Dark SU(2) Antecedents of the $U(1)$ Higgs Model

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Abstract

The original spontaneously broken $U(1)$ gauge model with one complex Higgs scalar field has been known in recent years as a possible prototype dark-matter model. Its antecedents in the context of $SU(2)$ are discussed. Three specific examples are described, with one dubbed "quantum scotodynamics".

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Consider the addition of the $U(1)_D$ Higgs model [1] to the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge model (SM) of quarks and leptons. The former may be used for dark matter [2, 3, 4, 5, 6, 7, 8] because it has the built-in Z_2 symmetry where the massive gauge boson Z_D after spontaneous symmetry breaking is odd and the one physical real scalar boson h_D is even. However, $U(1)_D$ may mix kinetically [9] with $U(1)_Y$, in which case the above Z_2 symmetry would be violated. To avoid this problem, it is suggested here that $U(1)_D$ be replaced with an $SU(2)$ antecedent, with an enriched dark-matter sector. Three explicit examples will be discussed. Note that this version of dark $SU(2)$ requires that it be broken to $U(1)$, in contrast to the case where a local or global $SU(2)$ dark symmetry remains [10].

To break $SU(2)_D$ to $U(1)_D$, the simplest choice is a real scalar triplet

$$\chi = (\chi_1, \chi_2, \chi_3) \tag{1}$$

with $\langle \chi_3 \rangle = v_3$. In that case, the vector gauge bosons

$$W_D^\pm = \frac{D_1 \pm i D_2}{\sqrt{2}} \tag{2}$$

acquire mass given by $m^2_{W_D} = 2g_D^2 v_3^2$. Note that the superscript $\pm$ refers to dark charge, the details of which will be discussed later.

To break $U(1)_D$ in the context of $SU(2)_D$ so that $D_3 = Z_D$ acquires mass, a complex scalar doublet

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{3}$$

is used. Moreover, a global $U(1)_\Phi$ symmetry is imposed, i.e.

$$\Phi \to e^{i\theta}\Phi, \tag{4}$$

which prevents the coupling of $\vec{\chi}$ to the triplet $\phi_i \epsilon_{ij} \vec{\sigma}_{jk} \phi_k$. The scalar potential consisting of χ and Φ is then given by

$$\begin{aligned} V &= m_2^2 \Phi^\dagger \Phi + \frac{1}{2} m_3^2 (\vec{\chi} \cdot \vec{\chi}) + \mu_0 \Phi^\dagger (\vec{\sigma} \cdot \vec{\chi}) \Phi \\ &+ \frac{1}{2}\lambda_2 (\Phi^\dagger \Phi)^2 + \frac{1}{2}\lambda_3 (\vec{\chi} \cdot \vec{\chi})^2 + \lambda_4 (\Phi^\dagger \Phi)(\vec{\chi} \cdot \vec{\chi}). \end{aligned} \tag{5}$$

Note that the triplet combination of two identical real scalar triplets is zero. The minimum of V admits a solution

$$\langle \chi_{1,2} \rangle = 0, \quad \langle \chi_3 \rangle = v_3, \quad \langle \phi_1 \rangle = 0, \quad \langle \phi_2 \rangle = v_2/\sqrt{2}, \tag{6}$$

where v_2 is assumed real without any loss of generality, and

$$0 = v_3[m_3^2 + 2\lambda_3 v_3^2 + \lambda_4 v_2^2] - \mu_0 v_2^2/2, \tag{7}$$

$$0 = v_2[m_2^2 + \lambda_2 v_2^2/2 + \lambda_4 v_3^2 - \mu_0 v_3], \tag{8}$$

provided that

$$m_2^2 + \lambda_4 v_3^2 + \mu_0 v_3 > 0, \tag{9}$$

$$m_2^2 + \lambda_4 v_3^2 - \mu_0 v_3 < 0. \tag{10}$$

As a result

$$m^2_{W_D} = 2g_D^2 v_3^2 + \frac{1}{4} g_D^2 v_2^2, \quad m^2_{Z_D} = \frac{1}{4} g_D^2 v_2^2, \quad m^2_{\phi_1} = 2\mu_0 v_3, \tag{11}$$

and the 2×2 mass-squared matrix spanning $h_D = \sqrt{2}Re(\phi_2) - v_2$ and $H_D = \chi_3 - v_3$ is given by

$$\mathcal{M}^2_{h_D, H_D} = \begin{pmatrix} \lambda_2 v_2^2 & v_2(2\lambda_4 v_3 - \mu_0) \\ v_2(2\lambda_4 v_3 - \mu_0) & 4\lambda_3 v_3^2 + \mu_0 v_2^2 / 2v_3 \end{pmatrix}. \tag{12}$$

A global residual symmetry remains, under which

$$W_D^+, \phi_1 \sim +1, \quad W_D^-, \phi_1^* \sim -1, \quad Z_D, h_D, H_D \sim 0. \tag{13}$$

This comes from $I_{3D} + S_\Phi$, where $S_\Phi = 1/2$ for Φ and zero for all other fields. It is possible because of the imposed global $U(1)_\Phi$ symmetry. Whereas $\langle \phi_2 \rangle = v_2/\sqrt{2}$ breaks both I_{3D} and S_Φ, the linear combination $I_{3D} + S_\Phi$ is zero for ϕ_2, so it remains as a residual dark symmetry.

An important consequence of this structure is the emergence of a dark charge conjugation symmetry as in the original Higgs model [1], i.e.

$$W_D^+ \leftrightarrow W_D^- \ (D_2 \leftrightarrow -D_2), \ \phi_1 \leftrightarrow \phi_1^*, \ Z_D(D_3) \leftrightarrow -Z_D(D_3). \tag{14}$$

This comes from the gauge-invariant terms

$$-\frac{1}{4}(\partial_\mu \vec{D}_\nu - \partial_\nu \vec{D}_\mu + g_D \vec{D}_\mu \times \vec{D}_\nu)^2 + |\partial_\mu \Phi - \frac{i g_D}{2} \vec{\sigma} \cdot \vec{D}_\mu \Phi|^2. \quad (15)$$

It means that Z_D is stable if its mass is less than twice that of ϕ_1, in complete analogy to the $U(1)_D$ model of Ref. [8]. This makes it possible in principle to implement the inception of self-interacting dark matter, i.e. ϕ_1 or W_D of order 100 GeV with Z_D as the light stable mediator of order 10 to 100 MeV, to explain [11] the observed core-cusp anomaly in dwarf galaxies [12]. If Z_D is unstable and decays to SM particles, as is the case for the light mediator proposed in most models, then very strong constraints exist [13] from the cosmic microwave background (CMB) which basically rule out [14] this scenario. On the other hand, h_D must also be light and decay quickly through its mixing with the SM Higgs boson h before big bang nucleosynthesis (BBN). In that case, the elastic scattering of W_D or ϕ_1 off nuclei through h_D exchange is much too large to be acceptable with the present data. In Ref. [8], this is not a problem because the dark matter is a Dirac fermion which couples to Z_D but not h_D.

As it is, this specific $SU(2)_D$ antecedent of the $U(1)$ Higgs model may still be a model of dark matter without addressing the core-cusp anomaly in dwarf galaxies. Assuming that W_D is heavy enough to decay into $\phi_1 h_D$ and Z_D heavy enough to decay into $\phi_1\phi_1^*$, then the complex scalar ϕ_1 may be considered dark matter. Assuming that h_D is lighter than ϕ_1, the annihilation cross section of $\phi_1\phi_1^*$ at rest $\times$ relative velocity is given by

$$\sigma(\phi_1\phi_1^* \to h_D h_D)\, v_{rel} = \frac{\lambda_2^2\sqrt{1-r_1}}{64\pi m_{\phi_1}^2}\left[1 + \frac{r_1(2+r_1)}{(2-r_1)(4-r_1)}\right]^2, \quad (16)$$

where $r_1 = m_{h_D}^2/m_{\phi_1}^2$. Assuming as an example $m_{\phi_1} = 150$ GeV and $m_{h_D} = 100$ GeV, the above may be set equal to $4.4 \times 10^{-26}\ cm^3/s$ for $\lambda_2 = 0.126$.

There is always the allowed quartic λ_{2h} coupling between the $SU(2)_D$ Higgs doublet and the $SU(2)_L \times U(1)_Y$ Higgs doublet of the SM, so that ϕ_1 interacts with quarks through the SM Higgs boson h in direct-search experiments. Using present data [15], it has been shown [16] that $\lambda_{2h} < 4.4 \times 10^{-4}$. This is also the mixing between h_D and h. Even with this limit on λ_{2h}, it can still be large enough so that h_D decays promptly to $b\bar{b}$ in the early Universe. This interaction [8] also keeps h_D in thermal equilibrium with the particles of the SM.

Consider the addition of a fermion doublet

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}_L \quad (17)$$

to the $SU(2)_D$ model discussed in the previous section. It has the allowed interactions

$$i\bar{\Psi}\gamma^\mu(\partial_\mu - \frac{i g_D}{2}\vec{\sigma}\cdot\vec{D}_\mu)\Psi + [f\tilde{\Psi}(\vec{\sigma}\cdot\vec{\chi})\Psi + H.c.], \quad (18)$$

where $\tilde{\Psi} = (\psi_2, -\psi_1)_L$. Since $\langle\chi_3\rangle = v_3$, this shows that $\psi_{1,2}$ combine to form a Dirac fermion of mass fv_3. To be specific, let ψ_{1L} be the left-handed component of the Dirac fermion ψ, and ψ_{2L} redefined as the conjugate of its right-handed component, i.e. $\psi_{2L} \sim \bar{\psi}_R$. Now ψ_1 has dark charge 1/2 and ψ_2 has dark charge $-1/2$. Together they form a Dirac fermion ψ of charge 1/2, which interacts vectorially with $D_3 = Z_D$. Note that $\bar{\psi}\gamma_\mu\psi$ is odd under dark charge conjugation as expected. Note also that ψ has no direct coupling to h_D because of $SU(2)_D$ gauge invariance. This allows the inception of self-interacting dark matter as described below.

Consider the elastic scattering of ψ with $\bar{\psi}$ through the exchange of the light mediator Z_D. Its cross section in the limit of zero momentum is

$$\sigma(\psi\bar{\psi} \to \psi\bar{\psi}) = \frac{g_D^4 m_\psi^2}{64\pi m_{Z_D}^4} = \frac{m_\psi^2}{4\pi v_2^4}. \quad (19)$$

For the benchmark value of $\sigma/m_\psi \sim 1\ cm^2/g$ for self-interacting matter, this is satisfied for example with

$$m_\psi = 100\ \text{GeV}, \quad v_2 = 200\ \text{MeV}. \quad (20)$$

This low-energy effective theory consisting of ψ, Z_D and h_D may be dubbed quantum scotodynamics, from the Greek 'scotos' meaning darkness.

Consider now the annihilation of $\psi\bar{\psi} \to Z_D Z_D$. Since Z_D is much lighter than ψ, this cross section $\times$ relative velocity is given by

$$\sigma(\psi\bar{\psi} \to Z_D Z_D)\, v_{rel} = \frac{g_D^4}{256\pi m_\psi^2}. \quad (21)$$

For $m_\psi = 100$ GeV, and setting $\sigma v_{rel} = 4.4 \times 10^{-26}\ cm^3/s$,

$$g_D = 0.42 \quad (22)$$

is obtained, which implies from Eq. (11) that

$$m_{Z_D} = 42\ \text{MeV}. \quad (23)$$

As shown in Ref. [8], the light mediator Z_D is stable but annihilates quickly to h_D which decays. The cross section $\times$ relative velocity is given by

$$\sigma(Z_D Z_D \to h_D h_D)\, v_{rel} = \frac{g_D^4\sqrt{1-r}}{64\pi m_{Z_D}^2} \times \left[\frac{4[r^2+4(2-r)^2]}{(4-r)^2} - \frac{24r(2+r)}{9(2-r)(4-r)} + \frac{8(2+r)^2}{9(2-r)^2}\right], \quad (24)$$

where $r = m_{h_D}^2/m_{Z_D}^2$. Assuming $m_{h_D} = 21$ MeV as an example so that $r = 0.25$, the above is equal to $4 \times 10^{-18}\ cm^3/s$, which is orders of magnitude greater than what is required for Z_D to be a significant component of dark matter. It may re-emerge at late times by $\phi_1\phi_1^*$ annihilation through Sommerfeld enhancement, but its fraction as dark matter remains negligible. Since Z_D is stable, it would also not disturb [13, 14] the cosmic microwave background (CMB).

As for h_D, it is allowed to mix with the SM Higgs boson h in the 2×2 mass-squared matrix

$$\mathcal{M}^2_{h_D,h} = \begin{pmatrix} \lambda_2 v_2^2 & \lambda_{2h} v_2 v_h \\ \lambda_{2h} v_2 v_h & m_h^2 \end{pmatrix}, \tag{25}$$

where $v_h = 246$ GeV and $m_h = 125$ GeV. For $m_{h_D} << m_h$, the $h_D - h$ mixing is $\theta_{2h} = \lambda_{2h} v_2 v_h / m_h^2$. Assuming

$$\lambda_{2h} = 0.01, \tag{26}$$

then $\theta_{2h} = 3.15 \times 10^{-5}$ and the h_D lifetime for $e^- e^+$ decay is given by

$$\Gamma^{-1}(h_D \to e^- e^+) = \frac{8\pi v_h^2}{m_{h_D} m_e^2 \theta_{2h}^2} = 0.184\, s, \tag{27}$$

which is short enough not to affect big bang nucleosynthesis (BBN). The decay of the SM Higgs boson to $h_D h_D$ is given by

$$\Gamma(h \to h_D h_D) = \frac{\lambda_{2h}^2 v_h^2}{16\pi m_h} = 0.963 \text{ MeV}, \tag{28}$$

which is less than 25% of the SM width of 4.12 MeV and allowed by present data. Note that $\lambda_2 = 0.0114$ in Eq. (25) for $m_{h_D} = 21$ MeV. Note also the important fact that ψ does not couple directly to h_D, otherwise Eq. (26) would be impossible, as discussed in the previous section.

In summary, a successful description of self-interacting fermion dark matter (ψ with $m_\psi = 100$ GeV) through a stable light vector gauge boson (Z_D with $m_{Z_D} = 42$ MeV) in an $SU(2)_D$ gauge model has been rendered. The Higgs scalar h_D associated with Z_D is also light (21 MeV), but it decays away quickly before the onset of BBN. Other heavier particles in the dark sector are $W_D^\pm$ (which decays to $\psi_1\psi_1/\psi_2\psi_2$), ϕ_1 (which decays to $W_D^+ h_D$), and H_D which mixes slightly with h and h_D.

In the previous two examples, an imposed symmetry of the $SU(2)_D$ scalar doublet Φ, i.e. Eq. (4), is necessary for obtaining a dark symmetry. Hence the latter is not predestined [17], i.e. not the automatic consequence of gauge symmetry and particle content. To have a predestined dark Z_2 symmetry, the simpler scalar triplet is now replaced with a scalar quintet. This is analogous to having a fermion quintet [18] in the SM for minimal dark matter, i.e. for simplicity.

Consider thereby the real scalar quintet

$$\zeta = (\zeta^{++}, \zeta^+, \zeta^0, \zeta^-, \zeta^{--}) \tag{29}$$

with $\langle \zeta^0 \rangle = v_5$, then $W_D^\pm$ obtains a mass given by $m_{W_D}^2 = 6 g_D^2 v_5^2$ from absorbing $\zeta^\pm$. This leaves $\zeta^{\pm\pm}$ as physical scalar bosons with two units of dark charge, interacting with Z_D. The scalar potential consisting of ζ and Φ is then given by

$$\begin{aligned} V &= m_2^2 \Phi^\dagger \Phi + \frac{1}{2} m_5^2 \zeta^\dagger \zeta + \frac{1}{2}\lambda_2 (\Phi^\dagger \Phi)^2 + \lambda_5 (\Phi^\dagger \Phi)(\zeta^\dagger \zeta) \\ &+ V_3 + V_4, \end{aligned} \tag{30}$$

where V_3 contains the one cubic invariant formed out of 3 scalar quintets and V_4 contains two quartic invariants. To show this explicitly, consider first the decomposition $5 \times 5 = 1 + 3 + 5 + 7 + 9$. Since 5 is assumed real, only the symmetric combinations of 1, 5, and 9 are possible. Now the product $(5 \times 5) \times 5$ contains 5, 5×5, and 9×5. Only 5×5 contains 1, hence there is just one cubic invariant. The product $(5 \times 5) \times (5 \times 5)$ contains 1×1, 5×5, and 9×9, which all contain 1, but only two are independent, resulting thus in two quartic invariants. As for a possible term connecting ζ with Φ, consider the triplet $\phi_i \epsilon_{ij} \vec{\sigma}_{jk} \phi_k$ pointed out earlier. Whereas it is obvious that ζ cannot couple to it because it is a quintet, but if the product $\zeta \times \zeta$ contains a triplet, then a quartic term would exist which violates $U(1)_\phi$. As it is, such a triplet is identically zero as explained in the above. Note that if a quartet were used, 4×4 would contain a triplet, because 4 is necessarily complex. Similarly, a sextet would not work. A real septet is possible as well as any real odd-dimensional representation higher than 5. Thus the scalar potential of Eq. (30) has automatically the necessary extra $U(1)_\Phi$ symmetry, so that $I_{3D} + S_\Phi$ remains unbroken as ϕ_2 acquires a vacuum expectation value $v_2/\sqrt{2}$ as explained previously.

Assuming that

$$m_\zeta < 2m_{\phi_1} < m_{Z_D} < m_{W_D}, \tag{31}$$

then W_D^+ decays to $\phi_1 h_D$, Z_D decays to $\phi_1 \phi_1^*$, but both ϕ_1 and ζ are stable. Hence this is an explicit example of two-component dark matter under one dark $U(1)$ symmetry. Let

$$m_\zeta = 200 \text{ GeV}, \quad m_{\phi_1} = 150 \text{ GeV}, \quad m_{h_D} = 100 \text{ GeV}, \tag{32}$$

then using Eq. (16) for $\sigma_1(\phi_1\phi_1^* \to h_D h_D) v_{rel}$ and the analogous

$$\begin{aligned} \sigma_2(\zeta\zeta^* \to h_D h_D, \phi_1\phi_1^*)\, v_{rel} &= \frac{\lambda_5^2 \sqrt{1-r_2}}{64\pi m_\zeta^2} \times \\ \left[1 + \frac{2(\lambda_5/\lambda_2) r_2}{2 - r_2} - \frac{3r_2}{4 - r_2}\right]^2 &+ \frac{\lambda_5^2\sqrt{1-r_3}}{32\pi m_\zeta^2}\left[1 - \frac{r_2}{4 - r_2}\right]^2, \end{aligned} \tag{33}$$

where $r_2 = m_{h_D}^2/m_\zeta^2$ and $r_3 = m_{\phi_1}^2/m_\zeta^2$, the condition for the correct relic abundance is roughly given by

$$\langle \sigma_1 v_{rel} \rangle^{-1} + \langle \sigma_2 v_{rel} \rangle^{-1} = (4.4 \times 10^{-26}\, cm^3/s)^{-1}. \tag{34}$$

It has for example the reasonable solution $\lambda_5 = \lambda_2 = 0.173$, in which case ϕ_1 is 53% and ζ 47% of dark matter. Again the mixing of ζ with the SM Higgs boson h must be small as it is for ϕ_1 to satisfy direct-search limits as discussed previously.

In this scenario, the addition of the fermion doublet of Eq. (17) could also provide a low-energy effective theory of quantum scotodynamics with light Z_D and h_D. In that case, $\zeta^{\pm\pm}$ would decay into $W_D^\pm W_D^\pm$, ϕ_1 would decay into $W_D^+ h_D$, and $W_D^\pm$ would decay into $\psi_1\psi_1/\psi_2\psi_2$.

Exploring the possible $SU(2)$ antecedents of the famous $U(1)$ Higgs model for a nontrivial application to dark matter, three interesting examples have been identified and discussed. The minimal version with one real scalar triplet χ and one complex scalar doublet Φ admits ϕ_1 as dark matter, but a global $U(1)$ symmetry has to be imposed. With the addition of a fermion doublet ψ, the inception of self-interacting dark matter may be implemented successfully, avoiding all potential astrophysical and laboratory constraints. A third example replaces χ with the real scalar quintet ζ, in which case the dark $U(1)$ symmetry becomes predestined, i.e. automatic from the gauge symmetry and particle content.

Since all these scenarios are based on an extended dark $SU(2)_D$ sector, they are only confirmed if the heavier gauge bosons $W_D^{\pm}$ and heavier scalar bosons H_D or $\zeta^{\pm\pm}$ could be observed. This is a very challenging phenomenological question because the only connection between the dark sector and the SM is through the SM Higgs boson. It is unfortunately, a generic feature of all dark-matter proposals using this so-called Higgs portal. The focus here is rather on a possible theoretical understanding of where dark matter may come from.

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