Implications of fast radio bursts for superconducting cosmic strings

Yun-Wei Yu, Kwong-Sang Cheng, Gary Shiu and Henry Tye

Abstract. Highly beamed, short-duration electromagnetic bursts could be produced by superconducting cosmic string (SCS) loops oscillating in cosmic magnetic fields. We demonstrated that the basic characteristics of SCS bursts such as the electromagnetic frequency and the energy release could be consistently exhibited in the recently discovered fast radio bursts (FRBs). Moreover, it is first showed that the redshift distribution of the FRBs can also be well accounted for by the SCS burst model. Such agreements between the FRBs and SCS bursts suggest that the FRBs could originate from SCS bursts and thus they could provide an effective probe to study SCSs. The obtained values of model parameters indicate that the loops generating the FRBs have a small length scale and they are mostly formed in the radiation-dominated cosmological epoch.

Keywords: Cosmic strings, domain walls, monopoles, gravitational waves / sources

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1 Introduction

Cosmic strings, which have many small-scale wiggles, are formed as linear topological defects during symmetry breaking phase transition in the very early universe [1]. As a result of string interactions, a large number of closed loops could detach from the string network and the length scale of the loops is comparable to that of the string wiggles. In a wide class of grand unified models, cosmic strings are predicted to behave as superconducting wires [2]. Therefore, as a superconducting cosmic string (SCS) moves through the cosmic magnetic fields of strength $B$, it is able to develop an electric current at a rate of $\frac{dI}{dt} \sim \left(\frac{ce^2}{\hbar}\right)B$ [3], where $c$ is the speed of light, $e$ the electron charge, and $\hbar$ the Planck constant. Furthermore, the superconducting loops oscillating in the magnetic fields can act as an alternating current generator. Consequently, some highly beamed, short electromagnetic (EM) bursts could be produced by the loops at some special points (i.e., cusps where the speed of the string segment is very close to $c$) [4–7]. Due to the EM and probably much stronger gravitational wave (GW) radiations, the loops would shrink with time.

The EM bursts of SCS loops could provide a valuable probe to discover SCSs. As relics from the early universe, the discovery of SCSs would give insight into the physics of fundamental interactions that governed cosmic evolution. Specifically, at very high redshifts, the EM energy released from SCS bursts could be absorbed by the dense surrounding medium to form a fireball, which could subsequently generate a gamma-ray burst (GRB) through internal dissipations [8–13]. It is undoubtedly encouraged to try finding signatures of SCSs from GRB observations [14]. Unfortunately, the present sample of high-redshift GRBs is very small [15–18] and, more seriously, it is not easy to identify SCS-produced GRBs from typical GRBs originating from collapsars and compact binary mergers [19]. In contrast, at relatively low redshifts, the SCS burst emission could successfully penetrate through the intergalactic medium (IGM) and be detected by radio telescopes [20–22]. Particularly, as suggested by Vachaspati [20], such a radio transient signal is very likely to have been reported by Lorimer et al. [23] in a survey with the 64-m Parkes radio telescope, because the basic features of the Lorimer burst, if it has a cosmological distance, can be reasonably explained by the SCS burst model [20].

Very recently, after the High Time Resolution Universe (HTRU) survey with the Parkes telescope, Thornton et al. [24] reported four new Lorimer burst-like radio transients (presently
called fast radio bursts; FRBs), the parameters of which are listed in table 1. First of all, as claimed by Thornton et al., the anomalously high dispersion measures (DMs) of all four FRBs coupled with their high Galactic latitudes confirm the cosmological origin with a redshift $z \sim 0.5–1$. Secondly, their basic properties are identical to the Lorimer burst and thus these new Lorimer burst-like events provide further support of the consistency between the FRBs and SCS bursts. Finally and most importantly, the accumulated number of FRBs could make it more stringent to constrain the event rate of SCS bursts and even its redshift evolution. Although the present sample is not large, such an attempt may still effectively substantiate the role of the FRBs as observational signatures of SCSs.

2 SCS burst model

2.1 Basic characteristics of the bursts

For a length of a SCS loop of $l$ at redshift $z$, the duration of the loop transient radiation could in principle be determined by the period of the loop oscillation as $T_l \sim l/c$, if the moving velocity is subrelativistic. However, the closer to the cusp, the higher the speed of the string segments. Therefore, as analyzed by Babul & Paczyński [8, 9], the duration of the EM burst for an observer should be corrected to $\Delta t_{\text{burst}} \sim f_z T_l / \gamma^3$, where $f_z \equiv (1 + z)$ is introduced due to the cosmological time dilation and $\gamma$ is the Lorentz factor of the string segments near the cusp.

Following Reference [5], the power of the EM radiation of a subrelativistically oscillating loop can be calculated with the use of the magnetic dipole radiation formula as $P_0 \sim m^2 \omega^4 / c^3 \sim I^2 / c$, where $m \sim I^2 / c$ is the magnetic moment of the loop and $\omega \sim 1 / T_l$ is the oscillation frequency. In terms of the string tension $\mu$ (i.e., mass per unit length of the string), the maximum value of the current is found to be $I_{\text{max}} \sim \mu^{1/2} c^2$ which follows from the equation $I_{\text{max}}^2 / c \sim \mu c^2 / T_l$. Then, the highest possible Lorentz factor of the cusp can be determined by $\gamma_{\text{max}} = I_{\text{max}} / I$ [25]. For a string segment moving at a Lorentz factor of $\gamma$, its energy release will be boosted by the Lorentz factor and beamed within an angle of $\theta \sim \gamma^{-1}$. Therefore, the angular distribution of the energy release of a SCS burst can be written as [5, 7]

$$\frac{dE}{d\Omega} \sim \frac{k_{\text{em}} \gamma (P_0 T_l)}{\theta^2} \sim \frac{k_{\text{em}} I^2 l}{c^2 \theta^3}, \quad \text{for } \theta > \theta_c,$$

where the numerical coefficient $k_{\text{em}} \sim 10$ [5, 12, 13] and $\theta_c \sim \gamma_{\text{max}}^{-1}$. Here $\theta = 0$ is defined at the direction of the string motion. For an observer at the light of sight of $\theta$, who cannot see the radiation from the segments with $\gamma > \theta^{-1}$, the isotropically-equivalent energy release can be written as

$$E_{\text{iso}} = 4\pi \frac{dE}{d\Omega} \sim 4\pi k_{\text{em}} \frac{I^2 l}{c^2 \theta^3}.$$  \hspace{1cm} (2.2)

This result could usually be much higher than the real total energy release of the SCS burst as $E_{\text{tot}} \sim k_{\text{em}} I^2 l / (c^2 \theta_c) \sim k_{\text{em}} I \mu^{1/2} l$ [5, 6, 12, 13], which is obtained by integrating eq. (2.1) over the whole solid angle.

Finally, the observational frequency of the EM burst can be estimated by [7, 21, 22]

$$\nu \sim \frac{1}{f_z} \frac{c}{\theta^3 l} \sim \frac{1}{\Delta t_{\text{burst}}},$$

which indicates that the intrinsic duration of SCS bursts can be simply inferred from the observational frequency.
2.2 Loop density

The timescale of the loop shrinkage due to the EM radiation can be estimated by $\tau_{\text{em}} \sim (\mu lc^2/E_{\text{tot}}) T_1 = \mu^{1/2} lc/(k_{\text{em}} I)$. In contrast, the shrinking timescale due to GW radiation can be written as $\tau_{\text{gw}} \sim lc/(k_{\text{gw}} G \mu)$ with a numerical coefficient $k_{\text{gw}} \sim 50\left[\frac{12}{13}\right]$, where $G$ is the Newton’s gravitational constant. Comparing $\tau_{\text{em}}$ with $\tau_{\text{gw}}$, we can get a critical current of $I_* = (k_{\text{gw}}/k_{\text{em}}) G \mu^{3/2} / (10^{19} \mu_{17}^{3/2} \text{ esu s}^{-1})$, below which the loop shrinkage is dominated by the GW radiation. Hereafter the conventional notation $Q_x = Q/10^x$ is adopted in the cgs units. Denoting the shrinking rate of the loop by $\Gamma \equiv -dl/dt$, we have $\Gamma = \Gamma_{\text{em}} + \Gamma_{\text{gw}} = \mu l (\tau_{\text{em}}^{-1} + \tau_{\text{gw}}^{-1})$. Then for a SCS loop having a length of $l$ at redshift $z$, its initial length before the radiation shrinkage is given by

$$l_i = l(z) + \Gamma [t(z) - t_i], \quad (2.4)$$

where the birth-time of the loop, $t_i$, is usually much smaller than $t(z)$ of interest here.

Numerical simulations showed that the string network scales with the horizon, i.e., the typical curvature radius of long SCSs and the distance between them are both on the order of the horizon size [26–30]. Therefore, the differential density of the SCS loops as a function of their initial length can be written as $dn/dl_i \sim [l_i^{3/2} (ct)^{3/2}]^{-1}$ and $dn/dl_i \sim (l_i ct)^{-2}$, for the loops formed in the radiation- and matter-dominated cosmological epochs, respectively [21, 22, 31]. More specifically, by considering the contribution of the loops survived from the radiation-dominated era, the distribution function in the matter-dominated era of interest here should be taken as follows [21, 22]:

$$dn/dl_i \sim \left(1 + \sqrt{\frac{ct_{\text{eq}}}{l_i}}\right) \frac{1}{l_i^2 (ct)^2}, \quad (2.5)$$

where $t_{\text{eq}} \sim 2 \times 10^{12} \text{ s}$ is the time of radiation-matter equality.

3 Implications from FRBs

3.1 Constraining the length of loops

Eq. (2.3) indicates that the intrinsic duration of SCS bursts at the frequency $\nu \sim 1 \text{ GHz}$ is extremely short, i.e., $\Delta t_{\text{burst}} \sim \nu^{-1} \sim 10^{-9} \text{ s}$, which is dramatically shorter than the observed widths of FRBs. This is because the observed duration of radio transient emission can be significantly influenced by the scattering by the turbulent IGM and the time resolution of telescopes. Therefore, we have [33]

$$\Delta t_{\text{obs}} = \max \left\{ (\Delta t_{\text{burst}}^2 + \Delta t_{\text{scat}}^2)^{1/2}, \Delta t_{\text{res}} \right\}. \quad (3.1)$$

In view of the time resolution of $\Delta t_{\text{res}} \sim 64 \mu\text{s}$ of the HTRU survey [34], the observed duration of FRB 110220 as $\Delta t_{\text{obs}} = 5.6 \text{ ms}$ indicates that the value of $\Delta t_{\text{scat}}$ is probably on the order of milliseconds and $\Delta t_{\text{obs}} \approx \Delta t_{\text{scat}}$. In addition, $\Delta t_{\text{scat}}$ is theoretically considered to evolve with redshift (e.g., in References [20–22]), which however has not been exhibited in the observational sample due to its small size.

In any case, the isotropically-equivalent energy release of the observed FRBs can be calculated by

$$E_{\text{iso}} \approx 4\pi d_*^2 \Delta t_{\text{obs}} \Delta \nu S_{\nu} f_z, \quad (3.2)$$
where the frequency bandwidth is taken to $\Delta \nu = 0.4$ GHz [24] and the values of the duration $\Delta t_{\text{obs}}$ and the flux density $S_{\nu}$ are listed in table 1. As shown in the last column of table 1, the calculated energies are on the order of magnitude of $\sim 10^{39}$–$10^{41}$ erg. Here the comoving distances of the sources $d_c = (c/H_0) \int_0^z (f_0^3 \Omega_m + \Omega_\Lambda)^{-1/2} dz'$ can be calculated with the redshifts derived from the measured DMs. The cosmological parameters read $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.27$, and $\Omega_\Lambda = 0.73$.

By attributing the observed FRBs to the EM bursts of SCS loops, the parameters of the SCS burst model can be constrained. To be specific, by taking $\nu \sim 1$ GHz and $E_{\text{iso}} \sim 10^{40}$ erg as reference values and adopting the following relationship [3]

$$I \sim (e^2/\hbar) B l,$$

we can solve for the length of the SCS loops from eqs. (2.2) and (2.3) as

$$l \sim 8 \times 10^{13} f^3 z^{-5/4} B_{0, -6}^{1/2} E_{\text{iso}, 40}^{1/4} \mu_0^{-1/4} \text{ cm}.\quad (3.4)$$

The above value is about $10^3$ times longer than that found by Vachaspati [20],\footnote{In reference [20] the intrinsic duration of SCS bursts is incorrectly overestimated by a factor of $\gamma^2$ (see [21, 22]) which leads to the underestimation of $l$.} but it is still very small on a cosmological scale. Here the cosmic magnetic fields are assumed to be frozen in the cosmic plasma, at least, for relatively low redshifts. This yields $B(z) = B_0 f_z^2$. However, more complicatedly, the fields probably distribute inhomogeneously and the field strength could vary on different field coherent lengths [32]. Here an upper limit value of the present strength $B_0 \sim 1 \mu \text{G}$ is adopted self-consistently corresponding to the short length of the loops.

The current on the loops can be derived from eq. (3.4),

$$I \sim 2 \times 10^{16} f^3 z^{-5/4} B_{0, -6}^{1/2} E_{\text{iso}, 40}^{1/4} \mu_0^{-1/4} \text{ esu s}^{-1},$$

which is much lower than the critical current $I_*$ except for a very small $\mu$. Strictly speaking, on one hand, the above current could be increased by an increasing loop length. On the other hand, however, the magnetic fields on longer length scales and farther away from galaxy clusters could become much lower, e.g. $10^{-9}$–$10^{-8}$ G [32]. Hence, the increased current could still not exceed $I_*$. In the following calculations, the shrinkage of the loops is considered to be dominated by the GW radiation. This yields

$$l \sim \Gamma_{\text{gw}} t \sim \frac{k_{\text{gw}} G \mu}{c} t \sim 5 \times 10^{18} f_z^{-3/2} \mu_{17} \text{ cm},\quad (3.6)$$

where, for an analytical expression, the time is approximated by $t(z) \approx (1/H_0)f_z^{-3/2}$. It should be noted that the redshift here corresponds to the FRB generation but not to the loop formation.

![Table 1](image)
3.2 Fitting to the accumulated numbers of FRBs

Following eqs. (2.4) and (2.5), the observational burst rate of the SCS loops of a length $l$ at redshift $z$ can be written as

$$\dot{R}(z) \sim \frac{\theta^2}{4 \Gamma l} \int_{l+\Gamma l}^{\infty} \frac{dn}{dl'} dl' \sim \frac{\theta^2 c}{4l} \left( 1 + \frac{2}{3} \frac{ct_{\text{eq}}}{l + \Gamma l} \right) \frac{1}{(l + \Gamma l)(ct)^2}. \tag{3.7}$$

By substituting eqs. (3.4) and (3.6) into the above equation and considering $l \ll l_i \sim \Gamma_{\text{gw}} t \ll ct_{\text{eq}}$, we can approximate the observational burst rate by

$$\dot{R}(z) \sim \frac{\theta^2 t_{eq}^{1/2}}{6c^{1/2} \Gamma^{3/2} \Delta z^{1/2}} \sim 4 \times 10^4 f_z^{20/3} B_{0_{\text{th}}}^{5/6} \mu_{17}^{-3/2} E_{\text{iso},40}^{-5/12} \nu_9^{-1/4} \text{ Gpc}^{-3} \text{yr}^{-1}, \tag{3.8}$$

where the viewing angle reads $\theta \sim (\nu f_z l/c)^{-1/3} \sim 7 \times 10^{-5} f_z^{1/12} B_{0_{\text{th}}}^{1/6} E_{\text{iso},40}^{-1/12} \nu_9^{-1/4}$.

Specifically, for the HTRU survey with the Parkes telescope, the observational threshold at $\nu = 1.3$ GHz with a bandwidth $\Delta \nu = 0.4$ GHz can be estimated by

$$E_{\text{iso}, \text{th}} = 4\pi d_c^2 \Delta \nu \Delta \nu S_{\nu_{\text{th}}, FZ} \approx 2 \times 10^{40} (f_z^{1/2} - 1)^2 \text{ erg}, \tag{3.9}$$

where the comoving distance is approximated analytically by $d_c \approx (3c/H_0)f_z^{-1/2}(f_z^{1/2} - 1)$ [12, 13], $\Delta \nu_0 \approx \Delta \nu_{\text{th}} \sim 1$ ms, and the flux sensitivity is taken to be $S_{\nu_{\text{th}}} = 0.3$ Jy as a reference value.\(^2\) Substituting eq. (3.9) into (3.8), we can get the event rate of SCS bursts for the HTRU single-pulse search as

$$\dot{R}_{\text{HTRU}}(z) \sim 3 \times 10^4 f_z^{20/3} (f_z^{1/2} - 1)^{-5/6} B_{0_{\text{th}}}^{5/6} \mu_{17}^{-3/2} \text{ Gpc}^{-3} \text{yr}^{-1}, \tag{3.10}$$

Then the observed accumulated number of SCS bursts can be calculated by

$$N(z) = \mathcal{T} \frac{A}{4\pi} \int_0^z \frac{\dot{R}_{\text{HTRU}}(z')}{f_{z'}} dV_p(z') \sim 311 B_{0_{\text{th}}}^{5/6} \mu_{17}^{-3/2} \int_1^{f_z} \left[ x(x^{1/2} - 1)^7 \right]^{1/6} dx, \tag{3.11}$$

where $\mathcal{T} = 270$ s is the duration of each pointing observation, $A = 4500 \text{ deg}^2 = 1.4 \text{ sr}$ is the area of the survey [24], the factor $f_{z'}$ is due to the cosmological time dilation of the observed rate, and the proper volume element is given by $dV_p \approx 54\pi (c/H_0)^3 f_z^{-11/2}(f_z^{1/2} - 1)^2 dx$ [12, 13].

Finally, in figure 1 we present the accumulated numbers of FRBs by the solid squares. In order to avoid the complicity due to the different telescope parameters in different surveys, here we only invoke the Thornton et al.’s data [24], but exclude FRB 010824 [23], FRB 010621 [36], and the first non-Parkes FRB 121102 discovered in the 1.4-GHz Pulsar ALFA survey with the Arecibo Observatory [37]. The best fitting to the data by eq. (3.11) is shown by the solid line, which corresponds to

$$\mu \sim 5.5 \times 10^{17} B_{0_{\text{th}}}^{5/9} \text{ g cm}^{-1} \sim 6.1 \times 10^{27} B_{0_{\text{th}}}^{5/9} \text{ GeV}^{-2}. \tag{3.12}$$

\(^2\)The determination of $S_{\nu_{\text{th}}}$ actually is not trivial, which depends on the sky region, the time resolution, and the DM etc. Miyamoto et al. [33] suggested a value of $S_{\nu_{\text{th}}} = 0.61$ mJy for the HTRU survey by following Reference [34], which, however, is appropriate for a pulsar survey but may not be for a single-pulse search. Here we take the reference value of $S_{\nu_{\text{th}}}$ according to figure 3 in Reference [35] with a rebinned time resolution of 0.512 ms [24].
Figure 1. Fitting to the redshift distribution of the observed FRBs (solid squares) by the SCS burst model (eq. 3.11; solid line).

Such a result is typical and well consistent with some previous cosmological and astrophysical constraints on SCSs [33]. With the above result, the previously used condition of \( l \ll l_i \sim \Gamma_{gw} t \ll c_{eq} \) can be confirmed. In such a case, it can be known that the loops responsible for the FRBs are mostly formed in the radiation-dominated era. Moreover, the comparison between \( l_i \) and \( c_{eq} \) indicates that the ratio of \( \alpha = l_i/(ct_i) \) is higher than \( \sim 3 \times 10^{-4} \).

4 Conclusion and discussions

By ascribing the observed FRBs to SCS EM bursts and using the EM frequency, duration, energy release, and number of the FRBs, we constrain the most important parameters of the SCS burst model such as \( l(z), l_i, \) and \( \mu \). The obtained typical values of the parameters reconfirm the possible connection between the FRBs and SCS bursts. More importantly, we first investigate the redshift distribution of the FRBs, both the normalization and the profile of which are found to be well accounted for by the SCS burst model. Such an excellent consistency provides a new and substantial evidence for the role of FRBs as observational signatures of SCSs. Furthermore, as implied by our results, the observed FRBs are probably associated with the loops formed in the radiation-dominated cosmological era.

An open issue remains as to why no high-redshift FRB has been detected. On the contrary, the model prediction is that about 6 and 9 FRBs would appear within the redshift ranges of \( 1 \leq z \leq 1.5 \) and \( 1.5 \leq z \leq 2 \), respectively, if the event rate of SCS bursts monotonously increase with an increasing redshift as shown. Here we note that this contradiction could be resolved if there exist some suppression effects at relatively high redshifts. Firstly, such a suppression could arise from the decrease of the magnetic fields surrounding SCS loops. On one hand, the cosmic fields are usually considered to be amplified at a certain redshift, above which the frozen field assumption could not be extended to. On the other hand, SCS loops could be continuously captured and accreted by growing matter perturbations. Therefore, at earlier times, the loops could be much farther away from galaxy clusters, where the diffuse magnetic fields are weaker. Secondly, there could be a cutoff on the loop density at a certain short \( l_i \). In other words, above a certain redshift, the shrinking timescale
of all SCS loops could be longer than the age of the universe at that time. Hence, above that redshift, there is no SCS loop short enough to generate FRBs. Thirdly, it could be easier to absorb FRBs by the relatively denser IGM at higher redshifts. Finally, in any case, it also cannot be ruled out that the absence of high-redshift FRBs could be partially caused by some observational selection effects in the relevant range of flux and duration or by the statistical bias due to the extremely small size of the present data sample.

Besides the SCS burst explanation for the FRBs, many astrophysical scenarios have also been proposed to account for the energy scale of $\sim 10^{40}$ erg and the millisecond duration, e.g., hyperflares of soft gamma-ray repeaters [38], collapses of super-massive neutron stars (NSs) to black holes at several-thousands to million years old [39] or at its baby time [40], mergers of double NSs [41] or binary white dwarfs [42]. However, all of these astrophysical models have not been confronted with the redshift distribution of the FRBs. Additionally, some counterparts in other EM energy bands could usually be predicted by most astrophysical models, which however do not exist in the SCS burst model. At present, there is indeed no counterpart reported to be associated with the FRBs, although this could just be caused by the low angular resolutions of the radio surveys. Another interesting point to note is that SCS loops could cluster and form a halo about the galaxies [49, 50]. Thus if FRBs originate from the SCS loops, we could expect to find some associated galaxies nearby the FRBs. Moreover, a characteristic anisotropic distribution could appear, when more FRB events are accumulated, e.g., through the surveys of future radio telescopes such as SKA. This could provide some clues to distinguish the SCS burst model from the other FRB models. Of course, a larger sample of FRB events is also crucial for confirming that their redshift distribution matches well with the SCS burst model.

If the observed FRBs indeed originated from SCS bursts, one would expect bursts of other light degrees of freedom such as GW bursts, and possibly neutrino bursts to be radiated from cusps and kinks of SCSs as well. Interestingly, with the cosmic string tension of the order of $G\mu/c^2 \sim 4.1 \times 10^{-11} B_{5/9}^{5/9}$ inferred from the FRBs, GW bursts radiated from SCS loops may be detectable by the planned GW detectors such as LIGO/VIRGO and LISA [43–45]. Combining data from the various channels could enable us to distinguish between different types of cosmic strings. For example, while SCSs arise commonly in particle physics models of the early universe [2], cosmic strings produced in string theory models (see, e.g., [46–48] for some recent reviews) generically couple to the Standard Model degrees of freedom with only gravitational strength [47]. This is because gauge fields in string theory (typically) arise from “branes” and stability requires the cosmic strings to be separated from (most of) them. Models in which the cosmic superstrings couple more strongly (with gauge interaction strength) to the EM waves can be constructed but they constitute special cases. Turning this around, these observational probes taken together can yield valuable insights into the fundamental interactions of Nature. We hope to return to the above issues in future work.

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