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$[SU(2)]^3$ dark matter

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ABSTRACT

An extra $SU(2)_D$ gauge factor is added to the well-known left-right extension of the standard model (SM) of quarks and leptons. Under $SU(2)_L \times SU(2)_R \times SU(2)_D$, two fermion bidoublets (2, 1, 2) and (1, 2, 2) are assumed. The resulting model has an automatic dark $U(1)$ symmetry, in the same way that the SM has automatic baryon and lepton $U(1)$ symmetries. Phenomenological implications are discussed, as well as the possible theoretical origins of this proposal.

1. Introduction

In the standard model (SM) of quarks and leptons, the choice of the gauge symmetry, i.e. $SU(3)_C \times SU(2)_L \times U(1)_Y$, and the particle content, i.e. quarks and leptons:

$$(u,d)_L \sim (3,2,1/6), \quad u_R \sim (3,1,2/3), \quad d_R \sim (3,1,-1/3), \tag{1}$$

$$(\nu,l)_L \sim (1,2,-1/2), \quad l_R \sim (1,1,-1), \tag{2}$$

together with the one Higgs scalar doublet

$$\Phi = (\phi^+,\phi^0) \sim (1,2,1/2), \tag{3}$$

automatically imply the existence of two global $U(1)$ symmetries, i.e. baryon number (B) under which quarks have charge 1/3, and lepton number (L) under which leptons have charge 1. Is there a corresponding scenario for the existence of dark matter? Consider for example the conventional left-right extension of the SM. Because of the implied $U(1)_{(B-L)/2}$ gauge factor, a discrete Z_2 parity, i.e. $R = (-1)^{3B+L+2j}$, may be used to distinguish some new particles from those of the SM automatically. The importance of this observation is that this parity is not imposed, as is necessary in the minimal supersymmetric standard model, or in models of dark matter [1] assuming only the SM gauge symmetry. Whereas this idea of an automatic R parity has been implemented in some recent studies [2–7], I look instead in this paper for a dark $U(1)$ symmetry (and not just a dark parity) which is also unrelated to B or L, but on the same footing, i.e. its emergence as the result of gauge symmetry and particle content. In the following I show how it may be achieved by inserting an extra $SU(2)_D$ gauge factor to the well-known $SU(3)_C \times U(1)_{(B-L)/2} \times SU(2)_L \times SU(2)_R$ model. Its theoretical origin is a possible $SU(6)$ generalization of the Pati–Salam $SU(4)$ symmetry [8]. It may also be embedded in an $[SU(3)]^4$ gauge model [9] which is different from those considered in Refs. [5,7].

2. Particle content

Under $SU(3)_C \times U(1)_{(B-L)/2} \times SU(2)_L \times SU(2)_R \times SU(2)_D$, the quarks and leptons transform as expected, i.e. as singlets under $SU(2)_D$:

$$(u,d)_L \sim (3,1/6,2,1,1), \quad (u,d)_R \sim (3,1/6,1,2,1), \tag{4}$$

$$(\nu,l)_L \sim (1,-1/2,2,1,1), \quad (\nu,l)_R \sim (1,-1/2,1,2,1), \tag{5}$$

and the new fermions transform as bidoublets:

$$\begin{pmatrix} \psi_1^0 & \psi_2^+ \\ \psi_1^- & \psi_2^0 \end{pmatrix}_L \sim (1,0,2,1,2), \quad \begin{pmatrix} \psi_3^0 & \psi_4^+ \\ \psi_3^- & \psi_4^0 \end{pmatrix}_R \sim (1,0,1,2,2), \tag{6}$$

where $SU(2)_{L,R}$ act vertically, and $SU(2)_D$ horizontally. The electric charge is given by

$$Q = \frac{1}{2}(B-L) + I_{3L} + I_{3R} + I_{3D}. \tag{7}$$

The gauge symmetry is broken by one $SU(2)_R$ doublet, and two $SU(2)_L \times SU(2)_R$ bidoublets:

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$$\begin{pmatrix} \phi_R^+ \\ \phi_R^0 \end{pmatrix} \sim (1, 1/2, 1, 2, 1), \quad \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \sim (1, 0, 2, 2, 1),$$
$$\begin{pmatrix} \phi_3^0 & \phi_4^+ \\ \phi_3^- & \phi_4^0 \end{pmatrix} \sim (1, 0, 2, 2, 1). \tag{8}$$

Whereas the gauge $U(1)_{(B-L)/2}$ is broken, the global $U(1)$ symmetries of baryon number (B) and lepton number (L) remain. If an $SU(2)_R$ triplet is used to break the symmetry, a large Majorana mass for ν_R would act as the canonical seesaw anchor for small Majorana neutrino masses. In that case, lepton number L is broken to lepton parity $(-1)^L$. It will not affect the dark matter stability to be discussed below, but will affect the details of the gauge-boson phenomenology.

What about the extra fermion bidoublets? The crucial observation is that they have built-in invariant masses because of the allowed terms

$$\psi_1^0\psi_2^0 - \psi_1^-\psi_2^+, \quad \psi_3^0\psi_4^0 - \psi_3^-\psi_4^+. \tag{9}$$

At the same time, $\bar\psi_{1L}\psi_{3R}$ and $\bar\psi_{2L}\psi_{4R}$ acquire mass terms from the $\phi_{1,2,3,4}$ vacuum expectation values. This means that an automatic global $U(1)_D$ symmetry emerges, i.e.

$$\psi_{1L}, \ \psi_{3R} \sim -1, \quad \psi_{2L}, \ \psi_{4R} \sim 1, \tag{10}$$

whereas all particles which are singlets under $SU(2)_D$ are trivial under it. It thus serves as a possible dark $U(1)$ symmetry unrelated to B or L. The lighter of the two neutral Dirac fermion eigenstates is then a possible candidate for dark matter. Since $\psi_{1,2}$ have $SU(2)_L$ interactions, they may scatter off nuclei with a large elastic cross section and are thus ruled out by direct-search experiments. It is hence assumed that the dark matter is predominantly $\psi^0_{3,4}$. At this stage, $SU(2)_D$ remains unbroken. To break it, one $SU(2)_D \times U(1)_{(B-L)/2}$ Higgs triplet is added, i.e.

$$\begin{pmatrix} \phi_D^{++} \\ \phi_D^+ \\ \phi_D^0 \end{pmatrix} \sim (1, 1, 1, 1, 3). \tag{11}$$

This choice ensures that there is no coupling between Φ_D and the SM fermions, which would not be the case if it were a doublet. It also generates an accidental global $U(1)$ symmetry for Φ_D, which is crucial for $U(1)_D$ to be a dark symmetry, to be discussed later.

3. Gauge bosons

Masses of the gauge bosons come from the vacuum expectation values of the appropriate neutral scalar bosons. Let

$$\langle \phi^0_{R,D,1,2,3,4} \rangle = v_{R,D,1,2,3,4}. \tag{12}$$

The charged gauge bosons $W_D^\pm$ have mass $g_D^2 v_D^2$ and does not mix with $W^\pm_{L,R}$, the 2×2 mass-squared matrix of which is given by

$$\mathcal{M}^2_{W_L - W_R} = \begin{pmatrix} (1/2)g_L^2(v_1^2+v_2^2+v_3^2+v_4^2) & -g_L g_R(v_1v_2+v_3v_4) \\ -g_L g_R(v_1v_2+v_3v_4) & (1/2)g_R^2(v_R^2+v_1^2+v_2^2+v_3^2+v_4^2) \end{pmatrix}. \tag{13}$$

Since W_D^+ takes $\psi_{1,3}$ to $\psi_{2,4}$, it has charge $+2$ under $U(1)_D$ to conform with Eq. (10) and ϕ_D^{++} has charge $+4$. This shows that $U(1)_D$ is not broken by ϕ_D. Note that the mass degeneracy of ψ^0_{3R}/ψ^0_{4R} with ψ^-_{3R}/ψ^+_{4R} is broken by a small finite radiative correction [10] through the exchange of neutral gauge bosons. Hence ψ^-_{3R} decays to the invisible ψ^0_{3R} and a virtual W_R^- which may convert to $\bar u d$. Its lifetime is presumably quite long and the outgoing lepton has rather low momentum because of the kinematics. This kind of signature may be searched for at the Large Hadron Collider (LHC) as already pointed out [10].

There are four neutral gauge bosons, i.e. B from $U(1)_{(B-L)/2}$, W_{3L} from $SU(2)_L$, W_{3R} from $SU(2)_R$, W_{3D} from $SU(2)_D$, with couplings g_B, g_L, g_R, g_D respectively. Let them be rotated to the following four orthonormal states:

$$A = \frac{e}{g_B}B + \frac{e}{g_L}W_{3L} + \frac{e}{g_R}W_{3R} + \frac{e}{g_D}W_{3D}, \tag{14}$$

$$Z = \frac{e}{g_Y}W_{3L} - \frac{e}{g_L}\left(\frac{g_Y}{g_B}B + \frac{g_Y}{g_R}W_{3R} + \frac{g_Y}{g_D}W_{3D}\right), \tag{15}$$

$$Z_R = \frac{g_R}{\sqrt{g_R^2+g_B^2}}W_{3R} - \frac{g_B}{\sqrt{g_R^2+g_B^2}}B, \tag{16}$$

$$Z_D = \sqrt{1-\frac{g_Y^2}{g_D^2}}W_{3D} - \frac{g_Y}{g_D}\left(\frac{g_B}{\sqrt{g_R^2+g_B^2}}W_{3R} + \frac{g_R}{\sqrt{g_R^2+g_B^2}}B\right), \tag{17}$$

where

$$\frac{1}{e^2} = \frac{1}{g_L^2} + \frac{1}{g_Y^2}, \quad \frac{1}{g_Y^2} = \frac{1}{g_D^2} + \frac{1}{g_R^2} + \frac{1}{g_B^2}. \tag{18}$$

The mass terms are given by

$$\frac{1}{2}(g_B B - g_R W_{3R})^2 v_R^2 + 2(g_B B - g_D W_{3D})^2 v_D^2 + \frac{1}{2}(g_L W_{3L} - g_R W_{3R})^2(v_1^2+v_2^2+v_3^2+v_4^2). \tag{19}$$

It is easily shown that the photon A is massless and decouples from Z, Z_R, Z_D as it should. The 3×3 mass-squared matrix spanning the latter is given by

$$\mathcal{M}^2_{ZZ} = \frac{1}{2}(g_L^2+g_Y^2)(v_1^2+v_2^2+v_3^2+v_4^2), \tag{20}$$

$$\mathcal{M}^2_{Z_RZ_R} = \frac{1}{2}(g_R^2+g_B^2)v_R^2 + \frac{4g_B^4v_D^2 + g_R^4(v_1^2+v_2^2+v_3^2+v_4^2)}{2(g_R^2+g_B^2)}, \tag{21}$$

$$\mathcal{M}^2_{Z_DZ_D} = \frac{g_D^2g_R^2g_B^2}{2g_Y^2(g_R^2+g_B^2)}(4v_D^2) + \frac{g_Y^2g_R^2g_B^2}{2g_D^2(g_R^2+g_B^2)}(v_1^2+v_2^2+v_3^2+v_4^2), \tag{22}$$

$$\mathcal{M}^2_{ZZ_R} = -\frac{g_Lg_Yg_R^2}{2e\sqrt{g_R^2+g_B^2}}(v_1^2+v_2^2+v_3^2+v_4^2), \tag{23}$$

$$\mathcal{M}^2_{ZZ_D} = \frac{g_Lg_Y^2g_Rg_B}{2eg_D\sqrt{g_R^2+g_B^2}}(v_1^2+v_2^2+v_3^2+v_4^2), \tag{24}$$

$$\mathcal{M}^2_{Z_RZ_D} = \frac{g_Rg_B}{2(g_R^2+g_B^2)} \times \left[\frac{g_Dg_B^2}{g_Y}(4v_D^2) - \frac{g_Yg_R^2}{g_D}(v_1^2+v_2^2+v_3^2+v_4^2)\right]. \tag{25}$$

To ensure that $SU(2)_L$ is broken at a scale significantly lower than that of $SU(2)_R$ or $SU(2)_D$, it is assumed that

$$v_1^2 + v_2^2 + v_3^2 + v_4^2 << v_R^2,\ v_D^2. \tag{26}$$

Hence Z decouples effectively from Z_R and Z_D, with negligible mixing to the latter. In the remaining $Z_R - Z_D$ sector, if the $v_1^2 + v_2^2 + v_3^2 + v_4^2$ terms are neglected, then the 2×2 mass-squared matrix is of the form

$$\mathcal{M}^2_{Z_R - Z_D} = \begin{pmatrix} A+B & \sqrt{BC} \\ \sqrt{BC} & C \end{pmatrix}, \tag{27}$$

where

$$A = \frac{1}{2}(g_R^2 + g_B^2) v_R^2, \quad B = \frac{g_B^4}{2(g_R^2 + g_B^2)}(4v_D^2), \quad C = \frac{g_R^2 g_D^2}{g_Y^2 g_B^2} B. \tag{28}$$

There are two interesting limits.

- (1) $B, C << A$, then A and C are eigenvalues with Z_R and Z_D as eigenstates.
- (2) $A << B, C$, then $B + C$ and $AC/(B + C)$ are eigenvalues with $Z_1 = (g_Y g_B Z_R + g_R g_D Z_D)/\sqrt{g_Y^2 g_B^2 + g_R^2 g_D^2}$ and $Z_2 = (g_R g_D Z_R - g_Y g_B Z_D)/\sqrt{g_Y^2 g_B^2 + g_R^2 g_D^2}$ as eigenstates.

4. Gauge interactions

The neutral-current gauge interactions are given by

$$\begin{aligned}\mathcal{L}_{NC} &= eAj_{em} + g_Z Z(j_{3L} - \sin^2\theta_W j_{em}) \\ &+ \frac{1}{\sqrt{g_R^2 + g_B^2}} Z_R (g_R^2 j_{3R} - g_B^2 j_B) \\ &+ g_Y Z_D \left(\frac{g_D \sqrt{g_R^2 + g_B^2}}{g_R g_B} j_{3D} - \frac{g_R g_B}{g_D \sqrt{g_R^2 + g_B^2}} (j_{3R} + j_B) \right).\end{aligned} \tag{29}$$

In particular Z_2 couples to

$$\frac{g_R \sqrt{g_Y^2 g_B^2 + g_R^2 g_D^2}}{g_D \sqrt{g_R^2 + g_B^2}} j_{3R} - \frac{g_Y^2 g_D \sqrt{g_R^2 + g_B^2}}{g_R \sqrt{g_Y^2 g_B^2 + g_R^2 g_D^2}} (j_{3D} + j_B). \tag{30}$$

If $v_D^2 << v_R^2$, then Z_D is the much lighter mass eigenstate with mass given by Eq. (22). It couples to quarks and leptons according to Eq. (29) with

$$j_{3R} = \frac{1}{2}\bar{u}_R \gamma u_R - \frac{1}{2}\bar{d}_R \gamma d_R + \frac{1}{2}\bar{\nu}_R \gamma \nu_R - \frac{1}{2}\bar{l}_R \gamma l_R, \tag{31}$$

$$j_B = \frac{1}{6}(\bar{u}\gamma u + \bar{d}\gamma d) - \frac{1}{2}(\bar{\nu}\gamma \nu + \bar{l}\gamma l), \tag{32}$$

$$j_{3D} = 0. \tag{33}$$

For the dark Dirac fermion ψ_3/ψ_4,

$$j_{3R} = -j_{3D} = \frac{1}{2}\bar{\psi}_{3R}\gamma\psi_{3R} - \frac{1}{2}\bar{\psi}_{4R}\gamma\psi_{4R}, \quad j_B = 0. \tag{34}$$

At the LHC, Z_D may be observed through its production by u and d quarks, with its subsequent decay to lepton pairs. The $c_{u,d}$ coefficients [11,12] used in the data analysis are

$$c_u = (g_{uL}^2 + g_{uR}^2)B = \frac{g_Y^2 g_R^2 g_B^2}{g_D^2 (g_R^2 + g_B^2)} \left[\left(\frac{1}{6}\right)^2 + \left(\frac{2}{3}\right)^2 \right] B, \tag{35}$$

$$c_d = (g_{dL}^2 + g_{dR}^2)B = \frac{g_Y^2 g_R^2 g_B^2}{g_D^2 (g_R^2 + g_B^2)} \left[\left(\frac{1}{6}\right)^2 + \left(-\frac{1}{3}\right)^2 \right] B, \tag{36}$$

where B is the Z_D branching fraction to e^-e^+ and $\mu^-\mu^+$. To estimate $c_{u,d}$, let $g_D = g_R = g_L$, then

$$\frac{e^2}{g_B^2} = 1 - \frac{3e^2}{g_L^2} = 1 - 3(0.23) = 0.31. \tag{37}$$

Assuming that Z_D decays to 3 copies of the dark fermions of Eq. (6) in addition to all the quarks and leptons, B is estimated to be about 0.07, and $c_u = 1.8 \times 10^{-3}$, $c_d = 5.4 \times 10^{-4}$. Based on the 13 TeV LHC data from ATLAS [13], this translates to a bound of about 3.5 TeV on the Z_D mass.

If $v_R^2 << v_D^2$, then Z_2 is the much lighter mass eigenstate with mass given by

$$M_{Z_2}^2 = \frac{g_R^2 g_D^2 (g_R^2 + g_B^2)}{2(g_Y^2 g_B^2 + g_R^2 g_D^2)} v_R^2 = 0.304\ v_R^2. \tag{38}$$

It couples to fermions according to Eq. (30). The branching fraction B is then about 0.03, and the $c_{u,d}$ coefficients are 1.6×10^{-3} and 2.8×10^{-3} respectively. This translates to a bound of about 3.6 TeV on the Z_2 mass. Note that this bound depends on c_u more than c_d because the LHC is a proton collider.

5. Dark matter interactions

The particles beyond the conventional left-right model are the $SU(2)_D$ gauge bosons, the ψ fermions and the one Higgs scalar Φ_D triplet. Whereas $SU(2)_D$ is completely broken by Φ_D, a residual global $U(1)_D$ symmetry remains, under which

$$\psi_{1L}, \psi_{3R} \sim -1, \quad \psi_{2L}, \psi_{4R} \sim +1, \quad W_D^\pm \sim \pm 2, \quad \phi_D^{\pm\pm} \sim \pm 4, \tag{39}$$

and the neutral W_{3D} and the physical neutral scalar h_D are trivial, which allow them to mix with the other neutral gauge bosons and scalar bosons. In Eq. (39), the first three $U(1)_D$ charges coincide with $2I_{3D}$, but not that of $\phi_D^{\pm\pm}$. This is due to the fact that in the scalar potential, Φ_D always appears with its conjugate, so an extra global $U(1)$ factor is present which may be assigned the arbitrary value of 2. In that case, $\phi_D^{++}, \phi_D^+, \phi_D^0$ have $2 + 2I_{3D} = 4, 2, 0$. Hence $U(1)_D$ is a linear combination of I_{3D} and this accidental global $U(1)$ symmetry.

The dark Dirac fermion ψ is assumed to be dominantly composed of ψ_{3R} and ψ_{4R}. To be specific, the outgoing ψ_{4R} may be redefined as an incoming ψ_{3L}, in which case the Dirac fermion ψ has a vector coupling to $g_R W_{3R} - g_D W_{3D}$.

The elastic scattering of ψ off nuclei in underground direct-search experiments is possible through Z_D or Z_2. The spin-independent cross section σ_0 is enhanced by coherence and depends only on their vector couplings to the u and d quarks. For Z_D which couples to $0.547 j_{3D} - 0.233(j_{3R} + j_B)$,

$$u_V = -0.0971, \quad d_V = 0.0194, \quad \psi_V = -0.390. \tag{40}$$

For Z_2 which couples to $0.547 j_{3R} - 0.233(j_{3D} + j_B)$,

$$u_V = 0.0979, \quad d_V = -0.1756, \quad \psi_V = 0.390. \tag{41}$$

The cross section σ_0 is then given by

$$\sigma_0 = \frac{4\mu^2}{\pi A^2}[Zf_p + (A - Z)f_n]^2, \tag{42}$$

where μ is the reduced mass of the effective interaction and equal to the nucleon mass for large m_ψ. In the case of Z_D as the mediator,

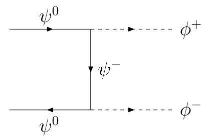

Fig. 1. Dark fermion annihilation to scalars.

$$f_p = \frac{\psi_V(2u_V + d_V)}{M_{Z_D}^2} = \frac{0.0682}{M_{Z_D}^2}, \quad f_n = \frac{\psi_V(u_V + 2d_V)}{M_{Z_D}^2} = \frac{0.0227}{M_{Z_D}^2}. \tag{43}$$

In the case of Z_2 as the mediator,

$$f_p = \frac{\psi_V(2u_V + d_V)}{M_{Z_2}^2} = \frac{0.0079}{M_{Z_2}^2}, \quad f_n = \frac{\psi_V(u_V + 2d_V)}{M_{Z_2}^2} = -\frac{0.0988}{M_{Z_2}^2}. \tag{44}$$

Assuming $m_\psi = 150$ GeV for example, σ_0 is bounded by the latest experimental result [14] to be below 2×10^{-46} cm^2. Using $Z = 54$ and $A = 131$ for xenon, this translates to $M_{Z_D} > 7.8$ TeV and $M_{Z_2} > 9.0$ TeV, which are stronger than the LHC bounds discussed earlier.

Instead of Z_D or Z_2, if the lightest new neutral gauge boson is $Z_3 = (g_B g_D Z_R + g_Y g_R Z_D)/\sqrt{g_Y^2 g_R^2 + g_B^2 g_D^2}$, then it is easily shown from Eq. (29) that it couples to

$$\frac{g_Y^2 g_D \sqrt{g_R^2 + g_B^2}}{g_R \sqrt{g_Y^2 g_R^2 + g_B^2 g_D^2}}(j_{3R} + j_{3D}) - \frac{g_B \sqrt{g_Y^2 g_R^2 + g_B^2 g_D^2}}{g_D \sqrt{g_R^2 + g_B^2}} j_B. \tag{45}$$

This means that $\psi_V = 0$ and there would be no interaction through Z_3 with nuclei and no bound on the mass of Z_3 from direct-search experiments. In other words, if the lightest new neutral gauge boson has a dominant Z_3 component, its bound may be lowered to a value comparable to that from the LHC.

Consider now the relic abundance of ψ. It has a dark $U(1)$ symmetry, so it could be asymmetric dark matter, depending on how it is produced. It is assumed here that it is not asymmetric and there is an equal abundance of both particles and antiparticles in thermal equilibrium in the early Universe. Their annihilation cross section through any new neutral gauge boson is much below 1 pb for a gauge-boson mass greater than 3.5 TeV. Hence a different process is required. Consider then the Yukawa sector. Note first that there is no scalar singlet, so if the dark fermion ψ is composed of only ψ_{3R}^0 and ψ_{4R}^0 with the invariant mass term $\psi_{3R}^0 \psi_{4R}^0$, it has no $\bar{\psi}\psi$ coupling to any scalar. However, as pointed out already, there are also the allowed $\bar{\psi}_{3R}^0(\bar{\phi}_1^0 \psi_{1L}^0 + \phi_1^+ \psi_{1L}^-) + \bar{\psi}_{4R}^0(\bar{\phi}_2^0 \psi_{2L}^0 + \phi_2^- \psi_{2L}^+)$ and $\bar{\psi}_{3R}^0(\bar{\phi}_3^0 \psi_{1L}^0 + \phi_3^+ \psi_{1L}^-) + \bar{\psi}_{4R}^0(\bar{\phi}_4^0 \psi_{2L}^0 + \phi_4^- \psi_{2L}^+)$ terms. Hence ψ annihilation to scalars is possible and it may remain in thermal equilibrium in the early Universe until the temperature drops below m_ψ.

There are several diagrams for ψ annihilation to scalars. As an estimate, consider Fig. 1 which depicts the process $\psi\bar{\psi} \to \phi^+\phi^-$ through ψ^- exchange. The cross section × relative velocity is given by

$$\sigma v_{rel} = \frac{f^4}{16\pi}\left(1 - \frac{m_\phi^2}{m_\psi^2}\right)^{3/2} \frac{m_\psi^2}{(M^2 + m_\psi^2 - m_\phi^2)^2}, \tag{46}$$

where f is the $\bar{\psi}^0\psi^-\phi^+$ coupling and M is the mass of the exchanged ψ^-. As an example, let $m_\psi = 150$ GeV, $m_\phi = 100$ GeV, and $M = 200$ GeV, then $\sigma v_{rel} = 1$ pb is obtained for $f = 0.442$. This shows that the proper relic abundance of dark matter in the Universe is possible within this framework.

6. Theoretical origin of $SU(2)_D$

As presented, the introduction of $SU(2)_D$ and the new fermions of Eq. (6) seems rather *ad hoc*. However, there is a possible unifying theoretical framework underlying their existence. Consider the well-known Pati–Salam partial unification symmetry $SU(4) \times SU(2)_L \times SU(2)_R$ [8], under which quarks and leptons are organized according to

$$\begin{pmatrix} u & u & u & \nu \\ d & d & d & l \end{pmatrix}_L \sim (4,2,1), \quad \begin{pmatrix} u & u & u & \nu \\ d & d & d & l \end{pmatrix}_R \sim (4,1,2), \tag{47}$$

where $SU(4)$ contains $SU(3)_C \times U(1)_{(B-L)/2}$. If this is extended to $SU(6) \times SU(2)_L \times SU(2)_R$, the new fermions introduced are naturally included, i.e.

$$\begin{pmatrix} u & u & u & \nu & \psi_1^0 & \psi_2^+ \\ d & d & d & l & \psi_1^- & \psi_2^0 \end{pmatrix}_L \sim (6,2,1), \quad \begin{pmatrix} u & u & u & \nu & \psi_3^0 & \psi_4^+ \\ d & d & d & l & \psi_3^- & \psi_4^0 \end{pmatrix}_R \sim (6,1,2). \tag{48}$$

This points to the possible unity of matter with dark matter, as discussed previously [5,7,15,16]. In such schemes, the stability of dark matter is akin to that of the proton. Whereas there are gauge bosons which connect quarks to leptons in $SU(4)$ thus allowing proton decay, there are also gauge bosons which connect dark matter to quarks and leptons in $SU(6)$ which render dark matter unstable. The lifetime in question is obviously orders of magnitude longer than the age of the Universe.

The only other possible (and very intriguing) $SU(6)$ assignment is

$$\begin{pmatrix} u & u & u & \nu & x_1 & x_2 \\ d & d & d & l & y_1 & y_2 \end{pmatrix}_L \sim (6,2,1), \quad \begin{pmatrix} u & u & u & \nu & x_3 & x_4 \\ d & d & d & l & y_3 & y_4 \end{pmatrix}_R \sim (6,1,2), \tag{49}$$

where x_i and y_i have charges 1/2 and −1/2 respectively, and $SU(2)_D$ is unbroken. This is a realization of an idea proposed many years ago [17], where color $SU(3)_q$ for quarks is matched with a parallel color $SU(3)_l$ for leptons. Whereas $SU(3)_q$ is unbroken, $SU(3)_l$ is broken to $SU(2)_l$, thereby confining only two components of the fundamental fermion triplet, leaving the third component free as the observed lepton. This notion of leptonic color may be unified [18] under $[SU(3)]^4$, with interesting predictions [7,19] for a future e^-e^+ collider.

Since $SU(4)$ is isomorphic to $SO(6)$ and $SU(2) \times SU(2)$ is isomorphic to $SO(4)$, it is well-known that $SU(4) \times SU(2)_L \times SU(2)_R$ may be embedded into $SO(10)$. As for $SU(6) \times SU(2)_L \times SU(2)_R$, it is not clear which simple group may be a possible unification symmetry. It must of course be at least rank 7.

Another possible unifying structure is $[SU(3)]^4$ without leptonic color. This version [9] is based on $SU(3)_C \times SU(3)_L \times SU(3)_D \times SU(3)_R$. The fermion content is

$$\begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix}, \quad \begin{pmatrix} \psi_1^0 & \psi_2^+ & \nu \\ \psi_1^- & \psi_2^0 & e \\ \chi_1^0 & \chi_2^+ & n \end{pmatrix}, \quad \begin{pmatrix} \psi_3^0 & \psi_3^+ & \chi_3^0 \\ \psi_4^- & \psi_4^0 & \chi_4^- \\ \nu^c & e^c & n^c \end{pmatrix}, \quad \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix}. \tag{50}$$

7. Concluding remarks

The notion is put forward that dark matter is intimately related to matter and the global $U(1)$ symmetry which allows it to be stable is an automatic consequence of gauge symmetry and particle content in the same way that baryon and lepton numbers are so in the standard model. A specific proposal is the addition of an $SU(2)_D$ gauge symmetry with new fermions which are bidoublets under $SU(2)_{L,R} \times SU(2)_D$. It is shown that with the complete breaking of the $SU(2)_D$ gauge symmetry by an $SU(2)_D \times U(1)_{(B-L)/2}$ scalar triplet, a global $U(1)_D$ symmetry remains for the new particles. Dark matter thus emerges naturally within this framework. Its phenomenology is discussed, noting especially that as Dirac fermion dark matter, the requirement of direct detection and relic abundance are markedly different from the case if the $U(1)_D$ symmetry is broken to Z_2 so that this Dirac fermion splits to two Majorana fermions [3]. There is also the intriguing possibility that $SU(2)_D$ may have a theoretical origin in $SU(6) \times SU(2)_L \times SU(2)_R$, where $SU(6)$ is a generalization of the well-known Pati–Salam $SU(4)$ which unifies quarks and leptons. Another possible unifying structure is $[SU(3)]^4$ but without leptonic color.

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