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Relativistic quantum information in detectors–field interactions

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Abstract

We review Unruh–DeWitt detectors and other models of detector–field interaction in a relativistic quantum field theory setting as a tool for extracting detector–detector, field–field and detector–field correlation functions of interest in quantum information science, from entanglement dynamics to quantum teleportation. In particular, we highlight the contrast between the results obtained from linear perturbation theory which can be justified provided switching effects are properly accounted for, and the nonperturbative effects from available analytic expressions which incorporate the backreaction effects of the quantum field on the detector behavior.

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(Some figures may appear in colour only in the online journal)

1. Introduction and background

1.1. Some basic issues in relativistic quantum information

We assume that the readers are somewhat familiar with the Unruh effect [1] on the one hand and the basic issues of quantum information on the other [2] and will only highlight the relativistic aspects of both of these topics here. First, the quantum field acting as an environment to the discrete (qubits) or continuous variables (oscillators) in quantum information processing—we will refer to these point-like physical objects with internal degrees of freedom as detectors (the Unruh–DeWitt (UD) detector being the familiar one [3])—will

necessarily exert environmental influences on the system. Second, the motional states of the detectors (e.g., inertial or accelerated, uniformly or otherwise) will affect both the quantum decoherence and entanglement dynamics of these detectors.

1.1.1. Quantum field effects. The presence of a quantum field is unavoidable, as it acts as a ubiquitous environment to the qubits or detectors in question. Two basic issues of quantum information need to be included in one's consideration are as follows.

Quantum decoherence. Coupling to a quantum field can induce decoherence of a single qubit or oscillator, but their mutual influences mediated by a field can lessen the degree of decoherence if the two qubits are placed in close range [4].

Entanglement dynamics. The entanglement between two qubits or oscillators changes in time as their reduced state (after coarse-graining over the field) evolves; it also depends on their spatial separation [4, 5].

1.1.2. Kinematical effects.

Unruh effect. A uniformly accelerated detector coupled with a quantum field in the Minkowski vacuum would experience a thermal bath of the field quanta at the Unruh temperature proportional to its proper acceleration. This was first discovered by Unruh using time-dependent perturbation theory (TDPT) [1]. Generalized considerations follow in the works of Higuchi *et al* [6] and Louko *et al* [7]. Exact solutions going beyond these test-field descriptions were found by Lin and Hu [8, 9] with interesting new physics.

Non-uniform acceleration. The kinematical viewpoint has proven to be more malleable and adaptable than the traditional geometrical (global concepts such as event horizon) viewpoint. We will mention how newer models in the 1990s such as the RSG model [10], especially those which incorporate open quantum system concepts such as the Raval–Hu–Anglin–Koks (RHAK) models [11, 12], have aided in treating non-uniform acceleration, in work from the 1990s (e.g., [13]) to now [14].

Mutual influences. The influence of one detector on the field will propagate in space and affect other detectors after some time. These causal mutual influences propagating back and forth are a source of non-Markovianity in multi-detector theories. They act to augment the quantum coherence between two detectors placed in close range. Another source of non-Markovianity is the long-range autocorrelation of the quantum field.

1.1.3. Relativistic effects. Furthermore, objects in a relativistic system may behave differently when observed in different reference frames, so we have the following.

Frame dependence. Quantum entanglement of two objects localized at different positions on a spatial hypersurface is a kind of spacelike correlation; the time evolution of their entanglement will depend on how the spacetime is foliated by spacelike hypersurfaces.

Time dilation. For moving objects with worldlines parameterized by their proper times, their time dilations observed in a reference frame will naturally enter the dynamics in that frame.

Projective measurement local in space. Quantum states make sense only in a given frame where a Hamiltonian is well defined [15]. Two quantum states of the same system with quantum fields in different frames are comparable only on those totally overlapping time-slices associated with certain moments in each frame. By a measurement local in space, e.g. on a point-like UD detector coupled with a quantum field, quantum states of the combined system in different frames can be interpreted as if they collapsed on different time-slices passing through the same measurement event. Nevertheless, the post-measurement states will evolve to the same state up to a coordinate transformation when they are compared at some time-slice in the future. In a two-detector system with the first detector being measured at some moment, the reduced state of the second detector collapsed in different frames will become consistent once it enters the future lightcone of the measurement event [4, 5, 16–18].

1.2. Unruh effect via perturbation theory and quantum information via exact solutions

TDPT was used by Unruh originally to show the detector response to uniform acceleration. For a comprehensive description of Unruh effect, see, e.g., the recent review of [19]. With the infusion of quantum open system ideas in the 1990s, these TDPT results were later found to hold only in the Markovian regime, corresponding to the limits of ultrahigh acceleration or ultraweak coupling. Discovery of exact solutions in the 2000s showed that the transition probability calculated from the infinite-time TDPT is valid only in transient under restricted conditions. We will develop the perturbative theory further in section 2 and comment on these developments in the last two sections. In section 3, we will introduce two other more general models for moving detector–quantum field interaction, namely the RSG and the RHAK models, treating the detectors as harmonic oscillators (HOs) rather than the two-level system as in the original Unruh derivation. We will also bring in the broader scope provided by the theory of open quantum systems exemplified by the quantum Brownian model, where the use of reduced density matrix and influence functionals opens the way to exploring the full parameter range of detector–field interaction including self-consistent backreaction. This opens the door for quantum information inquires. In section 4, we describe the detector–field dynamics from the exact solutions of one such model, and identify the limitations of TDPT. In section 5, we give an important example of relativistic quantum information, that of quantum teleportation which uses pretty much all of the relativistic and quantum information elements developed, such as frame dependence and entanglement dynamics. We end with some suggestions on further developments.

2. Nonstationary detector within first-order perturbation theory

In this section, we summarize recent results about the transition rate of a pointlike detector within linear perturbation theory, in situations where neither the detector trajectory nor the state of the quantum field is assumed stationary [7, 20–22]. The central issue is to isolate switch-on and switch-off effects from phenomena that are genuinely due to the acceleration and to the state of the field.

2.1. Transition probability

We consider a pointlike *two-level* detector that moves in a spacetime of dimension $d \geq 2$ along the worldline $z(\tau)$, where the parameter τ is the detector's proper time. The motion is driven by an external agent who is decoupled from the detector's internal degrees of freedom and from the quantum field to which the detector couples.

The detector's internal Hilbert space here is two dimensional, spanned by the orthonormal basis states $|0\rangle_d$ and $|\omega\rangle_d$ whose respective energy eigenvalues are 0 and ω , with $\omega \neq 0$. For $\omega > 0$, $|0\rangle_d$ is the ground state and $|\omega\rangle_d$ is the excited state; for $\omega < 0$, the roles of the states are reversed. A generalization to a countable number of nondegenerate energy eigenstates would be straightforward.

The spacetime contains a free real scalar field ϕ , whose mass and curvature coupling parameter may be arbitrary. The detector is coupled to ϕ linearly, by the interaction picture Hamiltonian

$$H_{\text{int}} = \lambda \chi(\tau) Q(\tau) \phi(z(\tau)), \quad (1)$$

where λ is the coupling constant and Q is the detector's monopole moment operator. The switching function χ specifies how the interaction is turned on and off. We assume χ to be smooth, nonnegative and of compact support. We also assume the trajectory $z(\tau)$ to be smooth.

We denote the initial state of the field by $|\psi_0\rangle$, and we assume $|\psi_0\rangle$ to be regular in the sense of the Hadamard property [23, 24]. The detector is initially prepared in the state $|0\rangle_d$.

We work within first-order perturbation theory in λ . After the interaction has ceased, the probability for the detector to be found in the state $|\omega\rangle_d$, regardless of the final state of the field, is [25, 26]

$$P(\omega) = \lambda^2 |{}_d\langle 0|Q(0)|\omega\rangle_d|^2 \mathcal{F}(\omega), \quad (2)$$

where the response function $\mathcal{F}(\omega)$ is given by

$$\mathcal{F}(\omega) = 2\text{Re} \int_{-\infty}^{\infty} du \chi(u) \int_0^{\infty} ds \chi(u-s) e^{-i\omega s} W(u, u-s), \quad (3)$$

and the correlation function $W(\tau', \tau'') := \langle \psi_0 | \phi(z(\tau')) \phi(z(\tau'')) | \psi_0 \rangle$ is the pull-back of the Wightman function to the detector's worldline. The prefactor $\lambda^2 |{}_d\langle 0|Q(0)|\omega\rangle_d|^2$ in (2) depends only on the detector's internal structure, while $\mathcal{F}(\omega)$ (3) encodes the dependence on $|\psi_0\rangle$, the trajectory and the switching. With minor abuse of terminology, we refer to $\mathcal{F}(\omega)$ as the transition probability.

2.2. Transition probability without distributional integrals

While formula (3) for the transition probability is as such well defined, it is not well suited for discussing how the probability depends on the switching function, especially when the switching becomes sharp. The correlation function W is not a genuine function but a distribution. When W is represented by a family W_ϵ of functions that converge to W as $\epsilon \rightarrow 0_+$, the sense of convergence entails that the limit $\epsilon \rightarrow 0_+$ is taken in (3) only *after* the integrals are evaluated [27–30]. The sharp switching limit may hence not necessarily be brought under the integrals and the $\epsilon \rightarrow 0_+$ limit in (3) [31–33].

What is needed is to re-express (3) in terms of the genuine function $W_0 := \lim_{\epsilon \rightarrow 0_+} W_\epsilon$, where the limit is understood pointwise. The results for $d = 2$, $d = 3$ and $d = 4$ are [7, 20–22]

$$\mathcal{F}_{d=2}(\omega) = 2\text{Re} \int_{-\infty}^{\infty} du \chi(u) \int_0^{\infty} ds \chi(u-s) e^{-i\omega s} W_0(u, u-s), \quad (4)$$

$$\mathcal{F}_{d=3}(\omega) = \frac{1}{4} \int_{-\infty}^{\infty} du [\chi(u)]^2 + 2 \int_{-\infty}^{\infty} du \chi(u) \int_0^{\infty} ds \chi(u-s) \text{Re}[e^{-i\omega s} W_0(u, u-s)], \quad (5)$$

$$\begin{aligned} \mathcal{F}_{d=4}(\omega) = & -\frac{\omega}{4\pi} \int_{-\infty}^{\infty} du [\chi(u)]^2 + \frac{1}{2\pi^2} \int_0^{\infty} \frac{ds}{s^2} \int_{-\infty}^{\infty} du \chi(u) [\chi(u) - \chi(u-s)] \\ & + 2 \int_{-\infty}^{\infty} du \chi(u) \int_0^{\infty} ds \chi(u-s) \text{Re} \left(e^{-i\omega s} W_0(u, u-s) + \frac{1}{4\pi^2 s^2} \right), \quad (6) \end{aligned}$$

and those for $d = 5$ and $d = 6$ can be found in [22] in the special case of a Minkowski space massless field in the Minkowski vacuum. The crucial point is that in addition to an expected integral term that involves W_0 , there are also additional terms that depend on the switching. These additional terms are remnants of the distributional singularity of W , and they are absent only for $d = 2$, where the singularity of W is merely logarithmic.

The Hadamard property of the Wightman function guarantees that the integrals in (4)–(6) are convergent at $s = 0$. We assume that any singularities of W_0 at $s > 0$ are integrable. Such singularities can occur for example when the spacetime has spatial periodicity so that points on the detector's trajectory can be joined by null geodesics that circumnavigate the space [22].

2.3. Transition rate

When both the detector trajectory and the quantum state of the field are stationary, in the sense that they are invariant under a Killing vector that is timelike in a neighborhood of the trajectory, a transition rate per unit time may be defined by making the switching function time independent and formally factoring out the infinite total time of detection [1, 3, 19, 25, 26, 34–39]. In time-dependent situations, this procedure is however not available, and separating the switching effects from the acceleration effects becomes delicate [6, 13, 37, 40–45].

To define a transition rate in the nonstationary setting, we consider the limit in which the detector is switched on an off sharply. We let the switching function χ take the value unity from the proper time τ_0 to the proper time τ , where $\tau_0 < \tau$, and we assume that the switch-on takes place over an interval of duration δ before τ_0 and the switch-off takes place over an interval of duration δ after τ , in a manner discussed in [7, 20]. The limit of sharp switching is $\delta \rightarrow 0$.

We regard the response function \mathcal{F} as a function of the switch-off moment τ , and we define $\dot{\mathcal{F}}_\tau := d\mathcal{F}/d\tau$. $\dot{\mathcal{F}}_\tau$ may be regarded as the detector's instantaneous transition rate per unit proper time, observationally meaningful in terms of consequent measurements in identical ensembles of detectors [7].

For $d = 2$ and $d = 3$, taking the $\delta \rightarrow 0$ limit in (4) and (5) is immediate and yields a finite result for the transition probability. For $d = 4$, the $\delta \rightarrow 0$ limit in (6) contains a divergent term proportional to $\ln \delta$ [7, 20]. This divergent term depends on the details of the switching but it is constant in time, and it is also independent of the trajectory and of the quantum state. The divergent term does hence not contribute to the transition rate. Physically, the $\delta \rightarrow 0$ limit means that we take the switching to be rapid compared with the overall duration of the interaction: focusing on the transition rate allows us to discard from the transition probability the numerically dominant piece that depends only on the details of the switching. Collecting, the $\delta \rightarrow 0$ transition rates for $d = 2$, and $d = 3$ and $d = 4$ are given by

$$d = 2 : \quad \dot{\mathcal{F}}_\tau(\omega) = 2\text{Re} \int_0^{\Delta\tau} ds e^{-i\omega s} W_0(\tau, \tau - s), \quad (7)$$

$$d = 3 : \quad \dot{\mathcal{F}}_\tau(\omega) = \frac{1}{4} + 2 \int_0^{\Delta\tau} ds \operatorname{Re}[e^{-i\omega s} W_0(\tau, \tau - s)], \quad (8)$$

$$d = 4 : \quad \dot{\mathcal{F}}_\tau(\omega) = -\frac{\omega}{4\pi} + 2 \int_0^{\Delta\tau} ds \operatorname{Re} \left(e^{-i\omega s} W_0(\tau, \tau - s) + \frac{1}{4\pi^2 s^2} \right) + \frac{1}{2\pi^2 \Delta\tau}, \quad (9)$$

where $\Delta\tau := \tau - \tau_0$. Equations (7) and (8) are valid as $\delta \rightarrow 0$ at fixed λ , provided λ is so small that the total transition probability remains within the validity domain of the perturbative treatment. Equation (9) is valid as $\delta \rightarrow 0$ provided λ simultaneously approaches zero so fast that it is bounded in absolute value by $k/\sqrt{|\ln \delta|}$, where the positive constant k is so small that the total transition probability remains within the validity domain of the perturbative treatment.

For $d = 5$ and $d = 6$, we specialize to a massless field in Minkowski spacetime in the Minkowski vacuum [21]. The transition probability contains again a term that diverges as $\delta \rightarrow 0$. For $d = 5$, the divergent term is constant in time, and the transition rate has the finite $\delta \rightarrow 0$ limit,

$$d = 5 : \quad \dot{\mathcal{F}}_\tau(\omega) = \frac{4\omega^2 + \ddot{z}^2(\tau)}{64\pi} + \frac{1}{4\pi^2} \int_0^{\Delta\tau} ds \left(\frac{\sin(\omega s)}{\sqrt{[-(\Delta z)^2]^3}} - \frac{\omega}{s^2} \right) - \frac{\omega}{4\pi^2 \Delta\tau}, \quad (10)$$

where $\Delta z := z(u) - z(u - s)$. For $d = 6$, by contrast, even the transition rate contains a term that diverges for generic trajectories as $\delta \rightarrow 0$, proportionally to $\ddot{z} \cdot \ddot{z} \ln \delta$. This means that the divergences due to the rapid switching cannot be isolated from the acceleration effects for $d = 6$. The sole exception occurs for trajectories whose scalar proper acceleration $\sqrt{\ddot{z}^2}$ is a constant, including as a special case all stationary trajectories. For such trajectories, the $d = 6$ transition rate remains finite as $\delta \rightarrow 0$ and is given by

$$d = 6 : \quad \dot{\mathcal{F}}_\tau(\omega) = -\frac{\omega(\omega^2 + \ddot{z}^2)}{24\pi^2} + \frac{1}{2\pi^3} \int_0^{\Delta\tau} ds \left(\frac{\cos(\omega s)}{[(\Delta z)^2]^2} - \frac{1}{s^4} + \frac{3\omega^2 + \ddot{z}^2}{6s^2} \right) + \frac{3\omega^2 + \ddot{z}^2}{12\pi^3 \Delta\tau} - \frac{1}{6\pi^3 \Delta\tau^3}. \quad (11)$$

2.4. Applications

When both the detector trajectory and the quantum state of the field are stationary, the transition rate formulas (7)–(11) reduce to the well-known formulas in which stationarity is assumed at the outset [25, 26, 38]. We re-emphasize, however, that formulas (7)–(11) apply in genuinely time-dependent situations.

A showcase example is a Minkowski spacetime trajectory that is asymptotically inertial at early times and of asymptotically uniform linear acceleration at late times, with the field in the Minkowski vacuum. Within the perturbative treatment, the transition rate is duly found [33] to interpolate between that in inertial motion and that in uniform linear acceleration, describing thus the onset of the Unruh effect [1].

Other applications can be found in [7, 20, 22, 33, 46]. Slowly varying acceleration is discussed in [47, 48].

2.5. Other definitions of the transition rate

To conclude this section, we mention two alternative definitions of the transition rate.

First, the transition rate of a pointlike detector in flat spacetime can be defined by first giving the detector a spatial size, specified covariantly in terms of the detector's instantaneous rest frame, and at the end taking the pointlike limit [31, 32, 38]. The results agree with those obtained via smooth switching in the common domain of validity [21, 31–33, 49, 50]. A related procedure that replaces spatial size by a pole prescription in proper time is discussed in [47].

Second, a spatially extended detector in flat spacetime can be reinterpreted as a pointlike detector with an energy cutoff that is specified covariantly in the detector's instantaneous rest frame [49, 50]. Definition of the transition rate via this energy cutoff can be generalized to curved spacetimes at least when the spacetime has a sufficient amount of symmetry [49, 50].

3. Detector–field interaction: UD, RSG, RHAK models

Quantum mechanics for the single particles in some nonlinear potentials such as anharmonic oscillators, Morse potential, etc are exactly solvable. But in field theory, since a field has infinitely many degrees of freedom, a small nonlinearity can create huge difficulty in calculations. One can at most perform perturbation theory or self-consistent approximations around some non-trivial background field configuration, where the calculation involves essentially Gaussian integrals. However, if the potential of the detector is that of a HO and the quantum state is in a Gaussian form, it is possible to solve the full dynamics of the combined system of the detectors and the field non-perturbatively.

3.1. Raine–Sciama–Grove model

A notable non-perturbative model is that of Raine, Sciama and Grove (RSG) [10], where they found the late-time expectation values for the stress tensor of a massless scalar field in (1+1)D Minkowski space. The method is generalized to the case with a point-like UD detector in a massless scalar field in (3+1)D Minkowski space, and the whole history of the combined system is solved in [8].

3.2. Proxy to quantum Brownian motion models

Suppose the internal degrees of freedom of the UD detector are HOs. Then the combined system of N UD detectors and a quantum field is an $(N + \infty)$ -HO system, which is linear and exactly solvable. Unruh and Zurek [51] have studied a model where a HO interacts with a massless scalar field in 2D. They derived the exact master equation for the reduced density matrix of the system (oscillator) at a temperature determined by the initial state of the field, and observed some general features different from the conventional Markovian results valid for an ohmic bath at ultrahigh temperature made known earlier in the famous paper of Caldeira and Leggett [52]. One feature is the dependence of the ultraviolet cut-off on the master equation and the reduced density matrix, and thus also on the von Neumann entropy of the system.

More general non-Markovian behavior was explored by Hu, Paz and Zhang [53] who derived an exact master equation with nonlocal dissipation and colored noise for the system of one HO (detector) interacting with a thermal bath of n -HOs. For quantum decoherence, they identified the low-temperature, supra-ohmic regime as a noticeable departure from the Markovian behavior (see followup in [54]). Using this model as a theoretical tool with the help of quantum open system ideas, many of the basic issues we listed in the beginning can be addressed effectively.

A bath of n -HOs of time-dependent frequencies to a quantum field was subsequently generalized by Hu and Matacz [55] for moving detectors–quantum field interactions. Because the treatment is given in quantum optics language—the quantum states described by the squeeze, rotation and displacement operators and the dynamics in terms of parametric amplification—their results are immediately applicable to ‘atomic-mirror’–optical systems [56]. It also served the intended purpose of bringing open system methods and concepts to quantum field theory. The influence functional treatment they used incorporates the backreaction of the environment on the system (which could be either the quantum field or the HO depending on what one is after) in a self-consistent way. In particular, they showed how the Unruh and Hawking temperatures can be identified from the noise kernel using this method.

Viewing the Unruh effect from this perspective, since in the QBM model there are nontrivial activities at zero temperature [51, 53, 55], we note that even for the zero acceleration $a = 0$ case, the detector is not just laying idle but has interesting physical features due to its interaction with the vacuum fluctuations in the quantum field.

3.3. RHAK models

A model of N detectors in arbitrary relativistic motion interacting with a common quantum field (but not with each other) was proposed by Raval, Hu, Anglin [11]. They calculated the influence of quantum fields on the detectors in motion, and the mutual influence of detectors by the action of fields via the Langevin equations derived from the influence functional. They introduced the notion of self- and mutual impedance, advanced and retarded noise, and the new relations between noise correlations and dissipation–propagation. They show the existence of general fluctuation–dissipation relations, and for trajectories without event horizons, correlation–propagation relations. Raval, Hu and Koks [12] used this model to explore different trajectories of the moving detectors in a quantum field and showed that this is a more feasible way (over the traditional global geometric view which relies on the existence of event horizons) to address situations where the spacetime possesses an event horizon only asymptotically, or none at all. Examples studied there include detectors moving at uniform acceleration only asymptotically or for a finite time, a moving mirror and a two-dimensional collapsing mass. They show that in such systems, radiance indeed is observed, albeit not in a precise Planckian spectrum. The setups in this model have been adopted in the study of charge particle motion by Johnson *et al* [57] in an electromagnetic field and by Galley *et al* [58] for the self-force of masses moving in a gravitational field.

3.4. Moving detectors–quantum field interaction

Consider a model with N identical point-like UD detectors with the internal degrees of freedom represented by HOs with mass m_0 and natural frequency Ω , moving in a quantum field in (3+1)D Minkowski space. Here we follow the treatment in [16]. The action of the combined system is given by

$$S = - \int d^4x \sqrt{-g} \frac{1}{2} \partial_\sigma \Phi(x) \partial^\sigma \Phi(x) + \sum_{\mathbf{d}} \int d\tau_{\mathbf{d}} \left\{ \frac{m_0}{2} [(\partial_{\mathbf{d}} Q_{\mathbf{d}})^2 - \Omega_0^2 Q_{\mathbf{d}}^2] + \lambda \int d^4x Q_{\mathbf{d}}(\tau_{\mathbf{d}}) \Phi(x) \delta^4(x - z_{\mathbf{d}}(\tau_{\mathbf{d}})) \right\}, \quad (12)$$

where $\sigma = 0, 1, 2, 3$, $g_{\sigma\sigma'} = \text{diag}(-1, 1, 1, 1)$, $\mathbf{d} = A, B, C, \dots$ denotes the names of the detectors, $\partial_{\mathbf{d}} \equiv \partial/\partial\tau_{\mathbf{d}}$, $\tau_{\mathbf{d}}$ is the proper time for the detector $Q_{\mathbf{d}}$ and $z_{\mathbf{d}}(\tau_{\mathbf{d}})$ is the trajectory of the detector \mathbf{d} . The scalar field Φ is assumed to be massless and λ is the coupling constant.

We consider a massless scalar field here because it is simpler and a good representation of the electromagnetic field. In fact all kinds of fields, massless or massive, bosonic or fermionic, can be considered, depending on the physics one aims at. The detectors do not have to be uniformly accelerated (e.g., [12]) or at rest. However, the motion of the detector here is assumed to be controlled by external agents, in other words, the trajectories or worldlines of the detectors are prescribed and not dynamical. If the motion of the detector becomes dynamical, it is extremely hard to obtain analytical results even in classical theory (for example, a relativistic charge in a field in classical [59] and quantum field theory [60]), since including backreaction of the field on the detector will alter its (test-field) prescribed trajectory. Trajectories of charged particles [57] and even extended objects [61] determined by their interplay with the quantum field have also been studied before using the influence functional method which is particularly suited to treating consistent backreaction effects.

The first noticeable attractive feature of (12) is that this model is linear and thus easy to treat. It is arguably the simplest model for an ‘atom’–field interacting system but complex enough to give nontrivial results and insights. By ‘atom’ here, we refer to a spatially localized physical object with internal degrees of freedom. By field, we categorically refer to dynamical variables which can be non-local in space⁶. In some simple setups, analytic results can be obtained in the whole parameter range, and backreaction to both the atom and the field can be fully studied with the help of quantum open system techniques.

Moreover, in a relativistic setting, such as for uniformly accelerated detectors or black holes, event horizons for the detectors can be sharply defined since the detectors are always localized. Also since the detectors are pointlike, they are allowed to be parametrized by their own proper times, which are invariants under coordinate transformations. This greatly simplifies the calculations in different reference frames when the related physics corresponds only to the two-point correlators of the detectors parametrized by two proper times. Note that this is even plausible for extended objects in the spirit of effective field theory [61, 62].

For a uniformly accelerated UD detector in (3+1)D with proper acceleration a , the Unruh effect [1, 3, 19, 25] attests that it should behave the same way as an inertial UD detector in contact with a thermal bath at the Unruh temperature T_U , or more precisely, as an inertial HO in contact with an Ohmic bath at T_U [63]. However, examining this from the vantage point of the exact solutions we obtained, we see that the above statements are accurate only at the initial moment. After the coupling is switched on, the quantum state of the field will be changed by the detector, so the field is no longer in the Minkowski vacuum and it does not make exact sense to say that the detector is immersed in a thermal state (or any state defined in the test-field description, i.e. where the field is assumed not to be modified by the presence of the detector).

A theorem by Bisognano and Wichmann (BW) [64] states that the Minkowski vacuum, which is uniquely characterized by its invariance under all Poincaré translations, is a Kubo–Martin–Schwinger (KMS) state with respect to all observables confined to a Rindler wedge. It does not apply here because the BW theorem refers to the vacuum state of a quantum field alone, not the combined detector–field system. Even when the combined system is in a steady state, the quantum state of the interacting field is not invariant under spatial translations in Minkowski space, hence does not subscribe to the assumption of the BW theorem pertaining to Poincaré invariance. Actually the Planck factor in, for example (equation (60) in [8]),

$$\langle Q(\eta)Q(\eta') \rangle_v \sim \frac{\lambda^2 \hbar}{(2\pi)^2 m_0^2} \int \frac{\kappa d\kappa}{1 - e^{-2\pi\kappa/a}} [\dots] \quad (13)$$

⁶ ‘Nonlocal’ is in the sense used by the atomic-optical quantum information community. Of course, quantum field theory is local.

is a consequence of the BW theorem. Nevertheless, it is derived from only the free-field-solution part of the complete interacting field. Here the factor is not distorted by the interaction simply because the field is linear and the coupling is bilinear. For nonlinear fields or couplings, it would have a non-Planckian spectrum and the departure from the conventional picture would be more pronounced.

4. Nonperturbative detector–field dynamics

Nonperturbatively solvable models such as (12) are particularly useful for examining the full features of a system which perturbative theories miss or misrepresent. They are essential for understanding new physics such as that associated with quantum entanglement whose dynamical behavior we do not really have a complete or accurate knowledge about. We now continue to develop model (12).

A quantum state of the combined detector–field system can be described by the density matrix $\bar{\rho}[(\mathbf{Q}, \Phi_{\mathbf{x}}), (\mathbf{Q}', \Phi'_{\mathbf{x}}); x^0]$ or equivalently, the Wigner function [51, 53],⁷

$$W[\mathbf{P}, \Sigma; x^0] = \int \mathcal{D}\left(\frac{\Delta}{2\pi}\right) e^{\frac{i}{\hbar}\mathbf{P}\cdot\Delta} \bar{\rho}\left[\Sigma - \frac{\Delta}{2}, \Sigma + \frac{\Delta}{2}; x^0\right]. \quad (14)$$

If we start with a Gaussian state, by virtue of the linearity of the combined system (12), the quantum state will always evolve in a Gaussian form in its entire history. Thus, solving the dynamical equations for the Wigner function boils down to solving the time-dependent factors in the Wigner function.

Since the field variables at some moment x^0 are defined on the whole time-slice associated with x^0 , the density matrix or Wigner functions at x^0 are also defined on that time-slice. In the Schrödinger picture, the evolution of a density matrix (Wigner function) is governed by the master equation (Fokker–Planck equation). It is possible to solve these equations for Gaussian states directly in simple cases (e.g. [65, 66]). However, when the degrees of freedom of the density matrix are large or even infinite, it becomes very difficult to solve the coupled equations, as the dynamics is often non-Markovian (the master equations for the reduced state) or nonlinear in appearance (the dynamical equations for the time-dependent factors in the Wigner functions). Moreover, even the solutions are obtained and can be expressed formally, the factors in the Wigner function are inverse matrices with infinite dimension, which are computationally challenging.

To get rid of these difficulties, it is convenient to apply the (K, Δ) -representation [51] (or called the Wigner characteristic function [67]), which is a double-Fourier-transformed function of (thus equivalent to) the usual Wigner function,

$$\rho[\mathbf{K}, \Delta; x^0] = \int \mathcal{D}\Sigma e^{\frac{i}{\hbar}\mathbf{K}\cdot\Sigma} \bar{\rho}\left[\Sigma - \frac{\Delta}{2}, \Sigma + \frac{\Delta}{2}; x^0\right] = \exp\left[\frac{i}{\hbar}(\langle\hat{\Phi}_{\mu}\rangle K^{\mu} - \langle\hat{\Pi}_{\mu}\rangle \Delta^{\mu}) - \frac{1}{2\hbar^2}(K^{\mu} \mathcal{Q}_{\mu\nu} K^{\nu} - 2\Delta^{\mu} \mathcal{R}_{\mu\nu} K^{\nu} + \Delta^{\mu} \mathcal{P}_{\mu\nu} \Delta^{\nu})\right], \quad (15)$$

where we denote $\hat{Q}_{\mathbf{d}}$ and $\hat{P}_{\mathbf{d}}$ by $\hat{\Phi}_{\mathbf{d}}$ and $\hat{\Pi}_{\mathbf{d}}$, respectively ($\hat{P}_{\mathbf{d}}$ and $\hat{\Pi}_{\mathbf{x}}$ are conjugate momenta to $\hat{Q}_{\mathbf{d}}$ and $\hat{\Phi}_{\mathbf{x}}$), $\mu, \nu = \{\mathbf{d}\} \cup \{\mathbf{x}\}$ run over all the detector and field degrees of freedom defined on the whole time-slice and the time-dependent factors $\mathcal{Q}_{\mu\nu}(x^0)$, $\mathcal{P}_{\mu\nu}(x^0)$ and $\mathcal{R}_{\mu\nu}(x^0)$ are exactly the symmetrized two-point correlators $\langle A, B \rangle \equiv \langle AB + BA \rangle / 2$ of the dynamical variables evaluated on the x^0 -slice, for they are obtained by, e.g.,

$$\langle \delta \hat{\Pi}_{\mu}(x^0), \delta \hat{\Phi}_{\nu}(x^0) \rangle = \frac{i\hbar\delta}{\delta\Delta^{\mu}} \frac{\hbar\delta}{i\delta K^{\nu}} \rho[\mathbf{K}, \Delta; x^0] \Big|_{\Delta=\mathbf{K}=0} = \mathcal{R}_{\mu\nu}, \quad (16)$$

⁷ Here we write $(\mathbf{Q}, \Phi_{\mathbf{x}}) = \Sigma - (\Delta/2)$ and $(\mathbf{Q}', \Phi'_{\mathbf{x}}) = \Sigma + (\Delta/2)$ with the boldface letters Σ and Δ denoting the vectors in the configuration space. $\int \mathcal{D}\Delta$ and $\int \mathcal{D}\Sigma$ are the functional integrals.

where $\delta\hat{\Phi}_\mu \equiv \hat{\Phi}_\mu - \langle\hat{\Phi}_\mu\rangle$ and $\delta\hat{\Pi}_\mu \equiv \hat{\Pi}_\mu - \langle\hat{\Pi}_\mu\rangle$. Note that in (15), $\mathcal{Q}_{\mu\nu}$ and $\mathcal{P}_{\mu\nu}$ are defined as symmetric matrices, but $\mathcal{R}_{\mu\nu} \neq \mathcal{R}_{\nu\mu}$ in general.

The reduced states of the system with the environment integrated out are simple for Gaussian states in the (K, Δ) -representation since a Gaussian integral gives another Gaussian function. For example, the reduced state of the detector A with the field and other detectors integrated out reads

$$\rho^R[K^A, \Delta^A; x^0] = \exp\left[\frac{i}{\hbar}(\langle\hat{Q}_A\rangle K^A - \langle\hat{P}_A\rangle \Delta^A) - \frac{1}{2\hbar^2}(K^A \mathcal{Q}_{AA} K^A - 2\Delta^A \mathcal{R}_{AA} K^A + \Delta^A \mathcal{P}_{AA} \Delta^A)\right]. \quad (17)$$

Thus, looking at the evolution of the Gaussian state (15) or (17) is equivalent to looking at the dynamics of those symmetrized two-point correlators, which would be obtained more easily in the Heisenberg picture.

4.1. Correlator dynamics for factorizable initial states

For mathematical convenience (and to a large extent reflective of not uncommon physical situations), one often assumes that the initial state at $x_0^0 = t_0$ in the Minkowski frame is a product state of the Minkowski vacuum of the field (which is Gaussian) and the Gaussian state of the detectors A, B, \dots . The detector part can be a product of the ground states and/or single-mode squeezed states, a multi-mode squeezed state, or any mixed state in the Gaussian form. The field part can be easily generated to a thermal state, which is Gaussian, too.

By virtue of linearity in (12), the operators of the detectors and the field in the Heisenberg picture will evolve to a linear combination of all the detector operators \hat{Q}_d, \hat{P}_d and the field operators $\hat{\Phi}_k, \hat{\Pi}_k$ defined at the initial moment t_0 . Then each symmetrized two-point correlator of the detectors for the factorizable initial state $\rho_{\Phi_x} \otimes \rho_d$ will split into a sum of the a-part and the v-part [8]. The a-part corresponds to the initial state of the detectors, while the v-part corresponds to the response to the field vacuum $|0_M\rangle$.

4.2. Divergences

There are two sources of the divergences for the correlators in this detector–field model.

- (1) In this linear system, the mode functions satisfy the classical equations of motion (the only difference is the initial conditions). Thus, they suffer the same divergences as the classical ones: in the UD detector theory in (3+1)D, the retarded field sourced by a pointlike detector diverges right at the position of the detector. One needs to introduce a cutoff Λ to regularize the δ -function in the interaction Hamiltonian between the detector and the field, expand the relevant mode functions of the detectors in series of Λ , absorb the divergent terms by some parameter of the model (in our model, it is the natural frequency of the detectors Ω [8]; in other model, it could be the mass of the detector m or the coupling constant λ), and then take the $\Lambda \rightarrow \infty$ limit to eliminate the $O(\Lambda^{-1})$ terms. The $O(\Lambda^0)$ terms will survive after taking the limit; it gives the radiation reaction the dissipation term ($\sim \gamma \dot{Q}$) such as those in our model or the higher derivative term such as the one in the Abraham–Lorentz–Dirac equation [59].⁸

⁸ Note that there is no such divergence in the RSG model in (1+1)D [10], where the retarded field is regular everywhere and simply offers the radiation reaction in the equation of motion of the detector.

- (2) The second kind of divergence is the UV or IR divergences arisen in the mode sum, or equivalently, the divergences arisen in the coincidence limit of Green's functions (UV), or in the integration over the whole position space (IR). The regularization of the UV divergences should be consistent with those for the first kind. These kinds of divergences are also a common feature for quantum Brownian motion.

4.3. Range of validity of perturbative results

For a single UD detector moving in (3+1)-dimensional Minkowski space, the total action is given by (12) with $\mathbf{d} = A$. Let us denote $Q \equiv Q_A$. Suppose the initial state of the system at τ_0 is a direct product of the ground state for Q and the Minkowski vacuum for Φ . It is straightforward to write down the reduced state of the detector in the (K, Δ) -representation. However, to compare with the perturbative results, we look at the reduced density matrix $\rho^R(Q, Q'; \tau)$ directly. Transformed to the representation in the basis of energy eigenstates for the free HO Q , the transition probability from the initial ground state to the first excited state then reads

$$\rho_{1,1}^R = \frac{\hbar[\langle P^2 \rangle \langle Q^2 \rangle - \langle P, Q \rangle^2 - (\hbar^2/4)]}{\{[\langle P^2 \rangle + (\hbar m_0 \Omega_r/2)][\langle Q^2 \rangle + (\hbar/2m_0 \Omega_r)] - \langle P, Q \rangle^2\}^{3/2}}, \quad (18)$$

where Ω_r is the renormalized frequency [8]. Expanding the symmetrized two-point correlators of the detector in terms of the coupling strength $\gamma \equiv \lambda^2/(8\pi m_0)$, the approximate value up to the first order of γ becomes

$$\rho_{1,1}^R|_{\gamma \eta \rightarrow 0} \xrightarrow{\eta \gg a^{-1}} \frac{\lambda^2}{4\pi m_0} \left[\frac{\eta}{e^{2\pi \Omega_r/a} - 1} + \frac{\Lambda_1 + \Lambda_0 - 2 \ln(a/\Omega_r)}{2\pi \Omega_r} \right] \quad (19)$$

when $\eta \equiv \tau - \tau_0 \gg a^{-1}$. Here Λ_0 and Λ_1 are large constants introduced by regularization (the second kind in section 4.2). We see that the first term of (19) gives the conventional transition probability from TDPT over infinite time. Only when $\Omega_r \eta \gg \Lambda_1, \Lambda_0$, or a is extremely large, can the second term in (19) be neglected. Hence, the conventional transition probability or transition rate is valid only in the limits of (a) ultrahigh acceleration ($a \gg \Omega_r$ and $\Lambda_1 \ll a\eta \ll a\gamma^{-1}$) or (b) ultraweak coupling ($a^{-1}, \Omega_r^{-1}\Lambda_1 \ll \eta \ll \gamma^{-1}$). Only in these limits, the effective temperature obtained by diagonalizing $\rho_{m,n}^R$ is very close to the Unruh temperature $T_U = \hbar a/2\pi k_B$ [9], and the thermal bath is only slightly affected by the backreaction from the detector to the field, namely the detector acts essentially as a test particle in the field.

Note that, in obtaining (19), we have assumed $a^{-1} \ll \eta \ll \gamma^{-1}$, when the system is still in transient. Indeed, in figure 1 TDPT works well only in the middle plot (with $O(10^{-1}) < \eta < O(10^{-2}/\gamma)$ for $\gamma = 10^{-6}$ and $a = 6$). If $a < \gamma$, the conventional transition probability has no chance to dominate at all. In particular, the $a = 0$ case is beyond the reach of TDPT over infinite time, and the conventional wisdom from perturbation theory that no transition occurs in an inertial detector is untenable. In contrast, the evolution of $\rho_{1,1}^R$ with $a = 0$ behaves qualitatively similar to those cases with nonzero acceleration [8]. This agrees with the expectation from the observation that the UD detector theory is a special case of quantum Brownian motion [51, 53, 55].

Λ_0 corresponds to the time scale of switching on the interaction, namely the 'ln δ ' in $d = 4$ case in section 2.3, so it could be finite in real processes. This implies that the Λ_0 terms in all two-point functions will be damped out so that Λ_0 will not be present in the late-time results. Actually (19) is formally identical to the first-order transition probability from TDPT for a UAD with finite duration of interaction and Λ_0 and Λ_1 are formally the same as the divergences found in [13]. In [6], it has been shown that these divergences can be tamed if one switches on and off the interaction smoothly, so can Λ_0 . Nevertheless, here we are looking at

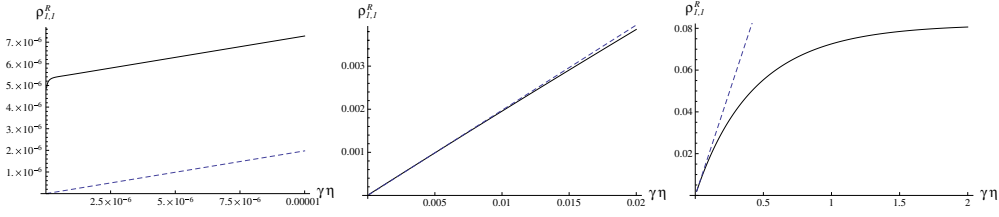


Figure 1. Comparison of the exact $\rho_{1,1}^R$ (solid curve, equation (18)) and the perturbative result (dashed line, the first term in (19)) in different time scales of $\eta \equiv \tau - \tau_0$ for a uniformly accelerated detector initially in its ground state and coupled to the Minkowski vacuum of the massless scalar field after τ_0 . The parameters in these plots are $\gamma = 10^{-6}$, $a = 6$, $\Omega = \sqrt{\Omega_r^2 - \gamma^2} = 2.3$, $\Lambda_0 = \Lambda_1 = 20$, $m_0 = 1$ and $\hbar = c = 1$.

the real-time causal evolution problem (‘in–in’ formulation) rather than a scattering transition amplitude (‘in–out’ formulation) problem, and we never turn off the coupling, so Λ_1 is a non-zero constant of time.

Λ_1 should not be absorbed by any parameter or subtracted from any physical quantity of this theory for more reasons: (a) the UD detector theory is not a fundamental theory to meet the renormalizability requirement, and the presence of cut-offs as physical parameters is an expected feature which characterizes the range of validity of this effective theory, just like the Compton wavelength of the electron serving as a cut-off in quantum optics; (b) Λ_1 is not present in the renormalized stress–energy tensor of the field induced by the detector [8], so that Λ_1 is not observable outside of the detector; (c) if Λ_1 was subtracted naively, the uncertainty principle will be violated at late times for a is small enough [9].

5. Applications to RQI: quantum teleportation

Quantum teleportation is not only of practical value but also of theoretical interest because it contains many illuminating manifestations of quantum physics, clarifying fundamental issues such as quantum information and classical information, quantum nonlocality and relativistic locality, spacelike correlations and causality, etc [2, 68, 69].

The first protocol of quantum teleportation is given by Bennett *et al* in [70], where an *unknown* state of a qubit C is teleported from one spatially localized agent Alice to another agent Bob using an entangled pair of qubits A and B prepared in one of the Bell states and shared by Alice and Rob, respectively. This idea is then adapted to the systems with continuous variables by Vaidman [71], who introduces an EPR state [72] for the shared entangled pair to teleport an unknown coherent state. Braunstein and Kimble (BK) [73] generalized Vaidman’s scheme from EPR states with exact correlations to squeezed coherent states. In doing so, the uncertainty of the measurable quantities reduces the degree of entanglement of the AB -pair as well as the fidelity of teleportation.

To explore how the Unruh effect affects teleportation, Alsing and Milburn made the first attempt of calculating the fidelity of quantum teleportation between two moving cavities in relativistic motions [74]—one is at rest (Alice), the other is uniformly accelerated (Rob) in the Minkowski frame—though their result is not quite reliable [75–77]. Then Landulfo and Matsas [78] considered quantum teleportation in the future asymptotic region in a two-level detector qubit model where Rob’s detector is uniformly accelerated and interacting with the quantum field only in a finite duration. Alternatively, Shiokawa [79] has considered quantum teleportation in the UD detector theory with the agents in similar motions but based on the BK

scheme in the interaction region. More recently, the relativistic effects of quantum information associated with the quantum field have been taken into account carefully in [18], as summarized below.

Consider a setup with the detectors A and C held by Alice, who is at rest in space with the worldline $\mathbf{z}_A = \mathbf{z}_C = (t, 1/b, 0, 0)$ in the Minkowski frame, and the detector B held by Rob, who is uniformly accelerated along the worldline $\mathbf{z}_B = (a^{-1} \sinh a\tau, a^{-1} \cosh a\tau, 0, 0)$, $0 < a < b$, where τ is Rob's proper time, namely $\tau_A = \tau_C = t$ and $\tau_B = \tau$. Suppose the initial state of the combined system at $t = \tau = 0$ is a product state $\rho_{\Phi_x} \otimes \rho_{AB} \otimes \rho_C^{(\alpha)}$ of the Minkowski vacuum of the field $\hat{\rho}_{\Phi_x} = |0_M\rangle\langle 0_M|$, a two-mode squeezed state ρ_{AB} of the detectors A and B , and a coherent state of the detector C , denoted $\hat{\rho}_C^{(\alpha)} = |\alpha\rangle_C\langle\alpha|$, which is the quantum state to be teleported. To concentrate on the best fidelity of quantum teleportation that the entangled AB -pair can offer, however, we assume that the dynamics of $\rho_C^{(\alpha)}$ is frozen. Also we design ρ_{AB} so that it goes to an EPR state with the correlations while $Q_A + Q_B$ and $P_A - P_B$ are totally uncertain as its squeezed parameter $r_1 \rightarrow \infty$.

In the Minkowski frame, at $t = 0$, the detectors A and B start to couple with the field, while the detector C is isolated from others. At $t = t_1$ when the reduced state of the three detectors continuously evolves to $\rho_{ABC}(\mathbf{K}, \mathbf{\Delta}; t_1)$, a joint Gaussian measurement by Alice is performed *locally in space* on A and C so that the post-measurement state right after t_1 collapses to $\tilde{\rho}_{ABC}(\mathbf{K}, \mathbf{\Delta}; t_1) = \tilde{\rho}_{AC}^{(\beta)}(K^A, K^C, \Delta^A, \Delta^C) \tilde{\rho}_B(K^B, \Delta^B)$, where $\tilde{\rho}_{AC}^{(\beta)}(K^A, K^C, \Delta^A, \Delta^C)$ is a two-mode squeezed state of the detectors A and C with displacement $\beta = \beta_R + i\beta_I$, which is the outcome Alice obtains.

5.1. Entanglement and pseudo-fidelities in different frames

Quantum entanglement of the detector pair A and B in a Gaussian state is fully determined by the symmetrized two-point correlators Q_{ij} , \mathcal{P}_{ij} and \mathcal{R}_{ij} , $i, j = A, B$, in the quantum state (15) [16, 80, 81]. Since the worldlines of the two detectors do not intersect, the entanglement between the detectors at some moment will be a kind of spacelike correlation and depend on the spatial hypersurface on which the entanglement is defined. The time evolution of the entanglement will hence depend on how the spacetime is foliated by spacelike hypersurfaces. In general, entanglement of the detectors is incommensurable with the physical fidelity of quantum teleportation, which is a kind of timelike correlation.

To compare these two correlations, let us first imagine that Rob receives the outcome β of Alice's joint measurement and make the proper operation on the detector B *instantaneously* at $\tau_1(t_1)$ when the worldline of B intersects the t_1 -slice (see figure 2). According to the outcome β obtained by Alice, the operation that Rob should perform on the detector B is a displacement by β in the phase space of B from $\tilde{\rho}_B$ to ρ_{out} , which is defined on a time-slice right after the one where the post-measurement state $\tilde{\rho}_{ABC}$ is defined. The 'pseudo-fidelity' of quantum teleportation from $|\alpha\rangle_C$ to $|\alpha\rangle_B$ is defined as $F(\beta) \equiv {}_B\langle\alpha|\hat{\rho}_{\text{out}}|\alpha\rangle_B / \text{Tr}_B \hat{\rho}_{\text{out}}$, where $\text{Tr}_B \hat{\rho}_{\text{out}} = P(\beta)$ has been normalized to be the probability of finding the outcome β . Then the *averaged* pseudo-fidelity is defined by

$$F_{\text{av}} \equiv \int d^2\beta P(\beta) F(\beta) = \int d\beta_R d\beta_I {}_B\langle\alpha|\hat{\rho}_{\text{out}}|\alpha\rangle_B. \quad (20)$$

$F_{\text{av}} = 1/2$ is known as the best fidelity of 'classical' teleportation using coherent states [73], without considering the coupling of the UD detectors with the environment.

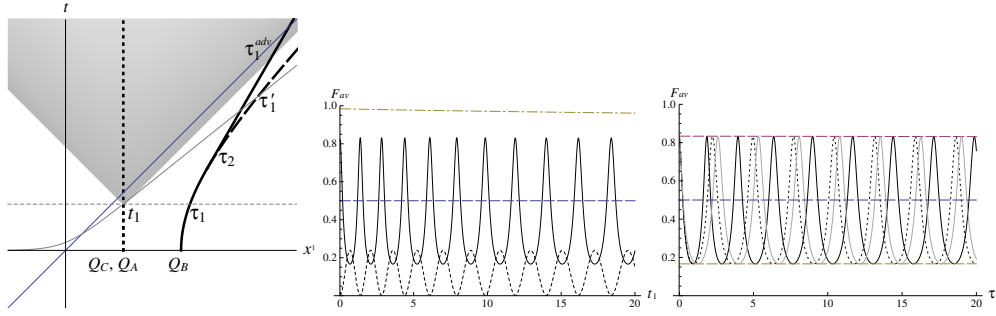


Figure 2. Left: setup for quantum teleportation from Alice (thick dotted line) to Rob (thick dashed for section 5.1 and thick solid for section 5.2). The gray solid curve and the horizontal line represent a τ_1 -slice in the quasi-Rindler frame and a t_1 -slice in the Minkowski frame, respectively. The shaded region is in the future lightcone of the measurement event by Alice, and the hypersurface $t = x^1$ is the event horizon of Rob for $\tau_2 \rightarrow \infty$. Middle: a comparison of the averaged pseudo-fidelity F_{av} (solid) in the Minkowski frame, the correlator $\langle Q_-^2 \rangle / 20$ (dotted), $Q_- \equiv Q_A - Q_B$ and the logarithmic negativity $E_{\mathcal{N}} / 3.5$ (dot-dashed) at early times in the ultraweak coupling limit. Right: F_{av} in the quasi-Rindler frame with the same parameters (solid) and with different a (dotted, gray). The upper and middle dashed lines represent F_{av}^+ and $1/2$.

Two results in the ultraweak coupling limit, one in the Minkowski frame and the other in the ‘quasi-Rindler frame’⁹, are shown in figure 2 (middle, right). One can see that the degree of entanglement (logarithmic negativity $E_{\mathcal{N}}$) evolves smoothly while the averaged pseudo-fidelity evaluated in whatever reference frame oscillates in t_1 or τ_1 . Even at very early times, F_{av} drops below $1/2$ frequently when $E_{\mathcal{N}}$ is still large. Clearly the oscillation of F_{av} here is mainly due to the distortion of the quantum state of AB -pair from their initial state (caused by the alternating squeeze–antisqueeze natural oscillations of their quantum state) rather than the disentanglement between them.

In our setup, when t_1 gets larger, the time dilation of the detector B becomes more significant and so the detector B appears to change extremely slowly in the Minkowski frame, while in the quasi-Rindler frame, the time dilation of the detector A becomes obvious and so the detector A looks frozen for the Rindler observer when τ_1 gets larger. In both cases, it makes the frequency of the oscillation of F_{av} approach Ω for larger times.

The peak values of F_{av} , denoted by F_{av}^+ , fall below the fidelity of classical teleportation $1/2$ (at the moment denoted by $t_{1/2}$ or $\tau_{1/2}$) always earlier than the disentanglement time t_{dE} or τ_{dE} when $E_{\mathcal{N}}$ become zero in both frames. Indeed, in appendix A of [18], it is shown that entanglement between the detectors A and B is necessary to provide the advantage of quantum teleportation, at least in the ultraweak coupling limit.

In the Minkowski frame, both F_{av}^+ and $E_{\mathcal{N}}$ are insensitive to a in the ultraweak coupling limit. In contrast, the dependence on the proper acceleration a is obvious in the quasi-Rindler frame, where the larger the a , the earlier the $\tau_{1/2}$ and the disentanglement time τ_{dE} . Here we see the frame dependence of entanglement dynamics.

Beyond the ultraweak coupling limit, both F_{av} and $E_{\mathcal{N}}$ are strongly affected by the environment. In most cases, quantum entanglement disappears quickly both in the Minkowski frame and the quasi-Rindler frame due to strong interplays with the environment, and the averaged pseudo-fidelity F_{av} drops below $1/2$ even quicker.

⁹ By a quasi-Rindler frame, we refer to the coordinate system in which each time-slice almost overlaps a Rindler time-slice in the R-wedge but the part in the L-wedge has been bent to the region with positive t to make the whole time-slice located after the initial time-slice for the Minkowski observer, as illustrated in figure 2.

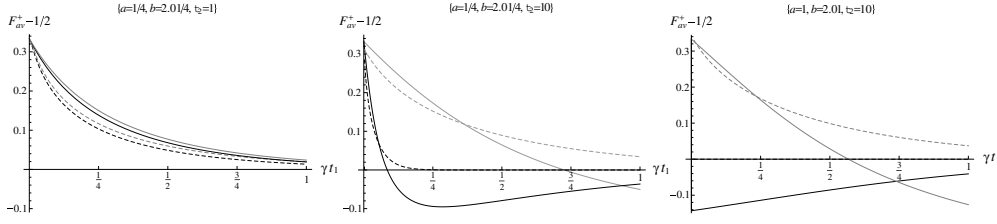


Figure 3. $F_{\text{av}}^+ - 1/2$ (solid) and $E_{\mathcal{N}}/10$ (dashed) in section 5.2 as a function of the moment of the joint measurement t_1 . The black curves represent those physical fidelities and the degrees of entanglement evaluated ‘on the lightcone’, while the gray curves represent those pseudo-fidelities and the degrees of entanglement evaluated on the t_1 -slice in the Minkowski frame.

5.2. Physical fidelity and ‘entanglement on the lightcone’

Suppose Rob stops accelerating at his proper time τ_2 ; after this moment Rob moves with constant velocity, while Alice stays at rest and performs the joint measurement on A and C at t_1 (see figure 2). In this setup, the classical information from Alice traveling at the speed of light can always reach Rob, though the acceleration of the detector B is not uniform—for the dynamics of the correlator similar situations, we refer to [14], and more results can be found in [82].

Suppose Rob performs the local operation on B at some moment $\tau_p > \tau_1^{\text{adv}}$ when he received the information traveling at light speed from Alice (see figure 2 for the definition of τ_1^{adv}). Then, similar to (20), the averaged physical fidelity should be given by $F_{\text{av}} = \int d^2\beta_B \langle \alpha | \hat{\rho}_{\text{out}}(\tau_p) | \alpha \rangle_B$, where $\rho_{\text{out}}(\tau_p)$ is obtained by performing a displacement on $\hat{\rho}_B(\tau_p)$ which started with the post-measurement state $\hat{\rho}_B(\tau_1)$ with $\tau_1 = a^{-1} \sinh^{-1} at_1$ (when the quantum state collapses in the Minkowski frame) and evolves from τ_1 to τ_p . Nevertheless, an analysis similar to [17] shows that the correlators in the reduced state of the detector B observed in all reference frames will become the same collapsed ones at the moment when the detector B enters the future lightcone of the measurement event by Alice ($\tau_B = \tau_1^{\text{adv}}$). So we are allowed to collapse the wave functional on a time-slice intersecting Alice’s worldline at $\tau_A = t_1$ and Rob’s at $\tau_B = \tau_1^{\text{adv}} - \epsilon$, $\epsilon \rightarrow 0+$.

If we further assume that mutual influences are small and Rob performs the local operation at $\tau_p = \tau_1^{\text{adv}} + \epsilon$ right after the classical information from Alice is received, then the continuous evolution of the reduced state of the detector B from τ_M to τ_p is negligible, and so we can directly compare the physical fidelity at $\tau_1^{\text{adv}} + \epsilon$ with entanglement between the detectors A at t_1 and B at $\tau_1^{\text{adv}} - \epsilon$. Again from appendix A of [18] with the proper time of the detector B substituted by τ_1^{adv} (actually $\tau_1^{\text{adv}} \pm \epsilon$, $\epsilon \rightarrow 0+$), quantum entanglement of AB -pair evaluated almost on the future lightcone of the measurement event by Alice is still a necessary condition of the best averaged physical fidelity of quantum teleportation beating the classical one in the ultraweak coupling limit.

The number of peaks of the physical F_{av} in the same duration of t_1 in this more realistic case is much more than the one for the averaged pseudo-fidelity, because it takes a long time from $\tau_1(t_1)$ to the moment τ_1^{adv} when the classical signal from Alice reaches Rob, during which the detector B has oscillated for many times. In figure 3, we see that the moment $t_1 = t_{1/2}$ when the best averaged physical fidelity of quantum teleportation F_{av}^+ drops to $1/2$ is earlier than any F_{av}^+ of pseudo-fidelity has. The larger the $a\tau_2$, the later the τ_1^{adv} Rob has, and so the lower value of the physical F_{av}^+ at that time due to the longer time of coupling with environment. When $a\tau_2$ is large enough, τ_1^{adv} is so large that $t_{1/2}$ is almost the moment when Alice enters the event horizon of Rob for $\tau_2 \rightarrow \infty$.

6. Outlook

Detector–field interaction is a very useful and versatile system to expound and explore many known and unknown effects of relativistic quantum information. Continuing the vein of quantum teleportation as an example, it is not difficult to generalize the systematics to curved spacetimes, from weak gravitational field as in the Earth’s environment to black hole spacetimes.

In another vein, learning from the new techniques and ideas in the study of detector–field interaction as was done for the Unruh effect [8, 9], and using the well-known correspondence between the Rindler and the Schwarzschild spacetimes, one may go beyond the test-field description of black hole physics and study how backreaction from the field impacts on the evolution of the black hole and the ‘information loss’ issues.

Acknowledgments

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