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Holographic gauge mediation via strongly coupled messengers

Paul McGuirk*

Department of Physics, University of Wisconsin,1150 University Avenue, Madison, Wisconsin 53706, USA

Gary Shiu† and Yoske Sumitomo‡

Department of Physics, University of Wisconsin-Madison,1150 University Avenue, Madison, Wisconsin 53706, USA
and Institute for Advanced Study, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China

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We consider a relative of semidirect gauge mediation where the hidden sector exists at large 't Hooft coupling. Such scenarios can be difficult to describe using perturbative field theory methods but may fall into the class of holographic gauge mediation scenarios, meaning that they are amenable to the techniques of gauge/gravity duality. We use a recently found gravity solution to examine one such case, where the hidden sector is a cascading gauge theory resulting in a confinement scale not much smaller than the messenger mass. In the original construction of holographic gauge mediation, as in other examples of semidirect gauge mediation at strong coupling, the primary contributions to visible sector soft terms come from weakly coupled messenger mesons. In contrast to these examples, we describe the dual of a gauge theory where there are significant contributions from scales in which the strongly coupled messenger quarks are the effective degrees of freedom. In this regime, the visible sector gaugino mass can be calculated entirely from holography.

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I. INTRODUCTION

A major driving force behind the considerations of physics beyond the standard model (BSM) is arguably the hierarchy problem. Though countless number of scenarios have been proposed over the past few decades, they can be broadly divided depending on whether the unknown physics at the TeV scale is weakly or strongly coupled. Supersymmetry (SUSY) is a flagship example of the former. While the dynamics of a strongly coupled hidden sector is typically assumed to be the trigger of SUSY breaking, its influence on the standard model and its supersymmetric extension can be parametrized by a collection of operators that softly break SUSY. The perturbativity of such weakly coupled models not only makes them appealing in light of LEP constraints, but also more amenable to quantitative studies. In comparison, strongly coupled scenarios such as technicolor involve strong coupling physics at the TeV scale, and thus a detailed precision analysis of such models becomes a highly formidable task.

In supersymmetric scenarios, one gains calculability by assuming that the BSM physics (i.e. the superpartners of the standard model or some extension) is weakly coupled, but the large number of operators that must be added makes it difficult to make unique predictions (see e.g. [1,2] for a review). The situation can be greatly ameliorated by studying the mechanism by which the effects of SUSY breaking are mediated to the visible sector. Of the different classes of mediation of SUSY breaking, gauge mediation [3–11] (see also [12] for a review and [13] for a very general discussion) has the advantage of suppressing the flavor-mixing effects that one would generically expect from the profusion of soft SUSY-breaking operators. In addition to the visible sector containing a supersymmetric extension of the standard model, such models possess fields that can be loosely divided into a hidden sector, which is neutral under the visible sector gauge group, and a messenger sector which is charged under the visible sector group. The hidden sector, either by design or by assumption, obtains a SUSY-breaking state via strong dynamics (see e.g. [14] for a review of dynamical SUSY breaking). The messenger sector, which couples to the hidden sector, communicates this effect to the visible sector fields via quantum effects.

Although in models of gauge mediation the messenger sector is often taken to be neutral with respect to hidden sector group responsible for the dynamical breaking of supersymmetry (in which case the coupling between the messengers and hidden sector typically occurs at the level of the superpotential), it is interesting to consider cases where this assumption is relaxed. In models of direct mediation, such as those in [15–18], the distinction between the messengers and hidden sector is less sharp, as the messengers are involved in the dynamical breaking of supersymmetry. Between these two extremes is semidirect gauge mediation [19] in which the messengers are charged under the hidden sector gauge group, as well as the visible sector gauge group, but do not participate in the SUSY breaking. When the messenger sector is weakly coupled, one can use the language and techniques of perturbative field theory to calculate the effects on the visible sector. However, since SUSY breaking is often taken to occur via

*mcguirk@physics.wisc.edu
†shiu@physics.wisc.edu
‡sumitomo@wisc.edu

strong dynamics, one may wish to consider scenarios in which the hidden sector has a large 't Hooft coupling. In this case the messengers, when charged under the hidden sector group, are themselves strongly coupled, and other techniques must be used. In recent years, our toolbox for handling strongly coupled gauge theories has expanded dramatically. Duality symmetries, such as Seiberg duality [20] and gauge/gravity duality [21–23], have enabled us to map strong coupling physics to their more tangible weak coupling duals. Armed with these tools, we can now explore new BSM scenarios and/or regions of model spaces which were previously overlooked or ignored because of the complications with strong coupling.

In this paper, we report on some progress in this direction by computing holographically the effects of semidirect gauge mediated supersymmetry breaking with *strongly coupled* messengers. Though these models are weakly coupled in the sense that the effects of SUSY breaking on the visible sector can be expressed in terms of a soft SUSY-breaking Lagrangian, the fact that the messengers are strongly coupled with respect to the hidden sector gauge group suggests that their contributions to soft terms are subject to large hidden sector loop corrections. As a result, the way that the messenger mass and SUSY-breaking scale appear in the soft SUSY Langrangian may differ from the usual perturbative expressions which assume weakly coupled messengers. Fortunately, holographic techniques become useful when the hidden sector gauge group has large 't Hooft coupling [24]. In examples where the holographic dual is known, a tree-level computation on the gravity side amounts to summing up all loop planar diagrams involving the messengers and the strongly coupled hidden sector (Fig. 3).

Our work is motivated by a related interesting scenario suggested in [26] and the gravity duals of SUSY-breaking large rank $SU(N+M)\times SU(N)$ gauge theories recently obtained in [27]. Utilizing the gravity background presented in [28], the authors of [26] constructed the holographic dual of a semidirect gauge mediation scenario where the masses of the messenger quarks are much higher than the hidden sector confinement scale. The large residual R symmetry preserved at high energies by the SUSY-breaking state suggests that contributions to the gaugino mass might be suppressed, and indeed to leading order in the SUSY-breaking order parameter, there is no contribution to the gaugino mass from physics at energies scales above the messenger quark mass in that scenario. Rather, the contributions to the gaugino mass come from below this scale where the effective degrees of freedom of the messenger sector are the *weakly* coupled mesonic bound states of the messenger quarks whose interactions are suppressed by the large 't Hooft coupling [29]. Therefore, even though the mesonic spectrum and effective F-terms require a holographic computation because the hidden sector is strongly coupled, the impact on the visible sector can be described by the usual perturbative expression. In contrast, in this paper we use the supergravity solutions obtained in [27] which allow us to work in a different kinematic regime where the messenger masses are comparable to the confinement scale. In this regime, we find a contribution from scales even above the messenger quark mass where the propagating degrees of freedom include strongly coupled quarks.

This paper is organized as follows. In Sec. II, we review the holographic approach to gauge mediation suggested in [26]. Generic arguments based on R symmetry motivate us to consider one of the nonsupersymmetric solutions presented in [27] which we briefly summarize in Sec. II A. In Sec. III, we compute the visible sector gaugino mass by considering gauginos living on a stack of probe D7-branes in this geometry. We end with some discussion in Sec. IV. Some useful details about the deformed conifold geometry and our conventions are relegated to the appendices.

II. HOLOGRAPHIC GAUGE MEDIATION

As in [26], we take the hidden sector at short distances to be an $\mathcal{N}=1$ $SU(N+M)\times SU(N)$ gauge theory with large 't Hooft couplings. The matter content of the hidden sector possesses an $SU(2)\times SU(2)$ flavor symmetry under which the bifundamental chiral multiplets $A_{i=1,2}$ and $B_{i=1,2}$ transform as $(\mathbf{2},\mathbf{1})$ and $(\mathbf{1},\mathbf{2})$, respectively. The superpotential for this chiral matter is

$$W_{\text{hidden}} = \lambda_1 \epsilon^{ij}\epsilon^{kl}(A_i B_k A_j B_l). \tag{1}$$

For nonvanishing M, as the theory flows to the IR, it undergoes a cascade of Seiberg dualities [20] ending with an $SU(M)$ gauge theory exhibiting confinement at a scale Λ_ε. At short distances, the theory possesses a $\mathbb{Z}_{2M}$ R symmetry that, in the IR, is spontaneously broken to $\mathbb{Z}_2$ by hidden sector gluino condensation.

At large 't Hooft coupling, the theory is strongly coupled and can be difficult to analyze. However, it is in this limit that the techniques of the gauge-gravity correspondence can be most reliably applied. The gravity dual for the high energy theory (the KT solution [31]) was constructed in IIB string theory [32] by placing N D3-branes and M fractional D3-branes (i.e. D5-branes that wrap collapsing two cycles) at a conifold singularity with the world volumes filling the four external spacetime dimensions. The failure of the Klebanov-Tseytlin (KT) solution to describe the IR behavior of the field theory is related to the presence of the naked conifold singularity in the geometry. The Klebanov-Strassler (KS) solution [33] provides the IR resolution by smearing M fractional D3-branes over the finite S^3 at the tip of the deformed conifold.

Although dynamical SUSY breaking in this theory is difficult to describe using standard field theory techniques, a holographic realization of a SUSY-breaking state can be constructed by adding $\overline{\text{D3}}$-branes to the tip. In the absence of D3-branes and as P, the number of $\overline{\text{D3}}$-branes, is much smaller than the amount of flux, the $\overline{\text{D3}}$-branes are perturbatively stable but will quantum mechanically tunnel into a SUSY-preserving vacuum [34]. The back reaction of the $\overline{\text{D3}}$-branes [the DeWolfe-Kachru-Mulligan (DKM) solution] was found in [28] for the KT region (i.e. at large radius). The presence of the $\overline{\text{D3}}$-branes in the geometry explicitly breaks SUSY on the gravity side, but the rapid falloff of the resulting non-SUSY perturbations to the bulk fields due to the $\overline{\text{D3}}$-brane indicates that this configuration is dual to a metastable SUSY-breaking state, rather than dual to a theory that explicitly breaks SUSY.

The DKM solution was used in [26] to provide a holographic realization of a scenario of gauge mediation. The standard model gauge group is taken as a subgroup of a global SU(K) symmetry that is introduced into the field theory dual by adding a stack of K probe D7-branes into the geometry and weakly gauged by adding a cutoff to the geometry (Fig. 1). These D7-branes fill the large four-dimensional spacetime and extend along the radial direction of the conifold while wrapping a three cycle in the angular directions. The matter content of the standard model is placed at the cutoff of the geometry, which on the gauge theory side, corresponds to taking the standard model fields to be elementary fields, rather than composites resulting from the strong dynamics of the hidden sector.

In particular, the (deformed) conifold inherits a complex structure through the defining equation

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2, \qquad z_i \in \mathbb{C}, \tag{2}$$

where the deformation parameter is related to the confining scale of the dual gauge theory by $\Lambda_\varepsilon = \varepsilon^{2/3}$. In terms of these holomorphic coordinates, we take the world volume

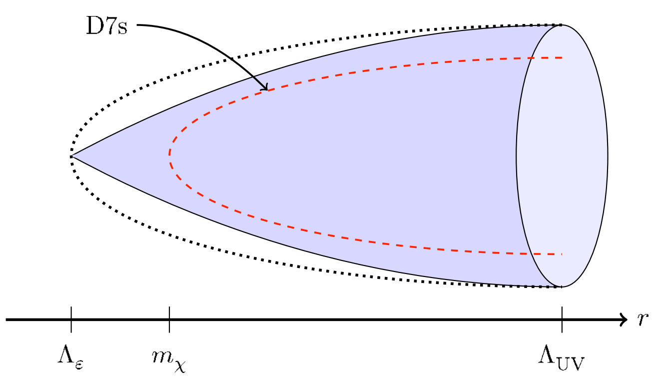

FIG. 1 (color online). The flavor branes dip down to a distance set by the messenger mass $r = m_\chi$ where the coordinate r is defined in (B2) and has mass dimension 1. Although we will consider a case where m_χ is not much larger than Λ_ε, because of the strong warping this corresponds to a large proper distance. The back reaction of the $\overline{\text{D3}}$-branes reintroduces a singularity at $r = \Lambda_\varepsilon$ so that unlike the KS solution (represented by the dotted line), it is no longer smooth at the tip.

of the D7-branes to be specified by the condition

$$z_4 = \mu. \tag{3}$$

The addition of K such D7-branes corresponds to the addition of the global SU(K) flavor symmetry [35] and matter fields χ and $\tilde{\chi}$ to the gauge theory with χ transforming as an antifundamental of the SU(N) factor of the gauge theory and a fundamental of SU(K) and $\tilde{\chi}$ as the conjugate representations (Fig. 2). The bifundamental fields A_i and B_i couple to the quarks through the superpotential [36,37]

$$W_{\rm mess} = \tilde{\chi}^a (A_1 B_1 + A_2 B_2 - \mu)\chi_a + \lambda_2 \tilde{\chi}\chi\tilde{\chi}\chi. \tag{4}$$

The fields χ, which have mass dimension $\frac{3}{4}$, have mass $m_\chi = \mu^{2/3}$. This choice of embedding of the D7 world volumes is made, since although any holomorphically embedded D7 is supersymmetric in the (deformed) conifold, D7-branes satisfying a condition other than (3) typically require the existence of nontrivial world volume flux to preserve the same supersymmetry as the KS solution [38].

Since χ and $\tilde{\chi}$ are charged under both the hidden sector and the visible sector, they are natural candidates for the messengers of the effect of SUSY breaking. Additionally, the SUSY-breaking state exists independently of the presence of the D7-branes, implying that these messengers do not actively participate in the dynamical breaking of supersymmetry [39]. Thus, the setup is closely related to semidirect gauge mediation [19], although χ and $\tilde{\chi}$ do have additional superpotential couplings to the hidden sector chiral matter. Because the messenger quarks are charged under the large 't Hooft hidden sector, there are potential contributions to the visible sector gaugino mass from all planar diagrams (Fig. 3).

Since the analysis of [26] was performed at large radius on the gravity side, the dual field theory is in a regime where the messengers are much heavier than the confining scale of the strongly coupled hidden sector, $m_\chi \gg \Lambda_\varepsilon$, and for many parts of that analysis, it is appropriate to neglect ε. In the absence of the deformation of the conifold singularity, the R symmetry preserved by the geometry is $\mathbb{Z}_{2M}$ [40,41]. This large amount of R symmetry suppresses contributions to the gaugino mass from scales above the messenger mass [42]. A nonvanishing messenger mass μ,

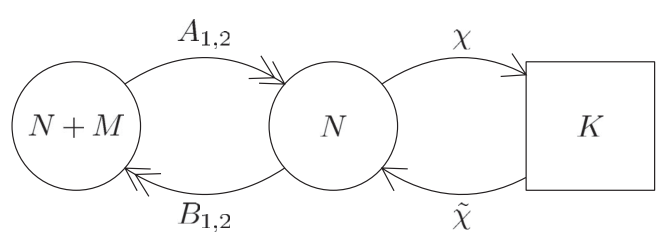

FIG. 2. Quiver for the high energy theory. The standard model gauge group is a subgroup of the global SU(K).

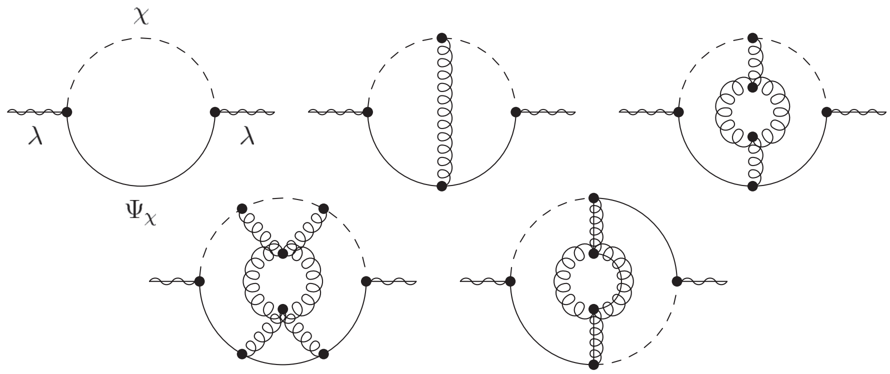

FIG. 3. A very small sample of the infinite number of loops that might contribute to the visible sector gaugino mass. The gaugino couples to the messenger quarks Ψ_χ and squarks χ which also couple to the large 't Hooft coupling hidden sector gluons and gluinos. Since the hidden sector has large 't Hooft coupling, there are leading order contributions from planar diagrams with arbitrary numbers of loops. The calculation can be done holographically and, to leading order in the SUSY-breaking parameter, the loops cancel for $m_\chi \gg \Lambda_\varepsilon$. However, for $m_\chi \approx \Lambda_\varepsilon$, the cancellation no longer occurs.

which has unit R charge, breaks R symmetry altogether. However, the R-symmetry breaking effect seems to be small, and indeed in [26], the messenger quarks χ do not directly contribute to the gaugino mass until higher order in perturbation theory. As a result of the strong dynamics, the messenger χ fields bind into mesons Φ_n which are neutral under the hidden sector gauge group but transform as adjoints under the visible SU(K). The spectrum of mesons includes states whose masses are below m_χ, and the SUSY-breaking dynamics of the hidden sector cause these meson superfields to feel R symmetry breaking effective F-terms which lead to a nonvanishing gaugino mass. Because the hidden sector gauge group has large rank, the mesons are weakly coupled [43–45], and their physics can be described using standard field theory techniques, though the spectra $\{M_n, F_n\}$ of masses and F-terms do require a holographic calculation [which was performed via a Dirac-Born-Infeld (DBI) analysis in [26]]. The mesons effectively act as messengers in a minimal gauge mediation scenario (Fig. 4), and the result of [26] is that the visible sector gauginos receive a mass

$$m_{1/2} = \frac{g_{\rm vis}^2 K}{16\pi^2}\sum_n \frac{F_n}{M_n} \sim \frac{g_{\rm vis}^2 K}{16\pi^2}\frac{\Lambda_{\mathcal{S}}^4}{m_\chi^3\sqrt{4\pi\lambda_{\rm eff}(m_\chi)}}\sum_n n e^{i\theta_n}, \quad (5)$$

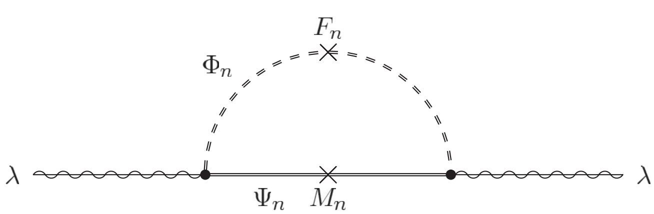

FIG. 4. Contribution to the visible sector gaugino mass from a messenger mesons Φ_n and the superpartner mesinos Ψ_n. In the 't Hooft limit, the mesons and mesinos are weakly coupled and this diagram gives the leading order contribution from the mesons, giving (5).

where $g_{\rm vis}$ is the visible sector gauge coupling, $\Lambda_{\mathcal{S}}^4$ is the exponentially small vacuum energy of the SUSY-breaking state (which on the gravity side is set by the warped tension of the $\overline{\rm D3}$-branes), $\lambda_{\rm eff}(m_\chi)$ is the 't Hooft coupling of the hidden sector (dual to the amount of effective D3 charge) evaluated at the energy scale m_χ, and θ_n are uncalculated phases. The summation is over a range of n such that the effective field theory of weakly coupled mesons is appropriate. The effects of SUSY breaking is communicated to the remaining visible sector fields via gaugino mediation [46,47].

A non-SUSY deformation of Klebanov-Strassler

In the far IR of the field theory, R symmetry is broken down to $\mathbb{Z}_2$ by hidden sector gluino condensation. One would expect then that for $m_\chi \sim \Lambda_\varepsilon$, there will be contributions to the gaugino mass even from energies above m_χ. Indeed, it was estimated in [26] that there should be a contribution to the gaugino mass from a finite deformation given by

$$\delta m_{1/2} \sim \frac{\Lambda_\varepsilon}{m_\chi}\frac{\Lambda_{\mathcal{S}}^4}{m_\chi^3\sqrt{4\pi\lambda_{\rm eff}(m_\chi)}}. \quad (6)$$

For $\mu \gg \varepsilon$ where the DKM solution is valid, this is much smaller than Eq. (5). In order for this contribution to be important compared to that of the meson messengers, it is necessary that [48]

$$\frac{\Lambda_\varepsilon}{m_\chi} \gtrsim \frac{g_{\rm vis}^2 K}{16\pi^2}. \quad (7)$$

As an estimate, we can suppose that the global symmetry has $K = 5$ and forms an SU(5) GUT with $\alpha_{\rm GUT} \sim \frac{1}{25}$. This gives

$$m_\chi \lesssim 60\Lambda_\varepsilon. \quad (8)$$

Clearly, as the hierarchy between Λ_ε and m_χ is reduced, the more important the R-breaking effects of confinement become. However, there is a possible concern with taking m_χ to be too small. On the gravity side of the calculation, decreasing m_χ corresponds to allowing the probe D7s to dip further into the throat, reaching smaller values of τ. The presence of a $\overline{\text{D3}}$-brane introduces a curvature singularity into the back-reacted geometry at $\tau = 0$ [27]. Such a singularity indicates the supergravity approximation of string theory breaks down, and so the solution should be modified at distances below the string length. Thus, in order to trust our analysis of the gaugino mass, the D7-branes must not extend too deeply into the throat. For small radial distances, the KS metric (16) takes the approximate form [49]

$$ds_{10}^2 \approx h_0^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h_0^{1/2}(\tfrac{1}{2}d\tau^2 + d\Omega_3^2 + \tfrac{1}{4}\tau^2[g_1^2 + g_2^2]), \qquad (9)$$

where $d\Omega_3^2$ is the line element for a unit S^3, g_i are other angular 1-forms, and $h_0 \sim (g_sM)^2$.

We can estimate the string length for strings stretching along the radial direction at small τ by considering the world-sheet action,

$$S_\sigma = -\frac{1}{2\pi\alpha'}\int_{\mathcal{M}} d^2\sigma\sqrt{-\gamma}\gamma^{ab}g_{MN}\partial_a X^M\partial_b X^N \sim -\frac{g_sM}{2\pi\alpha'}\int_{\mathcal{M}} d^2\sigma\sqrt{-\gamma}\gamma^{ab}\partial_a X^\tau\partial_b X^\tau. \qquad (10)$$

This implies that the effective string length for strings stretching along the holographic or internal angular directions is

$$\sim\sqrt{\frac{2\pi\alpha'}{g_sM}}. \qquad (11)$$

The unknown stringy modification of the geometry can be neglected if the D7-branes remain much further than a string length from the location of the $\overline{\text{D3}}$-branes. Temporarily setting $2\pi\alpha' = 1$, this condition becomes

$$\tau_{\min} \gg \frac{1}{g_sM}. \qquad (12)$$

In order for the supergravity approximation to be valid away from the singularity, g_sM must be large, so the stringy resolution is important only for very small values of $\tau_{\min}$.

For a world volume specified by the embedding condition (3), the D7-brane will extend to a minimum τ given by [37]

$$\tau_{\min} = 2\,\text{arccosh}\frac{\mu}{\varepsilon}. \qquad (13)$$

Combining (8) with (12) and using the relationships $\Lambda_\varepsilon = \varepsilon^{2/3}$ and $m_\chi = \mu^{2/3}$, we get the expectation that there will be important and calculable contributions to the gaugino mass when the D7-branes reach a minimum value $\tau_{\min}$ satisfying

$$\frac{1}{g_sM} \ll \tau_{\min} \lesssim 14. \qquad (14)$$

Given the relative complexity of the KS solution itself, an exact solution corresponding to the addition of an $\overline{\text{D3}}$-brane would be difficult to find. Instead, we will limit ourselves to a small τ expansion and take

$$\frac{1}{g_sM} \ll \tau_{\min} < 1. \qquad (15)$$

In terms of the dual field theory variables, this means that we are taking the confining scale Λ_ε and the messenger mass m_χ to be very near each other, but still requiring that latter be slightly larger. For simplicity, we take both ε and μ to be real. The final result of our calculation will be a contribution to the gaugino mass that differs from (6) (which is not necessarily a contradiction since the result was obtained in a regime where $\Lambda_\varepsilon/m_\chi$ is a good expansion parameter while it is not for the calculation presented here). Nevertheless, (6) provides a good motivation to consider deformations to KS at small radius, especially since our calculation will yield a contribution that is enhanced by the hidden sector 't Hooft coupling relative to the estimate (6).

In [27], we found small τ expansions for nonsupersymmetric perturbations to the KS solution. For a choice of parameters, one of these solutions corresponds to the addition of P D3-$\overline{\text{D3}}$-brane pairs smeared over the finite S^3 at the tip and is the small radius analogue of the DKM solution [28,50]. In terms of the angular 1-forms g_i (which are reviewed in Appendix B), the metric of this solution is of the warped type

$$ds_{10}^2 = h^{-1/2}(\tau)\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(\tau)d\tilde{s}_6^2, \qquad (16a)$$

where $\mu = 0, 1, 2, 3$ and where the radial and internal angular part of the metric is

$$d\tilde{s}_6^2 = p(\tau)d\tau^2 + u(\tau)g_5^2 + q(\tau)(g_3^2 + g_4^2) + s(\tau)(g_1^2 + g_2^2). \qquad (16b)$$

Similar to what was found in [28], the presence of the $\overline{\text{D3}}$-branes "squashes" the unwarped six-dimensional (6D) space so that it is no longer the geometry of the deformed conifold. Expanding the perturbation to leading order in τ gives

$$p(\tau) = p_{\text{KS}}, \qquad u(\tau) = u_{\text{KS}}\left(1 + \frac{u_0}{\tau}\right), \qquad q(\tau) = q_{\text{KS}}\left(1 + \frac{q_0}{\tau}\right), \qquad s(\tau) = s_{\text{KS}}\left(1 + \frac{s_0}{\tau}\right), \qquad (17)$$

where the Klebanov-Strassler solution has

$$p_{\rm KS}(\tau) = u_{\rm KS}(\tau) = \frac{\varepsilon^{4/3}}{6K^2(\tau)},$$
$$q_{\rm KS}(\tau) = \frac{\varepsilon^{4/3}}{2}K(\tau)\cosh^2\frac{\tau}{2}, \tag{18}$$
$$s_{\rm KS}(\tau) = \frac{\varepsilon^{4/3}}{2}K(\tau)\sinh^2\frac{\tau}{2},$$

with

$$K(\tau) = \frac{(\sinh 2\tau - 2\tau)^{1/3}}{2^{1/3}\sinh\tau}. \tag{19}$$

The presence of the $\overline{\rm D3}$-branes perturbs the geometry so that

$$u_0 \sim q_0 \sim s_0 \sim \mathcal{S}, \tag{20}$$

where $\mathcal{S}$ is proportional to the number of D3-$\overline{\rm D3}$ pairs

$$\mathcal{S} \sim \frac{P\tau_{\rm D3}\kappa_{10}^2}{(g_sM\alpha')^2\tilde{\mathcal{V}}_\Omega}, \tag{21}$$

where $\tau_{\rm D3}$ is the tension of a D3-brane, and $\tilde{\mathcal{V}}_\Omega$ is the unwarped volume of the S^3 at the tip [51]. Since our interest will be only in the parametric dependence of the gaugino mass, we will not need the more detailed expressions for the solution found in [27], Sec. III C.

The fractional D3-branes of the KS solution, together with the additional D3-$\overline{\rm D3}$ pairs, produce nontrivial warping

$$h(\tau) = h_{\rm KS} + \frac{h_0}{\tau}, \tag{22}$$

where

$$h_{\rm KS}(\tau) = (g_sM\alpha')^2 2^{2/3}\varepsilon^{-8/3}I(\tau),$$
$$I(\tau) = \int_\tau^\infty dx \frac{x\coth x - 1}{\sinh^2 x}(\sinh 2x - 2x)^{1/3}, \tag{23}$$

and

$$h_0 \sim (g_sM\alpha')^2\varepsilon^{-8/3}\mathcal{S}. \tag{24}$$

The geometry exhibits a curvature singularity at $\tau = 0$, where the Ricci scalar behaves as

$$R \sim \frac{\mathcal{S}}{g_sM\tau}. \tag{25}$$

The lower bound (12) also follows from demanding that the Ricci scalar remains small in string units.

The fluxes are

$$B_2 = \frac{g_sM\alpha'}{2}[f(\tau)g_1 \wedge g_2 + k(\tau)g_3 \wedge g_4], \tag{26a}$$
$$H_3 = \frac{g_sM\alpha'}{2}\left[d\tau \wedge (f'(\tau)g_1 \wedge g_2 + k'(\tau)g_3 \wedge g_4) + \frac{1}{2}(k(\tau) - f(\tau))g_5 \wedge (g_1 \wedge g_3 + g_2 \wedge g_4)\right], \tag{26b}$$
$$F_3 = \frac{M\alpha'}{2}[(1 - F(\tau))g_5 \wedge g_3 \wedge g_4 + F(\tau)g_5 \wedge g_1 \wedge g_2 + F'(\tau)d\tau \wedge (g_1 \wedge g_3 + g_2 \wedge g_4)], \tag{26c}$$

with

$$f(\tau) = f_{\rm KS} + f_0, \qquad k(\tau) = k_{\rm KS} + \frac{k_0}{\tau^2},$$
$$F(\tau) = F_{\rm KS} + \frac{F_0}{\tau}, \tag{27}$$

where the KS solution is

$$f_{\rm KS}(\tau) = \frac{\tau\coth\tau - 1}{2\sinh\tau}(\cosh\tau - 1),$$
$$k_{\rm KS}(\tau) = \frac{\tau\coth\tau - 1}{2\sinh\tau}(\cosh\tau + 1), \tag{28}$$
$$F_{\rm KS}(\tau) = \frac{\sinh\tau - \tau}{2\sinh\tau},$$

and again

$$f_0 \sim k_0 \sim F_0 \sim \mathcal{S}. \tag{29}$$

These source the Ramond-Ramond 5-form,

$$F_5 = (1 + *_{10})\mathcal{F}_5,$$
$$\mathcal{F}_5 = \frac{g_sM^2\alpha'^2}{4}\ell(\tau)g_1 \wedge g_2 \wedge g_3 \wedge g_4 \wedge g_5, \tag{30a}$$

where

$$\ell(\tau) = f(1 - F) + kF. \tag{30b}$$

For the choice of parameters implicitly considered here, this solution, like that in [28], does not introduce a net amount of charge localized at the tip, since the D3- and $\overline{\rm D3}$-branes are added in pairs. However, H_3 and F_3 give rise to an effective D3 charge which is dual to the scale dependent effective 't Hooft coupling

$$g_sN_{\rm eff}(\tau) \sim (g_sM\alpha')^2\ell(\tau). \tag{31}$$

Finally, the SUSY-breaking 3-form fluxes give a nontrivial source for the dilaton,

$$\Phi(\tau) = \log g_s + \Phi_0\tau, \tag{32}$$

where $\Phi_0 \sim \mathcal{S}$. The axion in both KS and this perturbation is trivial, $C = 0$.

The $\mathbb{Z}_2$ R symmetry is realized geometrically as a shift in an angle $\psi \to \psi + 2\pi$ (as briefly reviewed in Appendix B, ψ ranges from 0 to 4π). Since the D3-$\overline{\text{D3}}$ pairs are smeared over the angular directions, the expressions for the bulk fields respect this shift symmetry. It is thus reasonable to assume that the SUSY-breaking state in the dual theory preserves the $\mathbb{Z}_2$ R symmetry.

In [26,52,53], it was argued that the existence of a nonvanishing gaugino mass for the world-volume gauge theory living on a D7-brane is related to the existence of 3-form flux with Hodge type (0, 3). Using the relations between the 1-form g_i and the holomorphic coordinates (2) reviewed in Appendix B, one can show that indeed the 3-form flux G_3 picks up such components in the above perturbation of KS. Using (B9) and (B11), the only nonvanishing component for KS is the (2, 1) component,

$$G^{(2,1)}_{3(\mathrm{KS})} = \frac{M\alpha'}{2\varepsilon^6}\bigg[\frac{\sinh 2\tau - 2\tau}{\sinh^5\tau}(\bar{z}_m dz_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j dz_k \wedge d\bar{z}_\ell) + \frac{2(1-\tau\coth\tau)}{\sinh^4\tau}(z_m d\bar{z}_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j dz_k \wedge dz_\ell)\bigg]. \tag{33}$$

However, the above perturbation includes nonvanishing values for all components

$$\delta G^{(2,1)}_3 \sim \frac{\mathcal{S}M\alpha'}{\varepsilon^6\tau^5}[c_1(\bar{z}_m dz_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j dz_k \wedge d\bar{z}_\ell) + c_2(z_m d\bar{z}_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j dz_k \wedge dz_\ell)], \tag{34a}$$

$$\delta G^{(1,2)}_3 \sim \frac{\mathcal{S}M\alpha'}{\varepsilon^6\tau^5}[c_3(z_m d\bar{z}_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j dz_k \wedge d\bar{z}_\ell) + c_4(\bar{z}_m dz_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j d\bar{z}_k \wedge d\bar{z}_\ell)], \tag{34b}$$

$$\delta G^{(3,0)}_3 \sim \frac{c_5\mathcal{S}M\alpha'}{\varepsilon^6\tau^3}(\bar{z}_m dz_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j dz_k \wedge dz_\ell), \tag{34c}$$

$$\delta G^{(0,3)}_3 \sim \frac{c_6\mathcal{S}M\alpha'}{\varepsilon^6\tau^3}(z_m d\bar{z}_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j d\bar{z}_k \wedge d\bar{z}_\ell), \tag{34d}$$

where the c_i are nonvanishing $\mathcal{O}(1)$ coefficients whose exact values we will not need. In contrast, only (1, 2) and (2, 1) components appeared in the large radius solution of [28].

It was shown in [26] that if the complex structure of the space changes, then the existence of g_{zz} and $g_{\bar{z}\bar{z}}$ components of the metric can give rise to additional contributions to the gaugino mass. Such components exist in this perturbation. Using (B12), the unwarped metric for the holographic and internal radial directions for the KS metric is Calabi-Yau

$$d\tilde{s}^2_6 = (\partial_i \partial_{\bar{j}} \mathcal{F}) dz_i d\bar{z}_j, \qquad \mathcal{F}'(\varepsilon^2\cosh\tau) = \varepsilon^{-2/3}K(\tau), \tag{35}$$

while the perturbation to the metric is not even Hermitian with respect to the original complex structure

$$(\varepsilon^4\sinh^2\tau)\delta(d\tilde{s}^2_6) \sim \frac{\mathcal{S}\varepsilon^{4/3}}{\tau}[d_1((\bar{z}_i dz_i)^2 + (z_i d\bar{z}_i)^2) + d_2(\bar{z}_i dz_i)(z_i d\bar{z}_i)] + \mathcal{S}\varepsilon^{10/3}\tau[d_3(dz_i dz_i + d\bar{z}_i d\bar{z}_i) + d_4 dz_i d\bar{z}_i], \tag{36}$$

where the d_i are another set of $\mathcal{O}(1)$ coefficients.

For the purposes of calculating the gaugino mass, it is useful to introduce another set of holomorphic 1-forms dZ_i (B14). Using (B17) and (B18), the components of G_3 can be written in these coordinates as

$$G_{3(\mathrm{KS})} = -\frac{M\alpha'}{16\sinh^2\tau}[4(\sinh\tau - \tau\cosh\tau) d\bar{Z}_1 \wedge dZ_2 \wedge dZ_3 + (\sinh 2\tau - 2\tau)(dZ_1 \wedge dZ_2 \wedge d\bar{Z}_3 - dZ_1 \wedge d\bar{Z}_2 \wedge dZ_3)], \tag{37}$$

while the perturbation to G_3 has components

$$\delta G^{(2,1)}_3 \sim -\frac{\mathcal{S}M\alpha'}{4\tau^2}[c_1(dZ_1 \wedge dZ_2 \wedge d\bar{Z}_3 - dZ_1 \wedge d\bar{Z}_2 \wedge dZ_3) + c_2 d\bar{Z}_1 \wedge dZ_2 \wedge dZ_3], \tag{38a}$$

$$\delta G^{(1,2)}_3 \sim +\frac{\mathcal{S}M\alpha'}{4\tau^2}[c_3(d\bar{Z}_1 \wedge d\bar{Z}_2 \wedge dZ_3 - d\bar{Z}_1 \wedge dZ_2 \wedge d\bar{Z}_3) + c_4 dZ_1 \wedge d\bar{Z}_2 \wedge d\bar{Z}_3], \tag{38b}$$

$$\delta G^{(3,0)}_3 \sim -(c_5\mathcal{S}M\alpha') dZ_1 \wedge dZ_2 \wedge dZ_3, \tag{38c}$$

$$\delta G^{(0,3)}_3 \sim +(c_6\mathcal{S}M\alpha') d\bar{Z}_1 \wedge d\bar{Z}_2 \wedge d\bar{Z}_3. \tag{38d}$$

The metric in these coordinates is (B15). In KS, this becomes

$$d\tilde{s}^2_6 = \frac{\varepsilon^{4/3}}{6K^2} dZ_1 d\bar{Z}_2 + \frac{\varepsilon^{4/3}K}{2}\sinh^2\frac{\tau}{2} dZ_2 d\bar{Z}_2 + \frac{\varepsilon^{4/3}K}{2}\cosh^2\frac{\tau}{2} dZ_3 d\bar{Z}_3, \tag{39}$$

while the perturbation to the metric is

$$\delta(d\tilde{s}^2_6) \sim \mathcal{S}\varepsilon^{4/3}\bigg[\frac{\hat{d}_1}{\tau} dZ_1 d\bar{Z}_1 + \hat{d}_2\tau dZ_2 d\bar{Z}_2 + \frac{\hat{d}_3}{\tau} dZ_3 d\bar{Z}_3 + \frac{\hat{d}_4}{\tau}(dZ_1 dZ_1 + d\bar{Z}_1 d\bar{Z}_1) + \hat{d}_5\tau(dZ_2 dZ_2 + d\bar{Z}_2 d\bar{Z}_2) + \frac{\hat{d}_6}{\tau}(dZ_3 dZ_3 + d\bar{Z}_3 d\bar{Z}_3)\bigg], \tag{40}$$

where the $\hat{d}_i$ are $\mathcal{O}(1)$ coefficients that can be written in terms of d_i.

III. GAUGINO MASSES FROM HOLOGRAPHY

Using the above SUSY-breaking gravity solution, we can now proceed to calculate the mass of a gaugino living on a stack of K probe D7-branes in this geometry. In order to neglect the back reaction of the D7-branes, we take $K \ll P \ll M$. Although it would be interesting to calculate the back reaction as in [36,54,55], such a calculation would lead to a self-energy problem when we try to calculate the mass of a gaugino living on the D7s. The calculation here is similar that of [26] though because of the reduced isometry of the geometry (which in the dual field theory corresponds to reduced R symmetry in the hidden sector), it leads to a nonvanishing result even to leading order in $\mathcal{S}$.

The starting point is the Dirac-like action for a D7-brane presented in [56] based largely on [57,58] and reviewed in Appendix A. This action is strictly speaking only valid in the Abelian (i.e. $K = 1$) case, but in the supergravity limit, we do not expect any deviations for the gaugino mass from the Abelian result [59]. The strategy is to find the effective mass for the gaugino that results from a dimensional reduction of the world-volume action to $\mathbb{R}^{1,3}$, which in the dual field theory, corresponds to calculating the mass resulting from all planar diagrams in the 't Hooft limit. We begin with an analysis of contributions to the gaugino mass from 3-form flux (some of which are nonvanishing). Similar considerations were performed in [26,52]. However, *a priori* there could be additional contributions from other bulk fields which we consider towards the end of this section.

In the KS background, the 3-form flux is imaginary self-dual (2, 1), and the gaugino remains massless [52]. Thus contributions to the gaugino mass will come from the non-SUSY perturbations to KS. Since the solution is known only to leading order in $\mathcal{S}$, we will be interested only in contributions to the gaugino mass that are also linear in $\mathcal{S}$.

A. Contributions from the 3-form flux

The fermionic action is written in terms of a bispinor

$$\Theta = \begin{pmatrix} \theta \\ \tilde{\theta} \end{pmatrix}, \tag{41}$$

where θ and $\tilde{\theta}$ are ten-dimensional (10D) Majorana-Weyl spinors of positive chirality. For the probe D7-branes, the contribution to the Dirac action from the 3-form flux can be written as a trace over gauge indices

$$S^{(3)}_{\rm D7} = \frac{i\tau_{\rm D7} g_s^{-1/2}}{8} \int d^8\xi e^{3\Phi/2}\sqrt{|\det \mathcal{M}|}\,\mathrm{tr}\bigg\{\bar{\Theta} P^{\rm D7}_{-}\bigg[2\mathcal{G}_3^{+} + (\hat{\mathcal{M}}^{-1})^{\alpha\beta}\Gamma_\beta\bigg(\mathcal{G}_3^{-}\Gamma_\alpha + \frac{1}{2}\Gamma_\alpha(\mathcal{G}_3^{-} - \mathcal{G}_3^{+})\bigg)\bigg]\Theta\bigg\}, \tag{42}$$

where in the absence of world-volume flux,

$$\mathcal{M}_{\alpha\beta} = \gamma_{\alpha\beta} + g_s^{1/2} e^{-\Phi/2} b_{\alpha\beta}, \tag{43}$$

with γ and b the pullbacks of the metric and the Neveu-Schwarz–Neveu-Schwarz (NS-NS) 2-form and

$$\hat{\mathcal{M}}_{\alpha\beta} = \begin{pmatrix} \mathcal{M}_{\beta\alpha} & 0 \\ 0 & \mathcal{M}_{\alpha\beta} \end{pmatrix}, \tag{44}$$

where ξ^α are the world-volume coordinates, and tensors with indices α, β denote pullbacks onto the world volumes of the branes. x^μ denotes a coordinate in the four large spacetime dimensions, while x^a are coordinates on the radial direction and the internal angular directions. When acting on the gaugino in the supersymmetric case, the projection operator can be written as $P^{\rm D7}_{\pm} = \frac{1}{2}(1 \mp \Gamma_{\rm D7})$ with [56,60]

$$\Gamma_{\rm D7} = \begin{pmatrix} 0 & i\Gamma_{(8)} \\ -i\Gamma_{(8)} & 0 \end{pmatrix}, \tag{45}$$

where $\Gamma_{(8)}$ is the usual eight-dimensional chirality operator. The solution presented in Sec. II A is no longer supersymmetric, but the deviation from Eq. (45) essentially gives a mixing term and so contributes to the gaugino mass at higher order in $\mathcal{S}$. Finally, the contribution from the 3-form flux is

$$\mathcal{G}_3^{\pm} = \frac{1}{3!}(\tilde{F}_{MNP}\sigma_1 \pm e^{-\Phi} H_{MNP}\sigma_3)\Gamma^{MNP}. \tag{46}$$

As it is well known, the fermionic part of the action has a redundant description of the fermionic degrees of freedom known as κ symmetry. We choose to eliminate the redundancy by taking the particular κ-fixing condition $\tilde{\theta} = 0$,

$$\Theta = \begin{pmatrix} \theta \\ 0 \end{pmatrix}. \tag{47}$$

To leading order in $\mathcal{S}$, we can take the gaugino wave function to be unperturbed by the addition of the $\overline{\rm D3}$-branes in which case it is given by [60]

$$\theta(x^\alpha) = \lambda(x^\mu) \otimes h^{3/8}\eta(x^a), \tag{48}$$

where η is covariantly constant with respect to the underlying Calabi-Yau (i.e. deformed conifold) metric and is annihilated by the holomorphic Γ matrices Γ_z [61]. Taking θ to have negative 6D chirality, the four-dimensional (4D) chirality is also negative,

$$\Gamma_{(4)}\theta = i\Gamma^{0123}\theta = -\theta. \tag{49}$$

As shown in [60], the gaugino has positive chirality with respect to the chirality operator for the internal four cycle wrapped by the D7-brane, $\Gamma^{\rm extra}$. This gives a positive eight-dimensional chirality.

$$\Gamma_{(8)}\theta = -\Gamma_{(4)}\Gamma^{\rm extra}\theta = \theta. \tag{50}$$

Using the above choice of κ-fixing, we find that the action (42) is

$$S_{\rm D7}^{(3)} = -\frac{\tau_{\rm D7} g_s^{-1/2}}{8\cdot 3!}\int d^8\xi e^{3\Phi/2}\sqrt{|\det\mathcal{M}|} \times \mathrm{tr}\Big\{\bar\theta\Big[G_{MNP}\Gamma^{MNP} + (\mathcal{M}^{-1})^{(\alpha\beta)}\Gamma_\beta \times\Big(G^*_{MNP}\Gamma^{MNP}\Gamma_\alpha + \frac{1}{2}\Gamma_\alpha(G^*_{MNP} - G_{MNP})\Gamma^{MNP}\Big)\Big] + (\mathcal{M}^{-1})^{[\alpha\beta]}\Gamma_\beta\Big(G_{MNP}\Gamma^{MNP}\Gamma_\alpha + \frac{1}{2}\Gamma_\alpha(G_{MNP} - G^*_{MNP})\Gamma^{MNP}\Big)\Big]\theta\Big\}, \quad (51)$$

with

$$G_3 = \tilde F_3 - ie^{-\Phi}H_3, \qquad G_3^* = \tilde F + ie^{-\Phi}H_3, \quad (52)$$

and $(\alpha\beta)$ and $[\alpha\beta]$ indicate symmetrization and antisymmetrization over the indices.

To find contributions to the gaugino mass, we must consider perturbations to the fields in (51). The perturbations to consider are those of the measure $e^{3\Phi/2}\sqrt{|\det\mathcal{M}|}$, the metric g_{MN}, $\mathcal{M}_{\alpha\beta}$, and the 3-form flux G_3 (as discussed below, perturbations of the Γ matrices contribute only higher order terms to the gaugino mass). Moreover, to leading order in $\mathcal{S}$, we need only consider the perturbations to one of these at a time.

1. Contributions from the perturbed 3-form flux

We first consider the contributions from the perturbed flux but unperturbed metric and, in particular, consider the term

$$\mathrm{tr}\{\bar\theta G_{MNP}\Gamma^{MNP}\theta\}. \quad (53)$$

G_3 has legs only on the holographic and internal directions, so

$$\mathrm{tr}\{\bar\theta G_{MNP}\Gamma^{MNP}\theta\} = \mathrm{tr}(\lambda^2)\tilde g^{mn}\tilde g^{sr}\tilde g^{pq}G_{msp}\eta^T\tilde\Gamma_{nrq}\eta, \quad (54)$$

where we have used (48) and related the warped Γ matrices to the unwarped ones

$$\Gamma_m = h^{1/4}\tilde\Gamma_m, \quad (55)$$

and $\tilde g$ is the unwarped bulk metric. Since $\tilde\Gamma_z\eta = 0$, this becomes

$$\mathrm{tr}\{\bar\theta G_{MNP}\Gamma^{MNP}\theta\} = \mathrm{tr}(\lambda^2)\tilde g^{i\bar i'}\tilde g^{j\bar j'}\tilde g^{k\bar k'}G_{ijk}\eta^T\tilde\Gamma_{\bar i'\bar j'\bar k'}\eta, \quad (56)$$

where i, j, k are holomorphic indices, and $\bar i$, $\bar j$, $\bar k$ are antiholomorphic. Terms that involve Γ matrices of mixed types (e.g. $\Gamma_{\bar i}\Gamma_j\Gamma_{\bar k}$) give rise to mixing terms and so contribute to the gaugino mass at higher order in the perturbation. Equation (56) implies that in addition to the (0, 3) contribution to the gaugino mass argued to exist in [52] and coming from (75), there is a contribution from the (3, 0) component. η is covariantly constant with respect to the Calabi-Yau metric, which allows us to write [62]

$$\eta^T\tilde\Gamma_{\bar i\bar j\bar k}\eta = \bar\Omega_{\bar i\bar j\bar k}, \quad (57)$$

where Ω is the holomorphic 3-form of the underlying Calabi-Yau. Thus, there is a contribution to the gaugino mass of the form

$$-\frac{\tau_{\rm D7} g_s^{-1/2}}{8\cdot 3!}\int_{\mathrm{R}^{1,3}} d^4x\,\mathrm{tr}(\lambda^2)\int_{\Sigma_4} d^4x e^{3\Phi/2}\sqrt{|\det\mathcal{M}|}\,\widetilde{\bar\Omega^{ijk}}G_{ijk}, \quad (58)$$

where $\tilde i$, etc. denote indices raised with the unwarped metric $\tilde\gamma$, and Σ_4 denotes the four cycle wrapped by the D7.

Since we have only calculated the perturbations due to the $\overline{\rm D3}$-branes as a small τ expansion, we cannot calculate the gaugino mass exactly and will therefore only be interested in a parametric dependence. Using (34c) and (B13), we find

$$\frac{1}{3!}\widetilde{\bar\Omega^{ijk}}G_{ijk} \sim \frac{M\alpha'}{\varepsilon^2\tau}\mathcal{S} + \mathcal{O}(\tau). \quad (59)$$

The terms that are higher order in τ have been omitted, since the integral in (58) will receive contributions only for small τ.

Because we are expanding to linear order in $\mathcal{S}$, all other fields are set to their background (KS) values. Expressions for the pullbacks of the metric and NS-NS 2-form are given in Appendix C. Even though it is possible to write an exact expression for $\det\mathcal{M}$, it is relatively complex, and because we are interested only in the parametric dependence, we will consider only the behavior for small τ. To illustrate the approximation we use for $\det\mathcal{M}$ and other fields, we first consider the determinant of the induced metric γ which has a simpler exact expression. In [26], it was shown that the determinant of the pulled-back metric is

$$\gamma = \frac{K^4(\mu^2-\varepsilon^2)^4}{16\varepsilon^{8/3}}K_2^2\cosh^2\frac{\rho}{2}\sinh^2\frac{\rho}{2}, \quad (60)$$

where

$$\varepsilon^2\cosh\tau = (\mu^2-\varepsilon^2)\cosh\rho + \mu^2, \quad (61a)$$

$$K(\tau) = \frac{(\sinh 2\tau - 2\tau)^{1/3}}{2^{1/3}\sinh\tau}, \quad (61b)$$

$$K_2(\tau) = \cosh\rho - \frac{(\mu^2-\varepsilon^2)\sinh^2\rho}{\varepsilon^2\sinh^2\tau}\Big(\cosh\tau - \frac{2}{3K^3}\Big). \quad (61c)$$

The stack of D7s extends to a minimum value of τ given by

$$\tau_{\min} = 2\,\mathrm{arccosh}\frac{\mu}{\varepsilon}, \quad (62)$$

and the integral (58) should be dominated by contributions from τ near this value since the SUSY-breaking fluxes are peaked at small τ. Expanding (60) about $\tau = \tau_{\min}$, we find

$$\gamma = \frac{1}{64 \cdot 2^{1/3} \varepsilon^{2/3}} \frac{(\mu^2 - \varepsilon^2 \cosh\tau_{\rm min})^3}{\sinh^3 \tau_{\rm min}} \times (\sinh 2\tau_{\rm min} - 2\tau_{\rm min})^{4/3} (\tau - \tau_{\rm min}) + \mathcal{O}((\tau - \tau_{\rm min})^2). \tag{63}$$

To trust that the small τ expansion is good, we must have that $\tau_{\rm min} < 1$, which implies that μ cannot be much larger than ε. For small $\tau_{\rm min}$,

$$\mu \approx \varepsilon\left(1 + \frac{1}{8}\tau^2_{\rm min}\right). \tag{64}$$

Then γ takes the approximate form

$$\gamma \approx \frac{2^{5/6}}{3^{4/3}} \varepsilon^{16/3} \tau^7_{\rm min} (\tau - \tau_{\rm min}), \tag{65}$$

where higher order terms in $\tau_{\rm min}$ have been dropped. Following a similar process for $\mathcal{M}$ gives the same parametric dependence

$$\sqrt{|\det \mathcal{M}|} \sim \varepsilon^{8/3} \tau^{7/2}_{\rm min} (\tau - \tau_{\rm min})^{1/2}. \tag{66}$$

Combining this with (59) and (58) becomes

$$\sim -\tau_{\rm D7} g_s \int_{\mathbb{R}^{1,3}} d^4x\, {\rm tr}(\lambda^2) \int_{\tau_{\rm min}}^{\tau_{\rm max}} d\tau \varepsilon^{8/3} \tau^{7/2}_{\rm min} (\tau - \tau_{\rm min})^{1/2} \times \frac{M\alpha'}{\varepsilon^2 \tau} \mathcal{S}, \tag{67}$$

where we have omitted the angular integrals since they do not contribute to the parametric dependence and where $\tau_{\rm max}$ represents some UV cutoff for the field theory. Defining $t = \tau/\tau_{\rm min}$, we find

$$\sim -\tau_{\rm D7} g_s M \alpha' \varepsilon^{2/3} \tau^4_{\rm min} \mathcal{S} \int_{\mathbb{R}^{1,3}} d^4x\, {\rm tr}(\lambda^2). \tag{68}$$

In order to extract the mass, the field needs to be canonically normalized. The 4D kinetic term is given by

$$i\tau_{\rm D7} g_s^{-1} \int d^8\xi e^{\Phi} \sqrt{|\det \mathcal{M}|} \bar{\Theta} P_-^{\rm D7} g^{\mu\nu} \Gamma_\mu \partial_\nu \Theta = \frac{i\tau_{\rm D7}}{2} \int_{\mathbb{R}^{1,3}} d^4x\, {\rm tr}(\lambda \not{\partial} \lambda) \int_{\Sigma_4} d^4x \sqrt{|\det \mathcal{M}|} h. \tag{69}$$

The 4D gauge coupling, which follows from dimensional reduction of the bosonic part of the D7 action and is identified with the visible sector SU(K) coupling, is given by [63]

$$\frac{1}{g^2_{\rm vis}} = \tau_{\rm D7} (2\pi\alpha')^2 \int_{\Sigma_4} d^4x \sqrt{|\det \mathcal{M}|} h, \tag{70}$$

so the kinetic term can be expressed as

$$\frac{1}{8\pi^2 \alpha'^2 g^2_{\rm vis}} \int_{\mathbb{R}^{1,3}} d^4x\, {\rm tr}(\lambda \not{\partial} \lambda). \tag{71}$$

Canonically normalizing the field amounts to dividing by the prefactor of (71) so (67) gives a contribution to the gaugino mass

$$\delta m_{1/2} \sim g_s M \varepsilon^{2/3} \tau^4_{\rm min} g^2_{\rm vis} \mathcal{S}. \tag{72}$$

In [28], the parameter $\mathcal{S}$ was related to the vacuum energy in the dual field theory

$$\mathcal{S} \sim \mathcal{S}^{\rm DKM} \varepsilon^{-8/3} \sim \left(\frac{\Lambda_{\mathcal{S}}}{\Lambda_\varepsilon}\right)^4, \tag{73}$$

so that

$$\delta m_{1/2} \sim g^2_{\rm vis} \lambda(\Lambda_\varepsilon) \frac{\Lambda^4_{\mathcal{S}}}{\Lambda^3_\varepsilon} \left(\left(\frac{m_\chi}{\Lambda_\varepsilon}\right)^{3/2} - 1\right)^2, \tag{74}$$

where $\lambda(\Lambda_\varepsilon) = g_s M$ is the 't Hooft coupling of the hidden sector SU(M) in the far IR, $m_\chi = \mu^{2/3}$ is the messenger mass, and $\Lambda_\varepsilon = \varepsilon^{2/3}$ is the confining scale.

Another contribution resulting from perturbing only the 3-form flux potentially comes from

$$ {\rm tr}\{\bar{\theta}(\mathcal{M}^{-1})^{(\alpha\beta)} \Gamma_\beta (G^*_{MNP} \Gamma^{MNP} \Gamma_\alpha + \tfrac{1}{2}\Gamma_\alpha (G^*_{MNP} - G_{MNP}) \Gamma^{MNP}) \theta\}. \tag{75}$$

After some manipulation of the Γ matrices, this can be written as

$$ - {\rm tr}\{\bar{\theta}(\tfrac{1}{2}(\mathcal{M}^{-1})^{(\alpha\beta)} \gamma_{\alpha\beta} (G_{MNP} + G^*_{MNP}) \Gamma^{MNP} - 6(\mathcal{M}^{-1})^{(\alpha\beta)} \Gamma_\alpha \Gamma_{MN} G^{*MN}_\beta) \theta\}. \tag{76}$$

Since γ and $\mathcal{M}^{-1}$ are unperturbed, they satisfy

$$(\mathcal{M}^{-1})^{(\alpha\beta)} \gamma_{\alpha\beta} = 8 - \frac{2^{2/3} \tau^2_{\rm min}}{3^{4/3} a_0} + \mathcal{O}(\tau^4_{\rm min}), \tag{77}$$

where we have used the pullbacks presented in Appendix C. Thus to leading order in $\tau_{\rm min}$, the first two terms of (76), which couple to the (3, 0) and (0, 3) parts of G_3 respectively, result in contributions to the gaugino mass that are parametrically the same as (74). The third term of (76) can be cast as

$$ {\rm tr}\left\{\bar{\theta} \frac{\partial x^p}{\partial \xi^a} \frac{\partial x^q}{\partial \xi^b} (\mathcal{M}^{-1})^{(ab)} g^{mn} g^{st} G^*_{pnt} \Gamma_q \Gamma_{ms} \theta\right\}, \tag{78}$$

where the D7-brane world volumes are specified by $x^M = x^M(\xi^\alpha)$. Using the gaugino wave function and the relation between the warped and unwarped Γ matrices, this becomes (up to combinatorial factors)

$$ {\rm tr}(\lambda^2) \frac{\partial x^p}{\partial \xi^\alpha} \frac{\partial x^q}{\partial \xi^\beta} (\tilde{\mathcal{M}}^{-1})^{\alpha\beta} \tilde{g}^{mn} \tilde{g}^{st} G^*_{pnt} \bar{\Omega}_{qms}, \tag{79}$$

where we have defined the "unwarped" NS-NS tensor

$$\tilde{M}_{ab} = \tilde{\gamma}_{ab} + h^{-1/2} g_s^{1/2} e^{-\Phi/2} b_{ab}. \tag{80}$$

Because of the nontrivial embedding, this term is more difficult to compute. However, a similar computation was considered in [26] in the KT region where it was useful to introduce holomorphic 1-forms that are analogous to

(B14). Since the world volumes of the D7-branes are specified by the holomorphic condition (3), the unperturbed induced metric is Hermitian. Similarly, the fact that the D7-branes are supersymmetric in the KS background implies that b_2 is (1, 1) [64,65], and therefore $\mathcal{M}$ has nonvanishing values only for the components with one holomorphic and one antiholomorphic index. Then, (78) can be written as

$$\mathrm{tr}\,(\lambda^2)\frac{\partial Z^I}{\partial \zeta^\sigma}\frac{\partial \bar{Z}^{\bar{I}'}}{\partial \bar{\zeta}^{\bar{\rho}}}(\tilde{\mathcal{M}}^{-1})^{(\sigma\bar{\rho})}\tilde{g}^{J\bar{J}'}\tilde{g}^{K\bar{K}'}G^*_{IJK}\bar{\Omega}_{\bar{I}'\bar{J}'\bar{K}'}, \quad (81)$$

where I, J, K indicate the coordinates used in (B14), and ζ are some complex coordinates on Σ_4 whose exact form we will not need. To leading order,

$$\tilde{g}^{J\bar{J}'}\tilde{g}^{K\bar{K}'}G^*_{IJK}\bar{\Omega}_{\bar{I}'\bar{J}'\bar{K}'} \sim \frac{M\alpha'\mathcal{S}}{\varepsilon^{2/3}}\begin{pmatrix}\tau^{-1} & & \\ & \tau & \\ & & \tau^{-1}\end{pmatrix}, \quad (82)$$

where $I = 1, 2, 3$ and where terms higher order in τ have been dropped.

To precisely calculate (81), we would need to transform from these coordinates to Z^I, taking into account the nontrivial pullback. However, since we are only interested in the leading parametric dependence, it will suffice to consider the component of the symmetrized $\mathcal{M}^{-1}$ which has the leading $(\tau - \tau_{\mathrm{min}})$ and τ_{min} behavior. Using the pullbacks presented in C, this component is

$$(\tilde{\mathcal{M}}^{-1})^{h_2h_2} \sim \frac{1}{\tau_{\mathrm{min}}(\tau - \tau_{\mathrm{min}})\varepsilon^{4/3}}. \quad (83)$$

One can show that in addition to complicated angular dependence, the coordinate transformation is parametrically effected by multiplication by τ^2_{min}. Putting these together, we find that the leading order behavior is

$$\frac{\partial x^p}{\partial \xi^\alpha}\frac{\partial x^q}{\partial \xi^\beta}(\tilde{\mathcal{M}}^{-1})^{(\alpha\beta)}\tilde{g}^{mn}\tilde{g}^{st}G^*_{pnt}\bar{\Omega}_{qms} \sim \frac{M\alpha'}{\varepsilon^2(\tau - \tau_{\mathrm{min}})}. \quad (84)$$

Comparing this to (59) which results in (74), we find that (84) contributes to the gaugino mass an amount that is parametrically the same as (74).

Further contributions due to perturbed 3-form flux possibly come from

$$\mathrm{tr}\{\bar{\theta}(\mathcal{M}^{-1})^{[\alpha\beta]}\Gamma_\beta(G_{MNP}\Gamma^{MNP}\Gamma_\alpha + \tfrac{1}{2}\Gamma_\alpha(G_{MNP} - G^*_{MNP})\Gamma^{MNP})\theta\}. \quad (85)$$

Following similar steps that lead to (76), this becomes

$$-\,\mathrm{tr}\{\bar{\theta}(\mathcal{M}^{-1})^{[\alpha\beta]}\Gamma_\beta(\Gamma_\alpha(G_{MNP} + G^*_{MNP})\Gamma^{MNP} - 6G_{\alpha NP}\Gamma^{NP})\theta\}. \quad (86)$$

Using the results in [60], when acting on the gaugino

$$(\mathcal{M}^{-1})^{ab}\Gamma_a\Gamma_b\theta = (\mathcal{M}^{-1})^{ba}\Gamma_a\Gamma_b\theta. \quad (87)$$

Since $(M^{-1})^{[\mu\nu]} = 0$, this implies (when acting on the gaugino)

$$(\mathcal{M}^{-1})^{[\alpha\beta]}\Gamma_\alpha\Gamma_\beta\theta = 0. \quad (88)$$

Thus (85) becomes

$$\mathrm{tr}\{\bar{\theta}(\mathcal{M}^{-1})^{[\alpha\beta]}\Gamma_\beta G_{\alpha NP}\Gamma^{NP}\theta\}. \quad (89)$$

This term involves a contraction similar to (B14)

$$\tilde{g}^{J\bar{J}'}\tilde{g}^{K\bar{K}'}G_{IJK}\bar{\Omega}_{\bar{I}'\bar{J}'\bar{K}'} \sim \frac{M\alpha'\mathcal{S}}{\varepsilon^{2/3}}\begin{pmatrix}\tau^{-1} & & \\ & \tau & \\ & & \tau^{-1}\end{pmatrix}. \quad (90)$$

The remaining indices are contracted with the antisymmetric part of $\mathcal{M}^{-1}$. For the KS background, the most leading part is (using the pullbacks in Appendix C)

$$(\mathcal{M}^{-1})^{[\rho h_2]} \sim \frac{1}{\varepsilon^{4/3}\tau^{1/2}_{\mathrm{min}}(\tau - \tau_{\mathrm{min}})^{1/2}}. \quad (91)$$

Comparing to (83), this is subleading in $(\tau - \tau_{\mathrm{min}})$ and τ_{min} and thus will give a subleading contribution to the gaugino mass.

2. Contributions from the perturbed metric

We next need to take into account terms that result from perturbing the metric while leaving the flux unperturbed. The perturbations to the metric that are of the form $\delta g_{z\bar{z}}$ or $\delta g_{\bar{z}z}$ will not contribute to the gaugino mass since, as shown above, when the metric is Hermitian, the unperturbed (2, 1) component of G_3 does not contribute. However, as pointed out in [26], when the perturbed metric is no longer Hermitian with respect to the original complex structure, there are in general contributions to the gaugino mass from the components of the 3-form flux with mixed holomorphic and antiholomorphic indices.

We again consider the first term in (42),

$$\mathrm{tr}\{\bar{\theta}G_{MNP}\Gamma^{MNP}\theta\} = (\lambda^2)\tilde{g}^{mn}\tilde{g}^{sr}\tilde{g}^{pq}G_{mnp}\eta^T\tilde{\Gamma}_{nrq}\eta. \quad (92)$$

If the metric is no longer Hermitian, then there is a contribution of the form

$$\mathrm{tr}\,(\lambda^2)\tilde{g}^{\bar{i}\bar{i}'}\tilde{g}^{j\bar{j}'}\tilde{g}^{k\bar{k}'}G_{\bar{i}jk}\bar{\Omega}_{\bar{i}'\bar{j}'\bar{k}'}. \quad (93)$$

We could of course consider the contractions with even more non-Hermitian parts (i.e. terms with $\tilde{g}^{\bar{i}\bar{i}'}\tilde{g}^{\bar{j}\bar{j}'}\tilde{g}^{k\bar{k}'}$), but since the nonperturbed metric is Calabi-Yau, these are higher order in $\mathcal{S}$. Using the solution in Sec. II A, we find

$$\tilde{g}^{\bar{i}\bar{i}'}\tilde{g}^{\bar{j}j'}\tilde{g}^{\bar{k}k'}G_{\bar{i}'j'k'}\bar{\Omega}_{\bar{i}\bar{j}\bar{k}} \sim \frac{M\alpha'\mathcal{S}}{\varepsilon^2\tau}. \quad (94)$$

Comparing to (59), we see that this term contributes an amount that is parametrically the same as (74).

Since we are for now neglecting the change in $\mathcal{M}^{-1}$, the next group of terms (75) can again be written as (76) but now considering the 3-form flux to be unperturbed and the non-Hermitian perturbations to the bulk metric. The first term in (76) again contributes parametrically the same as (92) since making use of (77) it is of a closely related form. The third term of (76) potentially has a contribution from the non-Hermitian perturbations of g,

$$\mathrm{tr}\,(\lambda^2)\frac{\partial Z^I}{\partial \zeta^\sigma}\frac{\partial \bar{Z}^{\bar{I}'}}{\partial \bar{\zeta}^{\bar{\rho}}}(\mathcal{M}^{-1})^{\sigma\bar{\rho}}\tilde{g}^{\bar{J}\bar{J}'}\tilde{g}^{K\bar{K}'}G^*_{I\bar{J}K}\bar{\Omega}_{\bar{I}'\bar{J}'\bar{K}'}, \tag{95}$$

where $\mathcal{M}^{-1}$ is unperturbed. Writing $G^*_{I\bar{J}K} = (G_{\bar{I}J\bar{K}})^*$, we see that this is a coupling to the (1, 2) component of G_3. The unperturbed flux for the KS solution is purely imaginary self-dual (2, 1), so this term vanishes and does not contribute to the gaugino mass. This argument which also applies to the second term of (76) G^*_{MNP} when the metric is perturbed but the flux is not.

Next, we consider contributions resulting from the perturbation of the symmetric part of $\mathcal{M}^{-1}$ in (76). Again, only the perturbations to the purely holomorphic and purely antiholomorphic parts could possibly contribute to a gaugino mass (perturbations to the components of mixed type, for example $\delta\mathcal{M}_{z\bar{z}}$, do not contribute to the gaugino mass to leading order in $\mathcal{S}$). The first two terms couple to the (3, 0) parts of G_3 and G_3^*, and the third term couples to the (1, 2) part of G_3. Since the unperturbed flux G_3 is (2, 1), all of these contributions vanish to leading order in $\mathcal{S}$.

There could additionally be contributions from the purely holomorphic and purely antiholomorphic perturbations to the antisymmetric part of $\mathcal{M}^{-1}$ in (86). The first two terms of (76) give a coupling to the (0,3) and (3, 0) parts of G_3 which vanish. However, the third term gives a coupling to the (2, 1) component of G_3 which is nonvanishing in KS. To leading order in τ, the contraction of the unperturbed fields gives

$$\tilde{g}^{\,J\bar{J}'}\tilde{g}^{K\bar{K}'}G_{\bar{I}JK}\bar{\Omega}_{\bar{I}'\bar{J}'\bar{K}'} \sim \frac{M\alpha'}{\varepsilon^{2/3}}\begin{pmatrix} 1 & & \\ & \tau^2 & \\ & & 1\end{pmatrix}. \tag{96}$$

The remaining indices are again contracted with the antisymmetric part of $\mathcal{M}^{-1}$. Since we are interested in only the parametric dependence, we consider the component of the antisymmetrized $\mathcal{M}^{-1}$ with the most singular dependence in $\tau_{\min}$ and $(\tau - \tau_{\min})$, focusing on the part proportional to $\mathcal{S}$ (since the parts not proportional to $\mathcal{S}$ cannot contribute here). The leading component is (see Appendix C)

$$(\tilde{M}^{-1})^{[\rho h_2]} \sim \frac{\mathcal{S}}{\tau_{\min}^{7/2}(\tau-\tau_{\min})^{1/2}\varepsilon^{4/3}}. \tag{97}$$

Taking into account the coordinate transformations, to leading order

$$\sqrt{|\det\mathcal{M}|}\frac{\partial x^p}{\partial \xi^\alpha}\frac{\partial x^q}{\partial \xi^\beta}(\tilde{\mathcal{M}}^{-1})^{[\alpha\beta]}\tilde{g}^{mn}\tilde{g}^{st}G_{pnt}\bar{\Omega}_{qms} \sim M\alpha'\varepsilon^{2/3}\tau_{\min}^2\mathcal{S}. \tag{98}$$

Comparing to (67) which yielded (74), we get the contribution to the gaugino mass

$$\delta m_{1/2} \sim g_s M\varepsilon^{2/3}\tau_{\min}^3 g_{\mathrm{vis}}^2\mathcal{S}. \tag{99}$$

In terms of the parameters of the dual field theory,

$$\delta m_{1/2} \sim g_{\mathrm{vis}}^2\lambda(\Lambda_\varepsilon)\frac{\Lambda_{\mathcal{S}}^4}{\Lambda_\varepsilon^3}\left(\left(\frac{m_\chi}{\Lambda_\varepsilon}\right)^{3/2} - 1\right)^{3/2}, \tag{100}$$

which parametrically contributes more significantly than the previous contributions.

3. Other 3-form contributions

For each of the above terms, we have neglected the fact that the measure $e^{3\Phi/2}\sqrt{|\det\mathcal{M}|}$ and the Γ matrices should also be modified in the new geometry. However, since we are working to first order in $\mathcal{S}$, perturbing the measure means to consider the Dirac-like operator (i.e. $\bar{\theta}\cdots\theta$) to be unperturbed. Since the unperturbed operator does not give a mass to the gaugino, perturbing the measure will not contribute to $m_{1/2}$ to linear order in $\mathcal{S}$.

Perturbations to the antiholomorphic Γ matrices are of the form

$$\delta\Gamma_{\bar{z}} \sim a\Gamma_{\bar{z}} + b\Gamma_z. \tag{101}$$

The perturbations proportional to the antiholomorphic matrices $\Gamma_{\bar{z}}$ will not give any new contribution to the gaugino mass. Since the metric is unperturbed and therefore Hermitian, the perturbations proportional to $\Gamma_{\bar{z}}$ give terms like

$$a\widetilde{\bar{\Omega}^{ijk}}(G_{\bar{i}\bar{j}\bar{k}})^*, \qquad a\widetilde{\bar{\Omega}^{ijk}}(G_{ijk}), \tag{102}$$

both of which vanish for KS. The perturbations proportional to the holomorphic matrices Γ_z will simply annihilate the gaugino, since to leading order in perturbation theory, the gaugino function is unchanged and so will only give rise to mixing terms. Similar arguments apply when considering perturbations to the holomorphic Γ matrices.

In addition to the above effects, one must take into account the fact that the $\overline{\mathrm{D3}}$-branes will interact with the D7-branes, though the consideration is very closely related to the above discussions. That is, $z_4 = \mu$ is a volume minimizing condition in the KS geometry, but when we perturb the geometry, this condition will no longer hold. The world volumes will be slightly perturbed so that the embedding is specified by

$$\mathcal{F}(z_i, \bar{z}_i; \mu, \varepsilon, \mathcal{S}) = 0, \tag{103}$$

for some function $\mathcal{F}$. To leading order in perturbation theory, we can consider a stack of D7-branes satisfying (103) in the original KS geometry. With this new condition, in general the D7-branes will no longer have a complex structure that is compatible with that of the bulk geometry [66]. For example, the pullback of a (1, 0)-form will not in general be a (1, 0)-form with respect to any world-volume complex structure. Since the existence of the gaugino mass depends on the Hodge types of the fluxes, this might result in a nonvanishing mass for the gaugino (which is simply the statement that if the D7-brane is not holomorphically embedded into the geometry, then it is not supersymmetric). However, the relative change in Hodge type is $\mathcal{O}(\mathcal{S})$. That is, if w_σ are complex coordinates on the D7 world volumes, then

$$P[dz] \sim dw + \mathcal{S} d\bar{w}. \tag{104}$$

A possible gaugino mass could arise from the term [67]

$$(\mathcal{M}^{-1})^{(ab)} g^{mn} g^{st} \bar{\theta} \Gamma_{ams} G^*_{bnt} \theta \sim (\mathcal{M}^{-1})^{(ab)} g^{mn} g^{st} \bar{\Omega}_{ams} G^*_{bnt}. \tag{105}$$

Since the complex structure of the world volumes may be different than that of the bulk, this is generally of the form

$$(\mathcal{M}^{-1})^{(\sigma\bar{\rho})} g^{i\bar{i}'} g^{j\bar{j}'} (\bar{\Omega}_{\sigma\bar{i}'\bar{j}'} G^*_{\bar{\rho}ij} + \bar{\Omega}_{\bar{\rho}\bar{i}'\bar{j}'} G^*_{\sigma ij}), \tag{106}$$

where we have chosen the world-volume complex structure such that the $\mathcal{M}$ is nonvanishing only for components with one holomorphic and one antiholomorphic index. In the KS background, however, both of these terms vanish. Considering the first term, since $\bar{\Omega}_{\sigma\bar{i}'\bar{j}'}$ has a change in the complex structure [σ is a holomorphic world-volume index but $\bar{\Omega}$ is (0, 3) in the bulk], it is proportional to $\mathcal{S}$. Therefore, the only part of $G^*_{\bar{\rho}ij}$ that contributes is such that the Hodge type is compatible with the bulk complex structure; that is, the part that contributes is the part of G_3^* that is (2, 1) with respect to the bulk complex structure as well. Since the (2, 1) part of G_3^* is essentially the (1, 2) part of G_3 and to this order in perturbation, the flux is unperturbed, $G^*_{\bar{\rho}ij} = 0$. Similarly, the second term couples to the (3, 0) and (2, 1) parts of G_3^* (with respect to the bulk complex structure). Both of these vanish in KS, and so the second term vanishes as well.

The arguments for the vanishing of these terms were very similar to those for the perturbations to the symmetrized part of $\mathcal{M}$ considered in Sec. III A 2. An analogous argument for the antisymmetric part of $\mathcal{M}^{-1}$ would show that there is a coupling to the bulk (2, 1) part of G_3 when the world volumes are perturbed. Although it would be necessary to calculate $\mathcal{F}$ appearing in (103) to calculate this exactly, we expect that it should be parametrically similar to (100).

B. Contributions from the 5-form flux

All of the above subsections focused on the contributions related to the 3-form flux and were similar to discussions in [26,52]. However, in principle there could be additional contributions from other SUSY-breaking bulk fields.

For example, there is the possibility of a mass arising from the 5-form flux. In the SUSY case, the 5-form flux is related to the warp factor so we must consider the spin connection as well. The action contains

$$i\tau_{D7} g_s^{-1} \int d^8\xi e^{\Phi} \sqrt{|\det \mathcal{M}|} \times \mathrm{tr}\Big\{ \bar{\Theta} P_-^{D7} (\hat{\mathcal{M}}^{-1})^{\alpha\beta} \Gamma_\beta \Big(\nabla_\alpha + \frac{g_s}{16 \cdot 5!} \tilde{F}_{NPQRT} \times \Gamma^{NPQRT} \Gamma_\alpha (i\sigma_2) \Big) \Theta \Big\}. \tag{107}$$

∇_α is the pullback of the covariant derivative which has components

$$\nabla_\mu = \partial_\mu - \tfrac{1}{8} \Gamma_\mu \not{\partial} \log h, \tag{108}$$

$$\nabla_m = \tilde{\nabla}_m + \tfrac{1}{8} \Gamma_m \not{\partial} \log h - \tfrac{1}{8} \partial_m \log h, \tag{109}$$

where $\tilde{\nabla}$ is the covariant derivative with respect to the unwarped 6D metric, which in this subsection, we take to be unperturbed. Following [60], and using the κ-fixing condition (47), this becomes (when acting on the gaugino)

$$\frac{i\tau_{D7}}{2g_s} \int d^8\xi e^{\Phi} \sqrt{|\det \mathcal{M}|} \,\mathrm{tr}\Big\{ \bar{\theta} \Big[(\mathcal{M}^{-1})^{ab} \Gamma_a \Big(\tilde{\nabla}_b - \frac{1}{8} \partial_b \log h \Big) + \frac{(g_s M \alpha')^2}{16} \ell(\tau) \frac{\sqrt{p}}{\sqrt{u} s q h} (\mathcal{M}^{-1})^{ab} (\partial_a \tau) \Gamma_b - \frac{1}{2} \Big(1 - \frac{1}{4} (\mathcal{M}^{-1})^{ab} \Gamma_a \Gamma_b \Big) \not{\partial} \log h + \frac{(g_s M \alpha')^2}{8} \ell(\tau) \frac{\sqrt{p}}{\sqrt{u} s q h} \Big(1 - \frac{1}{4} (\mathcal{M}^{-1})^{ab} \Gamma_b \Gamma_a \Big) \Gamma^\tau \Big] \theta \Big\}, \tag{110}$$

where we have omitted the 4D kinetic term since it does not contribute to a mass term. Since θ is a Majorana-Weyl spinor, any bilinear $\bar{\theta} \Gamma_M \theta$ vanishes. Therefore, since the gaugino wave function behaves as $h^{3/8}\eta$, where η is covariantly constant with respect to $\tilde{\nabla}$, (110) becomes

$$\frac{i\tau_{D7}}{16 g_s} \int d^8\xi e^{\Phi} \sqrt{|\det \mathcal{M}|} \,\mathrm{tr}\Big\{ \bar{\theta} (\mathcal{M}^{-1})^{ab} \Big[\Gamma_a \Gamma_b \not{\partial} \log h - \frac{(g_s M \alpha')^2}{4} \ell(\tau) \frac{\sqrt{p}}{\sqrt{u} s q h} \Gamma_b \Gamma_a \Gamma^\tau \Big] \theta \Big\}. \tag{111}$$

Following similar arguments for the 3-form flux above, to obtain the contribution to the gaugino mass to linear order in $\mathcal{S}$, we consider perturbations to one field at a time. Perturbations to h, ℓ, or any of the metric functions p, u, or s do not give a contribution as the unperturbed $\mathcal{M}^{-1}$ has nonvanishing elements only for components with one holomorphic and one antiholomorphic index so that

$$\bar{\theta} (\mathcal{M}^{-1})^{ab} \Gamma_a \Gamma_b \Gamma^\tau \theta \tag{112}$$

consists of mixed holomorphic and antiholomorphic Γ

matrices. A similar argument applies if we consider perturbing the measure $e^{\Phi}\sqrt{|\det \mathcal{M}|}$.

The next potential contribution is from considering the perturbation to $\mathcal{M}^{-1}$. Since the perturbed $\mathcal{M}^{-1}$ contains pieces that are non-Hermitian, this may *a priori* contribute to the gaugino mass. The remaining fields are not perturbed from their KS values which satisfy

$$h'(\tau) = -\frac{(g_s M\alpha')^2}{4}\ell\frac{\sqrt{p}}{\sqrt{u}sq}, \tag{113}$$

so that (111) gives

$$\frac{i\tau_{\mathrm{D7}}}{16}\int d^8\xi\sqrt{|\det\mathcal{M}|}\,\mathrm{tr}\{\bar{\theta}(\mathcal{M}^{-1})^{ab}\{\Gamma_a,\Gamma_b\}\not{\partial}\log h\theta\}. \tag{114}$$

Using the Clifford algebra, the term in the trace is

$$(\mathcal{M}^{-1})^{ab}\gamma_{ab}\bar{\theta}\not{\partial}\log h\theta = 0, \tag{115}$$

where we have again used the fact that $\bar{\theta}\Gamma_M\theta = 0$. Perturbations to the Γ matrices will give the same form (115) except γ, rather than $\mathcal{M}^{-1}$ is perturbed, and so the term will also vanish.

Similar to the consideration of the 3-form fluxes, we must also consider the effect of the deformation of the world volumes. However, for the 3-form fluxes, the important aspect was the change in complex structure as a result of the pullback. In this case, the complex structure of the pullbacks of the 5-form flux and the spin connection are not important for arguing for the vanishing of the mass. Therefore, considering the effect of the perturbation of the world volumes is equivalent to considering perturbations to the fields and all of these contributions vanish.

C. Contributions from the perturbed spin connection

Additional contributions could potentially arise from the perturbed spin connection. The 6D manifold is perturbed from the deformed conifold geometry so that it is no longer conformally Calabi-Yau. Contained within the D7 Dirac-like action is the term

$$\frac{i\tau_{\mathrm{D7}}}{2g_s}\int d^8\xi\,\mathrm{tr}\{\bar{\theta}(\mathcal{M}^{-1})^{ab}\Gamma_a\tilde{\nabla}_b\theta\}, \tag{116}$$

where $\tilde{\nabla}$ is the pullback of the covariant derivative with respect to the unwarped 6D metric. When $\tilde{\nabla}$ is unperturbed, the fact that η is covariantly constant causes this term to automatically vanish. Therefore, in considering nonvanishing contributions, we need only to consider perturbations to $\tilde{\nabla}$. $\tilde{\nabla}$ is given by

$$\tilde{\nabla}_a = \partial_a + \tfrac{1}{4}\tilde{\omega}_a^{MN}\Gamma_{\underline{M}\,\underline{N}}, \tag{117}$$

where $\tilde{\omega}$ is the spin connection built from the unwarped 6D metric. Perturbations to the spin connection then give the contribution

$$(M^{-1})^{ab}g^{mn}g^{st}\delta\tilde{\omega}_{ams}\bar{\Omega}_{bnt}, \tag{118}$$

where we only consider the perturbations to ω, the unperturbed part being cancelled by the derivative ∂_a. A detailed calculation shows that this contraction vanishes for the isometry-preserving perturbation considered here. Some terms in the perturbed spin connection have the wrong Hodge type to contract with $\bar{\Omega}$. The sum over the remaining terms vanish based on the symmetries of $\delta\tilde{\omega}$. Thus there are no contributions to the visible sector gaugino mass coming from considering the perturbations to the 6D unwarped spin connection.

The components ∇_μ are also perturbed in this geometry. However, since this geometry is unperturbed from Minkowski space, the only perturbation to the covariant derivative comes from the perturbations to the warp factor which were considered in Sec. III B.

IV. CONCLUSION

In this paper, we have used to the language of the gauge-gravity correspondence to consider the effects of strong coupling dynamics on a relative of semidirect gauge mediation. In particular, we examined the holographic gauge mediation scenario of [26], where the hidden sector is a cascading $SU(N+M)\times SU(N)$ gauge theory, but considered the regime where the messenger mass m_χ was comparable (and in fact very close to) the confinement scale Λ_ε. In the gravity dual, this required use of one of the solutions presented in [27], which described the influence of a $\overline{\mathrm{D3}}$ on the near-tip geometry of the warped deformed conifold. The confining dynamics of the strongly coupled gauge theory breaks the R symmetry to $\mathbb{Z}_2$, which allows the gaugino to get a mass from physics above m_χ. To leading order in the SUSY-breaking order parameter,

$$\delta m_{1/2} \sim g_{\mathrm{vis}}^2\lambda(\Lambda_\varepsilon)\frac{\Lambda_{\mathcal{S}}^4}{\Lambda_\varepsilon^3}\left(\left(\frac{m_\chi}{\Lambda_\varepsilon}\right)^{3/2} - 1\right)^{3/2}, \tag{119}$$

where Λ_ε is the hidden sector confining scale, $\Lambda_{\mathcal{S}}$ is the vacuum energy, m_χ is the messenger mass, g_{vis} is the visible sector gauge coupling, and $\lambda(\Lambda_\varepsilon)$ is the hidden sector 't Hooft coupling evaluated at the scale Λ_ε [where the cascade has ended so the gauge group is the simple group $SU(M)$]. From the many possible terms that *a priori* could have given rise to a nonvanishing contributions, the only nonvanishing contributions come from the 3-form flux on the gravity side of the duality.

There are additional contributions from physics below the messenger mass. In particular, χ and $\tilde{\chi}$ bind into weakly interacting mesons, and the spectrum contains mesons with masses below m_χ. For $m_\chi \gg \Lambda_\varepsilon$, this contribution was calculated in [26] resulting in (5). Although we did not calculate the contribution from the mesons in this geometry, the mesons are weakly coupled in the large 't Hooft coupling limit, while χ and $\tilde{\chi}$ are strongly coupled. Thus we expect the contribution to the gaugino mass from any one meson to be highly suppressed by 't Hooft coupling compared to the contributions from χ.

The 't Hooft enhancement of (119) is quite different than the leading order contribution in the regime $m_\chi \gg \Lambda_\varepsilon$. In this regime, considered in [26], the large R symmetry at high energies suppresses contributions to the gaugino mass from physics above the messenger mass m_χ, and the leading order contribution comes from the 't Hooft suppressed interactions of mesonic bound states of the messenger quarks [68]. The fact that meson messengers are weakly coupled in the 't Hooft limit required the authors of [26] to use a combination of perturbative field theory and holographic techniques. In contrast, the reduced amount of R symmetry allowed us to compute the leading order contribution to the gaugino mass using only holography as the effective degrees of freedom are strongly coupled messengers.

Identification of the SUSY-breaking state in the dual gauge theory relies on the large radius behavior of the bulk gravitational fields. Since the solution used here is a small τ expansion, it is not useful for such an analysis. However, even using the large radius solution, it is not clear how to make the identification of the state in terms of dominant F-term or D-term breaking [28], and indeed at strong coupling, the distinction may not be sharp (though the weakly coupled mesonic states discussed in [26] do feel effective F-terms). However, to leading order in $\tau_{\min}$, the contribution (119) can expressed as

$$\delta m_{1/2} \sim g_{\rm vis}^2 \lambda(\Lambda_\varepsilon) \frac{\Lambda_S^4}{m_\chi^3} \left(\left(\frac{m_\chi}{\Lambda_\varepsilon} \right)^{3/2} - 1 \right)^{3/2}. \tag{120}$$

This suggests that there is some F-term component to the SUSY-breaking given by

$$F = \sqrt{\lambda(\Lambda_\varepsilon)} \Lambda_S^2. \tag{121}$$

The contribution to the gaugino mass from physics above m_χ is then

$$\delta m_{1/2} \sim g_{\rm vis}^2 \frac{F^2}{m_\chi^3} \left(\left(\frac{m_\chi}{\Lambda_\varepsilon} \right)^{3/2} - 1 \right)^{3/2}. \tag{122}$$

The fact that the gaugino mass occurs at higher order in F is similar to other examples of semidirect gauge mediation where $m_{1/2}$ vanishes to leading order in F [19,26,30].

This contribution to the gaugino mass naïvely vanishes for $m_\chi = \Lambda_\varepsilon$. We emphasize however that at this point, the supergravity description breaks down since the D7s reach the curvature singularity where stringy effects are important and there may be important corrections to (120). However, the fact that it decreases with $\tau_{\min}$ may not be surprising. The statement that the integral is dominated near $\tau_{\min}$ corresponds to the statement on the field theory side that the dominant contribution to the gaugino mass is from physics near m_χ. Decreasing $\tau_{\min}$ corresponds to taking m_χ closer to Λ_ε and so for smaller $\tau_{\min}$, the integral is dominated by physics at lower scales. Since the effective 't Hooft coupling decreases as the scale decreases, heuristically one might expect that this contribution to the gaugino mass also decreases.

The contribution (119) is a result of a calculation of a soft SUSY-breaking term that, unlike many other examples, at no point required the assumption of weak coupling in messenger or visible sectors aside from the gauge coupling to the standard model. Because the solutions presented in [27] were given only as a power series in τ, it was not possible to find an exact expression for the gaugino mass. However, we emphasize that this difficulty is a very distinct difficulty from that usually faced by strong coupling in that one could in principle use the gauge-gravity correspondence to find an exact result in the limit of large 't Hooft coupling. In contrast, without holography it is not clear how even to perform this calculation, even in principle.

The messenger mass parameter μ has unit R charge, so one would expect that for $\mu \neq 0$, even in the regime $m_\chi \gg \Lambda_\varepsilon$, there would be a contribution to $m_{1/2}$ from physics at all scales. However, for energies above m_χ, the messenger quark χ is no longer integrated out of the effective field theory, and the R-symmetry breaking effects are suppressed such that contributions to the gaugino mass occur only at subleading order in Λ_S. At least as far as the gaugino mass is concerned, the R-symmetry breaking effect of a nonvanishing μ is less important than the R-symmetry breaking effect of confinement, though it would be worthwhile to develop a clearer picture. One possible step in this direction would be to take into account the back reaction of the D7-branes, essentially moving away from the quenched approximation in the field theory. For general μ, such a back reaction would break the symmetry of the solution that is dual to the field theory R symmetry.

It is well known that for theories of semidirect or direct gauge mediation in which the hidden sector has large rank, one typically runs into a problem of visible sector Landau poles. Even in the regime of $m_\chi \gg \Lambda_\varepsilon$ discussed in [26], avoidance of visible sector Landau poles forced m_χ to be large. However, it was also suggested in [26] that the problem may be avoided by orbifolding the geometry. Although such a method might be needed to achieve realistic soft terms, we defer such analysis to future work.

An additional interesting future direction would be to holographically realize the visible sector matter fields as well. In the model of [26], the matter fields are taken to be elementary fields, living on the UV cutoff with other soft terms resulting from gaugino mediation. A more complete holographic realization of gauge mediation could involve a more detailed model in which the supersymmetric standard model (or some extension) is realized on a network of intersecting D7-branes carrying nonvanishing world-volume flux.

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APPENDIX A: CONVENTIONS

Our index notation is summarized in Table I.

We work in the type IIB supergravity limit of string theory where the low energy effective action in the 10D Einstein frame is [69]

$$S_{\rm IIB} = S^{\rm NS}_{\rm IIB} + S^{\rm R}_{\rm IIB} + S^{\rm CS}_{\rm IIB}, \tag{A1a}$$

$$S^{\rm NS}_{\rm IIB} = \frac{1}{2\kappa^2_{10}}\int d^{10}x\sqrt{-\det(g)} \times \left[R - \frac{1}{2}\partial_M\Phi\partial^M\Phi - \frac{g_s}{2\cdot 3!}e^{-\Phi}H_3^2\right], \tag{A1b}$$

$$S^{\rm R}_{\rm IIB} = -\frac{1}{4\kappa^2_{10}}\int d^{10}x\sqrt{-\det(g)} \times \left[e^{2\Phi}\partial_M C\partial^M C + \frac{g_s e^{\Phi}}{3!}\tilde F_3^2 + \frac{g_s^2}{2\cdot 5!}\tilde F_5^2\right], \tag{A1c}$$

$$S^{\rm CS}_{\rm IIB} = \frac{g_s^2}{4\kappa^2_{10}}\int C_4\wedge H_3\wedge F_3, \tag{A1d}$$

TABLE I. Index conventions. Exceptions should be clear from context. An index with a tilde (e.g. $\tilde\alpha$) indicates an index raised with the unwarped metric.

Label	Airection
M, N	Any direction
μ, ν	4D Minkowski
m, n	Radial and internal angular
α, β	D7 world-volume coordinate
a, b	D7 world-volume coordinate: radial or internal angular
i, j	Complex coordinate z_i defined in (B1)
I, J	Complex coordinates Z_I defined in (B14)
σ, ρ	Complex D7 world-volume coordinates

where in terms of the Ramond-Ramond potentials C, C_2, and C_4 and the NS-NS potential B_2,

$$\tilde F_3 = dC_2 - CH_3, \qquad \tilde F_5 = dC_4 + B_2\wedge dC_2. \tag{A2}$$

R is the Ricci scalar built from the metric g, and Φ is the dilaton such that $\langle\Phi\rangle = \log g_s$. The self-duality of the 5-form field strength is imposed at the level of the equations of motion, and we write

$$F_5 = (1 + *_{10})\mathcal{F}_5, \tag{A3}$$

where $*_{10}$ is the Hodge star built from g. The gravitational coupling is $2\kappa^2_{10} = (2\pi)^7\alpha'^4 g_s^2$, and the Einstein-frame metric g_{MN} is related to the string frame metric g^s_{MN} by the Weyl transformation $g_{MN} = g_s^{1/2}e^{-\Phi/2}g^s_{MN}$.

In the Einstein frame, the action for a Dp-brane is

$$S_{{\rm D}p} = S^{\rm DBI}_{{\rm D}p} + S^{\rm CS}_{{\rm D}p} + S^{\rm F}_{{\rm D}p}, \tag{A4}$$

where the bosonic part is

$$S^{\rm DBI}_{{\rm D}p} = -\tau_{{\rm D}p}g_s^{-((p-3)/4)\Phi}\int d^{p+1}\xi e^{-((p-3)/4)}\sqrt{|\det(\mathcal{M}_{\alpha\beta})|}, \tag{A5}$$

$$S^{\rm CS}_{{\rm D}p} = \tau_{{\rm D}p}\int P\left[\sum_n C_n\wedge e^{-B_2}\right]\wedge e^{2\pi\alpha' f_2}, \tag{A6}$$

with

$$\mathcal{M}_{\alpha\beta} = \gamma_{\alpha\beta} + g_s^{1/2}e^{-\Phi/2}b_{\alpha\beta} + g_s^{1/2}e^{-\Phi/2}(2\pi\alpha')f_{\alpha\beta}, \tag{A7}$$

where $\gamma_{\alpha\beta}$ and $b_{\alpha\beta}$ are the pullbacks of the metric and the NS-NS 2-form potential onto the world volume of the brane, $f_2 = dA_1$ is the field strength for the vector potential living on the world volume, $P[\cdot]$ indicates a pullback, and the tension of a Dp-brane satisfies $\tau^{-1}_{{\rm D}p} = (2\pi)^p\alpha'^{(p+1)/4}g_s^{-1}$. $\overline{{\rm D}p}$-branes are distinguished by an overall sign in front of the Chern-Simons piece. Although the generalization to the non-Abelian case is known [70], it is not needed for our purposes.

The fermionic action for a Dp-brane can be expanded out to quadratic order to give a Dirac-like action [56]. In the Einstein frame, this is given by [71]

$$S^{\rm (F)}_{{\rm D}p} = i\tau_{{\rm D}p}g_s^{-((p-3)/4)}\int d^8\xi e^{((p-3)/4)\Phi}\sqrt{\det(\mathcal{M})}\bar\Theta P_-^{{\rm D}p} \times\left[(\mathcal{M}^{-1})^{\alpha\beta}\Gamma_\beta\left(\mathcal{D}_\alpha + \frac{1}{4}\Gamma_\alpha\mathcal{O}\right) - \mathcal{O}\right]\Theta, \tag{A8}$$

where $\mathcal{D}_\alpha$ and $\mathcal{O}$ are the pullbacks of operators involved in the Einstein-frame SUSY transformations of the gravitino and dilatino,

$$\delta\Psi_M = \mathcal{D}_M\epsilon, \qquad \delta\lambda_\Phi = \mathcal{O}\epsilon, \tag{A9}$$

where these are related to string frame fields by

$$\epsilon = g_s^{1/8} e^{-\Phi/8} \epsilon^s, \qquad \lambda_\Phi = g_s^{-1/8} e^{\Phi/8} \lambda^s,$$
$$\Psi_M = g_s^{1/8} e^{-\Phi/8}\left(\Psi_M^s - \tfrac{1}{4}\Gamma_M^s \lambda_\Phi^s\right), \tag{A10}$$

and similarly $\Theta = g_s^{1/8} e^{-\Phi/8}\Theta^s$. $\Theta = (\theta_1 \theta_2)^{\mathrm{T}}$ is a doublet of 10D Majorana-Weyl spinors satisfying $\Gamma_{(10)}\theta_i = \theta_i$, where $\Gamma_{(10)}$ is the 10D chirality operator. $\bar{\Theta}$ is defined by

$$\bar{\Theta} = (\bar{\theta}_1 \quad \bar{\theta}_2). \tag{A11}$$

Following [56], we take the Γ matrices to be real implying $\bar{\theta} = \theta^T \Gamma^{\underline{0}}$, where underlined indices denote "flat" Γ matrices. $P_-^{\mathrm{D}p}$ is a projection operator defined by

$$P_\pm^{\mathrm{D}p} = \frac{1}{2}(1 \pm \Gamma_{\mathrm{D}p}) = \frac{1}{2}\begin{pmatrix} 1 & \pm\check{\Gamma}_{\mathrm{D}p}^{-1} \\ \pm\check{\Gamma}_{\mathrm{D}p} & 1 \end{pmatrix}, \tag{A12}$$

with

$$\check{\Gamma}_{\mathrm{D}p} = i^{(p-2)(p-3)}\Gamma_{\mathrm{D}p}^{(0)}\Lambda(\mathcal{F}), \tag{A13}$$

where $\Gamma_{\mathrm{D}p}^{(0)}$ is given by

$$\Gamma_{\mathrm{D}p}^{(0)} = \frac{1}{(p+1)!\sqrt{-\det(\gamma)}}\epsilon_{\alpha_1\ldots\alpha_{p+1}}\Gamma^{\alpha_1\ldots\alpha_{p+1}}, \tag{A14}$$

and

$$\Lambda(\mathcal{F}) = \frac{\sqrt{\det(\gamma)}}{\sqrt{\det(M)}}\sum_q \frac{(g_s e^{-\Phi/2})^{q/2}}{q! 2^q}\mathcal{F}_{\alpha_1\alpha_2}\cdots\mathcal{F}_{a_{2q-1}\alpha_{2q}} \times \Gamma^{\alpha_1\cdots\alpha_{2q}}, \tag{A15}$$

where $\mathcal{F}_2 = b_2 + 2\pi\alpha' f_2$. In terms of the usual $(2k+2)$-dimensional chirality matrix

$$\Gamma_{(2k+2)} = i^k \Gamma^{\underline{0\cdots 2k+1}}, \tag{A16}$$

where $\Gamma^{\underline{M}}$ is a flat Γ matrix, and we have

$$i^{(p-2)(p-3)}\Gamma_{\mathrm{D}p}^{(0)} = i^{(p-1)/2}\Gamma_{(p+1)}. \tag{A17}$$

Except for $\Gamma_{\mathrm{D}p}$, all Γ matrices act as identity on the doublet space,

$$\Gamma_M \Theta = \begin{pmatrix} \Gamma_M \theta_1 \\ \Gamma_M \theta_2 \end{pmatrix}. \tag{A18}$$

In IIB,

$$\hat{\mathcal{M}}_{\alpha\beta} = \gamma_{\alpha\beta} + g_s^{1/2} e^{-\Phi/2}\mathcal{F}_{\alpha\beta}\Gamma_{(10)} \otimes \sigma_3, \tag{A19}$$

where the Pauli matrices act on the doublet space so that we can effectively write

$$\bar{\Theta}\hat{\mathcal{M}}_{\alpha\beta} = \bar{\Theta}\begin{pmatrix} \mathcal{M}_{\beta\alpha} & \\ & \mathcal{M}_{\alpha\beta} \end{pmatrix}. \tag{A20}$$

In the IIB Einstein frame,

$$\mathcal{O} = \tfrac{1}{2}\Gamma^M \partial_M \Phi - \tfrac{1}{2}e^{\Phi}\Gamma^M \partial_M C(i\sigma_2) - \tfrac{1}{4}g_s^{1/2} e^{\Phi/2}\mathcal{G}_3^+, \tag{A21a}$$

$$\mathcal{D}_M = \nabla_M + \frac{1}{4}e^{\Phi}\partial_M C(i\sigma_2) + \frac{1}{8}e^{\Phi/2}g_s^{1/2}\left(\mathcal{G}_3^- \Gamma_M + \frac{1}{2}\Gamma_M \mathcal{G}_3^-\right) + \frac{1}{16\cdot 5!}g_s \tilde{F}_{NPQRT}\Gamma^{NPQRT}\Gamma_M(i\sigma_2), \tag{A21b}$$

where

$$\mathcal{G}_3^\pm = \frac{1}{3!}(\tilde{F}_{MNP}\sigma_1 \pm e^{-\Phi}H_{MNP}\sigma_3)\Gamma^{MNP}. \tag{A22}$$

$\sigma_{i=1,2,3}$ are the usual Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A23}$$

The non-Abelian generalization of the action in [56] is not known. However, to leading order in α', and as long as the transverse fluctuations are suppressed, the non-Abelian action should result from promoting θ to an adjoint-valued field and the regular derivative to a gauge covariant derivative, and tracing over gauge indices.

APPENDIX B: DEFORMED CONIFOLD GEOMETRY

Here, we briefly review the geometry of the conifold and its deformation following closely the discussion in [40] (though, see also [72]). The deformed conifold can be described as the locus of points satisfying

$$\sum_{i=1}^4 z_i^2 = \varepsilon^2, \tag{B1}$$

and the singular conifold is recovered for $\varepsilon = 0$. Equation (B1) is invariant under the $\mathbb{Z}_2$ transformation $z_i \to -z_i$ and the SO(4) transformation $z_i \to O_{ij}z_j$. The radial coordinates τ and r are defined by

$$z_i \bar{z}_i = \varepsilon^2 \cosh\tau = r^3. \tag{B2}$$

The angular space is an S^3 fibered over an S^2 and is frequently written in terms of angular coordinates $\theta_{i=1,2} \in [0, \pi]$, $\phi_{i=1,2} \in [0, 2\pi)$, and $\psi \in [0, 4\pi)$ related to the z_i by

$$\begin{aligned}\frac{z_1}{\varepsilon} &= \cosh\left(\frac{S}{2}\right)\cos\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\phi_1+\phi_2}{2}\right) + i\sinh\left(\frac{S}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right)\sin\left(\frac{\phi_1+\phi_2}{2}\right),\\ \frac{z_2}{\varepsilon} &= -\cosh\left(\frac{S}{2}\right)\cos\left(\frac{\theta_1+\theta_2}{2}\right)\sin\left(\frac{\phi_1+\phi_2}{2}\right) + i\sinh\left(\frac{S}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right)\cos\left(\frac{\phi_1+\phi_2}{2}\right),\\ \frac{z_3}{\varepsilon} &= -\cosh\left(\frac{S}{2}\right)\sin\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\phi_1-\phi_2}{2}\right) + i\sinh\left(\frac{S}{2}\right)\sin\left(\frac{\theta_1-\theta_2}{2}\right)\sin\left(\frac{\phi_1-\phi_2}{2}\right),\\ \frac{z_4}{\varepsilon} &= -\cosh\left(\frac{S}{2}\right)\sin\left(\frac{\theta_1+\theta_2}{2}\right)\sin\left(\frac{\phi_1-\phi_2}{2}\right) - i\sinh\left(\frac{S}{2}\right)\sin\left(\frac{\theta_1-\theta_2}{2}\right)\cos\left(\frac{\phi_1-\phi_2}{2}\right),\end{aligned} \tag{B3}$$

with $S = \tau + i\psi$. The $\mathbb{Z}_2$ symmetry is then realized as $\psi \to \psi + 2\pi$. It is convenient to define

$$e_1 = -\sin\theta_1 d\phi_1, \tag{B4a}$$
$$e_2 = d\theta_1, \tag{B4b}$$
$$e_3 = \cos\psi \sin\theta_2 d\phi_2 - \sin\psi d\theta_2, \tag{B4c}$$
$$e_4 = \sin\psi \sin\theta_2 d\phi_2 + \cos\psi d\theta_2, \tag{B4d}$$
$$e_5 = d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2. \tag{B4e}$$

The metric for the deformed conifold is diagonal in the basis of 1-forms given by [73]

$$g_1 = \frac{1}{\sqrt{2}}(e_1 - e_3), \tag{B5a}$$
$$g_2 = \frac{1}{\sqrt{2}}(e_2 - e_4), \tag{B5b}$$
$$g_3 = \frac{1}{\sqrt{2}}(e_1 + e_3), \tag{B5c}$$
$$g_4 = \frac{1}{\sqrt{2}}(e_2 + e_4), \tag{B5d}$$
$$g_5 = e_5. \tag{B5e}$$

In terms of the complex coordinates, there are relatively simple expressions available for the SO(4) invariant 1-forms

$$\begin{aligned} d\tau &= \frac{1}{\varepsilon^2 \sinh\tau}(z_i d\bar{z}_i + \bar{z}_i dz_i),\\ g_5 &= \frac{i}{\varepsilon^2 \sinh\tau}(z_i d\bar{z}_i - \bar{z}_i dz_i). \end{aligned} \tag{B6}$$

Similarly, for the SO(4)-invariant 2-forms

$$g_1 \wedge g_2 = \frac{i(1+\cosh\tau)}{2\varepsilon^4 \sinh^3\tau}\epsilon_{ijkl}(2z_i\bar{z}_j dz_k \wedge d\bar{z}_l - z_i\bar{z}_j dz_k \wedge dz_l - z_i\bar{z}_j d\bar{z}_k \wedge d\bar{z}_l), \tag{B7a}$$
$$g_3 \wedge g_4 = \frac{i\tanh(\frac{\tau}{2})}{2\varepsilon^4 \sinh^2\tau}\epsilon_{ijkl}(2z_i\bar{z}_j dz_k \wedge d\bar{z}_l + z_i\bar{z}_j dz_k \wedge dz_l + z_i\bar{z}_j d\bar{z}_k \wedge d\bar{z}_l), \tag{B7b}$$
$$g_1 \wedge g_3 + g_2 \wedge g_4 = \frac{1}{\varepsilon^4 \sinh^2\tau}\epsilon_{ijkl}(-z_i\bar{z}_j dz_k \wedge dz_l + z_i\bar{z}_j d\bar{z}_k \wedge d\bar{z}_l), \tag{B7c}$$
$$g_2 \wedge g_3 + g_4 \wedge g_1 = -\frac{2i\cosh\tau}{\varepsilon^4 \sinh^3\tau}(\bar{z}_j dz_j) \wedge (z_i d\bar{z}_i) + \frac{2i}{\varepsilon^2 \sinh\tau} dz_i \wedge d\bar{z}_i. \tag{B7d}$$

The other 1-forms g_i do not seem to be as easily expressed in terms of the holomorphic coordinates. However, one can show that

$$g_1^2 + g_2^2 = -\frac{1}{2\varepsilon^4 \sinh^2(\tau/2)\sinh^2\tau}[(\bar{z}_i dz_i)^2 + (z_i d\bar{z}_i)^2 + 2\cosh\tau(\bar{z}_i dz_i)(z_i d\bar{z}_i) + \varepsilon^2\sinh^2\tau(dz_i dz_i + d\bar{z}_i d\bar{z}_i - 2dz_i d\bar{z}_i)], \tag{B8a}$$

$$g_3^2 + g_4^2 = \frac{1}{2\varepsilon^4 \cosh^2(\tau/2)\sinh^2\tau}[(\bar{z}_i dz_i)^2 + (z_i d\bar{z}_i)^2 - 2\cosh\tau(\bar{z}_i dz_i)(z_i d\bar{z}_i) + \varepsilon^2\sinh^2\tau(dz_i dz_i + d\bar{z}_i d\bar{z}_i + 2dz_i d\bar{z}_i)]. \tag{B8b}$$

Using these expressions and the metric and fluxes given in Sec. II A, the components of the 3-form flux having mixed holomorphic and antiholomorphic indices (with respect to the complex structure of the unperturbed KS solution) are

$$G_3^{(2,1)} = \frac{M\alpha'}{2\varepsilon^6}[2(a_1^+ + a_2^+)(\bar{z}_m dz_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j dz_k \wedge d\bar{z}_\ell) + (a_1^- - a_2^- - a_3^+)(z_m d\bar{z}_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j dz_k \wedge dz_\ell)], \quad \text{(B9a)}$$

$$G_3^{(1,2)} = \frac{M\alpha'}{2\varepsilon^6}[2(a_1^- + a_2^-)(z_m d\bar{z}_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j dz_k \wedge d\bar{z}_\ell) + (a_1^+ - a_2^+ - a_3^-)(\bar{z}_m dz_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j d\bar{z}_k \wedge d\bar{z}_\ell)], \quad \text{(B9b)}$$

with

$$a_1^\pm(\tau) = \frac{\tanh\frac{\tau}{2}}{2\sinh^3\tau}(\pm(1-F) + g_s e^{-\Phi} k'), \quad \text{(B10a)}$$

$$a_2^\pm(\tau) = \frac{1+\cosh\tau}{2\sinh^4\tau}(\pm F + g_s e^{-\Phi} f'), \quad \text{(B10b)}$$

$$a_3^\pm(\tau) = \frac{1}{\sinh^3\tau}\left(\pm F' + g_s e^{-\Phi}\frac{k-f}{2}\right). \quad \text{(B10c)}$$

The components with pure holomorphic and pure antiholomorphic indices are

$$G_3^{(3,0)} = \frac{M\alpha'}{2\varepsilon^6}\left[\left((1-F)\frac{\tanh\frac{\tau}{2}}{2\sinh^3\tau} - F\frac{1+\cosh\tau}{2\sinh^4\tau} - \frac{F'}{\sinh^3\tau}\right) + g_s e^{-\Phi}\left(-f'\frac{1+\cosh\tau}{2\sinh^4\tau} + k'\frac{\tanh\frac{\tau}{2}}{2\sinh^3\tau} + \frac{k-f}{2\sinh^3\tau}\right)\right] \times (\bar{z}_m dz_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j dz_k \wedge dz_\ell), \quad \text{(B11a)}$$

$$G_3^{(0,3)} = \frac{M\alpha'}{2\varepsilon^6}\left[-\left((1-F)\frac{\tanh\frac{\tau}{2}}{2\sinh^3\tau} - F\frac{1+\cosh\tau}{2\sinh^4\tau} - \frac{F'}{\sinh^3\tau}\right) + g_s e^{-\Phi}\left(-f'\frac{1+\cosh\tau}{2\sinh^4\tau} + k'\frac{\tanh\frac{\tau}{2}}{2\sinh^3\tau} + \frac{k-f}{2\sinh^3\tau}\right)\right] \times (z_m d\bar{z}_m) \wedge (\epsilon_{ijk\ell} z_i \bar{z}_j d\bar{z}_k \wedge d\bar{z}_\ell). \quad \text{(B11b)}$$

For general functions p, b, q, and s, the metric will no longer be Hermitian with respect to the complex structure of the deformed conifold. In general, the unwarped 6D metric takes the form

$$\begin{aligned}(\varepsilon^4\sinh^2\tau)d\tilde{s}_6^2 = {}& \left\{p(\tau) - b(\tau) + \frac{1}{2}\left[\frac{q(\tau)}{\cosh^2(\tau/2)} - \frac{s(\tau)}{\sinh^2(\tau/2)}\right]\right\}((\bar{z}_i dz_i)^2 + (z_i d\bar{z}_i)^2) \\ &+ \frac{1}{2}\varepsilon^2\sinh^2\tau\left[\frac{q(\tau)}{\cosh^2(\tau/2)} - \frac{s(\tau)}{\sinh^2(\tau/2)}\right](dz_i dz_i + d\bar{z}_i d\bar{z}_i) \\ &+ 2\left\{p(\tau) + b(\tau) - \frac{1}{2}\cosh(\tau)\left[\frac{q(\tau)}{\cosh^2(\tau/2)} + \frac{s(\tau)}{\sinh^2(\tau/2)}\right]\right\}(\bar{z}_i dz_i)(z_i d\bar{z}_i) \\ &+ \varepsilon^2\sinh^2\tau\left[\frac{q(\tau)}{\cosh^2(\tau/2)} + \frac{s(\tau)}{\sinh^2(\tau/2)}\right]dz_i d\bar{z}_i. \end{aligned} \quad \text{(B12)}$$

The holomorphic 3-form for the deformed conifold is

$$\begin{aligned}\Omega &= \frac{\varepsilon^2}{16\sqrt{3}}[-\sinh\tau(g_1\wedge g_3 + g_2\wedge g_4) + i\cosh\tau(g_1\wedge g_2 - g_3\wedge g_4) - i(g_1\wedge g_2 + g_3\wedge g_4)]\wedge(d\tau + ig_5) \\ &= \frac{1}{4\sqrt{3}\varepsilon^4\sinh^2\tau}(\epsilon_{ijkl} z_i \bar{z}_j dz_k \wedge dz_l)\wedge(\bar{z}_m dz_m). \end{aligned} \quad \text{(B13)}$$

It also convenient to introduce another set of holomorphic 1-forms:

$$dZ_1 = d\tau + ig_5, \quad \text{(B14a)}$$

$$dZ_2 = g_1 - i\coth\frac{\tau}{2}g_4, \quad \text{(B14b)}$$

$$dZ_3 = g_3 - i\tanh\frac{\tau}{2}g_2. \quad \text{(B14c)}$$

In these coordinates, the metric (16b) is written as

$$d\tilde{s}_6^2 = \frac{1}{2}(p(\tau) + b(\tau))dZ_1 d\bar{Z}_1 + \frac{1}{2}\left(s(\tau) + q(\tau)\tanh^2\frac{\tau}{2}\right)dZ_2 d\bar{Z}_2 + \frac{1}{2}\left(s(\tau)\coth^2\frac{\tau}{2} + q(\tau)\right)dZ_3 d\bar{Z}_3 + \frac{1}{4}(p(\tau) - b(\tau))(dZ_1 dZ_1 + d\bar{Z}_1 d\bar{Z}_1) + \frac{1}{4}\left(s(\tau) - q(\tau)\tanh^2\frac{\tau}{2}\right)(dZ_2 dZ_2 + d\bar{Z}_2 d\bar{Z}_2) + \frac{1}{4}\left(-s(\tau)\coth^2\frac{\tau}{2} + q(\tau)\right)(dZ_3 dZ_3 + d\bar{Z}_3 d\bar{Z}_3), \tag{B15}$$

and the holomorphic 3-form of the deformed conifold takes the simple form

$$\Omega = -\frac{\varepsilon^2}{16\sqrt{3}}\sinh\tau dZ_1 \wedge dZ_2 \wedge dZ_3. \tag{B16}$$

The components of G_3 with mixed holomorphic and antiholomorphic indices can then be written as

$$G_3^{(2,1)} = -\frac{M\alpha'\sinh^3\tau}{8}\{(a_1^+ + a_2^+)(dZ_1 \wedge dZ_2 \wedge d\bar{Z}_3 - dZ_1 \wedge d\bar{Z}_2 \wedge dZ_3) + (a_1^- - a_2^- - a_3^+) \times d\bar{Z}_1 \wedge dZ_2 \wedge dZ_3\}, \tag{B17a}$$

$$G_3^{(1,2)} = \frac{M\alpha'\sinh^3\tau}{8}\{(a_1^- + a_2^-)(d\bar{Z}_1 \wedge d\bar{Z}_2 \wedge dZ_3 - d\bar{Z}_1 \wedge dZ_2 \wedge d\bar{Z}_3) + (a_1^+ - a_2^+ - a_3^-) \times dZ_1 \wedge d\bar{Z}_2 \wedge d\bar{Z}_3\}, \tag{B17b}$$

while the components with pure holomorphic or pure antiholomorphic indices are

$$G_3^{(3,0)} = \frac{M\alpha'}{16}\left\{-(1 - F)\tanh\frac{\tau}{2} + F\coth\frac{\tau}{2} + 2F' + g_s e^{-\Phi}\left(f'\coth\frac{\tau}{2} - k'\tanh\frac{\tau}{2} - (f - k)\right)\right\} \times dZ_1 \wedge dZ_2 \wedge dZ_3, \tag{B18a}$$

$$G_3^{(0,3)} = \frac{M\alpha'}{16}\left\{-(1 - F)\tanh\frac{\tau}{2} + F\coth\frac{\tau}{2} + 2F' - g_s e^{-\Phi}\left(f'\coth\frac{\tau}{2} - k'\tanh\frac{\tau}{2} - (f - k)\right)\right\} \times d\bar{Z}_1 \wedge d\bar{Z}_2 \wedge d\bar{Z}_3. \tag{B18b}$$

APPENDIX C: PULLBACKS OF BULK FIELDS

We consider a stack of D7-branes satisfying the Kuperstein embedding condition [37]

$$z_4 = \mu, \tag{C1}$$

where z_i are holomorphic coordinates satisfying (B1). For the purpose of computing pullbacks onto the world volumes, it is useful to adopt coordinates in which the bulk geometry is seen as a foliation of Kuperstein divisors. Such coordinates $(\rho, \chi, \bar{\chi}, \phi, \theta, \xi)$ were given in [26]:

$$z_1 = i\eta(\chi)\left[\cos\phi\cosh\left(\frac{\rho + i\xi}{2}\right)\cos\theta - i\sin\phi\sinh\left(\frac{\rho + i\xi}{2}\right)\right], \tag{C2a}$$

$$z_2 = i\eta(\chi)\left[\sin\phi\cosh\left(\frac{\rho + i\xi}{2}\right)\cos\theta - i\cos\phi\sinh\left(\frac{\rho + i\xi}{2}\right)\right], \tag{C2b}$$

$$z_3 = i\eta(\chi)\cosh\left(\frac{\rho + i\xi}{2}\right)\sin\theta, \tag{C2c}$$

$$z_4 = \mu + \chi, \tag{C2d}$$

where the z_i still satisfy (B1). η and ρ are defined by

$$\eta(\chi) = \sqrt{(\mu + \chi)^2 - \varepsilon^2}, \tag{C3}$$

and ρ is given by

$$\varepsilon^2\cosh\tau = |\eta|^2\cosh\rho + |\mu + \chi|^2. \tag{C4}$$

A Kuperstein embedding is then specified by the simple condition $\chi = 0$. One can find the unwarped metric on the four cycle wrapped by the D7-branes by substituting the coordinates (C2) into the metric (B12). The result is

$$d\tilde{s}_4^2 = v(\tau)d\rho^2 + w(\tau)h_3^2 + \frac{\eta^2 s(\tau)(\cosh\rho + 1)}{2\varepsilon^2(\cosh\tau - 1)}h_1^2 + \frac{\eta^2 q(\tau)(\cosh\rho - 1)}{2\varepsilon^2(\cosh\tau + 1)}h_2^2, \tag{C5}$$

with

$$v(\tau) = \frac{\eta^2}{2\varepsilon^2}\left\{\frac{2\eta^2 p(\tau)\sinh^2\rho}{\sinh^2\tau} + q(\tau)\left[\frac{\varepsilon^2(\cosh\rho + 1)}{\cosh\tau + 1} - \frac{\eta^2\sinh^2\rho}{(\cosh\tau + 1)^2}\right] + s(\tau)\left[\frac{\varepsilon^2(\cosh\rho - 1)}{\cosh\tau - 1} - \frac{\eta^2\sinh^2\rho}{(\cosh\tau - 1)^2}\right]\right\}, \tag{C6a}$$

$$w(\tau) = \frac{\eta^2}{2\varepsilon^2}\left\{\frac{2\eta^2 b(\tau)\sinh^2\rho}{\sinh^2\tau} + q(\tau)\left[\frac{\varepsilon^2(\cosh\rho - 1)}{\cosh\tau + 1} - \frac{\eta^2\sinh^2\rho}{\sinh^2\tau}\right] + s(\tau)\left[\frac{\varepsilon^2(\cosh\rho + 1)}{\cosh\tau - 1} - \frac{\eta^2\sinh^2\rho}{\sinh^2\tau}\right]\right\}, \tag{C6b}$$

where as in [26], the 1-forms h_i are

$$h_1 = 2\left(\cos\frac{\gamma}{2} d\theta - \sin\frac{\gamma}{2} \sin\theta d\phi\right), \tag{C7}$$

$$h_2 = 2\left(\sin\frac{\gamma}{2} d\theta + \cos\frac{\gamma}{2} \sin\theta d\phi\right), \tag{C8}$$

$$h_3 = d\gamma - 2\cos\theta d\phi. \tag{C9}$$

For the KS geometry, this reduces to the metric given in [26],

$$d\tilde{s}_4^2 = \frac{K(\tau)\eta^2}{2\varepsilon^{2/3}}\left[K_2(\rho)(d\rho^2 + h_3^2) + \cosh^2\frac{\rho}{2} h_1^2 + \sinh^2\frac{\rho}{2} h_2^2\right]. \tag{C10}$$

The pullback of $B_{(2)}$ can also be written in these coordinates as

$$b_2 = -\frac{g_s M\alpha'}{2\varepsilon^4}\eta^3\mu\left[k(\tau)\,\mathrm{csch}\frac{\rho}{2}\,\mathrm{csch}^3\tau\sinh^2\rho\sinh^2\frac{\tau}{2} \times d\rho\wedge h_2 + \frac{1}{2}f(\tau)\cosh^3\frac{\rho}{2}\,\mathrm{csch}^3\frac{\tau}{2}\,\mathrm{sech}\frac{\tau}{2} h_1\wedge h_3\right]. \tag{C11}$$

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