Unifying Heaven and Earth

Essays in the History of Early Modern Cosmology

Miguel Á. Granada Patrick J. Boner Dario Tessicini (eds.)



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Edicions

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CONTENTS

Miguel Á. Granada – Patrick J. Boner – Dario Tessicini, Introduction	9
11010000000000 · · · · · · · · · · · · ·	2
Peter Barker – Tofigh Heidarzadeh, Copernicus, the Ṭūsī Couple and East-West Exchange in the Fifteenth Century	19
	19
DIDIER KAHN, Paracelsus' Ideas on the Heavens, Stars and Comets	59
Dario Tessicini, <i>Giordano Bruno on Copernican Harmony</i> ,	
Circular Uniformity and Spiral Motions	117
Pietro Daniel Omodeo, Metaphysics Meets Urania:	
Daniel Cramer and the Foundations of Tychonic Astronomy	159
PATRICK J. BONER, Written in the Stars: Astronomy and Chronology	
at the Bavarian Court	187
Édouard Mehl, "Novum struam mundum": Kepler's Rebuilding	
of the Copernican "symmetria mundi"	197
JONATHAN N. REGIER, An Unfolding Geometry: Appropriating Proclus	
in the "Harmonice Mundi" (1619)	217
MIGUEL Á. GRANADA, Michael Maestlin and the Comet of 1618	239

RIENK VERMIJ, Seventeenth-Century Dutch Natural Philosophers	
on Celestial Influence	291
Steven Vanden Broecke, From Cosmic Governance	
to Governmentality: Shaping Sublunary Order in Seventeenth-Century	
French Critiques of Astrology	317
Index of Names	343

An Unfolding Geometry: Appropriating Proclus in the "Harmonice mundi" (1619)

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I. INTRODUCTION

Johannes Kepler's Harmonices mundi libri V(1619) is capable of leaving the modern reader with an impression of grandeur and triviality. It is long, complicated, and at times impenetrable. It mixes heavy metaphysical concoctions with an at once technical and quixotic geometry. A single page announces Kepler's discoverv of the relationship between planetary distance and period, what would later be known as the "third law", yet most of the book has nothing to do with said relation. Nevertheless, the Harmonice can also be seen as among the most coherent and sophisticated works of mathematical natural philosophy in the early modern period. Kepler's celestial harmonies are grounded in his account of what mathematical entities are, how they are known, and how they inhabit the world. He was trying to find a single, finite ensemble of proportions that would determine the world's large- and small-scale architecture, as well as the behavior of its constituent parts (including humans and animals). More than this, he was operating from the very deep assumption that these proportions would be unified in the source of existence, God. Thus the philosopher or historian of philosophy should be interested in the Harmonice as an attempt to justify one of the crucial themes of mathematical physics: not only that the world's movement expresses proportionality, but that this world and its mathematical language are unified. Indeed, the book's broad diversity - loosely based on the *quadrivium* - evinces an almost elegant consonance when considered from this point of view.¹ From his

I. As early as 1599, Kepler had in mind the chapter layout for a "cosmographical dissertation" (*dissertatio cosmographica*) to be called the *Harmonices mundi*. See his December 1599 letter to Herwart, in Johannes KEPLER, *Gesammelte Werke*, ed. Max CASPAR et al., Munich, C. H. Beck, 1937- (henceforth *KGW*), xiv, p. 100 (letter n. 148). The plan does not follow the *quadrivium* exactly, because it anticipates five chapters, adding astrology to geometry, arithmetic, music, and

geometrical musings, to his astrology, to his interpretation of harmony, Kepler creates an eminently knowable world system that can be derived from a single and essentially unified form, the sphere or circle. But in doing so, Kepler must explain intellectual movement. He must explain how geometrical knowledge is unpacked from the soul and set moving in the body, as well as in the embodied mind. It is on this point that Kepler owes a debt to Proclus in the *Harmonice*. Proclean geometry *is* a kind of movement, an unfolding of dimensionless forms into quantity, and it is precisely this unfolding that Kepler adopts, recasting it in his own vitalist framework. Moreover, as Kepler aficionados know, we can only with difficulty draw lines that separate phenomena of the living body from phenomena of the wider world. Not to be disappointed, we find Kepler at the summit of his *Harmonice* turning the vast celestial gyrations into a vision of the unfolding Proclean mind.

2. Reading and Misreading Proclus

If there is one "ancient" who strides through the pages of *Harmonice*, it is Proclus, with Kepler appealing to him frequently to justify his mathematical approach to natural philosophy. Without overstatement, we can say that Kepler promotes his harmonies as the fruition of Proclus' philosophy. He goes so far as to include his own translation of a lengthy passage from Proclus' commentary on book I of Euclid's *Elements* – this translation fills over three pages in the *Gesammelte Werke* edition.² While he was likely familiar with other works by Proclus,³ it is exclusively through this commentary that Kepler draws from Proclus in the *Harmonice*.

The first printing of Proclus' commentary was appended to the 1533 Greek *editio princeps* of the *Elements*, edited by the Reformation humanist Simon Grynaeus. Grynaeus, a professor of Greek at Heidelberg and then Basle, is likewise remembered as a close friend of Philipp Melanchthon, and it was to Grynaeus that Melanchthon dedicated his oft-reprinted 1531 preface to Sacrobosco's *De*

astronomy. Also see Johannes KEPLER, *The Harmony of the World*, trans. E. J. AITON, A. M. DUNCAN, and J. V. FIELD, Philadelphia, American Philosophical Society, 1997, pp. xvi-xvii.

^{2.} KGW, vi, pp. 218-221.

^{3.} In 1607, Kepler asks Herwart to send him copies of Proclus' *Hypotyposis* and commentary on the *Timaeus*. He needs these books among others, he writes, to advance his astronomical work. *KGW*, xv, pp. 462-463 (letter n. 424).

Sphaera.⁴ Not incidentally, one of Melanchthon's most famous protégés, Georg Joachim Rheticus, took the Grynaeus edition to Frauenburg in 1539, offering it as a gift to his host Nicolaus Copernicus.⁵ (Copernicus probably learned from his new disciple that Proclus gives a variant of the "Tūsī-couple" in the commentary on Euclid.)6 While Kepler almost certainly read Proclus from Grynaeus's volume. it is not evident when he first did, nor when he seriously began integrating Proclus into his own philosophy.7 The first mention of Proclus in Kepler's hand appears in a long letter written to his patron Herwart von Hohenburg, one of several letters dating from 1599 where Kepler establishes the outlines of his harmonic theory.⁸ It seems, then, that his interest in Proclus was from the beginning linked to his interest in harmony. Much of the letter to von Hohenburg is devoted to showing that harmony should be based upon geometry rather than arithmetic. Kepler wants to ground harmonic proportions in geometrical figures rather than in the status of certain numbers, which means bucking a tradition that had lasted since the Pythagoreans. Quite reasonably then, Kepler sees fit to begin his letter with a philosophical consideration of geometry, and he opens his discussion with a quote from Proclus:

Philosophical proposition. Proclus, writing on Euclid's first book, makes a most beautiful point that I consider to be drawn from Plato's philosophy, τά μαθηματικὰ ἀπογεννᾶν ἐξάρκουσι μετ' ἀλλήλων τούς μέσους διακόσμους τῶν ὄντῶν, καὶ τὴν ἐν αὐτοῖς ποικιλίαν ["mathematicals, in cooperation with one another, suffice to generate

4. See Philip MELANCHTHON, *Philip Melanchthon: Orations on Philosophy and Education*, ed. Sachiko Kusukawa, trans. Christine Salazar, Cambridge, Cambridge University Press, 1999, pp. 105-112.

5. Noel SwerdLow and Otto NEUGEBAUER, *Mathematical Astronomy in Copernicus's* De Revolutionibus, New York, Springer, 1984, p. 24.

6. Copernicus refers to Proclus' commentary when applying the Tūsī-couple to Mercury. However, he had known of the device much earlier, considering that it is used in the *Commentariolus*. F. Jamil RAGEP, *From Tūn to Torun: The Twists and Turns of the Tūsī-Couple*, Preprint 457, Berlin, Max Planck Institute for the History of Science, 2014, pp. 30-31, and p. 34. Also, see André GODDU, *Copernicus and the Aristotelian Tradition: Education, Reading, and Philosophy in Copernicus's Path to Heliocentrism*, Leiden, Brill, 2010, pp. 156-157. For a review of the relevant literature and an account of Copernicus' debt to Islamic astronomy as "virtually undeniable", see the contribution by Peter BARKER and Tofigh HEIDARZADEH in this volume.

7. In the *Harmonice*, when Kepler refers to propositions in the *Elements*, he is referring to Grynaeus's Greek edition. See KEPLER, *The Harmony of the World* (ref. 1), p. 24, n. 23.

8. If we take 1599 as the date when Kepler began to read Proclus and also first developed his theory of harmony (based on the circle), we should also consider that Kepler claims in the *Harmonice* to have developed his theory of harmony *before* he had read Proclus. *KGW*, vi, p. 222. KEPLER, *The Harmony of the World* (ref. 1), p. 303.

the intermediate orders of things and the variety that they display"9]. For although the flower does not grow from the pentagon as from the root, nevertheless it rushes to meet a formal cause that the creator has envisioned. And so wrong is it that this consideration is $\dot{\alpha}\varphi_1\lambda\dot{\alpha}\sigma\varphi_0$ [unphilosophical] (as Aristotle tries to prove), that instead, through it, we become the most accomplished philosophers of all, for we see that $\mu\alpha\theta\eta\mu\alpha\tau_1\kappa\dot{\alpha}$ [mathematicals] are within things, nor do they enter these things by way of nature. From this, we conclude that the cause is the $\dot{\alpha}\rho\chi_1\tau\epsilon\kappa\tau_0\tau_1\kappa\dot{\gamma}$ [architectonic] and intelligible principle which attaches the regular figures to matter.¹⁰

The actual quote is from early in Proclus' prologue, part one. However, the citation is problematic. Kepler cuts several clauses from the passage, clauses present in Grynaeus's edition, as in the standard modern edition by Friedlein.¹¹ It is possible that Kepler misremembered or made a copying mistake. Citation practices in the late Renaissance were definitely not as they are now. But willingly or not, he misinterprets or misrepresents the sense of the original: that mathematicals spring from ($\check{\epsilon}\kappa\gamma\sigma\alpha\alpha$) secondary principles which follow from the *Limit* and *Unlimited*. Needless to say, Kepler's interpretation of Proclus is in synch with his own philosophy of mathematics, particularly insofar as geometry's status. For Kepler, geometrical entities go right to the top of the ontological hierarchy, which is sim-

9. I follow Morrow, subtracting the clauses that Kepler subtracted. PROCLUS, *A Commentary on the First Book of Euclid's Elements*, trans. Glenn R. MORROW, Princeton, Princeton University Press, 1992, p. 5. Many thanks to Bernard Vitrac and Helen Lang for checking that my coarse modification to Morrow's translation was just. It goes without saying that any errors are exclusively my own.

10. *KGW*, 14, p. 63: "Propositio Philosophica. Proclus super primum Euclidis pulcherrimam sententiam ponit, puto ex Platonis philosophia, τά μαθηματικά ἀπογεννᾶν ἐξάρκουσι μετ' ἀλλήλων τούς μέσους διακόσμους τῶν ὄντῶν, καὶ τὴν ἐν αὐτοῖς ποικιλίαν. Nam etsj non crescit ex quinquangulo flos, ut ex radice, tamen concurrit id ad causam formalem, quam spectavit creator. Itaque tantum abest ut ἀφιλόσοφον sit hoc dictum (quod quidem Aristoteles nititur probare) ut potius ex hoc ipso perfectissimj reddamur philosophj, quod videmus μαθηματικὰ inesse in rebus, nec tamen in eas venire via naturalj. Ex hoc enim colligimus, causam esse principem ἀρχιτεκτονικήν et intelligentem, quae connectat materiae figuras ordinatas etc.".

II. He also takes liberty with the word order. The original sentence reads, "τὰ δὲ μαθηματικὰ πέρατος μέν ἐστιν ἕκγονα καὶ ἀπειρίας, ἀλλ' οὐ τῶν πρωτίστων μόνων οὐδὲ τῶν νοητῶν καὶ κρυφίων ἀρχῶν, ἀλλὰ καὶ τούτων, αἶ προῆλθον μὲν ἀπ ἐκείνων εἰς δευτέραν τάζιν, ἀπογεννᾶν δὲ μετ ἀλλήλων ἐζάρκοῦσι τοὺς μέσους διακόσμους τῶν ὄντων καὶ τὴν ἐν αὐτοῖς ποικιλίαν…" PROCLUS, Procli Diadochi in primum Euclidis Elementorum librum commentarii, ed. Gottfried FRIEDLEIN, Hildesheim, Georg Olms, 1992 [1873], p. 6. For this sentence in Grynaeus's edition, see p. 2, lines 2-6, of Proklou Diadochou eis to Eukleidous stoicheion biblion proton, in EUCLID, Eukleidou stoicheion biblia, ed. Simon GRYNAEUS, Basel, Johannes Hervagius, 1533. ply not the case in Proclus' complex ladder of being.¹² This accounts for more than a superficial difference. The Proclean *Limit* and *Unlimited* stand over and above mathematical things,¹³ and, while the *Unlimited* is indeed present in lowly things (in matter's essential shapelessness and formlessness), it is also present in the "unlimited power" that moves the heavens; and at the loftiest heights of Being, we reach an essential infinity, a first chaos that "[...] is primally unlimited and solely unlimited, and is the fount of all infinity – intelligible, intellectual, psychic, corporeal, or material."¹⁴ Naturally, Kepler makes no turn to this kind of essential, generative and utterly varied infinity.¹⁵ Like Aristotle, he equates end ($\tau\epsilon\lambda o\varsigma$) with limit ($\pi\epsilon\rho\alpha\varsigma$),¹⁶ and he is not about to insist that God is the source of a first chaos. And like Aristotle, he considers infinity as the imperfect, as the always lacking, as the unfinished.¹⁷ Kepler's God is essentially limiting and ordering. Hence, one of

12. In the *Harmonice mundi*, Kepler writes, "...these formal ratios of [constructible] Geometric entities are nothing else but the Essence of God ...". *KGW*, vi, p. 55. KEPLER, *The Harmony of the World* (ref. 1), p. 74.

13. The best way to summarize their influence throughout the world would be to say that the former accounts for concordance among all beings, whereas the latter accounts for all diversity. See PROCLUS, *Commentary on Plato's* Timaeus. *Book I: Proclus on the Socratic State and Atlantis*, trans. Harold TARRANT, Cambridge, Cambridge University Press, 2007, p. 140.

14. PROCLUS, *Commentary on Plato's* Parmenides, trans. Glenn R. MORROW and John M. DILLON, Princeton, Princeton University Press, 1987, p. 463.

15. Twenty years later, in the *Harmonice mundi*, we see to what extent the Proclean *Unlimited* remains neutered. It is relegated to matter or continuous quantity and is utterly subservient to categorically finite geometrical designs. The first page of the *Harmonice* is, in fact, an homage to Proclus. Kepler writes that contemporary geometers would have been freed from their ignorance if only Proclus had left to posterity his commentary on Euclid's tenth book. Proclus, as Kepler explains, managed to get down to the most basic principles of mathematics: "limit or boundary as the form, the unlimited as the matter of geometrical objects". But what Kepler attributes to Proclus is only superficially the latter's doctrine. *KGW*, vi, p. 15. KEPLER, *The Harmony of the World* (ref. 1), p. 6. Deep into book five, we read the following about the form-matter, limit-unlimited distinction. Here matter is described as unlimited in itself, but form "limited, unified, and itself the boundary of matter". *KGW*, iv, p. 360. KEPLER, *The Harmony of the World* (ref. 1), p. 488.

16. As Aristotle writes, "Nothing is complete (*teleion*) which has no end (*telos*), and the end is a limit (*peras*)". See ARISTOTLE, *Physics*, 206b33-207a15. Translation by Jonathan Lear in Jonathan Lear, "Aristotelian Infinity", *Proceedings of the Aristotelian Society*, 80, 1979, pp. 187-210 (200).

17. For Kepler on infinity, see Miguel Á. GRANADA, "Kepler and Bruno on the Infinity of the Universe and of Solar Systems", *Journal for the History of Astronomy*, 39, 2008, pp. 469-495, and Jean SEIDENGART, *Dieu, l'univers et la sphère infinie: penser l'infinité cosmique à l'aube de la science classique*, Paris, Albin Michel, 2006, pp. 337-373. I consider Kepler rather Scholastic on the point of divine infinity. Divine infinity is more or less an intensive perfection – a perfection of virtue – that is for Kepler reflected in the virtues of certain geometrical figures, particularly the sphere. As readers of Kepler are aware, he first presents his famous 'sphere as Trinity' metaphor in the *Mysterium cosmographicum* (1596). There, when he speaks about the equality of the sphere's three parts

the foundations of his thought is that divine perfection ought to be reflected in the perfection of geometrical figures used in the world, that is, in their self-similarity and in the uniformity of their sides and angles. In the physical world, this divine perfection is represented by a beauty of boundaries, of completed proportion.¹⁸ "But as beauty flows from equality," Kepler writes to Herwart, "and as God created the best and most beautiful world, he sought [to establish] equality in all things. Equality consists, however, in few things. Inequality is infinite."¹⁹

Our reflections on the nature of finitude and infinity signal a related, fundamental difference between Proclus and Kepler. Kepler's geometry is organizing and boundary-giving – and it is, particularly in the *Harmonice*, inextricable from the divine. When causes are to be sought, then, they are geometrical and never arithmetical.²⁰ Proclus' point of view is quite nearly the opposite. Not only is arithmetic more fundamental than geometry, as he duly explains in his commentary on Euclid's first book, it is dominant over geometry and provides first principles to the latter.²¹ Kepler does not differ from Proclus by a little on this point.

21. Proclus refers to the $\tau \circ \delta \sigma \tau - \delta \delta \sigma \tau$ distinction, what is later called *quia – propter quid* by medieval and Renaissance logicians and *a posteriori – a priori* by Kepler, Michael Maestlin and others. *To hoti* or *quia* are demonstrations that merely establish a phenomenon or fact to be so. *Dioti* or *propter quid* demonstrations establish why the fact is so: they demonstrate that a phenomenon follows necessarily from a unique cause. For Proclus, arithmetic provides *dioti* demonstration, whereas geometry must content itself with the *to hoti*: "For one science is more accurate than another, as Aristotle says; that is, a science that starts from simpler principles than one whose starting-point is more complex, or one that states why a fact is so than one which says that it is so, or a science concerned with intelligibles than one that applies to objects in the sense world. According to these crite-

⁽surface, center and intermediate space), he is obviously not referring to quantity but rather referring to ontological equality. Each part is as important as the others. This is the ontological equality of the Three Persons. See Jonathan REGIER, "A Perfect Knowledge Means a Finite World", *Azimuth*, 4, 2014, pp. 41-53, pp. 48-50.

^{18.} For more on proportion and symmetry in Kepler's philosophy, see Edouard Mehl's contribution to this volume.

^{19.} *KGW*, xiv, p. 64: "Cum autem pulchritudo fluat ex aequalitate, et mundum Deus creaverit optimum et pulcherrimum, aequalitatem in omnibus rebus quaesivit. Aequalitas autem in paucis consistit, inaequalitas infinita est".

^{20.} It suffices to remember the questions that Kepler asked at the beginning of the *Mysterium*, "From whence did God draw his numbers? [...] Why did God create six circuits? Why these spaces between each orb? Why are Jupiter and Mars, which are among the first orbs, separated by such a vast space? Here is all that Pythagoras teaches you here, by way of these five figures". All of these questions pertain to quantities that need to be explained by proportion. Quantities alone cannot give sufficient reason. They have no inherent status or power, as Kepler explains. *KGW*, i, p. 4: "Vnde Deo numeri, quae tantae regula moli, / Quid faciat sex circuitus, quo quaelibet orbe / Internalla cadant, cur tanto Iupiter & Mars, / Orbibus haud primis, interstinguantur hiatu: / Hîc te Pythagoras docet omnia quinque figuris [...]".

His position is the opposite, and it is central to why harmonies must be founded in geometry rather than arithmetic. Any importance that numbers seem to have arises from the characteristics of certain geometrical forms. As he writes to Herwart: "... all capacity of numbers depends on geometrical figures: arithmetic (in view of the genuine and innate subject) is nothing but the rational part of geometry (*quam pars geometriae \dot{\rho}\eta\tau\dot{\eta}*)".²²

3. Unfolding Geometry, Unfolding Soul

I would now like to focus on how Kepler positively adopts Proclus. It would be best to start with the long translation that Kepler furnishes in the *Harmonice*. Kepler's highly competent translation encompasses the entirety of chapter six of

ria of exactness, arithmetic is more precise than geometry, for its principles are simpler. A unit has no position, but a point has; and geometry includes among its principles the point with position, while arithmetic posits the unit. Likewise geometry is superior to spherics and arithmetic to music, for in general they furnish the principles of the theorems subordinate to them. And geometry is superior to mechanics and optics, for the latter discourse about objects in the sense world", PROCLUS, *A Commentary on the First Book of Euclid's Elements* (ref. 9), pp. 47-48. On the terminology of *a priori – a posteriori* see Peter BARKER and Bernard R. GOLDSTEIN, "Realism and Instrumentalism in Sixteenth Century Astronomy: A Reappraisal", *Perspectives on Science*, 6, 1998, pp. 232-258.

^{22.} KGW, xiv, p. 64: "[...] cum aptitudo numerorum omnis ex figuris pendeat: sitque arithmetica nihil aliud (causa genuinj et proprij subjectj) quam pars geometriae ῥητή". A moment after asserting geometry's precedence over arithmetic, Kepler cites Joseph Caesar Scaliger as an ally, writing, "Jam in Geometria saepe cogitavj duas partes facere, ut in prima considerentur quantitates, ut sunt in quarta specie qualitatis: in secunda, ut sunt in Relatione. Primae partis titulus esset de figuris, secundae de rationibus. Id consilium video et I. C. Scaligero probarj." The editors of the Gesammelte Werke provide the passage in question from Scaliger's Exotericarum exercitationum lib. XV (Frankfurt, 1557, p. 1026). In fact, when we look at the entire exercitatio, the waters are slightly muddied. Scaliger begins by attacking Cardano, who had had the temerity to write that geometry was the most subtle of all the sciences. Not so, writes Scaliger. From the point of view of abstraction or complexity, arithmetic is more subtle, *ibidem*, 1025. Yet geometry, writes Scaliger, is prior to arithmetic. Why? Because geometry's subject is the continuum, whereas arithmetic's subject is the multitude. The continuum, or oneness, is prior to multiplicity, both in nature and mind. ("Non enim pendet continua quantitas à discreta quantitate: sed contra. Geometrae namque subjectum continuum est. At continuum qua tale, unum est. Arithmeticae subiectum est multitudo. At haec, uno, est posterior. Cur ergo si Geometrae subiectum prius est, non & prior erit cognitio? Est sane prior tempore, & natura", *ibidem*, 1026.) Kepler's early debt to Scaliger on the question of planetary movement is well known. It is, on the other hand, strange to think that Scaliger might have inspired Kepler here, on this relatively unorthodox position. Arithmetic was almost always considered the more fundamental science in the 16th century. Among 16th-century natural philosophers, besides Scaliger and Kepler, Patrizi also took geometry to be the more fundamental.

Proclus' prologue, part one.²³ This chapter deals with "The Being of Mathematical Genera and Species; the Mode of their Subsistence."²⁴ Most of the chapter is simply Platonic, offering well-known arguments against Aristotelian abstraction and for a mathematical knowledge housed in the soul. Kepler seems to be using the passage to reinforce his beliefs about a geometrical soul and a God of geometrical ideas – not to mention the fact that he inserts a few parenthetical remarks reminding his readers that Platonic philosophy is consistent with Christian doctrine.²⁵ However, at the very end of the translation, we find an important idea that is unique to Proclus.

For all that moves, the basic principle is the circle, and circular motion. Therefore, the concepts of mathematical things, which integrate souls, are essential and self-moving; and Soul putting them forward and propagating and unfolding them, causes the whole variety of mathematical knowledge to persist. For it will never happen that it ceases to engender and bring to light one thing after another continuously, while it uncovers its concepts which are indivisible in their simplicity.²⁶

What we have here, in a nutshell, is the Proclean concept of unfolding. And it is particularly this apparatus of unfolding, hitched to a circular soul, that marks the vitalist philosophy of the *Harmonice*. Before moving on to Kepler's appropriation of this concept, we would do well to revisit it in the context of Proclus' philosophy. Unfolding is how Proclus explains the character of both geometrical entities and of geometrical thought. It constitutes the philosophical

23. Francesco Barozzi's translation of Proclus had existed since the middle of the 16th century. A cursory comparison of Kepler's translation with Barozzi's makes it hard to imagine that Kepler had even consulted the latter. PROCLUS, *Procli Diadochi Lycii philosophi platonici ac mathematici probatissimi in primum Euclidis Elementorum librum commentariorum ad universam mathematicam disciplinam...*, trans. Franciscus BAROCIUS [Francesco BAROZZI], Padua, G. Perchacinus, 1560. "It should be emphasized that Barozzi's translation was a masterpiece of scholarship. Rejecting the very poor *editio princeps* of the Greek text [Grynaeus's edition], he established his own text from several manuscripts. Although he did not publish his text, modern editors have used the Latin translation to infer Barozzi's excellent readings of the Greek", Robert GOULDING, *Defending Hypatia: Ramus, Savile, and the Renaissance Rediscovery of Mathematical History*, Dordrecht, Springer, 2010, p. 168.

24. PROCLUS, A Commentary on the First Book of Euclid's Elements (ref. 9), pp. 10-15.

25. For example, he pauses to make the following comment: "[...] that the mathematical reasons for the creation of bodies were coeternal with God, and that God is pre-eminently soul and mind, whereas human souls are images of God the Creator [...] is known to C hristians". *KGW*, vi, p. 219. KEPLER, *The Harmony of the World* (ref. 1), p. 299.

26. KGW, vi, p. 221. KEPLER, The Harmony of the World (ref. 1), p. 302.

core of his commentary on Euclid, where he explains it, and its implications, thoroughly. $^{\rm \scriptscriptstyle 27}$

Following Plato in the *Republic*. Proclus recognizes that forms of knowing are hierarchized according to the status of what they can grasp.²⁸ Mathematical entities are ontological intermediates, occupying a place between the unchanging and changing. On the one hand, they are separate from sensible things and known apart from sensible things;²⁹ on the other hand, they are not perfectly indivisible. In their immateriality and stability, they do indeed resemble the forms; vet even in the mind they are pictured, extended, cut-up, intersected, and so on.³⁰ In Proclus's scheme, this intermediateness entails a movement between higher and lower poles of being: geometry is not static but characterized by a process of descent and ascent. In other words, geometry emerges from act or movement. Geometrical objects are essentially partless in the understanding, but in order for the understanding to consider them, to "see" them, they must be unfolded into quantity. This unfolding is accomplished in the imagination.³¹ At times, Proclus talks of understanding "projecting" the ideas onto imagination as if onto a screen.³² The screen is best thought of as three-dimensional, and Proclus generally refers to it as a kind of matter.³³ Elsewhere, he calls imagination an immaterial substratum.³⁴ Whatever the case, imagination is a faculty of the soul participating in continuous quantity. This

27. Especially in chapter one, part two of the prologue. See PROCLUS, A Commentary on the First Book of Euclid's Elements (ref. 9), pp. 39-46.

28. Intelligibles (νοητά) are known by intellection (νόησις), understandables (διανοητά) by understanding (διάνοια), perceptibles (αἰσθητά) by belief (πίστις), and likenesses (εἰκαστά) by conjecture (εἰκασία). *Ibidem*, p. 9.

29. Here Proclus repeats the shopworn argument that asks where we see around us a line without thickness, a plane without breadth, or perfect equality; he asks how the certainty of geometrical demonstration could derive from material things, changeable and indeterminate as they are, *ibidem*, pp. 39-40.

30. "All these things indicate that the subject-matter of geometry is divisible and not composed of partless ideas", *ibidem*, p. 40.

31. ARISTOTLE, De anima, 430a10-13.

32. "... the elements latent in the form are produced distinctly and individually on the screen of imagination. What projects the images is the understanding; the source of what is projected is the form in the understanding; and what they are projected in is this 'passive *nous*' that unfolds in revolution about the partlessness of genuine *Nous*..." Proclus, *A Commentary on the First Book of Euclid's Elements* (ref. 9), p. 45.

33. "[Imagination] thinks the circle as extended, and although this circle is free of external matter, it possesses an intelligible matter provided by the imagination itself", *ibidem*.

34. Ibidem, p. 43.

participation in quantity is, for Proclus, the mark of all space, body, and substrate.³⁵

It goes without saying that Proclus believes the soul to contain knowledge of universals within itself without any recourse to the sensible world, even if the soul, because of its presence in a body, is obliged to understand the universals *via* division and multiplicity.³⁶ If *Nous* is characterized by its perfectly inward contemplation, human understanding is characterized by a mixture of inwardness and outwardness. Hence Proclus' analogy between the imagination and a mirror. Understanding, although it stores within itself the undivided forms, must look outward upon the mirror of imagination to understand its inner content.³⁷ To summarize, we can offer the following reconstruction of mathematical knowing for Proclus: spurred on by sense data, the soul, a "generatrix of mathematical forms and ideas", recalls the forms that are the essence of understanding; these forms are unfolded in the imagination and discursively studied by the understanding. By a process of taking the partless forms within itself and complexifying them into multiplicity, understanding discovers the theorems of mathematics.³⁸

4. Kepler's Circular Knowledge

In the *Harmonice*, Kepler defines the soul as a circle. This analogy had been present at least from the *Timaeus*, where the *anima mundi* is circular and moves circularly. Proclus picks it up and accentuates it in his commentary on the *Elements*, identifying the unfolding-enfolding process of understanding with the soul's circular motion.³⁹ But what could this mean for Kepler? He writes in the *Harmonices* that the mind is "like a kind of irradiation shed from the divine face onto the body and drawing thence its more noble nature".⁴⁰ The outrush of divine light forms a sphere and, when it is cut by a plane surface, forms a circle. The plane

35. Space is an extension and so must be a body, according to Proclus. However, he makes a distinction between the corporeal and the material (bulk), which gives resistance whereas space does not. See Lawrence P. SCHRENK, "Proclus on Corporeal Space", *Archiv für Geschichte der Philosophie*, 76, 1994, pp. 151-167 (159-160).

36. PROCLUS, A Commentary on the First Book of Euclid's Elements (ref. 9), p. 42.

37. "... when the soul is looking outside herself at the imagination [...] she is admiring her own ideas from which they are derived; and though she adores their beauty, she dismisses it as something reflected and seeks her own beauty", *ibidem*.

^{38.} Ibidem, p. 11.

^{39.} *Ibidem*, pp. 14-15.

^{40.} KGW, vi, p. 224. KEPLER, The Harmony of the World (ref. 1), p. 305.

surface represents the created body; the resulting circle represents the soul. The soul stands in the same relation to the body "as the curved to the straight, which are incompatible and incommensurable".⁴¹ On one level, we should read all this as natural light doctrine made geometrical, natural light being a kind of Platonic epistemology that, formulated by Augustine, anchored itself in Thomist and Melanchthonian philosophy.⁴² That said, I would like to show how, in the context of the *Harmonice*, Kepler's doctrine of the soul has a Proclean twist.

Kepler is extremely explicit in the *Harmonice* about circularity being the basis of human knowledge. The circle, he writes, is a "form of the soul itself" and is a "single storehouse of all geometrical and arithmetic knowledge" (*ut promptuarium unicum omnis Geometricae et Arithmeticae scientiae*).⁴³ He then describes the circle as a kind of "calculating machine" (*abacus*) of all the multiplications and divisions that can ever take place.⁴⁴ Since the earliest moment of his career, Kepler had been trying to show how all knowable and archetypal geometrical objects must be rooted in the sphere, with their status (as more or less perfect) determined by their closeness to spherical equalities.⁴⁵ For created souls, all knowledge must come out of the circle. A surprising and quite lovely aspect of the *Harmonice* is how thoroughly Kepler develops the definition of *anima* as *circulus* into the basis of mathematical knowability. At the beginning of the *Harmonice mundi*, during his initial discussion of geometrical knowledge, he provides the following uber-definition:

Definition VII: In geometrical matters, to know is to measure by a known measure, which known measure in our present concern, the inscription of Figures in a circle, is the diameter of the circle.⁴⁶

In fact, Kepler spends much of the *Harmonice's* Book I showing that the various lines found in Book X of the *Elements* can be classified according to de-

43. KGW, vi, p. 277. KEPLER, The Harmony of the World (ref. 1), p. 373.

44. Ibidem.

45. See Jonathan REGIER, "Method and the *A Priori* in Keplerian Metaphysics", *Journal of Early Modern Studies*, 2, 2013, pp. 147-62.

46. KGW, vi, p. 21. KEPLER, The Harmony of the World (ref. 1), p. 18.

^{41.} Ibidem.

^{42.} For Kepler's use of natural light doctrine, see Peter BARKER, "Kepler's Epistemology", in Daniel A. DI LISCIA, Eckhard KESSLER, and Charlotte METHUEN, eds., *Method and Order in Renaissance Philosophy of Nature: The Aristotle Commentary Tradition*, Hampshire, Ashgate, 1997, pp. 355-368. Also, see Peter HARRISON, *The Fall of Man and the Foundations of Science*, Cambridge, Cambridge University Press, 2007, pp. 103-106.

grees of knowability – where these degrees amount (somewhat vaguely) to the number of construction steps that separate a line from the diameter of on initial circle. Doing so is part of Kepler's effort to give a Proclean interpretation of Euclid's infamous Book X.⁴⁷ Kepler's effort is not as quixotic as it may seem. After all. Proclus himself declares that "the whole class of mixed lines, both those in planes and those about solids, come from the circle and the straight line. For this reason the soul contains in advance the straight and the circular in her essential nature [...]".⁴⁸ The question we must now ask is why Kepler would go to such an effort of grounding all knowable lines according to their relationship with "the diameter"? The reader will notice that if geometrical quantities are knowable, they can be constructed by compass and straightedge alone. This is perfectly Euclidean. But we must understand how Kepler puts the emphasis on constructibility. As I have just mentioned, a quantity or figure is constructible *when* it is the result of a certain number of elaborations of the diameter and the circumference, of the straight and its curve. It is important to note that construction, in the strict Greek sense that Kepler adopts, means for Kepler that *no* arbitrary quantities are employed (to use an arbitrary quantity would be, for example, to use a marked straightedge in the construction of a heptagon). Thus a construction begins with a first circle, then follows by unpacking various other circles from that first one and using the intersection of these precisely determined circumferences and diameters. Kepler is thinking, concretely, about how a circular soul can know; even more, he is explicitly making the circular soul the foundation of his mathematics.

Proclean geometry, as I mentioned, involves movement. And the fundamental goal of the *Harmonice* is to explain behavior, living and celestial, according to harmonic rules that are grounded in geometry. To do so, Kepler must explain how the circular soul, the only substantial form that he recognizes, exerts itself on the living body, whether it be the body of the earth, or of a blade of grass, or of a human.⁴⁹ Because the soul is essentially immaterial and partless – just like its geometrical knowledge – explaining embodiment is equivalent to asking how

47. "If [Proclus] had left to us his commentaries on the tenth book of Euclid as well, he would have freed our geometers from ignorance [...] and relieved me totally from this toil of explaining the distinguishing features of geometrical objects". *KGW*, vi, p. 15. KEPLER, *Harmony of the World* (ref. 1), p. 9.

48. PROCLUS, A Commentary on the First Book of Euclid's Elements (ref. 9), p. 87.

49. A comprehensive and very clear study of Kepler's vitalism is Patrick J. BONER's recent *Kepler's Cosmological Synthesis: Astrology, Mechanism and the Soul*, Leiden, Brill, 2013.

geometrical ideas are received into quantity.⁵⁰ Here, we can glimpse the uniquely Proclean influence. It is in Book IV of the *Harmonice* where Kepler, dealing with astrological perception, provides his most comprehensive account of the relationship between body and soul. He is obliged to do so. After all, his intent in the *Harmonice* is to explain definitively how geometry is inscribed in the world and how proportion governs change. To begin with, the soul is embodied as a dimensionless point: "First the soul has actually acquired the nature of a point (at least for the purposes of binding it to its body), potentially the shape of a circle; and as it is activity [*energia*], it spreads itself from that seat at a point into a circle".⁵¹ The soul's initial virtue is circular, writes Kepler, expanding then into a spherical emanation: "how would it have any other manner of coming out, existing as light and flame, than other lights coming out of their sources, that is on straight lines?"⁵²

Kepler takes the soul's first bodily manifestation to be an elemental flame.³³ This influence of the soul rushes outward as light, sensing and controlling changes in the body. In line with mainstream Galenic medicine, Kepler associates a basic activity of memory and thought with the vital faculty, that is, with the animal spirits that he identifies with light.⁵⁴ These animal spirits first shine forth at

50. On the distinction between considering geometrical entities "formally" and "materially", see the 1621 edition of the *Mysterium cosmographicum*, the third author's note to chapter eleven. *KGW*, viii, p. 63. I take it as given that to think of a geometrical object materially – i.e., as having parts, as extended in space – is what Kepler would consider a function of the imagination, just as Proclus does.

51. KGW, vi, p. 275. KEPLER, Harmony of the World (ref. 1), pp. 370-371.

52. *KGW*, vi, p. 275. KEPLER, *Harmony of the World* (ref. 1), p. 371. This doctrine of luminary emanation is essential to Kepler's explanation of how the earth and humans perceive celestial aspects. What Kepler proposes is quite simple. The celestial rays that affect the earth are those that intersect its center. But how can a ray intersect the center of the earth, or the center (that is, the heart) of a human? A ray of celestial light "intersecting" the center of the earth must meet head-on one of the infinite rays of the soul. The soul disregards oblique rays as unimportant. "For the soul's ray is brought into action and, so to speak, illuminated, by the star's ray which coincides with it on the same straight line, just as the color of visible things is brought into action by the advent of light, and just as vision is brought into action when we are aware and take into account that we are seeing". *KGW*, vi, pp. 275-276. KEPLER, *Harmony of the World* (ref. 1), pp. 371-372. It should be remembered that the vital faculty does not "see" in the same sense that the eyes do but perceives by a kind of extramission. Such a power is present in all living beings, whether or not they might be otherwise equipped with sight.

53. See Jonathan REGIER, "Kepler's Theory of Force and His Medical Sources", *Early Science and Medicine*, 19, 2014, pp. 1-27 (17-20).

54. Hence, he writes in *De fundamentis astrologiae certioribus* "that every animal faculty is the image of God practicing geometry in creation, and is roused to action by this celestial Geometry

the moment of birth, when the soul's fire is kindled in the heart. And it is at the moment of birth, Kepler specifies, when the vital faculty "constructs the harmonies".⁵⁵ It is likewise this moment when light from the natal zodiac "stamps" the baby's vital faculty.⁵⁶ The construction of the harmonies happens exactly as a baby receives its initial astrological imprinting, simply because the baby's animal spirits have been switched on, as it were, and are operating for the sake of both intellection and perception. Suffice to say that Kepler has adapted his doctrine of Proclean unfolding to Galenic anatomy: animal spirits play the same role as the imaginative material does in Proclean philosophy. On the other hand, Kepler does not make any real effort to localize imagination or differentiate it from other faculties. The fact that Kepler would make imagination a function of *spiritus* is not surprising in the context of 16th- and 17th-century medicine. Animal spirits travelling in the brain were typically mustered to explain what we would call higher functions, including imagination. Kepler's scheme explains, as well, how Proclean imagination, starkly geometrical, is altered in the *Harmonice*. Kepler's imagination retains its Aristotelian-Galenic capacity as a storehouse of incoming sensory information.⁵⁷ It is probably most accurate to say that Kepler understands Proclean imagination within an Aristotelian-Galenic context rather than vice versa, considering the standard references to imagination and common sense that he makes elsewhere in the *Harmonice*, as well as in earlier works.⁵⁸ Not incidentally, Kepler

58. In Book IV of the *Harmonice*, under the marginal heading "The imagination of the Earth's soul", we read that the formative faculty of the earth, "in the manner of a pregnant woman, portrays in cleft stones human affairs which are presented to it from outside, as if it saw them, such as new and unusual appearances of soldiers, monks, priests, kinds, and whatever is in men's mouths". *KGW*, vi, p. 269. KEPLER, *Harmony of the World* (ref. 1), p. 365. In the *Paralipomena*, imagination is equivalent to Aristotelian *sensus communis*. See, for example, the role of imagination in perceiving the depth of refracted and reflected objects: "Atqui cuiuslibet oculi radius visorius (linea lucida ex oculo per punctum repercussus, vel refractionis in continuum per imaginationem educta) est in eadem superficie cum superficie refractionis vel repercussus, per def. 2". For more on this topic, see Delphine BELLIS, "The Perception of Spatial Depth in Kepler's and Descartes' Optics: A Study of an Epistemological Reversal", in Jonathan REGIER and Koen VERMEIR, eds., *Boundaries, Extents and Circulations: Space and Spatiality in the Early Modern Natural Philosophy*, Dordrecht, Springer, forthcoming.

or Harmony of Aspects", *KGW*, iv, p. 23 (proposition XL). Translation in J. V. FIELD, "A Lutheran Astrologer: Johannes Kepler", *Archive for History of Exact Sciences*, 31, 1984, pp. 189-272 (252).

^{55.} KGW, vi, p. 278. Kepler, Harmony of the World (ref. 1), p. 375.

^{56.} KGW, vi, p. 278. KEPLER, Harmony of the World (ref. 1), p. 374.

^{57.} Guy Claessens makes the excellent point that Kepler does not conceive of the imagination as closed to sensory information. Guy CLAESSENS, "Imagination as Self-Knowledge: Kepler on Proclus' 'Commentary on the First Book of Euclid's Elements", *Early Science and Medicine*, 16, 2011, pp. 179-199.

turns imagination (or common sense) into something of the perfect weapon in his project of mathematizing nature *by way of* life that is mathematical at its source (literally, all the way to its circular soul). This is because imagination, being the intermediary between sensible and insensible, will instinctively apply the soul's knowledge to incoming perceptions gleaned from the senses.

As I have indicated, Kepler has reduced the soul's primary animal spirits to light. In doing so, he has deliberately reinforced the causal similarity of microcosm and macrocosm. The soul unfolds its mathematical knowledge and activity *via* animal spirits. By way of spirits, the center directs the periphery and at the same time perceives in the periphery the elaboration of its own internal nature. "For it seems that (if there is some such world soul) it resides in the center of the world, which for me is the Sun, and from there it is propagated over the length and breadth of [the world] by the agency of the rays of light, which are equivalent to spirits in the animate body".⁵⁹ We will now turn to the final scene of the *Harmonice*, where Kepler turns celestial movement into the folding-unfolding of the Proclean mind.

5. The World as Proclean Mind

The modern reader cannot help but notice a point of irony in the *Harmonice*. The whole work is built on top of the circle, yet its objective is to unveil the archetypal causes behind the eccentric – or, more precisely, elliptical – planetary paths.⁶⁰ As Kepler concedes in chapter three of Book V, the polyhedral hypothesis that he had laid out in his *Mysterium cosmographicum* (1596) cannot perfectly account for planetary distances.⁶¹ This is a problem that he would have known about for a long time. Having incorporated Brahe's tables, he had discovered that there were gaps and overlaps needing explanation. Although Kepler never abandoned the regular solids, he felt compelled to show why the distances did not line up perfectly. The Creator must have used some other fundamental principles in His plan for the world, along with the regular solids. Kepler's conclusion is that polyhedral distances do not precisely match because, if they did, the harmonies would not have a chance to emerge. If the solids explain the world's grand outlines, the harmonies should explain the details made available by Brahe's observa-

61. On this point, see also Edouard Mehl's contribution in this volume.

^{59.} KGW, vi, p. 265. Kepler, Harmony of the World (ref. 1), p. 359.

^{60.} Also see J. Bruce BRACKENRIDGE, "Kepler, Elliptical Orbits, and Celestial Circularity: A Study in the Persistece of Metaphysical Commitment", *Annals of Science*, 39, 1982, 265-295.

tions. Book V of the *Harmonice* narrates Kepler's discovery of how exactly the harmonies attach to nature. The infamous chapter nine of this book presents a network of *a priori* and *a posteriori* demonstrations for how and why the harmonies *must* manifest as they do.⁶² I will simply note that chapter nine aims to make two distinct yet related statements:

- The first is that the harmonies do indeed determine the discrepancies in the polyhedral model. Although Kepler does not explicitly say so, elliptical orbits can in this light be understood as a synthesis between the harmonic constraints and the polyhedral constraints.
- 2) The planetary movements at perihelia and aphelia, while not perfectly harmonic, are the best possible, given all the other constraints that need to be satisfied. On this second point, Kepler notes that what matters is not the perfect harmonic exactitude between two neighboring planets, but the harmony of the overall system. Such a statement is a reflection of the first *axiom* of chapter nine, that all kinds of harmonies were established "so that such variety [*varietas*] should adorn the world".⁶³ This axiom of variety means that the individual proportions should be smoothed out to favor the unity of the whole. Harmony *is* a kind of unity, as it always had been in Platonic philosophy: "For harmony is a certain relationship of unity: therefore they [the pairs] are united if they are all at one at the same time rather than if each pair separately agree in pairs of harmonies".⁶⁴

It is in order to justify these two points that Kepler gives a fully animate sense to the harmonies. Why *are* there harmonies rather than merely the polyhedra? The answer is, naturally, that the world is the best possible. But what does this entail? It entails the unity of a living being: "... harmonic decoration is as far above the simple geometrical as life is above the body, or form above matter".⁶⁵ Kepler's gist is that coordinated motion is the mark and indeed final cause of all animate bodies. Comparing the world with a sculpture, Kepler writes that the polyhedral relations yield the general bulk and shape of the world, whereas the harmonies

^{62.} My goal is not to attempt a close-up analysis of this chapter, a daunting task that Bruce Stephenson has already performed. See Bruce STEPHENSON, *The Music of the Heavens: Kepler's Harmonic Astronomy*, Princeton, Princeton University Press, 1994, pp. 185-236.

^{63.} KGW, vi, p. 331. KEPLER, The Harmony of the World (ref. 1), p. 452. Axiom 1.

^{64.} KGW, vi, p. 362. KEPLER, The Harmony of the World (ref. 1), p. 490. Epiphonema 49.

^{65.} KGW, vi, p. 360. KEPLER, The Harmony of the World (ref. 1), p. 488. Epiphonema 49.

furnish the details, the "nose and eyes and other limbs of the statue".⁶⁶ What he means precisely is that the original bulk was worked over so that the world's body could take on "organs necessary to life" ("*corpus organa vitae necessaria*").⁶⁷ The height of mathematical creation, then, is to reproduce the feel and movement of a living being, a notion that comes very close to the Renaissance fascination with automata – which Kepler was wont to appreciate, even as he disliked the "mechanical" character of algebra and rote calculation.⁶⁸ The production of life and the lifelike marks the most impressive sort of creation specifically because it requires the greatest harmonizing of disparate elements.⁶⁹

66. KGW, vi, p. 361. Kepler, The Harmony of the World (ref. 1), p. 490.

67. *KGW*, vi, p. 362. Kepler, *The Harmony of the World* (ref. 1), p. 490. Kepler gives a narrative sense to his own discovery of the harmonies by saying that, early on, he had been looking for nothing but the stones "in this house of the cosmos", without understanding that the Architect had given the world the shape of a living body. *KGW*, vi, p. 361. Kepler, *The Harmony of the World*, p. 490. This theme – a best possible world must be filled with life and, in some sense, itself alive – is present from very early in Kepler's writing. Consider what Kepler writes in his *De fundamentis astrologiae certioribus*: "The most wise Founder had decided to construct a corporeal World. Therefore, at the first, when He conceived Creation He conceived Matter, which from Moses we know to have been water, that is, wet, yielding and capable of being moulded. Therefore this one thing is natural, that is Fluid (Humor). But the Founder was not satisfied with the body of the world until He had made it somewhat like its Founder, in being capable of life and motion. Therefore there exists a second thing, namely Life". *KGW*, iv, pp. 15-16. FIELD, "A Lutheran Astrologer" (ref. 53), pp. 237-238.

68. For Kepler and his "clockwork metaphor", see Patrick J. BONER, "Kepler's Vitalistic View of the Heavens", in Miguel Á. GRANADA, ed., *Novas y cometas entre 1572 y 1618: Revolución cos-mológica y renovación política y religiosa*, Barcelona, Universitat de Barcelona, 2012, pp. 165-193 (166-187). For more on the relationship between vital and mechanical, see Miguel Á. GRANADA, "A quo moventur planetae?': Kepler et la question de l'agent du mouvement planétaire après la disparition des orbes solides", *Galilaeana: Journal of Galilean Studies*, 7, 2010, 111-141 (128-130).

69. Here, to emphasize what is meant by unity or utmost harmonization, we might consider how Kepler's best-possible world foreshadows Leibniz's. It is perfect, as it is the best; but it is also a compromise. The Leibnizian world is one collection (and the richest collection) of compossibles among infinitely many other possible collections. For Leibniz the compromise has as much to do with logical as with ethical necessities. Leibniz appeals to a specific image of God as making choices that are dependent upon his other choices, allowing evil when it leads to greater good. This is one of the themes of the *Essais de théodicée* (1710). "... l'univers, quel qu'il puisse être, est tout d'une pièce comme un océan; le moindre mouvement y étend son effet à quelque distance que ce soit, quoique cet effet devienne moins sensible à proportion de la distance; de sorte que Dieu y a tout réglé par avance une fois pour toutes, ayant prévu les prières, les bonnes et les mauvaises actions, et tout le reste; et chaque chose a contribué *idéalement* avant son existence à la résolution qui a été prise sur l'existence de toutes les choses". G. W. LEIBNIZ, *Discours de métaphysique; Essais de théodicée; Monadologie*, Paris: Flammarion, 2008, pp. 225-226 (point 9). For an analysis of Kepler's The ellipse should thus be understood as the expression of life, of organized and optimized movement, as Fernand Hallyn has pointed out. Kepler in his *Astronomia nova* (1609) had explained elliptical motion by physical cause, particularly by the action of a magnetic sun on magnetic planets. One of his stated goals in the *Astronomia* had been to render planetary minds unnecessary: planetary movement was to be explained solely by bodily force and by the position and direction of force.⁷⁰ Pure circular motion belongs to the province of the mind, writes Kepler in the *Astronomia*, whereas rectilinearity is a manifestation of bodily forces:

For I deny that God has created any perpetual non-rectilinear motion that is not ruled by a mind. Even in the human body, all the muscles move according to the principles of rectilinear motions. [...] The bending of the head, feet, arms, and tongue is expressed in certain mechanical devices by the contraction of many straight muscles carried across from one place to another. [...] Surely, then, if there had been any way of so constructing some moving facility that some body might be able to rotate, it would not have been neglected in the human body.⁷¹

Yet if the ellipse remains a sign of bodily force, it is in the *Harmonice* given a teleological import, as Hallyn also notes.⁷² The forces that cause the ellipse also bring about the harmonies. The ellipse becomes a compromise allowing for a maximum of harmonic richness. It therefore represents more than brute bodily movement. As we will see in a moment, the ellipse comes to signal the Proclean movement of discursive reason.

harmony as religious and political unity, see Aviva Tova ROTHMAN, "Far from Every Strife: Kepler's Search for Harmony in an Age of Discord", Princeton University, 2012, pp. 244-296.

^{70. &}quot;As a result, every detail of the celestial motion is caused and regulated by faculties of a purely corporeal nature, that is, magnetic, with the sole exception of the whirling of the solar body as it remains fixed in its space. For this, a vital faculty seems required". *KGW*, iii, p. 35. Johannes KEPLER, *New Astronomy*, trans. William H. DONAHUE, Cambridge, Cambridge University Press, 1992, p. 68. The sun's magnetic power, however, is almost always depicted by Kepler as issuing from an animate faculty, too. At the beginning of chapter 34, he writes, "For it may appear that there lies hidden in the body of the sun a sort of divinity, which may be compared to our soul, from which flows that *species* driving the planets around...". *KGW*, iii, p. 242. KEPLER, *New Astronomy*, p. 385.

^{71.} *KGW*, iii, p. 69. KEPLER, *New Astronomy* (ref. 68), p. 127. Hallyn signals this quotation. See Fernand HALLYN, *La structure poétique du monde: Copernic, Kepler*, Paris, Éditions du Seuil, 1987, p. 223.

^{72.} Ibidem, pp. 229-231.

We find in the "Epilogue on the Sun", the heady final chapter of the Har*monice*, a fitting denouement to the Proclean dream. In the "Epilogue", Kepler speculates on what kind of soul the sun might have. He has already at several moments insisted that the sun must be alive and capable of enjoying the harmonic spectacle in an immediate way, as if there would be no reason for the planetary harmonies unless an observer were there to appreciate them. This observer, then, is the sun. From its throne in the world's hall, it has over millennia watched the planets speeding up and slowing down, where their fastest and slowest points register like the plucking of so many visual strings.⁷³ The necessity of a solar soul is partly due to Kepler's insistence on what we might call "receptivity": signs and patterns are present because an animate being is there to perceive them. It was to this principle that Kepler turned when he explained to Galileo why the moons of Jupiter were not visible to the naked eve from the earth: the moons were there for the benefit of creatures on Jupiter. If we say that there is a music in the heavens. and if we say that this visual music is centered on the body of the sun, then there must be a creature for whom that music is meant.

But that is not the whole story. Kepler wonders about the nature of the solar mind. What kind of mind is it, exactly? To arrive at the answer, we must "follow the thread of analogy [*analogiae filo ducente*] and pass through the laby-rinths of the mysteries of nature".⁷⁴ The thread of analogy leads us to the following conclusion:

... it would not, I think, be absurd for someone to argue that the disposition of the six spheres towards their common center, and therefore the center of the whole world, is just the same as that of "thought" [$\delta tavo(ta\zeta)$] to "mind" [$vo\tilde{v}v$], as these faculties are distinguished by Aristotle, Plato, Proclus, and others; and again, that the disposition of the local revolutions of the individual planets round the Sun towards the Sun's "immutable" gyration in the central space of the whole system is the same as the dispositions of the "thinkable" [$\delta tavo\eta\tau tko\tilde{v}$] to the "understandable" [$vo\epsilon\rho \dot{v}$], of the manifold process of reasoning towards the completely simple understanding of the mind.⁷⁵

73. We should remember that the planets' lines of apsides do not correspond, meaning that a harmonic conjunction is not like an actual conjunction. During a harmonic conjunction, two or more planets are not on the same line of sight. Rather, they are at the same moment on their own apsides – they are swooping along at their fastest or slowest.

74. KGW, vi, p. 366. KEPLER, The Harmony of the World (ref. 1), p. 495.

75. KGW, vi, p. 366. KEPLER, The Harmony of the World (ref. 1), p. 496.

In Keplerian physics, the sun has a simple revolution on its axis, from which a whirling force is emanated into the world, turning the planets. From the simple motion of the sun, then, arises the complex and multiple harmonies of the planets. In this process, Kepler sees a reflection of how a discursive or complexified intellectual movement begins in the unified, self-same, circular movement of the vovc. The rotation of the sun is unraveled into a pleasing and harmonious multiplicity. Thus the sun is like the Proclean mind, which "understanding itself and all that is in itself, stimulates the use of reason, and by spreading and unfolding its simplicity causes all things to be understood [suamque simplicitatem in illas diducens et explicans, omnia facit intelligi]".76 The simplicity of the solar mind is of a kind with the "simplicity of the understanding".⁷⁷ After all, because it is immobile in the center of the world, it cannot learn the distances as we earthlings can. The distances must be known completely a priori, without any reasoning involved.⁷⁸ From this perspective, then, the earth is bestowed with an extraordinary privilege: we earthlings would not be able to study the heavens and to gain a perfect knowledge of its real movements and dimensions if we were not spun through space. Humanity, with its best possible vantage point at the center of the planets, must possess "the most outstanding and the most absolute" of "reasoning faculties".⁷⁹ We are the geometers.

6. CONCLUSION

Kepler's lifelong work was to promote a natural philosophy that would explain why the world was mathematical and how it was mathematical. The "why" was easy enough. It sufficed to have a geometer as God. And even if Kepler went further, turning God into a repository of geometrical forms, this is nothing but Christianized Plato – although, no doubt, Kepler adds his own unique touch. But on the question of how the world follows mathematical order, or how we behave geometrically, here we notice the influence of Proclus. Kepler had found in the commentary on Euclid the ideal movement – the unfolding movement of geometry as it happened in the mind of the geometer. This movement, which *is*

^{76.} KGW, vi, p. 366. Kepler, The Harmony of the World (ref. 1), p. 496.

^{77.} KGW, vi, p. 366. Kepler, The Harmony of the World (ref. 1), p. 496.

^{78. &}quot;Thus if it has knowledge of the size of the spheres, that must necessarily belong to it *a priori*, without the labor of reasoning (*sine ratiocinationum labore*)...". *KGW*, vi, 367. KEPLER, *The Harmony of the World* (ref. 1), p. 496.

^{79.} KGW, vi, p. 367. KEPLER, The Harmony of the World (ref. 1), p. 496.

Proclean geometry, became for Kepler the instinctive intellectual movement of all ensouled beings. It became the image of the very world order itself, with the sun spinning out its circular order and the planets excogitating in turn, teasing out all the proportions hidden in the initial circle. For Copernicus and early Copernicans, the elevation of the earth corresponded to an intellectual promotion for mankind. Kepler did them all one better. He made his new celestial physics into the very image of mathematical intellect.