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Nuclear Physics B 842 (2011) 383–413

NUCLEAR PHYSICS B

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Non-supersymmetric infrared perturbations to the warped deformed conifold

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Received 11 May 2010; received in revised form 7 September 2010; accepted 16 September 2010

Available online 21 September 2010

Abstract

We analyze properties of non-supersymmetric, isometry-preserving perturbations to the infrared region of the warped deformed conifold, i.e. the Klebanov–Strassler solution. We discuss both perturbations that "squash" the geometry, so that the internal space is no longer conformally Calabi–Yau, and perturbations that do not squash the geometry. Among the perturbations that we discuss is the solution that describes the linearized near-tip backreaction of a smeared collection of $\overline{\text{D3}}$-branes positioned in the deep infrared. Such a configuration is a candidate gravity dual of a non-supersymmetric state in a large-rank cascading gauge theory. Although $\overline{\text{D3}}$-branes do not directly couple to the 3-form flux, we argue that, due to the presence of the background imaginary self-dual flux, $\overline{\text{D3}}$-branes in the Klebanov–Strassler geometry necessarily produce singular non-imaginary self-dual flux. Moreover, since conformally Calabi–Yau geometries cannot be supported by non-imaginary self-dual flux, the $\overline{\text{D3}}$-branes squash the geometry as our explicit solution shows. We also briefly discuss supersymmetry-breaking perturbations at large radii and the effect of the non-supersymmetric perturbations on the gravitino mass.

Keywords: D-branes; Supergravity; Supersymmetry breaking

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0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2010.09.008

1. Introduction

Among the challenges in connecting string theory to our observable universe is the difficulty in constructing controllable supersymmetry-breaking backgrounds. While supersymmetry (SUSY) breaking is a prerequisite in any phenomenological study of four-dimensional supersymmetric theories, the myriad of string theory moduli makes this a formidable task. Unless all moduli are stabilized at a hierarchically higher scale than the scale of SUSY breaking, one generically finds runaway directions that destabilize the vacuum, taking us away from the controllable background which describes the original supersymmetric state.

On top of this challenge, the observational evidence of an accelerating universe adds yet another layer of complication: in addition to the requirement that the SUSY-breaking background be (meta)stable, viable vacua must also have positive energy density. Motivated by this cosmological consideration, several mechanisms to "uplift" the vacuum energy of string vacua have since been suggested, e.g., by adding $\overline{\text{D3}}$-branes [1], by introducing D-terms from gauge fluxes [2], or by considering negatively curved internal spaces [3–6] (see also [7–9]). Though these mechanisms are often discussed in terms of 4D effective field theories, it is of interest for a variety of reasons discussed below to find backreacted supergravity solutions including such uplifting sources as full 10D backgrounds.

In this paper, we report on some properties of non-supersymmetric perturbations to the Klebanov–Strassler (KS) solution [10], a prototypical warped supersymmetric background which is dual to a cascading $SU(N+M) \times SU(N)$ gauge theory in the strong 't Hooft limit, and is ubiquitous in flux compactifications and in describing moduli stabilization. The backreaction of a collection of $\overline{\text{D3}}$-branes placed at the tip of the deformed conifold, should be described by such perturbations. Such a configuration is known to be metastable against brane/flux annihilation provided that the number of $\overline{\text{D3}}$-branes is sufficiently small in comparison to the background flux [11]. Though further instabilities generically arise upon compactification when the closed string degrees of freedom become dynamical and further stabilization mechanisms (e.g., fluxes, non-perturbative effects, etc.) are needed, this *local* construction represents progress towards a genuine metastable SUSY breaking background. Other than being an essential feature in [1] for vacuum uplifting to de Sitter space, the warped $\overline{\text{D3}}$ tension introduces an exponentially small supersymmetry breaking scale which can be useful for describing hidden sector dynamics (both in dimensionally reduced theories and in their holographic descriptions).

Although we are interested especially in modes related to $\overline{\text{D3}}$-branes, the analysis with more general modes brings us interesting features for the classification of near-tip perturbations. We analyze perturbations that are either singular or regular and those that either do or do not "squash" the geometry (i.e. those that do or do not leave the internal geometry as conformally the deformed conifold) in accordance with the equations of motion. We also identify which modes can break supersymmetry. The mode related to $\overline{\text{D3}}$s at the tip should have singular behavior, at least in the warp factor, in order to capture the localized tension. We show below however that the only singular, non-squashed, non-SUSY mode corresponds to a point source for the dilaton, and thus cannot be identified as an $\overline{\text{D3}}$-brane. Furthermore, the squashed backreaction of an $\overline{\text{D3}}$-brane is supported by a 3-form flux that is no longer imaginary self-dual (ISD). The fact that an $\overline{\text{D3}}$-brane squashes the geometry was observed in [12] where the $\overline{\text{D3}}$-brane backreaction was studied in the Klebanov–Tseytlin (KT) region [13]. However, due to the decreased complexity of the geometry, the squashing of the geometry in [12] is less dramatic than the squashing in the near-tip region. Likewise, the resulting non-ISD flux near the tip is more complex than the non-ISD flux supporting the solution of [12]. We also discuss these issues in the KT region.

Other than the consideration of $\overline{\text{D3}}$-branes, the existence of non-SUSY fluxes is interesting to consider for many reasons. It is well known (see for example [14–17]) that such non-SUSY fluxes can give rise to soft SUSY-breaking terms in 4D effective theories. Additionally, non-SUSY fluxes can play an important role in the context of D-brane inflation.[1] While the deformed conifold can support certain non-SUSY fluxes (at least to the level of approximation at which we work), we show below that in order to have any non-ISD flux, the geometry must be squashed so that it is no longer Calabi–Yau.

Perturbations to the KS solution appear in many other places in the literature (and indeed most of our solutions have appeared elsewhere though previously none had been identified as describing the presence of $\overline{\text{D3}}$-branes). Using an alternative parametrization [24,25] of the ansatz that we present below, the linearized equations of motion for perturbations to the KS geometry have been written elsewhere as a system of coupled first order equations, solutions for which can be written formally in terms of integrals [26,27]. Although writing the equations of motion in this way can be convenient, we choose to work directly with the linearized second order equations. The second derivative equations were also directly solved in [28,29], though we relax some of the assumptions made in those references. Analysis of perturbations to KS also arise in studies of the glueball spectrum of the dual theory [30–32].

There are several other reasons why we are interested in analyzing non-SUSY perturbations to the near-tip region of KS. First of all, being closest to the source of SUSY breaking, this is the region where the supergravity fields are most affected. Moreover, as is common in warped compactification, the wavefunctions of non-zero modes (e.g. the gravitino after SUSY breaking) tend to peak in the tip region. Thus, our perturbative solutions are useful in determining the low energy couplings (including soft masses) in the 4D effective action involving these infrared localized fields. Additionally, as discussed in a companion paper [33], the backreacted $\overline{\text{D3}}$ solution in the near tip region provides a holographic dual of gauge mediation in a different parameter space regime from that of [34]. As a result, strongly coupled messengers (and not only weakly coupled mesonic bound states) of the hidden sector gauge group can contribute significantly to visible sector soft terms. Given the aforementioned applications, it is of importance for us to consider warped geometries which are infrared smooth[2] before perturbations. Since we are focusing on the near tip region, our starting point is the KS solution which provides a more accurate description at small radius than KT. Although the KT background correctly reproduces the cascading behavior of the field theory, it becomes singular in the IR where the effective D3 charge (which is dual to the scale dependent effective 't Hooft coupling) becomes negative and the cascade must end. The appropriate IR modification is the KS solution which is built on the deformed conifold so that the solution is smooth even in the IR.

This paper is organized as follows. In Section 2, we discuss our solution ansatz and express the KS solutions in accordance with this ansatz. In Section 3, we describe how we obtain the perturbative solution corresponding to placing a $\overline{\text{D3}}$-brane point source in the warped deformed conifold. We also clarify that solutions where the internal space is unsquashed should satisfy the ISD condition and cannot describe the backreaction of an $\overline{\text{D3}}$-brane. In Section 4, we present regular solutions which also break supersymmetry, but do not correspond to the backreaction of a localized source. We also point out that even for regular solutions, squashing is needed to break the ISD condition. In Section 5, we present solutions in the KT region, both with and without

[1] For recent reviews, see, e.g., [18–23].

[2] By this we mean that, at least before the addition of 3-branes, the warp factor approaches a constant, or equivalently, the (minimal surface) dual Wilson loop [35,36] has a finite tension.

the ISD condition imposed. We calculate the gravitino mass in these SUSY-breaking warped backgrounds in Section 6 and end with some discussions in Section 7. Some useful details about our conventions and the complex coordinates of KS are relegated to Appendices A and B.

We note that after the completion of this paper, another preprint [37] that addresses the question of adding $\overline{\text{D3}}$-branes to the geometry was made available. Our treatment of the $\overline{\text{D3}}$-brane differs from [37] by the boundary conditions imposed in the IR[3] as elaborated in Section 3.4.

2. Supergravity ansatz

In this section we give the ansatz that we will use for the metric and other fields, working in the Einstein frame of IIB supergravity. Our conventions are presented in Appendix A. Since we are considering perturbations to the KS solution [10] (see also [38]), our ansatz will be based on that solution. In particular, since we are looking for perturbations that preserve the isometry of KS we take the metric

$$ds^2 = h^{-\frac{1}{2}}(\tau)\,dx_\mu^2 + h^{\frac{1}{2}}(\tau)\,d\tilde{s}_6^2, \tag{2.1a}$$

$$d\tilde{s}_6^2 = p(\tau)\,d\tau^2 + b(\tau)g_5^2 + q(\tau)\big(g_3^2 + g_4^2\big) + s(\tau)\big(g_1^2 + g_2^2\big), \tag{2.1b}$$

where τ is the radial coordinate and where the angular one-forms g_i are reviewed in Appendix A. This metric ansatz includes the warped deformed conifold as a special case by a certain choice of p, b, q, and s presented in the next section. For the dilaton and fluxes we take

$$\Phi = \Phi(\tau), \qquad C = 0, \tag{2.2a}$$

while for the fluxes,

$$\begin{aligned}
B_2 &= \frac{g_s M\alpha'}{2}\big[f(\tau)g_1\wedge g_2 + k(\tau)g_3\wedge g_4\big],\\
F_3 &= \frac{M\alpha'}{2}\big[\big(1-F(\tau)\big)g_5\wedge g_3\wedge g_4 + F(\tau)g_5\wedge g_1\wedge g_2\\
&\quad + F'(\tau)\,d\tau\wedge(g_1\wedge g_3 + g_2\wedge g_4)\big],\\
F_5 &= (1+*_{10})\mathcal{F}_5, \qquad \mathcal{F}_5 = \frac{g_s M^2\alpha'^2}{4}\ell(\tau)g_1\wedge g_2\wedge g_3\wedge g_4\wedge g_5.
\end{aligned} \tag{2.2b}$$

These choices of fluxes respect the isometries of the deformed conifold and satisfy the Bianchi identities

$$dF_3 = 0, \qquad dH_3 = 0. \tag{2.3}$$

2.1. Klebanov–Strassler solution and its expansion near the tip

The KS solution [10] corresponds to placing M fractional D3-branes at a deformed conifold point (i.e. wrapping M D5-branes around the collapsing two-cycle) and smearing these branes over the finite $\mathbb{S}^3$. It is recovered by the choice

[3] Additionally, the equations of motion in [37] are formally solved for all radii and would thus be useful for further analysis connecting the IR and UV regions.

$$
\begin{aligned}
&f_{KS}(\tau)=\frac{\tau\cosh\tau-\sinh\tau}{4\cosh^2(\tau/2)}, \qquad k_{KS}(\tau)=\frac{\tau\cosh\tau-\sinh\tau}{4\sinh^2(\tau/2)}, \qquad F_{KS}(\tau)=\frac{\sinh\tau-\tau}{2\sinh\tau},\\
&\ell_{KS}(\tau)=f_{KS}(1-F_{KS})+k_{KS}F_{KS},\\
&p_{KS}(\tau)=b_{KS}(\tau)=\frac{\varepsilon^{4/3}}{6K^2(\tau)}, \qquad q_{KS}(\tau)=\frac{\varepsilon^{4/3}}{2}K(\tau)\cosh^2\frac{\tau}{2}, \qquad (2.4)\\
&s_{KS}(\tau)=\frac{\varepsilon^{4/3}}{2}K(\tau)\sinh^2\frac{\tau}{2},\\
&\Phi(\tau)=\log g_s, \qquad h_{KS}(\tau)=\left(g_sM\alpha'\right)^2 2^{\frac{2}{3}}\varepsilon^{-\frac{8}{3}}I(\tau),\\
&K(\tau)=\frac{(\sinh(2\tau)-2\tau)^{1/3}}{2^{1/3}\sinh\tau}, \qquad I(\tau)=\int_\tau^\infty dx\,\frac{x\coth x-1}{\sinh^2 x}(\sinh 2x-2x)^{\frac{1}{3}}, \qquad (2.5)
\end{aligned}
$$

where ℓ_{KS} is determined by requiring $\mathcal{F}_5=B_2\wedge F_3$, which ensures that the solution is regular. However, if a regular D3-brane is added to the geometry, then ℓ must have an additional constant part. This introduces a τ^{-1} part to the warp factor and the solution becomes singular, which on the field theory side corresponds to the loss of confinement.

We are interested in solving for perturbations to the KS background. Since the geometry is already relatively complicated, we will consider perturbations to the geometry as a power expansion about $\tau=0$. It is therefore useful to note the series solutions for the KS solution near the tip,

$$
\begin{aligned}
f_{KS}&=\frac{\tau^3}{12}-\frac{\tau^5}{80}+\frac{17\tau^7}{10\,080}-\frac{31\tau^9}{145\,152}+\frac{691\tau^{11}}{26\,611\,200}+\cdots,\\
k_{KS}&=\frac{\tau}{3}+\frac{\tau^3}{180}-\frac{\tau^5}{5040}+\frac{\tau^7}{151\,200}-\frac{\tau^9}{4\,790\,016}+\cdots,\\
F_{KS}&=\frac{\tau^2}{12}-\frac{7\tau^4}{720}+\frac{31\tau^6}{30\,240}-\frac{127\tau^8}{1\,209\,600}+\frac{73\tau^{10}}{6\,842\,880}+\cdots, \qquad (2.6a)
\end{aligned}
$$

for the three-form fields, and

$$
\begin{aligned}
p_{KS}=b_{KS}&=\frac{\varepsilon^{4/3}}{2^{2/3}\,3^{1/3}\,2}+\frac{\varepsilon^{4/3}\tau^2}{2^{2/3}\,3^{1/3}\,10}+\frac{\varepsilon^{4/3}\tau^4}{2^{2/3}\,3^{1/3}\,210}\\
&\quad+\frac{(\frac{2}{3})^{1/3}\varepsilon^{4/3}\tau^8}{606\,375}-\frac{19\varepsilon^{4/3}\tau^{10}}{2^{2/3}\,3^{1/3}\,322\,481\,250}+\cdots,\\
q_{KS}&=\frac{\varepsilon^{4/3}}{2^{2/3}\,3^{1/3}}+\frac{(\frac{3}{2})^{2/3}}{20}\varepsilon^{4/3}\tau^2+\frac{17\varepsilon^{4/3}\tau^4}{2^{2/3}\,3^{1/3}\,2800}\\
&\quad+\frac{\varepsilon^{4/3}\tau^6}{2^{2/3}\,3^{1/3}\,10\,080}-\frac{83\varepsilon^{4/3}\tau^8}{2^{2/3}\,3^{1/3}\,155\,232\,000}+\cdots,\\
s_{KS}&=\frac{\varepsilon^{4/3}\tau^2}{4\,2^{2/3}\,3^{1/3}}-\frac{\varepsilon^{4/3}\tau^4}{2^{2/3}\,3^{1/3}\,240}+\frac{59\varepsilon^{4/3}\tau^6}{2^{2/3}\,3^{1/3}\,50\,400}\\
&\quad-\frac{\varepsilon^{4/3}\tau^8}{2^{2/3}\,3^{1/3}\,8960}+\frac{6401\varepsilon^{4/3}\tau^{10}}{2^{2/3}\,3^{1/3}\,558\,835\,200}+\cdots,\\
h_{KS}&=\left(g_sM\alpha'\right)^2 2^{\frac{2}{3}}\varepsilon^{-\frac{8}{3}}\left[a_0-\frac{3\tau^2}{6^{1/3}}+\frac{\tau^4}{6^{1/3}\,18}-\frac{37\tau^6}{6^{1/3}\,4725}+\frac{4\,2^{2/3}\tau^8}{3^{1/3}\,7875}+\cdots\right], \qquad (2.6b)
\end{aligned}
$$

for the squashing functions and warp factor. Note that each of these functions contains powers of τ with only one parity (e.g. h_{KS} contains only terms of the form τ^{2k}). The given expansions satisfy the dilaton equation of motion (A.3b) up to $O(\tau^9)$, the Einstein equation (A.3a) up to $O(\tau^7)$, the H_3 Eqs. (A.4a) and (A.4b) up to $O(\tau^8)$ and $O(\tau^{10})$, and the F_3 Eq. (A.5) up to $O(\tau^9)$. The leading constant of h can be calculated numerically, $a_0 = I(0) \approx 0.71805$.

3. Non-SUSY deformations from localized sources

In this section, we present perturbations to the KS geometry that are solutions to the supergravity equations of motion with singular behavior. One of these solutions corresponds to adding $\overline{\text{D3}}$-branes to the tip of the geometry. We first argue why a singular solution is necessary to describe the point source behavior of a $\overline{\text{D3}}$. We then discuss two solutions, the first in which the internal space remains the deformed conifold, and the second in which the geometry is "squashed" away from this geometry. We argue that squashing the geometry is necessary in order to have a flux that is not ISD. We also match the parameters in the latter solution to the tension of the 3-branes.

3.1. Effect of point sources

For simplicity, we seek solutions that retain the isometry of the KS solution. Because the $\mathbb{S}^3$ remains finite as $\tau = 0$, in general placing a point source into the internal geometry will break the angular isometry even at $\tau = 0$. Therefore, in order to retain the KS isometry, these point sources must be smeared over the finite $\mathbb{S}^3$. Alternatively, we can consider a collection of point sources that to good approximation are uniformly distributed over the $\mathbb{S}^3$.

The effect of such a localized source on the geometry can be estimated by considering the Green's function in the unperturbed background. Using the metric (2.1), an S-wave solution (i.e. dependent only on τ) to Laplace's equation $\nabla^2 H = 0$ can be written

$$H = \mathcal{P} - \int_\tau^\infty dx\, \frac{\phi}{q(x)s(x)}\sqrt{\frac{p(x)}{b(x)}}, \tag{3.1}$$

with arbitrary integration constants $\mathcal{P}$ and ϕ.

If the warp factor h is obtained by solving the Killing spinor equations (A.6) (more precisely (A.9)), then the form of h is similar to that of the Green's function,

$$h = \mathcal{P} + \frac{(g_s M\alpha')^2}{4}\int_\tau^\infty dx\, \frac{\ell(x)}{q(x)s(x)}\sqrt{\frac{p(x)}{b(x)}}. \tag{3.2}$$

A localized source of D3 charge gives a constant piece to ℓ, and this constant piece indeed generates a Green's function in h. The ansatz (2.1) admits solutions that are either asymptotically flat or AdS (up to possible log corrections) and setting $\mathcal{P} = 0$ corresponds to demanding the latter.

The integral (3.1) cannot be performed exactly for the KS solution. However, using the expansion about $\tau = 0$ given above, one can write for small τ,

$$H = \mathcal{P} + 2^{\frac{10}{3}} 3^{\frac{2}{3}} \phi \varepsilon^{-\frac{8}{3}}\left(-\frac{1}{\tau} - \frac{2\tau}{15} + \frac{\tau^3}{315} + \cdots\right). \tag{3.3}$$

Thus if we include a point source at the tip that respects the same supersymmetry as KS, then the geometry should become singular as $\tau \to 0$. In particular, the warp factor will depend as τ^{-1}.

Dropping the constant term, in the large radius region the Green's function takes the form (in terms of $r^2 = 2^{5/3} 3\varepsilon^{4/3} e^{2\tau/3}$)

$$H = -2^{1/3} 24 \varepsilon^{-\frac{8}{3}} \phi e^{-\frac{4\tau}{3}} + \cdots = -\frac{27\phi}{r^4} + \cdots. \tag{3.4}$$

If the object that is added to geometry breaks supersymmetry, then it needs not perturb the warp factor by simply adding a Green's function piece. Indeed, in the large radius region, where a Green's function behaves as r^{-4}, adding D3–$\overline{\text{D3}}$ pairs perturbs the warp factor by r^{-8} [12,39] (though there are also log corrections). Heuristically, the presence of the non-supersymmetric source adds a perturbation that scales as $\delta h \sim h_0 H$ where h_0 is the unperturbed warp factor (if the charge of the non-BPS source is non-vanishing, then there will be a Green's function contribution as well). Since the KS warp factor approaches a constant, this suggests that even for non-supersymmetric sources, we should look for perturbations to the warp factor that behave as τ^{-1}. This argument is only very heuristic, though we are able to check using boundary conditions that this behavior is indeed correct.

3.2. Unsquashed singular perturbations to KS

We consider perturbing the KS solution by taking the ansatz (2.1), (2.2a) and writing

$$\begin{aligned}
&f = f_{KS} + f_p(\tau), \qquad k = k_{KS} + k_p(\tau),\\
&F = F_{KS} + F_p(\tau), \qquad \ell = f(1-F) + kF,\\
&\Phi = \log g_s + \Phi_p(\tau), \qquad h = h_{KS} + h_p(\tau),\\
&p = p_{KS}, \qquad b = b_{KS}, \qquad q = q_{KS}, \qquad s = s_{KS},
\end{aligned} \tag{3.5}$$

where the KS solution is (2.4) and the subscript p indicates a perturbation to KS. Such an ansatz changes the warp factor and the fluxes, but leaves the internal unwarped geometry as the deformed conifold. We do not attempt to solve for the perturbations exactly, but write them as a power series about $\tau = 0$. We then solve the equations of motion (A.3) to first order in the perturbations and order-by-order in τ. To linear order in perturbations the coefficients for the odd powers of τ in h_p decouple from the coefficients for the even powers. As argued above, to capture the behavior of a point source, the warp factor ought to behave as τ^{-1}, implying that we should focus on the odd powers in τ in h_p. We find the solution

$$\begin{aligned}
\Phi_p &= \phi\left(\frac{1}{\tau} + \frac{2\tau}{15} - \frac{\tau^3}{315} + \frac{2\tau^5}{23\,625}\right),\\
F_p &= \phi\left(-\frac{1}{2} - \frac{23\tau^3}{720} + \frac{\tau^5}{1400}\right) + \mathcal{U}\left(\frac{1}{\tau} - \frac{\tau}{6} + \frac{7\tau^3}{360} - \frac{31\tau^5}{15\,120}\right),\\
f_p &= \phi\left(\frac{23}{12} + \frac{3\tau^2}{16} - \frac{\tau^4}{80} + \frac{61\tau^6}{26\,880}\right) + \mathcal{U}\left(-\frac{13}{6} - \frac{\tau^2}{8} + \frac{\tau^4}{48} - \frac{17\tau^6}{5760}\right) + \frac{\mathcal{H}}{6},\\
k_p &= \phi\left(\frac{1}{\tau^2} + \frac{9}{4} - \frac{3\tau^2}{80} - \frac{113\tau^4}{25\,200}\right) + \mathcal{U}\left(-\frac{2}{\tau^2} - \frac{5}{2} - \frac{\tau^2}{120} + \frac{\tau^4}{3024}\right) + \frac{\mathcal{H}}{6},\\
h_p &= \frac{(g_s M\alpha')^2 2^{2/3} \varepsilon^{-8/3}}{6^{1/3}}\left[\phi\left(\frac{11}{\tau} + \frac{206\tau^3}{1575} - \frac{487\tau^5}{23\,625}\right) + \mathcal{U}\left(-\frac{12}{\tau} - \frac{4\tau^3}{25} + \frac{208\tau^5}{7875}\right)\right.\\
&\quad \left. + \mathcal{H}\left(\frac{1}{\tau} + \frac{2\tau}{15} - \frac{\tau^3}{315} + \frac{2\tau^5}{23\,625}\right)\right].
\end{aligned} \tag{3.6}$$

This solution is valid to linear order in the parameters $\phi, \mathcal{U}$, and $\mathcal{H}$. It can be extended to higher order in τ by expressing the higher order coefficients in terms of $\phi, \mathcal{U}$, and $\mathcal{H}$ so that no additional parameters need to be introduced. These perturbations satisfy the dilaton Eq. (A.3b) up to $O(\tau^3)$, the Einstein equation (A.3a) up to $O(\tau^3)$, and the gauge equations (A.4a) up to $O(\tau^4)$, (A.4b) up to $O(\tau^4)$, and (A.5) up to $O(\tau^3)$.

It is worth noting that even if we allow a perturbation to $b(\tau)$, which describes a squashing of the internal space, the solution (3.6) does not change and b remains unperturbed ($b_p = 0$). The squashing of this direction was considered in [12] to obtain a non-SUSY deformation of KT space, but in the KS region, there is no solution in which only this direction is squashed.

To this order in the perturbations and in τ, the solution (3.6) respects the ISD condition (A.8) of the 3-form flux as well as the first derivative SUSY condition for the warp factor (A.9), even though the solution follows from solving second derivative equations. However, we expect (and indeed we have checked to several higher orders in τ) that the flux remains ISD to all orders in τ since the dilaton takes the form of a Green's function (3.3). If the flux had an IASD component as well, then in general the fluxes would provide a potential for the dilaton and Φ would no longer satisfy $\nabla^2\Phi = 0$. Indeed, since Φ does have the same form as (3.1), we can identify ϕ as corresponding to some point source for the dilaton smeared over the finite $\mathbb{S}^3$ at $\tau = 0$.

Some non-SUSY perturbations to the KS solutions, found by solving the first order differential equations given in [24], were analyzed in [26,27]. For $\phi = 0$ (i.e. constant dilaton), the solution (3.6) is a small τ expansion of the exact solution appearing in [27], the flux part of which is

$$\begin{aligned}
F(\tau) &= \frac{1}{2}\left(1 - \frac{\tau}{\sinh\tau}\right) + \frac{\mathcal{U}}{\sinh\tau} + \frac{5\mathcal{T}}{32}\left(\cosh\tau - \frac{\tau}{\sinh\tau}\right),\\
f(\tau) &= \frac{\tau\cosh\tau - \sinh\tau}{4\cosh^2(\tau/2)} - \frac{\mathcal{U}}{6\cosh^2(\tau/2)}(5 + 8\cosh\tau) + \frac{\mathcal{H}}{6}\\
&\quad + \frac{5\mathcal{T}}{128\cosh^2(\tau/2)}(2\tau + 4\tau\cosh\tau - 4\sinh\tau - \sinh 2\tau),\\
k(\tau) &= \frac{\tau\cosh\tau - \sinh\tau}{4\sinh^2(\tau/2)} - \frac{\mathcal{U}}{6\sinh^2(\tau/2)}(-5 + 8\cosh\tau) + \frac{\mathcal{H}}{6}\\
&\quad + \frac{5\mathcal{T}}{128\sinh^2(\tau/2)}(-2\tau + 4\tau\cosh\tau - 4\sinh\tau + \sinh 2\tau).
\end{aligned} \tag{3.7}$$

The solution is singular for non-vanishing $\mathcal{U}$ or $\mathcal{H}$ which are essentially the same parameters that appear in (3.6), though (3.7) is an exact solution of (A.3) to all orders in $\mathcal{U}$, $\mathcal{H}$, and $\mathcal{T}$. The remaining parameter $\mathcal{T}$ appears in another solution (4.1), and of the parameters of (3.7), only a non-vanishing $\mathcal{T}$ leads to supersymmetry breaking. The additional parameter ϕ appearing in (3.6) comes from relaxing the condition that the dilaton Φ is constant. Note also that the parameter $\mathcal{H}$ is related to the gauge symmetry $B_2 \to B_2 + d\Lambda_1$.

To check if supersymmetry is preserved, we consider the SUSY variations of the gravitino and dilatino (A.6), taking into account the non-trivial dilaton profile. Since G_3 is ISD, the last term of the dilatino variation (A.6b) vanishes. However, the terms involving the derivative of the dilaton do not. Indeed for small τ,

$$\delta\lambda \sim -i\frac{3^{1/6}\phi}{2^{1/3}a_0^{1/4}(g_s M\alpha')^{1/2}\tau^2}\hat{\Gamma}^\tau\epsilon + \cdots, \tag{3.8}$$

where $\hat{\Gamma}$ indicates an unwarped Γ-matrix. Since this variation is non-vanishing, the solution (3.6) breaks supersymmetry.

The variation for the gravitino is also non-vanishing since the solution includes a (0, 3) part of G_3. From (B.8), we see that for the solution (3.6),

$$G_3^{(0,3)} = \phi\left(\frac{1}{3\tau^3} + \frac{1}{15\tau} - \frac{86\tau}{1575} + \cdots\right)(z_i\, d\bar{z}_i) \wedge (\epsilon_{ijkl} z_i \bar{z}_j\, d\bar{z}_k \wedge d\bar{z}_l). \tag{3.9}$$

As shown in Section 4.1, the exact solution (3.7), for which $\phi = 0$, has an additional contribution to the (0, 3) part from $\mathcal{T}$. For the perturbation (3.6), the (3, 0) and (1, 2)-parts vanish, which is consistent with the fact that the ISD condition allows only for (2, 1) and (0, 3) components.

Both the variation of the dilatino and gravitino involve only ϕ. Therefore, even though the singular behavior seems to imply that $\mathcal{U}$ and $\mathcal{H}$ can be associated with a point source, they do not break supersymmetry (though (3.7) breaks supersymmetry for non-vanishing $\mathcal{T}$) and only ϕ is a possible candidate to describe the presence of a localized SUSY-breaking source. The parameter ϕ characterizes a localized source for the dilaton and therefore cannot correspond to the presence of $\overline{\mathrm{D3}}$-branes since $\overline{\mathrm{D3}}$s do not directly couple to the dilaton. Furthermore, it was shown in [12] that an $\overline{\mathrm{D3}}$ squashes the geometry so that it is no longer conformally Calabi–Yau. Extrapolating this result to short distances, the source associated with ϕ, which does not squash the geometry, should therefore not be identified with an $\overline{\mathrm{D3}}$-brane. Indeed, this mode is the small radius analogue of the r^{-4} mode for the dilaton that appeared in [12] (as well as the flat space analysis of non-BPS branes in [39]) which could be turned off independently of the existence of $\overline{\mathrm{D3}}$-branes as it does not contribute to the total mass of the solution.

Note that this solution possesses a curvature singularity at $\tau = 0$; at small τ the Ricci scalar behaves as

$$R = \frac{-45\, 2^{2/3}\mathcal{H} + (45\, 2^{2/3} - 3^{1/3}a_0)(12\mathcal{U} - 11\phi)}{30\, 3^{1/3} a_0^{5/2} g_s M \alpha' \tau} + \cdots. \tag{3.10}$$

The presence of the curvature singularity indicates a breakdown of the supergravity approximation, and so our solution is only expected to be valid for $1/(g_s M\alpha') \ll \tau < 1$ where the upper bound coming from the fact that we are performing a small τ expansion and the lower bound comes from assuming that R is small in string units.

3.3. Squashed singular perturbations to KS

We can generalize by considering solutions that "squash" the internal geometry so that the unwarped geometry is no longer that of the deformed conifold. At large distances where the DKM solution [12] is valid, the only non-trivial squashing that occurs due to the presence of a $\overline{\mathrm{D3}}$-brane is in the direction on which the $U(1)$ isometry acts.[4] However, as discussed in the previous section, at small radius the equations of motion do not admit a solution in which the only squashing is in this direction. Thus, we consider an ansatz of the form

$$\begin{aligned}
&\Phi = \log g_s + \Phi_p(\tau), \qquad h = h_{KS} + h_p(\tau),\\
&f = f_{KS} + f_p(\tau), \qquad k = k_{KS} + k_p(\tau), \qquad F = F_{KS} + F_p(\tau),\\
&\ell = f(1-F) + kF, \qquad b = b_{KS}\big(1 + b_p(\tau)\big), \qquad q = q_{KS}\big(1 + q_p(\tau)\big),
\end{aligned}$$

[4] This $U(1)$ isometry is actually broken down to $\mathbb{Z}_2$ in the deformed conifold.

$$s = s_{KS}\big(1 + s_p(\tau)\big), \qquad p = p_{KS}, \tag{3.11}$$

where we have used the freedom to redefine τ to keep p unperturbed but have allowed b, q, and s to be general so that the ansatz is the most general ansatz consistent with the isometry of KS. This more general ansatz will allow G_3 to have both ISD and IASD components. We are again interested in describing the effect of a localized source and since the even and odd powers of τ in the warp factor decouple from each other, we focus on odd powers of τ in h_p. We find a power series solution to (A.3) where the dilaton obtains a non-trivial profile that is regular at small τ

$$\Phi_p = \mathcal{S}\tau + \mathcal{Y}\tau^3. \tag{3.12a}$$

However, the squashing functions for the solution are singular

$$\begin{aligned}
b_p &= \mathcal{S}\left(\frac{7}{\tau} - \frac{3293\tau^3}{3150}\right) + \mathcal{Y}\left(\frac{70}{\tau} - \frac{404\tau^3}{45}\right) + \mathcal{B}\left(\tau - \frac{43\tau^3}{210}\right),\\
q_p &= \mathcal{S}\left(\frac{7}{4\tau} + \frac{103\tau}{48} - \frac{44\,129\tau^3}{100\,800}\right) + \mathcal{Y}\left(\frac{35}{2\tau} + \frac{70\tau}{3} - \frac{1673\tau^3}{360}\right) + \mathcal{B}\left(\frac{3\tau}{4} - \frac{71\tau^3}{560}\right),\\
s_p &= \mathcal{S}\left(\frac{73}{3\tau} - \frac{253\tau}{720} + \frac{29\,999\tau^3}{60\,480}\right) + \mathcal{Y}\left(\frac{1085}{6\tau} - \frac{56\tau}{9} + \frac{1049\tau^3}{216}\right)\\
&\quad + \mathcal{B}\left(\frac{5}{\tau} - \frac{7\tau}{12} + \frac{529\tau^3}{5040}\right).
\end{aligned} \tag{3.12b}$$

Similarly the fluxes are

$$\begin{aligned}
F_p &= \mathcal{S}\left(\frac{3193}{84\tau} + \frac{312}{35\tau}6^{1/3}a_0 + \left(-\frac{5959}{1008} - \frac{299\,3^{1/3}a_0}{70\,2^{2/3}}\right)\tau\right)\\
&\quad + \mathcal{Y}\left(\frac{760}{3\tau} + \frac{72\,6^{1/3}a_0}{\tau} + \left(-\frac{1555}{36} - \frac{69\,3^{1/3}a_0}{2\,2^{2/3}}\right)\tau\right) + \mathcal{B}\left(\frac{50}{7\tau} - \frac{65\tau}{84}\right),\\
f_p &= \mathcal{S}\left(-\frac{863\,221}{2520} - \frac{101\,247\,3^{1/3}a_0}{700\,2^{2/3}} + \frac{13\,3^{2/3}a_0^2}{5\,2^{1/3}} + \left(-\frac{265}{112} - \frac{321\,3^{1/3}a_0}{112\,2^{2/3}}\right)\tau^2\right.\\
&\quad \left. + \left(\frac{17\,209}{40\,320} + \frac{1037\,3^{1/3}a_0}{5600\,2^{2/3}}\right)\tau^4 + \left(\frac{143\,221}{2\,419\,200} - \frac{653a_0}{6400\,6^{2/3}}\right)\tau^6\right)\\
&\quad + \mathcal{Y}\left(-\frac{81\,913}{36} - \frac{5333\,3^{1/3}a_0}{5\,2^{2/3}} + \frac{21\,3^{2/3}a_0^2}{2^{1/3}} + \left(-\frac{125}{8} - \frac{165\,3^{1/3}a_0}{8\,2^{2/3}}\right)\tau^2\right.\\
&\quad \left. + \left(\frac{491}{288} + \frac{43\,3^{1/3}a_0}{40\,2^{2/3}}\right)\tau^4 + \left(\frac{3881}{4320} - \frac{413a_0}{640\,6^{2/3}}\right)\tau^6\right)\\
&\quad + \mathcal{B}\left(-\frac{3155}{84} - \frac{15\tau^2}{56} - \frac{47\tau^4}{672} + \frac{307\tau^6}{10\,080}\right),\\
k_p &= \mathcal{S}\left(-\frac{3193}{42\tau^2} - \frac{513\,3^{1/3}a_0}{35\,2^{2/3}\tau^2} - \frac{37\,454}{105} - \frac{104\,777\,3^{1/3}a_0}{700\,2^{2/3}} + \frac{13\,3^{2/3}a_0^2}{5\,2^{1/3}}\right.\\
&\quad \left. + \left(-\frac{11\,617}{5040} + \frac{1649\,3^{1/3}a_0}{2800\,2^{2/3}}\right)\tau^2\right)\\
&\quad + \mathcal{Y}\left(-\frac{1520}{3\tau^2} - \frac{39\,6^{1/3}a_0}{\tau^2} - \frac{28\,621}{12} - \frac{5503\,3^{1/3}a_0}{5\,2^{2/3}} + \frac{21\,3^{2/3}a_0^2}{2^{1/3}}\right.
\end{aligned}$$

$$+\left(-\frac{1321}{72}+\frac{197\,3^{1/3}a_0}{40\,2^{2/3}}\right)\tau^2\Bigg)+\mathcal{B}\left(-\frac{100}{7\tau^2}-\frac{1095}{28}-\frac{17\tau^2}{168}\right). \tag{3.12c}$$

The warp factor resulting from the fluxes exhibits the desired singular behavior

$$\begin{aligned}h_p=(g_sM\alpha')^2 2^{\frac{2}{3}}\varepsilon^{-\frac{8}{3}}\Bigg[&\mathcal{S}\left(-\frac{105\,907\,2^{2/3}}{105\,3^{1/3}\tau}-\frac{146\,913a_0}{350\tau}+\left(-\frac{182\,561}{900\,6^{1/3}}-\frac{5413a_0}{125}\right)\tau\right)\\&+\mathcal{Y}\left(-\frac{80\,393}{6\,6^{1/3}\tau}-\frac{30\,753a_0}{10\tau}+\left(-\frac{29\,414\,2^{2/3}}{45\,3^{1/3}}-\frac{7896a_0}{25}\right)\tau\right)\\&+\mathcal{B}\left(-\frac{3055}{14\,6^{1/3}\tau}-\frac{29\,2^{2/3}\tau}{3\,3^{1/3}}\right)\Bigg].\end{aligned} \tag{3.12d}$$

Perturbations that respect the ISD condition and were presented in the previous section have been omitted. Again, $\mathcal{S}$, $\mathcal{Y}$, and $\mathcal{B}$ are treated as perturbations and so the solution is valid to linear order in these parameters and can be extended to higher order in τ without introducing any new independent parameters.

Since the dilaton does not exhibit a τ^{-1} behavior, the nontrivial profile cannot be interpreted as resulting from a localized source for the dilaton. Instead it comes from the lift of the vacuum energy due to the presence of both ISD and IASD components of G_3 (A.3b),

$$H_3^2-e^{2\Phi}F_3^2=\frac{48\,6^{1/3}\mathcal{S}}{\sqrt{a_0}g_s^3M\alpha'\tau}+\frac{40\mathcal{S}-16\,6^{1/3}a_0(\mathcal{S}-90\mathcal{Y})\tau}{5a_0^{3/2}g_s^3M\alpha'}+\cdots. \tag{3.13}$$

The non-vanishing potential for the dilaton implies the existence of an IASD component since, for $C=0$,

$$\nabla^2\Phi=-\frac{g_se^{-\Phi}}{2\times 3!}\left[H_3^2-e^{2\Phi}F_3^2\right]\propto \mathrm{Re}\left(G^+_{mnp}G^{-mnp}\right) \tag{3.14}$$

where $G^\pm=iG_3\pm\tilde{*}_6G_3$. One can also see directly from (A.8) that G_3 is no longer purely ISD. The parameters controlling the deviation from the ISD condition (A.8) are $\mathcal{S}$ and $\mathcal{Y}$. Both of these are included in the squashing functions, implying that the squashing of the deformed conifold is needed to have non-vanishing $(3,0)$ or $(1,2)$ components of G_3.

As was the case for (3.6), the geometry exhibits a curvature singularity at $\tau=0$. Indeed at small τ the Ricci scalar behaves as

$$R\sim\frac{\mathcal{O}(\mathcal{B},\mathcal{S},\mathcal{Y})}{g_sM\alpha'\tau}, \tag{3.15}$$

where we have omitted numerical coefficients since the essential behavior is τ^{-1}. This singularity implies that the solution is valid only for $\mathcal{S}/(g_sM\alpha')\ll\tau<1$.

The solution (3.12) breaks supersymmetry, squashes the geometry, and introduces an IASD component of the flux. All of these properties are also shared by the DKM solution [12] which describes the large radius influence of D3–$\overline{\text{D3}}$ pairs at the conifold point (though the DKM solution contains squashing in fewer directions than this solution). The perturbations to the KT geometry in the DKM solution behave as r^{-4} and $r^{-4}\log r$ compared to the KT geometry itself (e.g. the KT warp factor included r^{-4} and $r^{-4}\log r$ while the perturbations behaved as r^{-8} and $r^{-8}\log r$). Similarly, the solution (3.12) involves perturbations that behave as τ^{-1} relative to the KS solution (2.4). We note that τ^{-1} and r^{-4} are the small and large radius expansions of the Green's function (3.1). This, together with the shared properties mentioned above, is a hint that the solution (3.12) may describe the backreaction of $\overline{\text{D3}}$-branes. We can confirm that this is the case by checking boundary conditions.

3.4. Boundary conditions

We now seek to match the parameters of this solution to the tension of the $\overline{\text{D3}}$-branes that are localized at $\tau = 0$. Since the solution is singular at $\tau = 0$, we expect the solution to be modified by undetermined stringy corrections at distances $\tau \lesssim \mathcal{S}/(g_s M\alpha')$. We will therefore not try to obtain the coefficients exactly.

Following from the behavior of the Green's function (3.3), the $O(1/\tau)$ behavior of the warp factor is tied to the existence of a localized source of tension. Indeed if there is a collection of D3-branes and $\overline{\text{D3}}$-branes located at $\tau = 0$ and angular positions Ω_i, then they contribute to the stress-energy tensor as

$$T^{\text{loc}}_{\mu\nu} = -\kappa_{10}^2 T_3 \frac{\delta(\tau)}{\sqrt{p_{KS} b_{KS} s_{KS} q_{KS}}} \sum_i \frac{\delta^5(\Omega - \Omega_i)}{\sqrt{\tilde{g}_5}} \eta_{\mu\nu}, \tag{3.16}$$

where $\tilde{g}_5$ is the angular part of the determinant of the unwarped metric and the other components of T^{loc}_{MN} vanish. We make the approximation that there are enough 3-branes that we can treat them as uniformly smeared over the finite $\mathbb{S}^3$ at the tip. Then integrating over the $\mathbb{S}^3$ gives

$$\int_{\mathbb{S}^3} d\text{vol}_{\mathbb{S}^3}\, T^{\text{loc}}_{\mu\nu} = -\kappa_{10}^2 T_3 (N_{\text{D3}} + N_{\overline{\text{D3}}}) \frac{\delta(\tau)}{\sqrt{p_{KS} b_{KS} s_{KS} q_{KS}}} \frac{\delta^2(\Omega)}{\sqrt{\tilde{g}_2}} \eta_{\mu\nu}, \tag{3.17}$$

where $\delta^2(\Omega)$ fixes the angular position on the vanishing 2-cycle, $\tilde{g}_2$ is the unwarped metric for that 2-cycle, and N_{D3} and $N_{\overline{\text{D3}}}$ are the numbers of D3- and $\overline{\text{D3}}$-branes added to the tip. This localized source of tension should cause a $1/\tau$ behavior in the warp factor. Tracing over the Einstein equation in the presence of the localized source, we have

$$-\frac{1}{4\tau^2} \partial_\tau \left(\tau^2 \partial_\tau h_p\right) \sim \frac{1}{2} \kappa_{10}^2 T_3 (N_{\text{D3}} + N_{\overline{\text{D3}}}) \delta(\tau) \sqrt{\frac{p_{KS}}{b_{KS}}} \frac{1}{q_{KS} s_{KS}} \frac{1}{V_2}, \tag{3.18}$$

where we have integrated over the angular directions and defined $V_2 = \int d^2x \sqrt{\tilde{g}_2}$. Integrating over τ, we find that near the tip of the deformed conifold,

$$h_p \sim \frac{(N_{\text{D3}} + N_{\overline{\text{D3}}}) \kappa_{10}^2 T_3 \varepsilon^{-8/3}}{V_2} \frac{1}{\tau}, \tag{3.19}$$

where $h_{KS} = h_0 + \mathcal{O}(\tau^2)$. That is, the τ^{-1} coefficient in the warp factor is proportional to the total tension of the 3-branes added to the tip. Using (3.12), we can use this relation to match the parameters to this tension.

Similarly, one can match to the total charge added to $\tau = 0$ by considering the constant part of ℓ,

$$\ell(\tau = 0) \propto N_{\text{D3}} - N_{\overline{\text{D3}}}. \tag{3.20}$$

Since the solution (3.12) involves ratios of relatively large numbers, we omit the detailed form of this expression, but by some choice of parameters, we can take the solution to correspond to adding negative charge. Thus, for some choice of $\mathcal{S}$, $\mathcal{B}$, and $\mathcal{Y}$ (as discussed below, one must additionally include the $\mathcal{U}$-mode of (3.6)), the solution (3.12a) corresponds to adding $\overline{\text{D3}}$-branes to the tip of KS. Another choice of parameters allows us to describe the influence of D3–$\overline{\text{D3}}$ pairs which adds tension, but no net charge to the solution and so is the small radius analogue of the solution presented in [12].

Alternatively, we might match the parameters to the tension of the 3-branes by calculating the analogue of the ADM mass. For spaces which do not necessarily asymptote to either flat space or AdS, a generalization of the ADM mass was given in [40]. However, this is applicable only at large distances (and indeed was used in the large radius solution [12]) while the solution (3.11) is valid for small τ. Although analogues of the ADM mass exist for arbitrary surfaces, and not just those at infinity [41,42], it is more efficient to match to the localized tension discussed above.

The behavior of the 3-form fluxes in (3.12) gives rise to divergent energy densities H_3^2 and F_3^2. In particular, the leading order behavior $F_p \sim \mathcal{S}\tau^{-1}$ (for the remainder of the section, $\mathcal{S}$ will be used as short hand for linear combinations of $\mathcal{S}$, $\mathcal{B}$, and $\mathcal{Y}$) leads to $F_3^2 \sim \mathcal{S}^2\tau^{-6}$. The contribution to the action then diverges since $\sqrt{g}F_3^2 \sim S^2\tau^{-4}$. Similarly, the τ^{-2} behavior of $k_p - f_p$ gives $H_3^2 \sim \mathcal{S}^2\tau^{-4}$ which also gives a divergent action. Since the $\overline{\mathrm{D3}}$-branes do not directly source these fields, one should impose that these very singular behaviors should be absent from the solution describing the backreaction of $\overline{\mathrm{D3}}$-branes. Since two of $\mathcal{S}$, $\mathcal{B}$, and $\mathcal{Y}$ are fixed by matching to the tension and charge of the 3-branes, there is not enough freedom to cancel both of these divergences using just the modes in (3.12). However, these divergences can be cancelled by additionally including the $\mathcal{U}$-modes given in (3.6). Imposing this additional condition on $\mathcal{S}$, $\mathcal{B}$, $\mathcal{Y}$, and $\mathcal{U}$ gives the leading order behavior

$$F_p \sim \mathcal{S}\tau, \qquad k_p - f_p \sim \mathcal{S}\tau^0. \tag{3.21}$$

From these,

$$F_3^2 \sim H_3^2 \sim \frac{\mathcal{S}^2}{\tau^2}. \tag{3.22}$$

That is, even after imposing that the most singular parts of the 3-form flux vanish, the energy densities H_3^2 and F_3^2 are divergent. Furthermore, these divergences cannot be removed by including any of the other modes discussed here without setting all of these constants to be zero. However, these do not lead to a divergent action since $\sqrt{g}F_3^2 \sim \sqrt{g}H_3^2 \sim \tau^0$.

The fact that F_3^2 and H_3^2 are divergent may be at first be surprising since the $\overline{\mathrm{D3}}$s do not directly couple to the 3-form flux and thus the singularities in H_3^2 and F_3^2 have no obvious physical interpretation.[5] Here, however, we suggest that such singular behavior might have been anticipated from the equations of motion and the boundary conditions. Indeed, the coupling between the 3-form and 5-form flux can be written as (see for example [43])

$$d\Lambda + \frac{i}{\mathrm{Im}(\tau)} d\tau \wedge (\Lambda + \bar{\Lambda}) = 0, \tag{3.23}$$

where the external part of the C_4 has been written[6]

$$C_4 = \alpha\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3, \tag{3.24}$$

and

$$\Lambda = \Phi^+ G^- + \Phi^- G^+ \tag{3.25}$$

[5] For example, it was argued in [37] that the resulting H_3 has the wrong orientation and dependence on the $\overline{\mathrm{D3}}$-charge to be due to the NS5-branes that were described in [11].

[6] We use notation which is slightly different than the remainder of the paper to match with the notation in [43] and to present (3.23) simply.

where $\Phi^{\pm} = h^{-1} \pm \alpha$ and $G^{\pm} = iG_3 \pm \tilde{*}_6 G_3$. Since in the KS background, τ is constant and both Φ^- and G^- vanish, the second term in (3.23) is higher order in the perturbations and the remainder of the equation implies $d\Lambda = 0$. To leading order in perturbations, this implies

$$\Phi^+ \delta G^- = -\delta\Phi^- G^+. \tag{3.26}$$

Although the $\overline{\text{D3}}$-branes do not directly couple to $G^{\pm}$, they do directly source Φ^-. Since the KS geometry has both Φ^+ and G^+ non-vanishing, this direct coupling implies that G^- must be non-vanishing when an $\overline{\text{D3}}$-brane is added. Furthermore, since $\delta\Phi^-$ will have singular behavior at small τ while Φ^+ and G^+ are regular, δG^- must have smaller powers of τ than G^+. For example, in the KS background $f - k \sim \tau$ while $\Phi^+ \sim \tau^0$. Since $\delta\Phi^- \sim \tau^{-1}$, one might expect $f_p - k_p \sim \tau^0$. Due to the presence of the Hodge-$*$ which will introduce the squashing functions into this analysis this argument alone is not conclusive and one must solve the equations of motion as we did above. Nevertheless, it provides a heuristic argument for why this singular behavior for H_3^2 and F_3^2 is present in this solution. The intuition that an $\overline{\text{D3}}$-brane should not result in such singular behavior comes partially from the flat space case where $G^+ = 0$ and this argument fails. Similarly, it fails for the addition of D3-branes to KS since D3s source only Φ^+ and not Φ^-.

The backreaction of $\overline{\text{D3}}$s was also addressed in [37]. In [37], the existence of the constant part of $k_p - f_p$ and the linear part of F_p, after imposing that the more singular parts vanish, was deduced from a slightly different logic. The authors used the fact that a probe D3-brane in the geometry will be attracted to $\overline{\text{D3}}$-branes at the tip.[7] Under the assumption that the backreaction of the $\overline{\text{D3}}$-branes could be described as a linear perturbation of the Klebanov–Strassler geometry with at least some non-normalizable modes absent, it was shown in [37] that the existence of this force implies such behavior. It was then argued that this may imply that treating the $\overline{\text{D3}}$s-branes as a perturbation to the Klebanov–Strassler background is not a valid procedure because the $\overline{\text{D3}}$-branes do not directly couple to H_3 and F_3 and that therefore the resulting singular H_3^2 and F_3^2 are unphysical. The point of view that we adopt is that although it is true that adding D3-branes to KS or $\overline{\text{D3}}$-branes to flat space will not result in such behavior, in light of (3.23) it is not surprising that such modes exist when adding $\overline{\text{D3}}$-branes to KS. Therefore, unlike the possibility discussed in [37], we do not impose that H_3^2 and F_3^2 are non-singular.

4. Regular non-supersymmetric perturbations

Here we present solutions which do not include a singular $O(1/\tau)$ behavior in the warp factor. In this case, the warp factor is a power series in τ consisting only of even powers.

4.1. Unsquashed regular perturbations to KS

The equations of motion (A.3) admit a solution that is regular and unsquashed. Taking the ansatz (3.5), we find for the dilaton and fluxes

$$\Phi_p = \mathcal{P},$$
$$F_p = \mathcal{P}\left(-\frac{\tau^2}{6} - \frac{\tau^4}{180} - \frac{13\tau^6}{15\,120}\right) + \mathcal{T}\left(\frac{5\tau^2}{48} + \frac{\tau^4}{288} + \frac{13\tau^6}{24\,192}\right),$$

[7] Of course, requiring the solution to exhibit non-SUSY behavior and that the warp factor behave as τ^{-1} will result in such a force.

$$f_p = \mathcal{P}\left(\frac{\tau^3}{12} - \frac{\tau^5}{240} + \frac{\tau^7}{1120}\right) + \mathcal{T}\left(-\frac{\tau^5}{192} + \frac{\tau^7}{2016}\right),$$

$$k_p = \mathcal{P}\left(-\frac{\tau}{3} - \frac{7\tau^3}{180} - \frac{11\tau^5}{5040}\right) + \mathcal{T}\left(\frac{5\tau}{12} + \frac{\tau^3}{36} + \frac{5\tau^5}{4032}\right), \tag{4.1a}$$

while the warp factor is

$$h_p = 2^{\frac{2}{3}}\left(g_s M\alpha'\right)^2 \varepsilon^{-\frac{8}{3}}\left[\mathcal{A} + \mathcal{T}\left(-\frac{5\tau^2}{6^{1/3}24} + \frac{5\tau^4}{6^{1/3}144}\right)\right]. \tag{4.1b}$$

Again, these solutions are valid up to linear order in the parameters $\mathcal{A}$, $\mathcal{P}$, and $\mathcal{T}$. The solution satisfies the dilaton equation (A.3b) up to $O(\tau^4)$, the gravitational equations (A.3a) up to $O(\tau^2)$, the H_3 Eqs. (A.4a) and (A.4b) up to $O(\tau^3)$ and $O(\tau^5)$, and the F_3 Eq. (A.5) up to $O(\tau^4)$. It can be easily extended to higher order in τ without introducing additional independent parameters (i.e. higher order coefficients can be expressed in terms of $\mathcal{P}$, $\mathcal{T}$, and $\mathcal{A}$). The solution related to the parameter $\mathcal{T}$ is the same solution appeared in the exact solution (3.7) after expanding around $\tau = 0$.

As was the case for the singular unsquashed perturbation given in Section 3.2, the fluxes in this solution respect the ISD condition. The solution has a non-vanishing (0, 3)-component

$$G_3^{(0,3)} = (8\mathcal{P} - 5\mathcal{T})\left(\frac{1}{8\tau^2} - \frac{1}{24} + \frac{\tau^2}{120} - \frac{\tau^4}{756} + \cdots\right)(z_i\, d\bar{z}_i) \wedge (\epsilon_{ijkl} z_i \bar{z}_j\, d\bar{z}_k \wedge d\bar{z}_l), \tag{4.2}$$

while the $(3,0)$ and $(1,2)$-components vanish. The existence of the $(0,3)$ part implies that the gravitino variation is non-vanishing for general choices of $\mathcal{P}$ and $\mathcal{T}$ and thus supersymmetry is broken even though the flux is ISD. However, taking $8\mathcal{P} = 5\mathcal{T}$ results in an $\mathcal{N} = 1$ supersymmetric solution. This special case is a generalization of KS, corresponding to a constant shift of the string coupling and a canceling shift in H_3 such that G_3 is unchanged. Indeed in this case, $F_p = 0$ while $f_p \propto f_{KS}$ and $k_p \propto k_{KS}$.

4.2. *Squashed regular perturbations to KS*

As was the case for the singular perturbations, it is possible to obtain solutions that break the ISD condition by adopting the more general squashed ansatz (3.11). We again find such a solution to (A.3) as a power series in τ. The dilaton profile is again non-trivial

$$\Phi_p = \varphi\left(-\frac{\tau^2}{16} + \frac{\tau^4}{96} - \frac{37\tau^6}{25\,200} + \frac{\tau^8}{5250}\right). \tag{4.3a}$$

The metric squashing functions are

$$b_p = \mathcal{D}\left(-3\tau^2 + \frac{13\tau^4}{70} - \frac{517\tau^6}{15\,750}\right) + \mathcal{M}\left(1 - \frac{\tau^2}{4} + \frac{\tau^4}{42} - \frac{82\tau^6}{23\,625}\right)$$
$$+ \mathcal{Q}\left(-2\tau^2 + \frac{\tau^4}{30} - \frac{121\tau^6}{15\,750}\right) + \varphi\left(-\frac{\tau^4}{560} + \frac{223\tau^6}{378\,000}\right),$$

$$q_p = \mathcal{D}\left(\frac{3\tau^4}{28} - \frac{1847\tau^6}{126\,000}\right) + \mathcal{M}\left(1 + \frac{\tau^4}{112} - \frac{589\tau^6}{378\,000}\right)$$

$$+\mathcal{Q}\left(\tau^2-\frac{\tau^4}{60}-\frac{101\tau^6}{126\,000}\right)+\varphi\left(-\frac{3\tau^4}{560}+\frac{1669\tau^6}{1\,512\,000}\right),$$

$$s_p=\mathcal{D}\left(\tau^2-\frac{29\tau^4}{300}+\frac{13\,817\tau^6}{882\,000}\right)+\mathcal{M}\left(-\frac{3\tau^4}{400}+\frac{157\tau^6}{98\,000}\right)+\mathcal{Q}\left(\frac{\tau^4}{100}+\frac{697\tau^6}{294\,000}\right)$$
$$+\varphi\left(\frac{\tau^4}{200}-\frac{1231\tau^6}{1\,176\,000}\right) \tag{4.3b}$$

and the warp factor is

$$h_p=\left(g_sM\alpha'\right)^2 2^{\frac{2}{3}}\varepsilon^{-\frac{8}{3}}\left[\mathcal{D}\left(-\frac{13\tau^2}{5\,6^{1/3}}+\frac{317\,2^{2/3}\tau^6}{7875\,3^{1/3}}\right)+\mathcal{M}\left(-\frac{13\tau^2}{40\,6^{1/3}}+\frac{1163\tau^6}{126\,000\,6^{1/3}}\right)\right.$$
$$\left.+\mathcal{Q}\left(-\frac{\tau^2}{2\,6^{1/3}}+\frac{13\tau^6}{1260\,6^{1/3}}\right)+\varphi\left(\frac{\tau^2}{40\,6^{1/3}}-\frac{23\tau^6}{18\,000\,6^{1/3}}\right)\right]. \tag{4.3c}$$

The perturbed fluxes are

$$F_p=\mathcal{D}\left(\frac{13\tau^2}{10}+\frac{17\tau^4}{75}-\frac{173\tau^6}{8400}+\frac{5921\tau^8}{1\,323\,000}\right)$$
$$+\mathcal{M}\left(\frac{139\tau^2}{240}+\frac{319\tau^4}{7200}-\frac{101\tau^6}{604\,800}+\frac{50\,087\tau^8}{127\,008\,000}\right)$$
$$+\mathcal{Q}\left(\frac{\tau^2}{4}+\frac{11\tau^4}{120}-\frac{27\tau^6}{5600}+\frac{6457\tau^8}{6\,350\,400}\right)$$
$$+\varphi\left(\left(-\frac{1}{80}-\frac{3^{1/3}a_0}{16\,2^{2/3}}\right)\tau^2+\left(\frac{19}{2400}-\frac{a_0}{160\,6^{2/3}}\right)\tau^4\right.$$
$$\left.+\left(-\frac{73}{67\,200}-\frac{13a_0}{13\,440\,6^{2/3}}\right)\tau^6+\left(\frac{2059}{14\,112\,000}+\frac{a_0}{19\,200\,6^{2/3}}\right)\tau^8\right),$$

$$f_p=\mathcal{D}\left(-\frac{3^{1/3}a_0\tau^3}{2\,2^{2/3}}+\left(\frac{11}{100}-\frac{3^{1/3}a_0}{40\,2^{2/3}}\right)\tau^5+\left(-\frac{29}{800}-\frac{a_0}{560\,6^{2/3}}\right)\tau^7\right.$$
$$\left.+\left(\frac{164\,063}{21\,168\,000}-\frac{a_0}{40\,320\,6^{2/3}}\right)\tau^9\right)$$
$$+\mathcal{M}\left(\left(-\frac{1}{8}+\frac{a_0}{8\,6^{2/3}}\right)\tau^3+\left(-\frac{13}{1600}+\frac{a_0}{160\,6^{2/3}}\right)\tau^5\right.$$
$$\left.+\left(-\frac{57}{22\,400}+\frac{a_0}{6720\,6^{2/3}}\right)\tau^7+\left(\frac{314\,123}{508\,032\,000}+\frac{a_0}{483\,840\,6^{2/3}}\right)\tau^9\right)$$
$$+\mathcal{Q}\left(-\frac{\tau^5}{80}-\frac{\tau^7}{700}+\frac{22\,003\tau^9}{42\,336\,000}\right)$$
$$+\varphi\left(\frac{a_0\tau^3}{32\,6^{2/3}}+\left(-\frac{17}{4800}-\frac{a_0}{128\,6^{2/3}}\right)\tau^5+\left(\frac{359}{201\,600}+\frac{5a_0}{5376\,6^{2/3}}\right)\tau^7\right.$$
$$\left.+\left(-\frac{227\,047}{508\,032\,000}-\frac{233a_0}{1\,935\,360\,6^{2/3}}\right)\tau^9\right),$$

$$k_p=\mathcal{D}\left(\left(\frac{26}{5}+3\,6^{1/3}a_0\right)\tau+\left(\frac{47}{150}+\frac{3^{1/3}a_0}{2\,2^{2/3}}\right)\tau^3+\left(\frac{11}{1050}+\frac{3^{1/3}a_0}{40\,2^{2/3}}\right)\tau^5\right.$$

$$+\left(\frac{289}{1\,764\,000}+\frac{a_0}{560\,6^{2/3}}\right)\tau^7\Big)$$
$$+\mathcal{M}\bigg(\left(\frac{149}{60}-\frac{3^{1/3}a_0}{2\,2^{2/3}}\right)\tau+\left(\frac{209}{900}-\frac{a_0}{8\,6^{2/3}}\right)\tau^3$$
$$+\left(\frac{979}{100\,800}-\frac{a_0}{160\,6^{2/3}}\right)\tau^5+\left(\frac{311}{1\,411\,200}-\frac{a_0}{6720\,6^{2/3}}\right)\tau^7\bigg)$$
$$+\mathcal{Q}\left(\tau+\frac{2\tau^3}{5}+\frac{109\tau^5}{8400}-\frac{19\tau^7}{496\,125}\right)$$
$$+\varphi\bigg(\left(-\frac{1}{20}+\frac{3^{1/3}a_0}{8\,2^{2/3}}\right)\tau+\left(-\frac{1}{50}+\frac{3^{1/3}a_0}{160\,2^{2/3}}\right)\tau^3$$
$$+\left(-\frac{1}{4800}+\frac{3^{1/3}a_0}{4480\,2^{2/3}}\right)\tau^5+\left(-\frac{1433}{63\,504\,000}+\frac{a_0}{44\,800\,6^{2/3}}\right)\tau^7\bigg), \tag{4.3d}$$

where we have again omitted terms presented in Section 4.1. This solutions is valid to linear order in the parameters φ, $\mathcal{M}$, $\mathcal{Q}$, and $\mathcal{D}$ which characterize the perturbation and again one could extend this to higher orders in τ.

The resulting G_3 is no longer purely ISD since

$$H_3^2-e^{2\Phi}F_3^2=\frac{6^{1/3}9\varphi}{a_0^{1/2}g_s^3M\alpha'}+\frac{3(-1+6^{1/3}4a_0)\varphi\tau^2}{2a_0^{3/2}g_s^3M\alpha'}+\cdots\neq 0. \tag{4.4}$$

This can also be checked more directly using (A.8). Although only the parameter φ appears in the potential for the dilaton, making any of these independent parameters non-zero leads to a non-ISD flux.[8]

5. Non-SUSY solutions in the KT region

It is also possible to find non-SUSY perturbations to the KT solution. We again will find that non-ISD fluxes can be found only if the conifold is squashed. As before, we consider solutions that are linear in the perturbations, though since we are working at large τ, we do not perform a power series expansion around $\tau=0$.

5.1. Klebanov–Tseytlin solution

The ansatz (2.1), (2.2a) also includes the KT solution [13]. This solution corresponds to adding N D3-branes and M fractional D3-branes (i.e. M D5-branes wrapping a collapsing 2-cycle) to the undeformed conifold singularity and is valid at large distances from the conifold point. It is recovered by

$$f_{KT}(r)=k_{KT}(r)=\frac{3}{2}\log\frac{r}{r_0},\qquad F_{KT}(r)=\frac{1}{2},$$
$$\ell_{KT}(r)=f_{KT}(1-F_{KT})+k_{KT}F_{KT}+\frac{\pi N}{g_sM^2},$$

[8] The vanishing of the potential (3.14) merely implies $\mathrm{Re}(G^+_{mnp}G^{-mnp})=0$, not that $G^+=0$.

$$p_{KT}(r) = 1, \qquad b_{KT}(r) = \frac{r^2}{9}, \qquad q_{KT}(r) = s_{KT}(r) = \frac{r^2}{6},$$
$$\Phi_{KT}(r) = \log g_s, \qquad h_{KT}(r) = \frac{27\pi}{4r^4}\left(g_s N\alpha'^2 + \frac{3}{8\pi}\left(g_s M\alpha'\right)^2 + \frac{3}{2\pi}\left(g_s M\alpha'\right)^2 \log\frac{r}{r_0}\right), \tag{5.1}$$

where $r^2 = 2^{5/3}3\varepsilon^{4/3}e^{2\tau/3}$. In contrast to the KS solution, ℓ is chosen to satisfy

$$\mathcal{F}_5 = 27\pi\alpha'^2 N \mathrm{vol}_{T^{1,1}} + B_2 \wedge F_3, \tag{5.2}$$

where $\mathrm{vol}_{T^{1,1}}$ is the volume form of the angular space. This reflects the fact that the effective D3 charge receives contributions from both the 3-form fluxes and the N regular D3-branes which provide a localized source for the charge.

5.2. Unsquashed perturbations to KT

In analogy with the analyses of unsquashed perturbations of KS in Sections 3.2 and 4.1, we first consider perturbations for which the unwarped 6D space is still the unsquashed conifold and the flux is ISD. We take the ansatz

$$\Phi = \log g_s + \Phi_p(r), \qquad h = h_{KT} + h_p(r), \qquad \ell = f(1-F) + kF + \frac{\pi N}{g_s M^2},$$
$$f = f_{KT} + f_p(r), \qquad k = k_{KT} + k_p(r), \qquad F = F_{KT} + F_p(r),$$
$$p = p_{KT}, \qquad b = b_{KT}, \qquad q = q_{KT}, \qquad s = s_{KT}. \tag{5.3}$$

Solving the ISD condition (A.8) and the first order equation (A.9) yields

$$\Phi_p = \mathcal{P} + \frac{\phi}{r^4},$$
$$F_p = \frac{\mathcal{G}}{r^3},$$
$$f_p = \mathcal{C} + \frac{\mathcal{G}}{r^3} - \frac{3\phi}{8r^4} + \frac{3\mathcal{P}}{2}\log\frac{r}{r_0},$$
$$k_p = \mathcal{C} - \frac{\mathcal{G}}{r^3} - \frac{3\phi}{8r^4} + \frac{3\mathcal{P}}{2}\log\frac{r}{r_0},$$
$$h_p = \left(g_s M\alpha'\right)^2\left[\mathcal{A} + \frac{27\mathcal{C}}{r^4} + \frac{81\mathcal{P}}{8r^4}\left(\frac{1}{4} + \log\frac{r}{r_0}\right) - \frac{81\phi}{64r^8}\right], \tag{5.4}$$

where we have retained only solutions that are regular as $r \to \infty$. The solution is valid to linear order in the parameters $\mathcal{P}$, ϕ, $\mathcal{C}$, $\mathcal{G}$, and $\mathcal{A}$ which characterize the perturbation. Note that some terms in the perturbation are sub-dominant to the corrections to the KT geometry coming from the full KS solution; however even if these corrections are included, the perturbations are not corrected until even higher order in $1/r$.

The parameters $\mathcal{P}$ and ϕ are essentially the same parameters that appear in the perturbations to KS in Sections 4.1 and 3.2 respectively. That is, $\mathcal{P}$ is a constant shift of the string coupling and the part including ϕ is a solution to Laplace's equation $\nabla^2\Phi = 0$. The parameters $\mathcal{G}$ and $\mathcal{U}$ are related to those appearing in (3.6) and (3.7) as $\mathcal{G} = 2\varepsilon^2\mathcal{U}$ and $\mathcal{C} = \frac{\mathcal{H}}{6} - \frac{8\mathcal{U}}{3}$ (the remaining parameter $\mathcal{T}$ appearing in (3.7) is not regular as $r \to \infty$).

The parameter ϕ is also the same parameter appearing in [12]. By calculating the Hawking–Horowitz mass [40] (the generalization of ADM mass), which is valid at large radius, the authors of [12] concluded that the relevant behavior of the perturbation to the warp factor due to the D3–$\overline{\text{D3}}$ pairs should include a term behaving as $r^{-8}\log r$. However no such a term appears in (5.4). Moreover there is no squashing and the flux remains ISD. Therefore, even though SUSY is broken in this solution, it does not correspond to the presence of $\overline{\text{D3}}$-branes.

5.3. Squashed perturbations to KT

A perturbation of KT which is no longer ISD was found in [12]. Based on a similar analysis of $\mathrm{AdS}_5 \times \mathbb{S}^5$, the authors of [12] assume the perturbations due to the $\overline{\text{D3}}$-branes behave as $O(r^{-4}, r^{-4}\log r)$ relative to the original KT solution and took an ansatz which squashes each of the $SU(2)$-isometry directions in the same way. However, it is interesting to relax this condition and take the more general ansatz (2.1), (2.2a) with

$$\begin{aligned}
&\Phi = \log g_s + \Phi_p(r), \qquad h = h_{KT} + h_p(r), \qquad \ell = f(1-F) + kF + \frac{\pi N}{g_s M^2},\\
&f = f_{KT} + f_p(r), \qquad k = k_{KT} + k_p(r), \qquad F = F_{KT} + F_p(r),\\
&b = b_{KT}\big(1 + b_p(r)\big), \qquad q = q_{KT}\big(1 + q_p(r)\big),\\
&s = s_{KT}\big(1 + s_p(r)\big), \qquad p = p_{KT}.
\end{aligned} \tag{5.5}$$

Such an ansatz in general squashes the spheres in different ways. Assuming perturbations that behave as $O(1, \log r, r^{-4}, r^{-4}\log r)$ relative to the KT solution,[9] the equations of motion (A.3) admit a solution

$$\begin{aligned}
&\Phi_p = -\frac{3\mathcal{S}\log(r/r_0)}{r^4},\\
&b_p = \mathcal{J} + \frac{\mathcal{S}}{r^4}, \qquad q_p = s_p = \mathcal{J},\\
&k_p = f_p = \frac{\mathcal{S}}{r^4}\left(\frac{33}{32} + \frac{3N\pi}{4g_s M^2} + \frac{9}{4}\log\frac{r}{r_0}\right), \qquad F_p = 0,\\
&h_p = -\frac{27\pi\mathcal{J}}{2r^4}\left(g_s N\alpha'^2 + \frac{3}{8\pi}\big(g_s M\alpha'\big)^2 + \frac{3}{2\pi}\big(g_s M\alpha'\big)^2\log\frac{r}{r_0}\right)\\
&\qquad + \frac{\mathcal{S}}{r^8}\left(\frac{27\pi}{32}g_s N\alpha'^2 + \frac{1053}{256}\big(g_s M\alpha'\big)^2 + \frac{81}{16}\big(g_s M\alpha'\big)^2\log\frac{r}{r_0}\right),
\end{aligned} \tag{5.6}$$

where $\mathcal{J}$ and $\mathcal{S}$ parameterize the perturbation and we omit the parameters which have appeared in the previous subsection. The parameter $\mathcal{S}$ is the same parameter appearing in [12] and breaks the ISD condition and thus breaks SUSY. It was shown in [12] that $\mathcal{S}$ contributes a finite amount to the ADM mass as one would expect from the addition of D3s or $\overline{\text{D3}}$s but since it does not contribute to the net charge, $\mathcal{S}$ characterizes the influence of D3–$\overline{\text{D3}}$ pairs.

Similarly, while turning on the parameter $\mathcal{J}$ preserves the ISD condition (A.8) and the first derivative equation for warp factor (A.9) and does not introduce a $(0,3)$-component to G_3, it causes the unwarped 6D space to no longer be Ricci flat (and therefore no longer Calabi–Yau)

[9] As was the case in the previous section, including the finite deformation corrections to KT will not change the form of the perturbations.

so that there is no spinor covariantly constant with respect to the unwarped metric, implying that supersymmetry is broken. In order for the flux part of the Killing spinor equations to vanish, any Killing spinor of the perturbed geometry would have to satisfy the same chirality conditions as the Killing spinor of the unperturbed geometry (i.e. $\Gamma_z \epsilon = 0$ where z is any holomorphic coordinate of KT). Therefore, while the flux part of the SUSY variation of the gravitino vanishes, the spin connection part does not and SUSY is broken. A priori, one might expect a cancellation between the flux and spin connection parts might be possible for a different choice of chirality, but one can show that this cannot occur (see e.g. [44] and references therein).

Although a non-vanishing $\mathcal{J}$ breaks supersymmetry, it does not describe the presence of $\overline{\text{D3}}$-branes. Note that while taking $\mathcal{J} \neq 0$ does not add any charge to the background, it still might describe the presence of D3–$\overline{\text{D3}}$-brane pairs. However, such a configuration would still provide a localized source of tension. The constant shift $\mathcal{J}$ in the squashing cannot be a result of a localized tension since such a source should cause a functional form that is singular as $r \to 0$. Similarly, the perturbed warp factor is not a result of additional localized sources of tension, but results as a solution of (A.9) with the perturbed squashing functions. Thus, the large radius backreaction of D3–$\overline{\text{D3}}$-pairs is found by setting $\mathcal{J} = 0$, reproducing the result of [12].[10]

6. Gravitino mass

In this section we calculate the effective 4D gravitino mass that results from dimensional reduction of the SUSY breaking solution. The gravitino can potentially obtain a mass from interactions with the 5-form flux F_5 and the 3-form flux G_3. This problem was addressed previously in [45], though in their analysis they considered a background for which the warp factor satisfied the condition (A.9). However, even when this condition is not satisfied, their method can still be applied and we follow it closely here. Note that although we are interested in the specific case of the warped deformed conifold, this discussion applies to any perturbation of a warped Calabi–Yau.

Since we work in the Einstein frame, we relate the Einstein frame spinors to those in the string frame

$$\begin{aligned}
&\Psi_M^E = g_s^{\frac{1}{8}} e^{-\frac{\phi}{8}} \Psi_M^s - \frac{i}{4} g_s^{\frac{1}{8}} e^{-\frac{\phi}{8}} \Gamma_M \lambda^{s*}, \qquad \lambda^E = g_s^{-\frac{1}{8}} e^{\frac{\phi}{8}} \lambda^s, \\
&\Gamma_M^E = g_s^{\frac{1}{4}} e^{-\frac{\phi}{4}} \Gamma_M^s, \qquad \epsilon^E = g_s^{\frac{1}{8}} e^{-\frac{\phi}{8}} \epsilon^s.
\end{aligned} \tag{6.1}$$

Up through bilinear terms, the action for the type IIB Einstein frame gravitino is

$$\begin{aligned}
&S_f = \frac{1}{\kappa^2} \int d^{10}x \sqrt{-g} (\mathcal{L}_1 + \mathcal{L}_2), \\
&\mathcal{L}_1 = i \bar{\Psi}_M \Gamma^{MNS} \left(D_N \Psi_S + \frac{i}{4} e^{\Phi} \partial_N C \Psi_S + i \frac{g_s}{192} \Gamma^{R_1 R_2 R_3 R_4} F_{S R_1 R_2 R_3 R_4} \Psi_N \right), \\
&\mathcal{L}_2 = -i \frac{g_s^{1/2} e^{\Phi/2}}{192} \bar{\Psi}_M \Gamma^{MNS} \left(\Gamma_S^{R_1 R_2 R_3} G_{R_1 R_2 R_3} - 9 \Gamma^{R_1 R_2} G_{S R_1 R_2} \right) \Psi_N^* + \text{h.c.},
\end{aligned} \tag{6.2}$$

where D_M is the covariant derivative which, when acting on Ψ_μ, is given by

[10] We thank S. Kachru and M. Mulligan for some useful comments related to this discussion.

$$D_{[\mu}\Psi_{\nu]} = \hat{D}_{[\mu}\Psi_{\nu]} - \frac{1}{8}\Gamma_{[\mu}\not{\partial}\log h\Psi_{\nu]},$$
$$D_{[m}\Psi_{\nu]} = \hat{D}_{[m}\Psi_{\nu]} + \frac{1}{8}\Gamma_{[m}\not{\partial}\log h\Psi_{\nu]} - \frac{1}{8}\partial_{[m}\log h\Psi_{\nu]}, \tag{6.3}$$

where $\hat{D}_\mu$ and $\hat{D}_m$ are the covariant derivatives built from the unwarped metrics $\hat{g}_{\mu\nu}$ and $\hat{g}_{mn}$.

The Ψ_μ part of the 10D gravitino is decomposed as a product of a 4D gravitino ψ_μ and a 6D spinor χ that is covariantly constant with respect to the unwarped metric

$$\Psi_\mu\left(x^\mu, x^m\right) = \psi_\mu\left(x^\mu\right) \otimes h^{-\frac{1}{8}}\chi\left(x^m\right), \tag{6.4}$$

where χ is normalized such that $\chi^\dagger\chi = 1$. The $h^{-1/8}$ factor of the warp factor comes from requiring that the spinor is covariantly constant with respect to the warped metric, $\hat{D}_m\Psi_\mu = 0$ [46].

The 4D kinetic term following from (6.2) can be evaluated by dimensional reduction

$$\frac{1}{\kappa^2}\int d^{10}x\sqrt{-g}\,i\bar{\Psi}_\mu\Gamma^{\mu\nu\rho}\hat{D}_\nu\Psi_\rho = \frac{1}{\kappa_4^2}\int d^4x\sqrt{-\hat{g}_4}\,i\bar{\psi}_\mu\hat{\Gamma}^{\mu\nu\rho}\hat{D}_\nu\psi_\rho, \tag{6.5}$$

where on the right hand the indices are contracted with the unwarped metric $\hat{g}_{\mu\nu}$ and where the 4D gravitational constant and the warped volume are

$$\frac{1}{\kappa_4^2} \equiv \frac{1}{\kappa^2}V_6^w, \qquad V_6^w \equiv \int d^6y\sqrt{\hat{g}_6}h. \tag{6.6}$$

If the supersymmetry condition on the warp factor (A.9) is satisfied, then the coupling to F_5 is canceled by the spin connection. However in general this interaction term could a priori contribute to the gravitino mass and we have

$$\begin{aligned}&\frac{1}{\kappa^2}\int d^{10}x\sqrt{-g}\,i\bar{\Psi}_M\Gamma^{MNR}\left(-\frac{1}{4}\omega_R^{AB}\hat{\Gamma}_{AB} + i\frac{g_s}{16}\not{F}_5\Gamma_R\right)\Psi_N \\ &\quad \ni -\frac{1}{\kappa^2}\int d^4x\sqrt{-\hat{g}_4}\bar{\psi}_\mu\hat{\Gamma}^{\mu\nu}\psi_\nu\int d^6y\sqrt{\hat{g}_6}\frac{i}{8h^{1/2}}\left(\frac{h'}{\sqrt{p}} + \frac{(g_sM\alpha'^2)}{4}\frac{\ell}{\sqrt{bqs}}\right)\chi^\dagger\hat{\Gamma}_\tau\chi,\end{aligned} \tag{6.7}$$

where ω_M^{AB} is the spin connection with letters from the beginning of the alphabet denoting tangent space indices and where on the right-hand side, terms involving the unwarped spin-connections have been omitted and indices are again contracted with the unwarped metric. The gravitino mass resulting from the 5-form flux is then

$$\frac{i}{8V_6^w}\int d^6y\sqrt{\hat{g}_6}h^{-1/2}\left(\frac{h'}{\sqrt{p}} + \frac{(g_sM\alpha'^2)}{4}\frac{\ell}{\sqrt{bqs}}\right)\chi^\dagger\hat{\Gamma}_\tau\chi. \tag{6.8}$$

However, this term vanishes as a result of the 6D chirality of χ and thus F_5 does not contribute to the gravitino mass.

The essential contribution to the gravitino mass comes from the 3-form flux. Dimensional reduction gives

$$\frac{1}{\kappa^2}\int d^4x\sqrt{-\hat{g}_4}\bar{\psi}_\mu\hat{\Gamma}^{\mu\nu}\psi_\nu^*\int d^6y\sqrt{\hat{g}_6}\left(\frac{i\sqrt{g_s}e^{\Phi/2}}{64}\chi^\dagger\hat{\Gamma}^{mnp}\chi^*G_{mnp} + \text{h.c.}\right). \tag{6.9}$$

Since $\hat{\Gamma}^{\bar{\imath}}\chi = 0$, we can write

$$\chi^\dagger \hat{\Gamma}^{mnp}\chi^* = \chi^\dagger \hat{\Gamma}^{\bar{\imath}\bar{\jmath}\bar{k}}\chi^* = \Omega^{\bar{\imath}\bar{\jmath}\bar{k}}, \tag{6.10}$$

where Ω is the holomorphic 3-form of the underlying Calabi–Yau whose explicit form for the deformed conifold is given in (B.11). Thus only the $(0,3)$-component of G_3 contributes to the gravitino mass.[11] This has been shown previously [45], but here we argued that it holds even when (A.9) is not satisfied. The 4D gravitino mass resulting from the 3-form flux is then

$$m_{3/2} = \frac{3\sqrt{g_s}}{i\mathcal{V}_6^w}\int e^{\Phi/2}\Omega \wedge G_3, \tag{6.11}$$

which is quite similar to what follows from the Gukov–Vafa–Witten superpotential [47]. With the explicit formula for the of Kähler potential[12] and restoring the Kähler modulus ρ, we can write the gravitino mass as [45]

$$m_{3/2} \propto \kappa_4^2 e^{\frac{\mathcal{K}}{2}} W_{GVW}, \tag{6.12}$$

where W_{GVW} is the GVW superpotential and $\mathcal{K}$ is the Kähler potential.

If we apply these expressions for the gaugino mass to (3.12), we find

$$m_{3/2} \sim \kappa_4^2 \frac{(\mathcal{S}+10\mathcal{T})\varepsilon^{2/3}}{a_0(g_s M\alpha')\tau_{\min}}. \tag{6.13}$$

In evaluating this, we have assumed that most of the contribution to the gravitino mass should come from small τ, close to where the source of SUSY breaking is located, and cut the integral at some lower bound $\tau_{\min}$. The lower bound must be introduced because for sufficiently small τ, the supergravity approximation breaks down. For the singular solutions of Section 3 where the warp factor behaves at small τ as $O(1/\tau)$, the Ricci scalars of these backgrounds behave as $R \sim \mathcal{S}/(g_s M\alpha'\tau)$ where $\mathcal{S}$ stands for any of the parameters characterizing the perturbation (which we expect to be all of the same order for a given solution). Thus, the solutions are valid for τ satisfying $1/(g_s M\alpha') \ll \tau < 1$. If we naïvely take $\tau_{\min}$ to be this lower bound then

$$m_{3/2} \sim \kappa_4^2 \mathcal{S}\varepsilon^{2/3}. \tag{6.14}$$

This is a finite value even if $g_s M$ is large. A more precise calculation of the gravitino mass would require extending the integral to smaller τ where the stringy corrections to the geometry become important.

We also found solutions which behaves regularly at $\tau = 0$. The result of the calculation for the solution in Section 4.2 is

$$\begin{aligned} m_{3/2} \sim \kappa_4^2 \frac{\varepsilon^{2/3}}{g_s M\alpha'}\big[&\big(-318 + 20\,6^{1/3}a_0\big)\mathcal{M} - 120\mathcal{Q} \\ &- \big(624 + 240\,6^{1/3}a_0\big)\mathcal{D} + \big(6 + 5\,6^{1/3}a_0\big)\varphi\big]. \end{aligned} \tag{6.15}$$

This is a finite value, but since $\mathcal{S}$ is taken to be perturbatively small, and $g_s M$ is large, the mass of the gravitino is highly suppressed.

The solutions (3.6) and (4.1) yield values for the gravitino mass that are similar to (6.13) and (6.15) respectively.

[11] We are treating the background as a non-SUSY perturbation to a warped Calabi–Yau. More generally, when the Calabi–Yau is squashed there will be additional potential contributions from terms such as $g^{ij}g^{kl}g^{\bar{m}n}G_{ik\bar{m}}\Omega_{jln}$, but these are higher order in perturbations since the unperturbed metric has $g_{ij} = g_{\bar{\imath}\bar{\jmath}} = 0$.

[12] Here we continue to follow [45], but in the presence of strong warping, the Kähler potential should be modified from the expression used there [48–54].

7. Discussion

In this paper, we analyze several solutions to type IIB supergravity, corresponding to non-supersymmetric perturbations to the warped deformed conifold. Of particular interest are the solutions presented in Section 3.3 which capture some key properties of a solution describing the backreaction of $\overline{\text{D3}}$-branes smeared over the finite $\mathbb{S}^3$ at $\tau = 0$. In particular, we discussed the *necessary* boundary conditions in the IR for the solution to describe a localized $\overline{D3}$ source and how these IR boundary conditions lead to the constant component of H_3 that was discussed in [37]. These solutions are thus related to a small τ expansion of a background whose large radius behavior was found in [12] and is dual to a metastable SUSY breaking state.

For all of the above solutions, we have assumed the validity of a linearized approximation. For a small number $\overline{\text{D3}}$-branes, it is natural to expect that the linearized approximation is valid at least at large distances where the background flux largely dominates the effects of the $\overline{\text{D3}}$, though an extrapolation to larger radii would be necessary to confirm this. For small distances, one can ensure that the linearized approximation is good for τ above some particular value determined by the parameters of the solution. The linearized approximation requires, for example that $F_p \ll F_{KS}$. Using the perturbations of Section 3.3 and taking $\mathcal{S} \sim \mathcal{B} \sim \mathcal{Y}$, this gives the condition $\tau \gg \mathcal{S}^{1/3}$ where $\mathcal{S} \sim \kappa_{10}^2 T_3 (N_{\text{D3}} + N_{\overline{\text{D3}}})/(V_2 g_s^2 M^2 \alpha'^2)$. Similar or less restrictive conditions follow by considering the other functions in the perturbation. As discussed above, a similar constraint is imposed by demanding that the curvature (3.15) is small in string units.[13] Note that for large M, τ is allowed to be quite small. For the other solutions presented above for which there is not always an obvious boundary condition to impose, the validity of the linearized approximation is more difficult to check.

There are several remaining open lines of research. A particularly important remaining open problem is to find a solution that interpolates between the small and large radius regions.[14] Such a solution would be important for many reasons. For example, all of the above solutions should admit a dual description as either deformations of the KS gauge theory or states in the (possibly deformed) KS gauge theory. Although for some of the solutions the field theory interpretation has been studied (for example, the dual of the $\overline{\text{D3}}$ solution was considered in [12]), analysis of the remaining solutions would clearly require extrapolating them to the UV. Additionally, the boundary conditions discussed in Section 3.4 do not seem to be sufficient to fix all of the integration constants. Having a solution that is valid at all distances would allow for a calculation of quantities such as the Hawking–Horowitz mass or the asymptotic charge which could provide other conditions to fix the integration constants. Finally, an interpolating solution would allow for a more precise calculation of the flux-induced gravitino mass and similar quantities. Unfortunately, even the linearized equations of motion are likely too complex to solve analytically in which case the solution could only be presented numerically or formally in terms of integrals, an analysis that we leave for future work.

The solutions could be improved in other ways. For example, the solutions presented in Sections 3.2 and 3.3 exhibit curvature singularities as $\tau \to 0$ and it is an interesting, though difficult, problem to understand the stringy modifications of those backgrounds. More modestly, it would be interesting to relax the assumption that the solutions retain the same isometry as the KS so-

[13] The additional requirement that $h_p \lesssim h_p^2$ can be satisfied if ε is not too small.

[14] As mentioned in the introduction, some progress was made in this direction after this paper was completed [37].

lution by, for example, not smearing the $\overline{\text{D3}}$-branes over the $\mathbb{S}^3$.[15] One can also consider similar perturbations to the baryonic branch solution [56,57].

Along similar lines, the solution (3.12a), which, for some choice of parameters, would describe the effect of $\overline{\text{D3}}$-branes on the near tip geometry of KS, has been argued to be a metastable background [11]. However, it would be interesting to use the explicit solution to analyze fluctuations about this geometry to confirm the perturbative stability, though this would require moving beyond the linearized approximation.

Our solutions have potential applications to model building in warped compactifications. For example, the addition of $\overline{\text{D3}}$-branes into the warped deformed conifold was an important step in the construction of stabilized de Sitter vacua [1] and in the modeling of inflation (see [19–22,58] and references therein). It would be interesting to understand the impact of the backreaction of the $\overline{\text{D3}}$-branes on these scenarios. The construction in [1] further inspired the scenario of mirage mediation [59] and one might use the solutions given here to provide a more string theoretical understanding of this scheme.

A related though conceptually distinct application is in the context of gauge-gravity duality. The large radius solution [12] was used in [34] as a holographic dual of a metastable SUSY breaking state. The large amount of isometry in this large radius region was found to suppress gaugino masses in their construction. However, the small radius solution presented in Section 3.3, has reduced isometry, and should result in more significant contributions. Details of the application to holographic gauge mediation will be discussed in a companion paper [33].

Acknowledgements

We have benefited from discussions with A. Dymarsky, A. Ishibashi, S. Kachru, I. R. Klebanov, F. Marchesano, L. McAllister, M. Mulligan, P. Ouyang, Y. Tachikawa, and H. L. Verlinde. PM and GS also thank T. Liu for collaboration and preliminary discussions on this and related topics. We would like to thank the Institute for Advanced Study and the Hong Kong Institute for Advanced Study, Hong Kong University of Science and Technology for hospitality and support. PM and GS also thank the Standford Institute for Theoretical Physics and SLAC for hospitality while some preliminary discussions were held. YS appreciates the KEK Theory Center and the International Visitor Program for hospitality and the opportunity to present this work. PM and GS were supported in part of NSF CAREER Award No. Phy-0348093, DOE grant DE-FG-02-95ER40896, a Cottrell Scholar Award from Research Corporation, a Vilas Associate Award from the University of Wisconsin, and a John Simon Guggenheim Memorial Foundation Fellowship. YS was supported by Nishina Memorial Foundation. GS also would like to acknowledge support from the Ambrose Monell Foundation during his stay at the Institute for Advanced Study.

Appendix A. Conventions

We work in the type IIB supergravity limit where the bosonic part of the Einstein frame action is [60]

$$S = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{-g}\bigg[R - \frac{1}{2}\partial_M\Phi\partial^M\Phi - \frac{1}{2}e^{2\Phi}\partial_M C\partial^M C$$

[15] The localization of three-branes was considered in [55] in different context.

$$- \frac{g_s}{2 \times 3!} e^{-\Phi} H_3^2 - \frac{g_s}{2 \times 3!} e^{\Phi} \tilde{F}_3^2 - \frac{g_s^2}{4 \times 5!} \tilde{F}_5^2 \Big] - \frac{g_s^2}{4\kappa^2} \int C_4 \wedge H_3 \wedge F_3, \tag{A.1}$$

where we use

$$\tilde{F}_3 = dC_2 - CH_3, \qquad \tilde{F}_5 = dC_4 + B_2 \wedge F_3 = (1 + *_{10})\mathcal{F}_5, \qquad 2\kappa^2 = (2\pi)^7 \alpha'^4 g_s^2, \tag{A.2}$$

and the self-duality of $\tilde{F}_5$ is imposed at the level of the equations of motion. The string frame metric is related to the Einstein frame metric by $g^E_{MN} = g_s^{\frac{1}{2}} e^{-\frac{\Phi}{2}} g^s_{MN}$.

The equations of motion resulting from (A.1) are

$$\begin{aligned} &R_{MN} - \frac{1}{2} g_{MN} R - \frac{1}{2} \partial_M \Phi \partial_N \Phi - \frac{1}{2} e^{2\Phi} \partial_M C \partial_N C \\ &\quad - \frac{g_s}{2 \times 2!} e^{-\Phi} H_{MR_1R_2} H_N{}^{R_1R_2} - \frac{g_s}{2 \times 2!} e^{\Phi} \tilde{F}_{MR_1R_2} \tilde{F}_N{}^{R_1R_2} \\ &\quad - \frac{g_s^2}{4 \times 4!} \tilde{F}_{MR_1R_2R_3R_4} \tilde{F}_N{}^{R_1R_2R_3R_4} \\ &\quad + \frac{1}{2} g_{MN} \left[\frac{1}{2} (\partial \Phi)^2 + \frac{1}{2} e^{2\Phi} (\partial C)^2 + \frac{g_s}{2 \times 3!} e^{-\Phi} H_3^2 + \frac{g_s}{2 \times 3!} e^{\Phi} \tilde{F}_3^2 + \frac{g_s^2}{4 \times 5!} \tilde{F}_5^2 \right] = 0, \end{aligned} \tag{A.3a}$$

$$\nabla^2 \Phi - e^{2\Phi} (\partial_M C)^2 + \frac{g_s e^{-\Phi}}{2 \times 3!} \left[H_3^2 - e^{2\Phi} \tilde{F}_3^2 \right] = 0, \tag{A.3b}$$

$$d * \left(e^{-\Phi} H_3 - C e^{\Phi} \tilde{F}_3 \right) + g_s F_5 \wedge F_3 = 0, \tag{A.3c}$$

$$d * \left(e^{\Phi} \tilde{F}_3 \right) - g_s F_5 \wedge H_3 = 0, \tag{A.3d}$$

$$d * \tilde{F}_5 - H_3 \wedge F_3 = 0. \tag{A.3e}$$

Note that imposing the self-duality of $\tilde{F}_5$ implies $\tilde{F}_5^2 = 0$. With the ansatz (2.1), (2.2a), and taking $\ell = f(1 - F) + kF$, the Bianchi identity for $\tilde{F}_5$ is automatically satisfied. With this ansatz, the equations for H_3 (A.3c) can be written

$$\frac{d}{d\tau} \left(e^{-\Phi} h^{-1} \sqrt{\frac{b}{p}} \frac{q}{s} f' \right) + \frac{e^{-\Phi}}{2h} \sqrt{\frac{p}{b}} (k - f) - \frac{g_s M^2 \alpha'^2}{4h^2} \sqrt{\frac{p}{b}} \frac{\ell (1 - F)}{qs} = 0, \tag{A.4a}$$

$$\frac{d}{d\tau} \left(e^{-\Phi} h^{-1} \sqrt{\frac{b}{p}} \frac{s}{q} k' \right) - \frac{e^{-\Phi}}{2h} \sqrt{\frac{p}{b}} (k - f) - \frac{g_s M^2 \alpha'^2}{4h^2} \sqrt{\frac{p}{b}} \frac{\ell F}{qs} = 0, \tag{A.4b}$$

while the equation for F_3 (A.3d) is

$$\frac{d}{d\tau} \left(e^{\Phi} h^{-1} \sqrt{\frac{b}{p}} F' \right) + \frac{e^{\Phi}}{2h} \sqrt{\frac{p}{b}} \left[(1 - F) \frac{s}{q} - F \frac{q}{s} \right] - \frac{g_s^3 M^2 \alpha'^2}{8h^2} \sqrt{\frac{p}{b}} \frac{\ell (k - f)}{qs} = 0. \tag{A.5}$$

The bosonic and fermionic actions together are invariant under the supersymmetric transformations for the gravitino Ψ_M and dilatino λ,

$$\delta\Psi_M = D_M\epsilon + i\frac{\sqrt{g_s}e^{\Phi/2}}{96}\big(\Gamma_M{}^{R_1R_2R_3}G_{R_1R_2R_3} - 9\Gamma^{R_1R_2}G_{MR_1R_2}\big)\epsilon^* + i\frac{g_s}{192}\Gamma^{R_1R_2R_3R_4}F_{MR_1R_2R_3R_4}\epsilon, \tag{A.6a}$$

$$\delta\lambda = \frac{i}{2}\Gamma^R\big(ie^{\Phi}\partial_R C + \partial_R\Phi\big)\epsilon^* - \frac{e^{\Phi}}{2}\Gamma^R\partial_R C\epsilon + \frac{\sqrt{g_s}e^{\Phi/2}}{24}\Gamma^{R_1R_2R_3}G_{R_1R_2R_3}\epsilon, \tag{A.6b}$$

together with accompanying bosonic transformations. Here,

$$G_3 \equiv F_3 - \tau_{ad}H_3, \qquad F_3 = dC_2, \qquad \tau_{ad} \equiv C_0 + ie^{-\Phi}. \tag{A.7}$$

We can use these transformations to check if supersymmetry is respected by the solution. Using the ansatz (2.1), (2.2a), the supersymmetry conditions $\delta\Psi_M = \delta\lambda = 0$ imply

$$1 - F - g_s e^{-\Phi}\sqrt{\frac{b}{p}}\frac{q}{s}f' = 0, \qquad F - g_s e^{-\Phi}\sqrt{\frac{b}{p}}\frac{s}{q}k' = 0,$$
$$F' - \frac{g_s}{2}e^{-\Phi}\sqrt{\frac{p}{b}}(k - f) = 0, \tag{A.8}$$

which impose that the flux G_3 is imaginary-self-dual (ISD). One can further show that supersymmetry requires that the flux be a primitive (2, 1)-form. The variation for the gravitino (A.6a) requires the warp factor to be related to F_5,

$$h' = -\frac{(g_s M\alpha')^2}{4}\frac{\ell}{qs}\sqrt{\frac{p}{b}}. \tag{A.9}$$

This condition implies that the BPS condition equates the tension and charge of 3-branes added to the geometry.

The conifold and its related geometries make use of the angular 1-forms

$$e_1 = -\sin\theta_1\, d\phi_1, \qquad e_2 = d\theta_1, \qquad e_3 = \cos\psi\sin\theta_2\, d\phi_2 - \sin\psi\, d\theta_2,$$
$$e_4 = \sin\psi\sin\theta_2\, d\phi_2 + \cos\psi\, d\theta_2, \qquad e_5 = d\psi + \cos\theta_1\, d\phi_1 + \cos\theta_2\, d\phi_2. \tag{A.10}$$

In terms of these it is also useful to define [61]

$$g_1 = \frac{e_1 - e_3}{\sqrt{2}}, \qquad g_2 = \frac{e_2 - e_4}{\sqrt{2}}, \qquad g_3 = \frac{e_1 + e_3}{\sqrt{2}}, \qquad g_4 = \frac{e_2 + e_4}{\sqrt{2}}, \qquad g_5 = e_5, \tag{A.11}$$

which satisfy

$$d(g_1\wedge g_3 + g_2\wedge g_4) = g_5\wedge(g_1\wedge g_2 - g_3\wedge g_4),$$
$$d(g_1\wedge g_2 - g_3\wedge g_4) = -g_5\wedge(g_1\wedge g_3 + g_2\wedge g_4),$$
$$d(g_1\wedge g_2 + g_3\wedge g_4) = 0,$$
$$dg_5\wedge g_1\wedge g_2 = dg_5\wedge g_3\wedge g_4 = 0,$$
$$d(g_5\wedge g_1\wedge g_2) = d(g_5\wedge g_3\wedge g_4) = 0. \tag{A.12}$$

Appendix B. Complex coordinates

The angular coordinates and radial coordinate of the deformed conifold are related to the complex coordinates z_i by [62]

$$W = L_1 \cdot W_0 \cdot L_2^\dagger \equiv \begin{pmatrix} z_3 + iz_4 & z_1 - iz_2 \\ z_1 + iz_2 & -z_3 + iz_4 \end{pmatrix}, \tag{B.1a}$$

$$L_j = \begin{pmatrix} \cos\frac{\theta_j}{2} e^{i(\psi_j+\phi_j)/2} & -\sin\frac{\theta_j}{2} e^{-i(\psi_j-\phi_j)/2} \\ \sin\frac{\theta_j}{2} e^{i(\psi_j-\phi_j)/2} & \cos\frac{\theta_j}{2} e^{-i(\psi_j+\phi_j)/2} \end{pmatrix},$$

$$W_0 = \begin{pmatrix} 0 & \varepsilon e^{\tau/2} \\ \varepsilon e^{-\tau/2} & 0 \end{pmatrix}, \tag{B.1b}$$

and the z_i satisfy

$$\sum_{i=1}^4 z_i^2 = \varepsilon^2. \tag{B.2}$$

The angles ψ_i always appear in the combination $\psi = \psi_1 + \psi_2$. For $\varepsilon \neq 0$, τ is defined by

$$R^2 = \sum_{i=1}^4 z_i \bar{z}_i = \frac{1}{2} \operatorname{Tr}\bigl(W \cdot W^\dagger\bigr) = \varepsilon^2 \cosh\tau. \tag{B.3}$$

The deformed conifold metric can be written as [62]

$$\begin{aligned}
ds_6^2 &= \partial_i \partial_{\bar{j}} \mathcal{F}\, dz_i\, d\bar{z}_j \\
&\quad = \frac{1}{4} \mathcal{F}''\bigl(R^2\bigr)\bigl|\operatorname{Tr}\bigl(W^\dagger\, dW\bigr)\bigr|^2 + \frac{1}{2} \mathcal{F}'\bigl(R^2\bigr) \operatorname{Tr}\bigl(dW^\dagger\, dW\bigr) = -i J_{i\bar{j}}\, dz_i\, d\bar{z}_{\bar{j}}, \\
J &= j_{dc}(\tau)(g_2 \wedge g_3 + g_4 \wedge g_1) + dj_{dc}(\tau) \wedge g_5, \\
\mathcal{F}'\bigl(R^2\bigr) &= \varepsilon^{-\frac{2}{3}} K(\tau), \qquad j_{dc}(\tau) = \frac{\varepsilon^2}{2} \sinh\tau\, \mathcal{F}'\bigl(R^2\bigr),
\end{aligned} \tag{B.4}$$

where $'$ indicates a derivative with respect to R^2 and J is the almost complex structure.

It is convenient to write G_3 in terms of these complex coordinates. Following [63], we consider the $SO(4)$ invariant 1-forms and 2-forms

$$\begin{aligned}
&\xi_1 = \bar{z}_i\, dz_i, \quad \xi_2 = z_i\, d\bar{z}_i, \\
&\eta_1 = \epsilon_{ijkl} z_i \bar{z}_j\, dz_k \wedge d\bar{z}_l, \qquad \eta_2 = \epsilon_{ijkl} z_i \bar{z}_j\, dz_k \wedge dz_l, \qquad \eta_3 = \epsilon_{ijkl} z_i \bar{z}_j\, d\bar{z}_k \wedge d\bar{z}_l, \\
&\eta_4 = (z_i\, d\bar{z}_j) \wedge (\bar{z}_j\, dz_j), \qquad \eta_5 = dz_i \wedge d\bar{z}_i.
\end{aligned} \tag{B.5}$$

In terms of these,

$$d\tau = \frac{1}{\varepsilon^2 \sinh\tau}(z_i\, d\bar{z}_i + \bar{z}_i\, dz_i), \qquad g_5 = \frac{i}{\varepsilon^2 \sinh\tau}(z_i\, d\bar{z}_i - \bar{z}_i\, dz_i), \tag{B.6a}$$

$$g_1 \wedge g_2 = \frac{i(1+\cosh\tau)}{2\varepsilon^4 \sinh^3\tau} \epsilon_{ijkl}(2 z_i \bar{z}_j\, dz_k \wedge d\bar{z}_l - z_i \bar{z}_j\, dz_k \wedge dz_l - z_i \bar{z}_j\, d\bar{z}_k \wedge d\bar{z}_l), \tag{B.6b}$$

$$g_3 \wedge g_4 = \frac{i \tanh\frac{\tau}{2}}{2\varepsilon^4 \sinh^2\tau} \epsilon_{ijkl}(2 z_i \bar{z}_j\, dz_k \wedge d\bar{z}_l + z_i \bar{z}_j\, dz_k \wedge dz_l + z_i \bar{z}_j\, d\bar{z}_k \wedge d\bar{z}_l), \tag{B.6c}$$

$$g_1 \wedge g_3 + g_2 \wedge g_4 = \frac{1}{\varepsilon^4 \sinh^2\tau} \epsilon_{ijkl}(-z_i \bar{z}_j\, dz_k \wedge dz_l + z_i \bar{z}_j\, d\bar{z}_k \wedge d\bar{z}_l), \tag{B.6d}$$

$$g_2 \wedge g_3 + g_4 \wedge g_1 = -\frac{2i\cosh\tau}{\varepsilon^4 \sinh^3\tau}(\bar{z}_j\,dz_j) \wedge (z_i\,d\bar{z}_i) + \frac{2i}{\varepsilon^2 \sinh\tau}\,dz_i \wedge d\bar{z}_i. \tag{B.6e}$$

The other remaining 1-forms cannot be as easily written in terms of the complex coordinates. However, we find

$$\begin{aligned} g_1^2 + g_2^2 &= -\frac{1}{2\varepsilon^4 \sinh^2(\tau/2)\sinh^2\tau}\big[(\bar{z}\cdot dz)^2 + (z\cdot d\bar{z})^2 + 2\cosh\tau(\bar{z}\cdot dz)(z\cdot d\bar{z}) \\ &\quad + \varepsilon^2\sinh^2\tau(dz\cdot dz + d\bar{z}\cdot d\bar{z} - 2dz\cdot d\bar{z})\big], \\ g_3^2 + g_4^2 &= \frac{1}{2\varepsilon^4 \cosh^2(\tau/2)\sinh^2\tau}\big[(\bar{z}\cdot dz)^2 + (z\cdot d\bar{z})^2 - 2\cosh\tau(\bar{z}\cdot dz)(z\cdot d\bar{z}) \\ &\quad + \varepsilon^2\sinh^2\tau(dz\cdot dz + d\bar{z}\cdot d\bar{z} + 2dz\cdot d\bar{z})\big]. \end{aligned} \tag{B.7}$$

In terms of these complex coordinates

$$\begin{aligned} G_3^{(3,0)} &= \frac{M\alpha'}{2\varepsilon^6}\bigg[\bigg\{(1-F)\frac{\tanh\frac{\tau}{2}}{2\sinh^3\tau} - F\frac{1+\cosh\tau}{2\sinh^4\tau} - \frac{F'}{\sinh^3\tau}\bigg\} \\ &\quad + g_s e^{-\Phi}\bigg\{-f'\frac{1+\cosh\tau}{2\sinh^4\tau} + k'\frac{\tanh\frac{\tau}{2}}{2\sinh^3\tau} + \frac{k-f}{2\sinh^3\tau}\bigg\}\bigg]\xi_1\wedge\eta_2, \end{aligned} \tag{B.8a}$$

$$\begin{aligned} G_3^{(0,3)} &= \frac{M\alpha'}{2\varepsilon^6}\bigg[-\bigg\{(1-F)\frac{\tanh\frac{\tau}{2}}{2\sinh^3\tau} - F\frac{1+\cosh\tau}{2\sinh^4\tau} - \frac{F'}{\sinh^3\tau}\bigg\} \\ &\quad + g_s e^{-\Phi}\bigg\{-f'\frac{1+\cosh\tau}{2\sinh^4\tau} + k'\frac{\tanh\frac{\tau}{2}}{2\sinh^3\tau} + \frac{k-f}{2\sinh^3\tau}\bigg\}\bigg]\xi_2\wedge\eta_3. \end{aligned} \tag{B.8b}$$

For the KS solution (2.4), these components vanish since each of the terms in braces vanishes independently. The remaining components of G_3 are

$$G_3^{(2,1)} = \frac{M\alpha'}{2\varepsilon^6}\big\{2\big(a_1^+ + a_2^+\big)\xi_1\wedge\eta_1 + \big(a_1^- - a_2^- - a_3^+\big)\xi_2\wedge\eta_2\big\}, \tag{B.9a}$$

$$G_3^{(1,2)} = \frac{M\alpha'}{2\varepsilon^6}\big\{2\big(a_1^- + a_2^-\big)\xi_2\wedge\eta_1 + \big(a_1^+ - a_2^+ - a_3^-\big)\xi_1\wedge\eta_3\big\}, \tag{B.9b}$$

where we have defined

$$a_1^\pm(\tau) = \frac{\tanh\frac{\tau}{2}}{2\sinh^3\tau}\big(\pm(1-F) + g_s e^{-\Phi}k'\big), \tag{B.10a}$$

$$a_2^\pm(\tau) = \frac{1+\cosh\tau}{2\sinh^4\tau}\big(\pm F + g_s e^{-\Phi}f'\big), \tag{B.10b}$$

$$a_3^\pm(\tau) = \frac{1}{\sinh^3\tau}\bigg(\pm F' + g_s e^{-\Phi}\frac{k-f}{2}\bigg). \tag{B.10c}$$

For the KS solution, the only non-vanishing term is the $(2,1)$-form. The 3-form flux for the KS solution can also be shown to satisfy the primitivity condition $G_3 \wedge J = 0$.

In calculating the gravitino mass, we make use of the holomorphic $(3,0)$-form of the deformed conifold. Explicitly [57,64,65],

$$\begin{aligned} \Omega &= \frac{\varepsilon^2}{16\sqrt{3}}\big[-\sinh\tau(g_1\wedge g_3 + g_2\wedge g_4) + i\cosh\tau(g_1\wedge g_2 - g_3\wedge g_4) \\ &\quad - i(g_1\wedge g_2 + g_3\wedge g_4)\big]\wedge(d\tau + ig_5) \\ &= \frac{1}{4\sqrt{3}\varepsilon^4\sinh^2\tau}(\epsilon_{ijkl}z_i\bar{z}_j\,dz_k\wedge dz_l)\wedge(\bar{z}_m\,dz_m). \end{aligned} \tag{B.11}$$

Ω is normalized so that $\Omega \wedge \bar{\Omega}/\|\Omega\|^2 = i\mathrm{vol}_6$ with $\|\Omega\|^2 \equiv \Omega_{ijk}\bar{\Omega}^{ijk}/3! = 1$ where the indices are contracted with the unwarped metric. The holomorphic 3-form and the almost complex structure (B.4) satisfy the algebraic constraints $\Omega \wedge \bar{\Omega} = -\frac{i}{48}J^3$, $J \wedge \Omega = 0$ and the sourceless calibration conditions, $d\Omega = dJ = d(J \wedge J) = 0$.

Some of these expressions simplify if we adopt an alternative basis of holomorphic 1-forms,

$$dZ_1 \equiv d\tau + i g_5, \qquad dZ_2 \equiv g_1 - i \coth\frac{\tau}{2} g_4, \qquad dZ_3 \equiv g_3 - i \tanh\frac{\tau}{2} g_2. \tag{B.12}$$

In these coordinates, G_3 is

$$\begin{aligned} G_3 = -\frac{M\alpha'}{16\sinh^2\tau}\big[&4(\sinh\tau - \tau\cosh\tau)\, d\bar{Z}_1 \wedge dZ_2 \wedge dZ_3 \\ &+ (\sinh 2\tau - 2\tau)(d\bar{Z}_2 \wedge dZ_1 \wedge dZ_3 + d\bar{Z}_3 \wedge dZ_1 \wedge dZ_2)\big], \end{aligned} \tag{B.13a}$$

while the holomorphic 3-form and metric for the deformed conifold are

$$\Omega = -\frac{\varepsilon^2}{16\sqrt{3}} \sinh\tau\, dZ_1 \wedge dZ_2 \wedge dZ_3, \tag{B.13b}$$

$$ds_6^2 = \frac{\varepsilon^{4/3}}{6K^2} dZ_1\, d\bar{Z}_1 + \frac{\varepsilon^{4/3}K}{2}\sinh^2\frac{\tau}{2}\, dZ_2\, d\bar{Z}_2 + \frac{\varepsilon^{4/3}K}{2}\cosh^2\frac{\tau}{2}\, dZ_3\, d\bar{Z}_3 \tag{B.13c}$$

where K is defined in (2.4).

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