $\mathcal{N}=4$ SYM: $C^T C = 0$

To see the IR-divergence, we need the integrated answer

In terms of the Grassmannian coordinates, ABJM is at the boundary of $\mathcal{N} = 4$ SYM: $C^T C = 0$

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All multiplicity integrands for ABJM Song He, Y-t Huang:

1-loop:

$$A_n^{1-\text{loop}} = \sum_{i<j<k} \left(C^{+}_{i,j,k} I^{+}(i,j,k) + C^{-}_{i,j,k} I^{-}(i,j,k) \right) - A_n^{\text{tree}} \sum_{i=1}^{n} (-)^i I(i-1,i,i+1)$$

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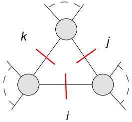
$$I^{\pm}(i,j,k) = \int_a \frac{-\epsilon(a,i,j,k,X)}{\sqrt{2}(a\cdot i)(a\cdot j)(a\cdot k)(a\cdot X)} \pm \frac{\sqrt{(i\cdot j)(j\cdot k)(k\cdot i)}}{(a\cdot i)(a\cdot j)(a\cdot k)}$$

$C^{\pm}_{i,j,k}$



$$\text{Res}^+_{i,j,k} I^+(i,j,k) = \text{Res}^-_{i,j,k} I^-(i,j,k) = 1, \quad \text{Res}^+_{i,j,k} I^-(i,j,k) = \text{Res}^-_{i,j,k} I^+(i,j,k) = 0.$$

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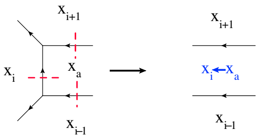
$$I(i-1,i,i+1) = \int_a \frac{-\epsilon(a,i-1,i,i+1,X)}{\sqrt{2}(a\cdot i)(a\cdot i-1)(a\cdot i+1)(a\cdot X)}$$



All multiplicity integrands for ABJM Song He, Y-t Huang:

2-loop: 1-loop⊗1-loop

$$\mathrm{I_A(i,j)} \equiv \int_{\mathrm{a,b}} \frac{\epsilon(\mathrm{a,i-,i,i{+}1,})\cdot\epsilon(\mathrm{b,j{-}1,j,j{+}1,}) - (\mathrm{a\cdot i})(\mathrm{b\cdot j})(\mathrm{i{-}1\cdot i{+}1})(\mathrm{j{-}1\cdot j{+}1})}{2(\mathrm{a\cdot i{-}1})(\mathrm{a\cdot i})(\mathrm{a\cdot i{+}1})(\mathrm{a\cdot b})(\mathrm{b\cdot j{-}1})(\mathrm{b\cdot j})(\mathrm{b\cdot j{+}1})},$$

$$\mathrm{I_B^{\pm}(r;i,j,k)} \equiv \int_{\mathrm{a,b}} \frac{\epsilon(\mathrm{a,r{-}1,r,r{+}1,})\cdot\epsilon(\mathrm{b,i,j,k,}) \pm \sqrt{2(\mathrm{i\cdot j})(\mathrm{j\cdot k})(\mathrm{k\cdot i})}\,\epsilon(\mathrm{a,r{-}1,r,r{+}1,b})}{2(\mathrm{a\cdot r{-}1})(\mathrm{a\cdot r})(\mathrm{a\cdot r{+}1})(\mathrm{a\cdot b})(\mathrm{b\cdot i})(\mathrm{b\cdot j})(\mathrm{b\cdot k})} + ..$$

$$I'_B(i;j,k) \equiv \int_{a,b} \frac{\epsilon(a,i{-}1,i,i{+}1,j)}{(i\cdot j)(a\cdot i{-}1)(a\cdot i)(a\cdot i{+}1)(a\cdot b)(b\cdot j)(b\cdot k)}.$$

All multiplicity integrands for ABJM Song He, Y-t Huang:

2-loop: 1-loop⊗1-loop

$$A_{n,\mathrm{soft}}^{2-\mathrm{loop}} = \sum_{i<j<k} \Big(\mathrm{C}^{+}_{\mathrm{i,j,k}} \times \big(\sum_{\mathrm{i<r<j}} \mathrm{I}^{+}_{\mathrm{B}}(\mathrm{r;i,j,k}) + \mathrm{I}'_{\mathrm{B}}(\mathrm{i;j,k}) + \{(\mathrm{i,j,k}) \to (\mathrm{j,k,i}),(\mathrm{k,i,j})\}\big)$$

$$+\mathrm{C}^{-}_{\mathrm{i,j,k}} \times \big(\sum_{\mathrm{i<r<j}} \mathrm{I}^{-}_{\mathrm{B}}(\mathrm{r;i,j,k}) - \mathrm{I}'_{\mathrm{B}}(\mathrm{i;j,k}) + \{(\mathrm{i,j,k}) \to (\mathrm{j,k,i}),(\mathrm{k,i,j})\}\big)\Big)$$

$$-A_n^{\mathrm{tree}} \sum_{\mathrm{i,j,j-i>1}} (-)^{\mathrm{i+j}} \mathrm{I}_{\mathrm{A}}(\mathrm{i,j}).$$

All multiplicity integrands for ABJM Song He, Y-t Huang:

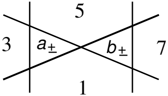
2-loop: 1-loop $\otimes$1-loop

$$A^{2-\mathrm{loop}}_{n,\mathrm{soft}} = \sum_{i<j<k} \Big(\mathrm{C}^{+}_{\mathrm{i,j,k}} \times \Big(\sum_{\mathrm{i<r<j}} \mathrm{I}^{+}_{\mathrm{B}}(\mathrm{r;i,j,k}) + \mathrm{I}'_{\mathrm{B}}(\mathrm{i;j,k}) + \{(\mathrm{i,j,k}) \to (\mathrm{j,k,i}),(\mathrm{k,i,j})\}\Big)$$

$$+\mathrm{C}^{-}_{\mathrm{i,j,k}} \times \Big(\sum_{\mathrm{i<r<j}} \mathrm{I}^{-}_{\mathrm{B}}(\mathrm{r;i,j,k}) - \mathrm{I}'_{\mathrm{B}}(\mathrm{i;j,k}) + \{(\mathrm{i,j,k}) \to (\mathrm{j,k,i}),(\mathrm{k,i,j})\}\Big)\Big)$$

$$-A^{\mathrm{tree}}_{n} \sum_{\mathrm{i,j,j-i>1}} (-)^{\mathrm{i+j}} \mathrm{I}_{\mathrm{A}}(\mathrm{i,j}).$$

Also need to include pure two-loop leading singularity



$$A^{2-\mathrm{loop}}_{8} = A^{2-\mathrm{loop}}_{8,\mathrm{soft}} + \sum_{i=1}^{4} C^{+}_{i,i+2,i+4;i+4,i-2,i} I^{+}_{C}(i,i+2,i+4;i+4,i-2,i) + (+\to -)$$

Integrating ....

All multiplicity integrands for ABJM Song He, Y-t Huang:

2-loop: 1-loop $\otimes$1-loop

$$A^{2-\mathrm{loop}}_{n,\mathrm{soft}} = \sum_{i<j<k} \Big(\mathrm{C}^{+}_{\mathrm{i,j,k}} \times \Big(\sum_{\mathrm{i<r<j}} \mathrm{I}^{+}_{\mathrm{B}}(\mathrm{r;i,j,k}) + \mathrm{I}'_{\mathrm{B}}(\mathrm{i;j,k}) + \{(\mathrm{i,j,k}) \to (\mathrm{j,k,i}),(\mathrm{k,i,j})\}\Big)$$

$$+\mathrm{C}^{-}_{\mathrm{i,j,k}} \times \Big(\sum_{\mathrm{i<r<j}} \mathrm{I}^{-}_{\mathrm{B}}(\mathrm{r;i,j,k}) - \mathrm{I}'_{\mathrm{B}}(\mathrm{i;j,k}) + \{(\mathrm{i,j,k}) \to (\mathrm{j,k,i}),(\mathrm{k,i,j})\}\Big)\Big)$$

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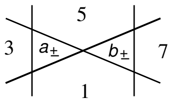

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Integrating ....

Conclusion:

- There are rich structures for scattering amplitudes in $D \neq 4$: D=3 Chern-Simons matter theory
- For $\mathcal{N} = 6$ on-shell diagrams are applicable: Grassmannian representation reflects the topology of kinematics.
- The Grassmannian representation exposes its close tie to $\mathcal{N} = 4$ SYM
- One- two-loop all multiplicity integrand under control.

Outlook

- Extension to $\mathcal{N} < 6$, no new physical singularities. Unlike $\mathcal{N} < 4$ SYM

$$A^{(\mathcal{N})}_{n,2,1} = \langle a_1 a_2\rangle^{4-\mathcal{N}} A^{(\mathcal{N}=4)}_{n,2,1} + (4-\mathcal{N})\langle a_1 a_2\rangle^{2-\mathcal{N}} A^{\text{chiral}}_{n,2,1},$$

$$\mathrm{A}^{\text{chiral}}_{\mathrm{n},2,1} = \mathrm{D}(\mathrm{a}_1,\mathrm{a}_2)\mathrm{D}(\mathrm{a}_2,\mathrm{a}_1)$$

$$\mathrm{D}_\mathrm{n}(\mathrm{a},\mathrm{b}) = \frac{1}{\langle \mathrm{AB}\rangle}\left(\sum_{\mathrm{i}=\mathrm{a}}^{\mathrm{b}-1}\frac{\langle \mathrm{a}\{\mathrm{i}\rangle\langle \mathrm{i}+1\}\mathrm{b}\rangle}{\langle \mathrm{ABii}+1\rangle} + \sum_{\mathrm{i}=\mathrm{a}+1}^{\mathrm{b}-1}\frac{\langle \mathrm{ai}\rangle\langle \mathrm{bi}\rangle\langle \mathrm{ABi}-1\mathrm{i}+1\rangle}{\langle \mathrm{ABi}-1\mathrm{i}\rangle\langle \mathrm{ABii}+1\rangle}\right)$$

$$UV : \mathrm{Res}_{AB\to I} A^{(\mathcal{N})}_{n,k,1} \equiv lim_{AB\to I}\langle AB\rangle^2 A^{(\mathcal{N})}_{n,k,1},$$

- What about $\mathcal{N} = 8$?

$$\int \frac{dC\delta^{2|3}(C\Lambda)}{(12)(23)} \to \int \frac{dC\delta^{2|4}(C\Lambda)}{(12)(23)(13)}$$

$$\int \frac{dC\delta^{2|3}(C\Lambda)}{(123)(234)(345)} \to \int \frac{dC\delta^{2|4}(C\Lambda)}{(123)(234)(345)(135)}$$

- What is the dual object?