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Cobimaximal neutrino mixing from $S_3 \times Z_2$

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ABSTRACT

It has recently been shown that the phenomenologically successful pattern of cobimaximal neutrino mixing ($\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, and $\delta_{CP} = \pm\pi/2$) may be achieved in the context of the non-Abelian discrete symmetry A_4. In this paper, the same goal is achieved with $S_3 \times Z_2$. The residual lepton Z_3 triality in the case of A_4 is replaced here by $Z_2 \times Z_2$. The associated phenomenology of the scalar sector is discussed.

1. Introduction

Present neutrino data [1,2] are indicative of $\theta_{13} \neq 0$, $\theta_{23} = \pi/4$ and $\delta_{CP} = -\pi/2$. Calling this cobimaximal mixing [3], it has been shown that it may be derived in the following two ways. (I) The Majorana neutrino mass matrix, in the basis where charged-lepton masses are diagonal, is *assumed* to be of the form [4–6]

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}, \quad (1)$$

where A, B are real. (II) The neutrino mixing matrix is *assumed* to be of the form [7–10]

$$U_{l\nu} = U_\omega \mathcal{O}, \quad (2)$$

where [11,12]

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (3)$$

with $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$, and $\mathcal{O}$ is any arbitrary real orthogonal matrix. This yields $|U_{\mu i}| = |U_{\tau i}|$ which leads to cobimaximal mixing. Using the fact that U_ω is derivable from A_4 [13], and the scotogenic generation of neutrino mass from a set of real scalars [9,14–16], Eq. (2) is naturally achieved. This conceptual shift from tribimaximal [17,18], i.e. $\theta_{13} = 0$, $\theta_{23} = \pi/4$, and $\sin^2\theta_{12} = 1/3$, to cobimaximal mixing may also be understood as the result of a residual generalized CP symmetry [6,19–24]. Here we show how cobimaximal mixing may be obtained from the soft breaking of $S_3 \times Z_2$ to $Z_2 \times Z_2$ instead of the soft breaking of A_4 to Z_3.

2. Soft breaking of S_3 with two Higgs doublets

Let $(\Phi_1, \Phi_2) \sim \underline{2}$ under S_3, then the most general S_3 invariant scalar potential for $\Phi_{1,2}$ is given by [25]

$$V_0 = \mu_0^2(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2) + \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2)^2 + \frac{1}{2}\lambda_2(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1). \quad (4)$$

This V_0 actually has an additional Z_2 symmetry, i.e.

$$\Phi_1 \to i\Phi_2, \quad \Phi_2 \to -i\Phi_1.$$

Consider now the addition of a soft term $i\mu_{12}^2(\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1)$, which breaks S_3 but preserves this additional Z_2 transformation inherent in V_0, with $\langle\phi_1^0\rangle = i\langle\phi_2^0\rangle = v/\sqrt{2}$. The minimum of the scalar potential is then easily shown to be given by $v^2 = -(\mu_0^2 + \mu_{12}^2)/[\lambda_1 + (\lambda_3/2)]$.

3. Two lepton families under S_3

Let $(\nu_1, l_1)_L, (\nu_2, l_2)_L \sim \underline{2}$ under S_3, and $l_{1R}, l_{2R} \sim \underline{1}', \underline{1}$ under S_3. The S_3 invariant Yukawa terms for charged-lepton masses are then

$$-\mathcal{L}_Y = f_\mu(\bar{l}_{1L}\phi_1^0 - \bar{l}_{2L}\phi_2^0)l_{1R} + f_\tau(\bar{l}_{1L}\phi_1^0 + \bar{l}_{2L}\phi_2^0)l_{2R} + H.c. \quad (5)$$

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The 2×2 mass matrix linking $(\bar{l}_{1L}, \bar{l}_{2L})$ to (l_{1R}, l_{2R}) is then given by

$$\mathcal{M}_l = \begin{pmatrix} f_\mu & f_\tau \\ if_\mu & -if_\tau \end{pmatrix} \frac{v}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \begin{pmatrix} f_\mu v & 0 \\ 0 & f_\tau v \end{pmatrix}. \quad (6)$$

4. Three lepton families under $S_3 \times Z_2$

A third lepton family may be added which transforms as $(\underline{1}, -)$ under $S_3 \times Z_2$, so that it couples to a third Higgs doublet which transforms as $(\underline{1}, +)$. The 3×3 unitary matrix linking the diagonal charged-lepton mass matrix to the neutrino mass matrix is then

$$U_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -i/\sqrt{2} \\ 0 & 1/\sqrt{2} & i/\sqrt{2} \end{pmatrix}. \quad (7)$$

This serves the same purpose as U_ω of Eq. (3), because

$$U_{l\nu} = U_2 \mathcal{O} \quad (8)$$

also yields $|U_{\mu i}| = |U_{\tau i}|$ which leads to cobimaximal mixing. In fact, since $U_{ei} = \mathcal{O}_{1i}$, the θ_{12} and θ_{13} angles are the same in both $U_{l\nu}$ and $\mathcal{O}$. This is the main point of my paper, i.e. U_ω of Eq. (3) may be replaced by U_2 of Eq. (7) to obtain cobimaximal mixing. The orthogonal matrix $\mathcal{O}$ is simply *assumed* here, but it may be implemented as shown in previous studies [9,14–16].

5. Soft breaking of S_3 to Z_2 with three Higgs doublets

Adding $\Phi_3 \sim \underline{1}$ under S_3, the scalar potential of our model becomes

$$\begin{aligned} V = {} & \mu_0^2(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2) + i\mu_{12}^2(\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1) + \mu_3^2\Phi_3^\dagger\Phi_3 \\ & + \left[\frac{1}{2}\mu_{30}^2\Phi_3^\dagger(\Phi_1 + i\Phi_2) + H.c.\right] + \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2)^2 \\ & + \frac{1}{2}\lambda_2(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{1}{2}\lambda_4(\Phi_3^\dagger\Phi_3)^2 \\ & + \lambda_5(\Phi_3^\dagger\Phi_3)(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2) + \lambda_6\Phi_3^\dagger(\Phi_1\Phi_1^\dagger + \Phi_2\Phi_2^\dagger)\Phi_3 \\ & + [\lambda_7(\Phi_3^\dagger\Phi_1)(\Phi_3^\dagger\Phi_2) + H.c.] \end{aligned} \quad (9)$$

The μ_{12}^2 and μ_{30}^2 terms break S_3 softly to Z_2, under which Φ_3 and $\Phi_+ = (\Phi_1 + i\Phi_2)/\sqrt{2}$ are even and $\Phi_- = (\Phi_1 - i\Phi_2)/\sqrt{2}$ is odd. The S_3 allowed quartic term $(\Phi_3^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) + (\Phi_3^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2)$ is forbidden by making Φ_3 odd under an extra Z_2' symmetry. This term does not have the required *accidental* residual symmetry inherent in the two-Higgs-doublet case and if allowed, would have changed U_2 of Eq. (7) and invalidated Eq. (8). The new symmetry Z_2' is then broken softly by the μ_{30}^2 term. With this modification, $(\nu_e, e)_L$ is even and e_R is odd under this extra Z_2' to allow the Yukawa coupling $f_e\bar{e}_L e_R\phi_3^0$, with $m_e = f_e v_3$. Assuming a small μ_{30}^2 term, v_3 is naturally much smaller than v. Hence $m_e << m_\mu, m_\tau$ and the charged leptons are distinguished from each other according to

$$e \sim (+,-), \quad \mu \sim (-,+), \quad \tau \sim (+,+). \quad (10)$$

The Z_3 triality [26,27] coming from A_4, i.e. $e \sim 1$, $\mu \sim \omega^2$, $\tau \sim \omega$, has now been replaced by the above under $Z_2 \times Z_2$. This serves to forbid $\mu \to e\gamma$, etc. as in the case of Z_3 lepton triality. The odd Higgs doublet Φ_- transforms as $(-,+)$ and couples to $\bar{\mu}_L\tau_R$ and $\bar{\tau}_L\mu_R$ as in Ref. [28].

Table 1
Particle content of leptons and scalars under $S_3 \times Z_2 \times Z_2'$.

Particles	S_3	Z_2	Z_2'
$(\nu_{1,2}, l_{1,2})_L$	2	+	+
$(\nu_3, l_3)_L$	1	−	+
l_{1R}	1′	+	+
l_{2R}	1	+	+
l_{3R}	1	−	−
$(\phi_{1,2}^+, \phi_{1,2}^0)$	2	+	+
(ϕ_3^+, ϕ_{13}^0)	1	+	−

Table 2
Particle content of leptons and scalars under $Z_2(S_3) \times Z_2$.

Particles	$Z_2(S_3)$	Z_2
$\tau_{L,R}$	+	+
$\mu_{L,R}$	−	+
$e_{L,R}$	+	−
$(\Phi_1 + i\Phi_2)/\sqrt{2}$, Φ_3	+	+
$(\Phi_1 - i\Phi_2)/\sqrt{2}$	−	+

6. Symmetry assignments of leptons and scalars

The symmetries of this model are $S_3 \times Z_2 \times Z_2'$ as shown in Table 1. The complete Lagrangian contains Eqs. (5) and (9) as well as terms from the third lepton doublet and singlet, to be identified as the electron family as shown in Eq. (7). Using Eq. (9), S_3 is broken *indirectly* to Z_2 (under which $\Phi_1 \to i\Phi_2, \Phi_2 \to -i\Phi_1$), whereas Z_2 (under which $e = l_3$ is odd) is unbroken, and Z_2' is softly broken by the μ_{30}^2 term. The residual symmetry is shown in Table 2. This shows explicitly how the charged leptons retain a $Z_2 \times Z_2$ symmetry in this model. Once a realistic neutrino mass matrix is considered, this symmetry will be *completely* broken, but the effects are small on the phenomenology to be discussed below.

7. Phenomenology of scalar interactions

The leptonic Yukawa interactions are given by

$$\begin{aligned} -\mathcal{L}_Y = {} & f_\tau(\bar{\tau}_L\phi_+^0 + \bar{\mu}_L\phi_-^0)\tau_R + f_\tau(\bar{\nu}_\tau\phi_+^+ + \bar{\nu}_\mu\phi_-^+)\tau_R \\ & + f_\mu(\bar{\mu}_L\phi_+^0 + \bar{\tau}_L\phi_-^0)\mu_R + f_\mu(\bar{\nu}_\mu\phi_+^+ + \bar{\nu}_\tau\phi_-^+)\mu_R \\ & + f_e\bar{e}_L\phi_3^0 e_R + f_e\bar{\nu}_e\phi_3^+ e_R + H.c. \end{aligned} \quad (11)$$

The scalar interactions are given by

$$\begin{aligned} V = {} & (\mu_0^2 + \mu_{12}^2)\Phi_+^\dagger\Phi_+ + (\mu_0^2 - \mu_{12}^2)\Phi_-^\dagger\Phi_- + \mu_3^2\Phi_3^\dagger\Phi_3 \\ & + \left[\frac{1}{\sqrt{2}}\mu_{30}^2\Phi_3^\dagger\Phi_+ + H.c.\right] + \left(\frac{1}{2}\lambda_1 + \frac{1}{4}\lambda_3\right)(\Phi_+^\dagger\Phi_+)^2 \\ & + \left(\frac{1}{2}\lambda_1 + \frac{1}{4}\lambda_3\right)(\Phi_-^\dagger\Phi_-)^2 + \left(\lambda_1 - \frac{1}{2}\lambda_3\right)(\Phi_+^\dagger\Phi_+)(\Phi_-^\dagger\Phi_-) \\ & + \left(\frac{1}{2}\lambda_2 - \frac{1}{4}\lambda_3\right)[(\Phi_+^\dagger\Phi_-)^2 + (\Phi_-^\dagger\Phi_+)^2] \\ & + \left(\lambda_2 + \frac{1}{2}\lambda_3\right)(\Phi_+^\dagger\Phi_-)(\Phi_-^\dagger\Phi_+) + \frac{1}{2}\lambda_4(\Phi_3^\dagger\Phi_3)^2 \\ & + \lambda_5(\Phi_3^\dagger\Phi_3)(\Phi_+^\dagger\Phi_+ + \Phi_-^\dagger\Phi_-) + \lambda_6\Phi_3^\dagger(\Phi_+\Phi_+^\dagger + \Phi_-\Phi_-^\dagger)\Phi_3 \\ & + \left(\frac{1}{2i}\lambda_7[(\Phi_3^\dagger\Phi_+)^2 - (\Phi_3^\dagger\Phi_-)^2] + H.c.\right) \end{aligned} \quad (12)$$

Assuming v, v_3 to be real, the conditions for minimizing V are

$$v[(\mu_0^2+\mu_{12}^2)+(\lambda_1+\frac{1}{2}\lambda_3)v^2+(\lambda_5+\lambda_6+Im(\lambda_7))v_3^2] + \frac{1}{\sqrt{2}}\mu_{30}^2 v_3 = 0, \quad (13)$$

$$v_3[\mu_3^2+\lambda_4 v_3^2+(\lambda_5+\lambda_6+Im(\lambda_7))v^2]+\frac{1}{\sqrt{2}}\mu_{30}^2 v = 0. \quad (14)$$

For $v_3 << v$, we obtain

$$v^2 \simeq \frac{-(\mu_0^2+\mu_{12}^2)}{\lambda_1+(\lambda_3/2)}, \quad (15)$$

$$v_3 \simeq \frac{-\mu_{30}^2 v}{\sqrt{2}[\mu_3^2+(\lambda_5+\lambda_6+Im(\lambda_7))v^2]}. \quad (16)$$

The states $\sqrt{2}[vIm(\phi_+^0)+v_3Im(\phi_3^0)]/\sqrt{v^2+v^3}$ and $[v\phi_+^\pm+v_3\phi_3^\pm]/\sqrt{v^2+v_3^2}$ are the would-be massless Goldstone modes for the Z and $W^\pm$ bosons. The states $A=\sqrt{2}[vIm(\phi_3^0)-v_3Im(\phi_+^0)]/\sqrt{v^2+v_3^2}$ and $H^\pm=[v\phi_3^\pm-v_3\phi_+^\pm]/\sqrt{v^2+v_3^2}$ have masses given by

$$m_A^2 = -Im(\lambda_7)(v^2+v_3^2)-\frac{\mu_{30}^2(v^2+v_3^2)}{\sqrt{2}vv_3} \simeq \mu_3^2+(\lambda_5+\lambda_6-Im(\lambda_7))v^2, \quad (17)$$

$$m_{H^\pm}^2 = -(\lambda_6+Im(\lambda_7))(v^2+v_3^2)-\frac{\mu_{30}^2(v^2+v_3^2)}{\sqrt{2}vv_3} \simeq \mu_3^2+\lambda_5 v^2. \quad (18)$$

The states $h=\sqrt{2}Re(\phi_+^0)$ and $H=\sqrt{2}Re(\phi_3^0)$ are approximate mass eigenstates with

$$m_h^2 \simeq (2\lambda_1+\lambda_3)v^2, \quad m_H^2 \simeq \mu_3^2+(\lambda_5+\lambda_6+Im(\lambda_7))v^2, \quad (19)$$

and $h-H$ mixing given by

$$\epsilon \simeq \frac{-v_3}{v}\left[\frac{\mu_3^2-(\lambda_5+\lambda_6+Im(\lambda_7))v^2}{\mu_3^2+(\lambda_5+\lambda_6+Im(\lambda_7))v^2}\right]. \quad (20)$$

The Φ_- doublet has odd Z_2 and does not mix with Φ_+ or Φ_3. The masses of its components are given by

$$m^2(\phi_-^\pm) \simeq \mu_0^2-\mu_{12}^2+\left(\lambda_1-\frac{1}{2}\lambda_3\right)v^2, \quad (21)$$

$$m^2(\sqrt{2}Re(\phi_-^0)) \simeq \mu_0^2-\mu_{12}^2+\left(\lambda_1-\frac{1}{2}\lambda_3+2\lambda_2\right)v^2, \quad (22)$$

$$m^2(\sqrt{2}Im(\phi_-^0)) \simeq \mu_0^2-\mu_{12}^2+\left(\lambda_1+\frac{1}{2}\lambda_3\right)v^2. \quad (23)$$

All physical masses squared are positive in a large region of parameter space, thus proving that Eqs. (15) and (16) correspond to a local minimum of V.

8. Phenomenology of lepton interactions

From Eq. (11), the lepton interactions of this model are given by

$$-\mathcal{L}_Y = \frac{m_\tau}{v\sqrt{2}}h\bar{\tau}\tau+\frac{m_\mu}{v\sqrt{2}}h\bar{\mu}\mu + \frac{m_e}{v_3\sqrt{2}}[(H+iA)\bar{e}_L e_R+H^+\bar{\nu}_e e_R+H.c.] + \left[\frac{m_\tau}{v}[\phi_-^0\bar{\mu}_L\tau_R+\phi_-^+\bar{\nu_\mu}\tau_R]+\frac{m_\mu}{v}[\phi_-^0\bar{\tau}_L\mu_R+\phi_-^+\bar{\nu_\tau}\mu_R]\right] + H.c. \quad (24)$$

to a very good approximation. Since $v_3 << v$ is assumed, the heavy H and A couple predominantly to e^-e^+. If they are produced, through a virtual Z for example, at the Large Hadron Collider (LHC), the $e^-e^+e^-e^+$ final state is very distinctive and potentially measurable. In the same way, $\sqrt{2}Re(\phi_-^0)$ and $\sqrt{2}Im(\phi_-^0)$ may be produced. They decay to $\mu^-\tau^+$ and $\mu^+\tau^-$ which are again rather distinctive if $\tau^\pm$ can be reconstructed experimentally. On the other hand, the decay of $\phi_-^\pm$ is predominantly to $\tau^+\nu_\mu$, $\tau^-\bar{\nu}_\mu$.

9. Conclusion

The notion of cobimaximal neutrino mixing, i.e. $\theta_{13}\neq 0$, $\theta_{23}=\pi/4$, and $\delta_{CP}=-\pi/2$, is shown to be a consequence of the residual $Z_2\times Z_2$ symmetry of an $S_3\times Z_2$ model of lepton masses. This is an alternative theoretical understanding from the usual A_4 realization. It has verifiable distinctive decay signatures (such as $\mu^\pm\tau^\mp$) in its three Higgs doublets.

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