Antenna on Null Polygon ABJM Wilson Loop

Soo-Jong Rey

Seoul National University, Seoul KOREA
Institute of Basic Sciences, Daejeon KOREA

Scattering Amplitudes at Hong Kong IAS

November 21, 2014

What Are We Studying?

- 3d ABJM theory
- null hexagon Wilson loop at planar, 2 loop 't Hooft coupling

  null tetragon [Henn Plefka Wiegandt]
- antenna (collinear/soft factorization) function of null polygons
- tetragon / hexagon → antenna recurrence relation → polygon
- circular Wilson loop

  check against exact SUSY localization results [SJR Suyama Yamaguchi, Marino Putrov]

Why Interesting?

- ABJ(M) = 3d $OSp(6|4)$ SCFT with 2 couplings k, N

  $k = 1, 2$: nonperturbative enhancement to $OSp(8|4)$
- curious parallels with 4d $SU(4^*|4)$ SYM theory
- details differ:

  scattering amplitude $\not\leftrightarrow$ wilson loop $\longleftrightarrow$ correlators
- string theory & M-theory quest: how closely "parallel"?
- exact / precision results are available for comparative study

  SUSY localization techniques

  conformal bootstrap approach

Previous Works

- $SU(4^*|4)$ SYM: MHV amplitudes ↔ null polygon Wilson loop [Drummond, Henn, Korchemsky, Sokatchev]
- ABJM: 1-loop, 4-pt amplitude =0; via AdS/CFT, relation to 4-pt SYM amplitude [Argawal, Beisert, McLoughlin]
- ABJM: 1-loop, Wilson loop = 0 [Drukker Plefka Young, SJR Suyama Yamaguchi, Chen Wu]
- ABJM: 1-loop, Wilson loop = correlator [Bianchi, Leoni, Mauri, Penati, Ratti, Santambrogio]
- ABJM: 2-loop, tetragon Wilson loop [Henn, Plefka, Wiegandt]
- ABJM: 2-loop, 4-pt amplitude = tetragon Wilson loop [Chen Huang; Bianchi Leoni Mauri Penati Santambrogio]
- ABJM: 2-loop, polygon Wilson loop partial results [Wiegandt]
- ABJM: 3-loop, 4-pt amplitude via DCI integrals + 2 particle cuts [Bianchi Leoni]

Results

Input Data

hexagon Wilson loop

cf. tetragon Wilson loop [Henn, Plefka, Wienhardt]

Recursion via Soft / Collinear Factorization

antenna function for n-gon to $(n-2)$-gon factorization

polygon Wilson loop recursion relation via antenna function

Polygon Wilson loop

$$W_n = -\frac{1}{2}\sum_{i=1}^{n} \frac{(x_{i,i+2}^2 \tilde{\mu}^2)^{2\epsilon}}{(2\epsilon)^2} + \mathrm{BDS}_n^{(2)}(x) + \left[n\left(\frac{\pi^2}{12} + \frac{3}{4}\log^2(2)\right) - \frac{\pi^2}{6}\right]$$

$n \to \infty$ limit = circular Wilson loop = exact localization result

4d from 3d Matter

- CS coupled to matter → induces nonvanishing 1-loop effect [SJR]

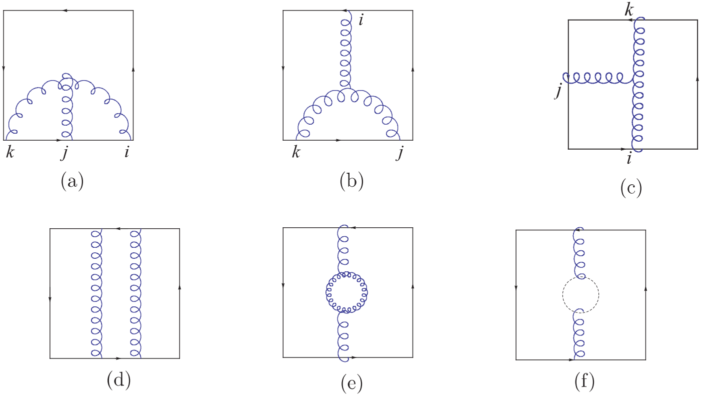
$$\Delta \mathcal{L}^{(1)} \simeq \frac{\lambda^2}{4} \mathrm{Tr} F_{mn} \frac{1}{\sqrt{D^2}} F_{mn}$$

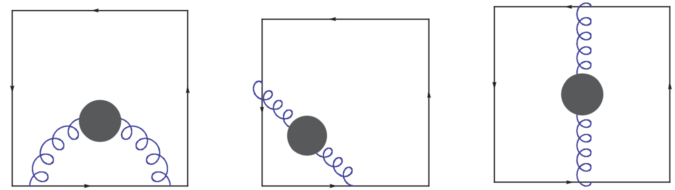
- 3d counterpart of 4d beta function contribution → finite
- coordinate space propagator

$$D_{mn} = -\frac{\lambda^2}{N} \pi^{-1+2\epsilon} \Gamma(1/2 - \epsilon) \frac{\eta_{mn}}{(-x^2)^{1-2\epsilon}}$$

- electric flux energy = scaling dimension of twist-1, high spin operators [Maldacena SJR]

Tetragon [Henn, Plefka, Wiegandt]





(a) (b) (c)

$$W_{4,\text{mat}}^{(2)} = -\frac{(-x_{13}^2 4\pi e^{\gamma_E}\mu^2)^{2\epsilon}}{(2\epsilon)^2} + (13 \leftrightarrow 24) + \frac{1}{2}\,\text{Log}^2\left(\frac{x_{13}^2}{x_{24}^2}\right) + \text{Rem}_4^{(2)}(u)\Big|_{\text{mat}}$$

$$W_{4,\text{CS}}^{(2)} = -\frac{\log(2)}{2}\sum_{i=1}^{4}\frac{(-x_{i,i+2}\pi e^{\gamma_E}\mu^2)^{2\epsilon}}{2\epsilon} + \text{Rem}_4^{(2)}(u)\Big|_{\text{CS}}$$

$\text{Rem}_n^{(2)}(u) :=$ IR finite part modulo the BDS finite part.

$$\text{Rem}_4^{(2)}\Big|_{\text{mat}} = \frac{\pi^2}{4}; \qquad \text{Rem}_4^{(2)}\Big|_{\text{CS}} = \frac{5\pi^2}{12} - 2\text{Log}^2(2); \qquad \tilde{\mu}^2 = 8\pi e^{\gamma_E}\mu^2$$

$$\Rightarrow W_{4,\text{ABJM}}^{(2)} = -\frac{(-x_{13}^2\tilde{\mu}^2)^{2\epsilon}}{(2\epsilon)^2} + (13 \leftrightarrow 24) + \frac{1}{2}\,\text{Log}^2\left(\frac{x_{13}^2}{x_{24}^2}\right) + \text{Rem}_4^{(2)}(u)$$

$$\text{Rem}_4^{(2)}(u) = \text{Rem}_4(u)\Big|_{\text{mat}} + \text{Rem}_4(u)\Big|_{\text{CS}} + 5\,\text{Log}^2(2) = 3\,\text{Log}^2(2) + \frac{2\pi^2}{3}$$

Strategy

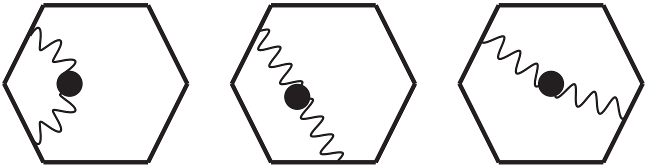
- Define functional relation

$$W^{(2)}_{n,\text{ABJM}} := W^{(1)}_{n,\text{SYM}} + \text{Rem}^{(2)}_n$$

- Remainder function

$$\text{Rem}^{(2)}_n := \text{Rem}^{(2)}_n\Big|_{\text{CS}} + \text{Rem}^{(2)}_n\Big|_{\text{mat}} + \frac{5}{4} n \,\text{Log}^2(2)$$

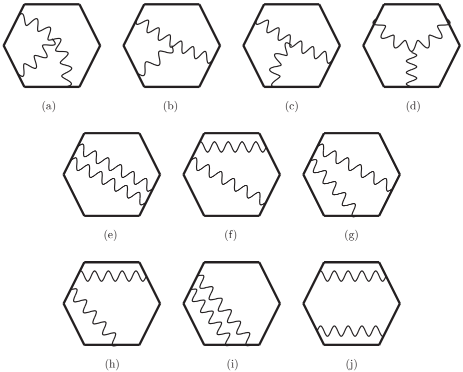
- divide & conquer: matter part easier + CS part complicated matter 1-loop contribution $\leftrightarrow$ 1-loop in SYM Wilson loop



Remainder from matter:

$$\boxed{\mathrm{Rem}^{(2)}_{n,\mathrm{mat}} = -\frac{1}{16} n\pi^2}$$

CS 2 loops ↔ 2-loop in SYM Wilson loop



Remainder from CS:

$$\boxed{\mathrm{Rem}^{(2)}_{n,\mathrm{CS}} = l_{\mathrm{CS}} + \frac{n}{2}\,\mathrm{Log}(2)}$$

Mandelstam & Gram

- moduli space of null polygon

$$\text{vertices}: \quad x_1 + \cdots + x_n = 0 \quad \leftrightarrow \quad \text{edges}: \quad y_1 + \cdots + y_n = 0$$

$$\mathcal{M}[C_n] = 3n - 15$$

- null edge conditions = nonzero Mandelstam invariants

$$\dim \mathcal{M}[C_n] = \frac{1}{2} n(n-3)$$

- projection to Gram condition subspace

$$\begin{aligned} \dim \Pi_G \mathcal{M}[C_n] &= \dim \mathcal{M}[C_n] - \frac{1}{2}(n-d)(n-d-1) \\ &= n(d-1) - \frac{1}{2} d(d+1) \end{aligned}$$

Euclid and Signs

- For Mellin-Barnes evaluation w/o spurious poles, choose all Mandelstam variables of equal sign
- In 4d, this is always possible. In 3d, this requires to have $n = 2\mathbb{Z}$
  purely kinematical and computational; nothing to do with ABJM
- Conformal null polygon

$$\dim \Pi_G \mathcal{M}_c[C_n] = (d-1)n - \frac{1}{2}(d+1)(d+2)$$

- Mellin-Barnes + PSLQ analytic result:
- CS contribution

$$
\begin{aligned}
\text{Rem}_{6,\text{CS}} &= -3.470168804 \\
&:= -\frac{17}{4}\zeta(2) + 3\ \text{Log}(2) + 3\ \text{Log}^2(2) = -3.4701692...
\end{aligned}
$$

- adding matter contribution

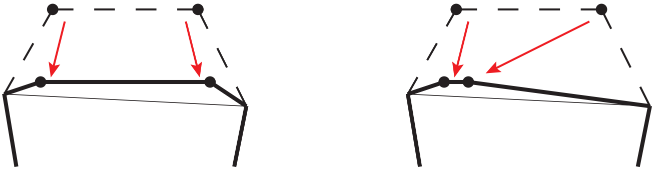
$$\text{Rem}_{6,\text{mat}} = -6\frac{1}{16}\pi^2$$

- ABJM remainder function reads

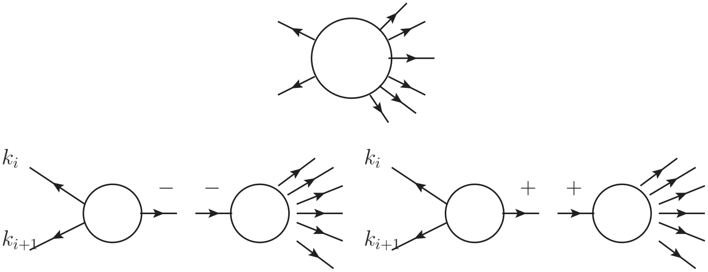
$$W^{(2)}_{6,\text{ABJM}} = -\sum_{i=1}^{3} \frac{(x^2_{i,i+2}\tilde{\mu}^2)^{2\epsilon}}{(2\epsilon)^2} + \text{BDS}_6(x) + \left(\frac{9}{2}\ \text{Log}^2(2) + \frac{\pi^2}{3}\right)$$

ABJM Antenna Function

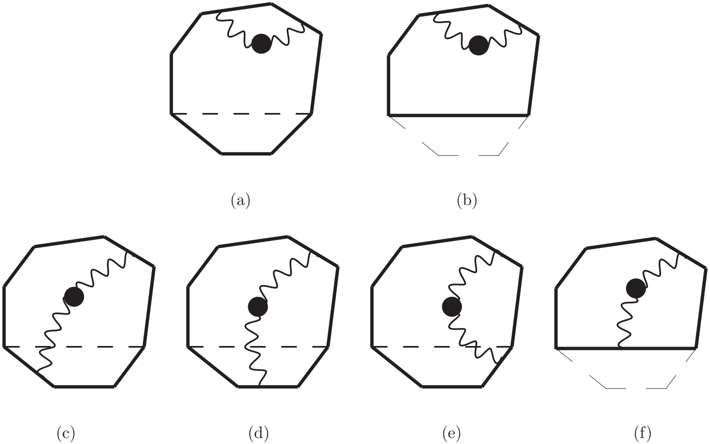
- To obtain remainder function for general polygon, look for recursive relation
- The soft or collinear limits of null segments relates different n's
- The 3d kinematics (Euclid for MB) requres to take simultaneous collinear+soft+collinear for $n \to (n-2)$
- Use the recurion relation with $n = 4, 6$ as inputs to solve for $n \geq 8$



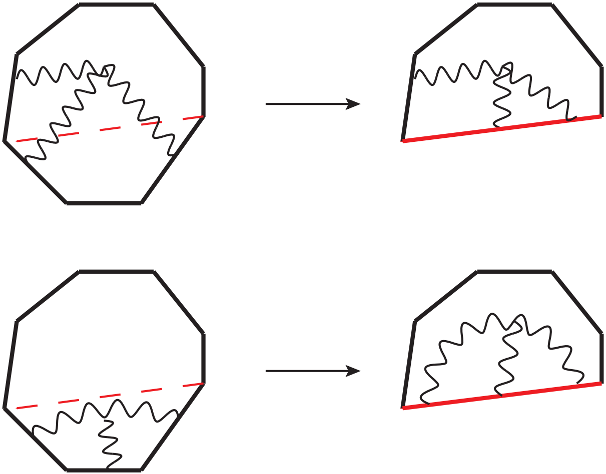
Analogous to the splitting function in (S)YM:



compute "antenna function" for (collinear)+(soft)+(collinear) factorization:
Matter contribution



CS contribution



Result for Antenna Function

$$\left[\frac{\mathrm{Ant}^{(2)}[C_n]}{\mathrm{Ant}^{(0)}[C_n]}\right]_{\rm CS} = \frac{\mathrm{Log}(2)}{2\epsilon} + \left[\frac{1}{2}\log(2)\mathrm{Log}(h_1) + \frac{1}{2}\,\mathrm{Log}(2)\,\mathrm{Log}(h_3)\right.$$
$$\left. + \frac{1}{2}\,\mathrm{Log}(2)\,\mathrm{Log}(x_{24}^2) + \frac{1}{2}\,\mathrm{Log}(2)\,\mathrm{Log}(x_{35}^2)\right]$$
$$- \frac{7\pi^2}{24} + \mathrm{Log}^2(2)$$

$$\left[\frac{\mathrm{Ant}^{(2)}[C_n]}{\mathrm{Ant}^{(0)}[C_n]}\right]_{\rm mat} = \frac{1}{4\epsilon^2} + \frac{1}{4\epsilon}\left[\mathrm{Log}(h_1) + \mathrm{Log}(h_3) + \mathrm{Log}(x_{24}^2)\right.$$
$$\left. + \mathrm{Log}(x_{35}^2)\right] + \frac{1}{2}\,\mathrm{Log}(h_1)\,\mathrm{Log}(x_{24}^2)$$
$$+ \frac{1}{2}\,\mathrm{Log}(h_3)\,\mathrm{Log}(x_{35}^2) + \frac{1}{2}\,\mathrm{Log}(x_{35}^2)\,\mathrm{Log}(x_{24}^2)$$
$$- \frac{1}{2}\,\mathrm{Log}(h_1)\,\mathrm{Log}(h_3)\Big] - \frac{\pi^2}{6}$$

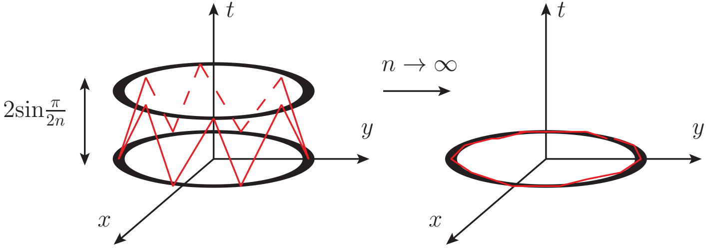
Recursion Relation Result

The recursion relation implied by the antenna function yields

$$
\begin{aligned}
\text{Rem}^{(2)}_{n,\mathrm{CS}} - \frac{n}{2}\,\text{Log}(2) &= \text{Rem}^{(2)}_{n-2,\mathrm{CS}} - \frac{n-2}{2}\,\text{Log}(2) + \text{Ant}^{(2)}_{\mathrm{CS}}[C_n]\Big|_{\mathrm{finite}} \\
&= \text{Rem}^{(2)}_{n-2,\mathrm{CS}} - \frac{n-2}{2}\,\text{Log}(2) - \frac{7}{4}\zeta(2) + \;\text{Log}^2(2) \\
&= \cdots \\
&= \text{Rem}^{(2)}_{6,\mathrm{CS}} - 3\,\text{Log}(2) \\
&\qquad + \frac{n-6}{2}\Big(- \frac{7}{4}\zeta(2) + \;\text{Log}^2(2)\Big) \\
&= \Big[\frac{1}{2}\,\text{Log}^2(2) - \frac{7\pi^2}{48}\Big] n + \frac{\pi^2}{6}.
\end{aligned}
$$

Circular Wilson Loop

We also considered special Euclid configuration that asymptotes to a circle:



The result is compared with the exact result from SUSY localization, whose weak coupling expansion yields

$$W_{\text{circle}} = 1 + \frac{5\pi^2}{6}\lambda^2 + \cdots$$