HKUST Jockey Club Institute for Advanced Study
Hong Kong University of Science and Technology

Graphs and Groups, Tessellations and Transformations

Hong Kong, China, July 12–26, 2026

Abstracts

THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

20TH IAS HKUST JOCKEY CLUB INSTITUTE FOR ADVANCED STUDY

G2 T2

Hong Kong, China – 2026

GRAPHS AND GROUPS, TESSELLATIONS AND TRANSFORMATIONS, 2026: Abstracts of the G2T2 – the International Conference and PhD-Master Summer School on Graphs and Groups, Tessellations and Transformations, Hong Kong: HKUST Jockey Club Institute for Advanced Study of Hong Kong University of Science and Technology, 2026. – p. 88.

Editors
Elena V. Konstantinova, China Three Gorges University
Roman Nedela, University of West Bohemia
Yaokun Wu, Shanghai Jiao Tong University
Min Yan, Hong Kong University of Science and Technology

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General Information

Graphs and Groups, Tessellations and Transformations, G2T2-2026, belongs to the G2-series including international conferences and summer schools. This is the eleventh event of the G2-series. Since 2014, the G2-events were held in Russia and China, and in Slovenia as satellite events of the 8th European Congress of Mathematics.

The main goal: G2T2-2026 aims to bring together experts, young researchers and students from different fields of mathematics and their applications mainly based on algebra and group theory, geometry and algebraic graph theory, topology and algebraic combinatorics, coding theory and theory of computational complexity, especially those involving tessellations and group actions on combinatorial objects.

Venue: The International Conference and PhD-Master Summer School on Graphs and Groups, Tessellations and Transformations is held at the Institute for Advanced Study building on the beautiful seaside campus of The Hong Kong University of Science and Technology. Inaugurated in 1991, HKUST is a university dedicated to the advancement of learning, with special emphasis on research education and close collaboration with business and industry. The University occupies an impressive 60–hectare site on the northern end of Clear Water Bay Peninsula. Situated on the slopes along the shore, the campus grounds are terraced to afford buildings on all levels with unobstructed panoramic views of the sea.

Language: The official language of the event is English.

Program: The program includes 50-minute plenary talks, 25-minute contributed talks and minicourses.

Organizers:
Elena V. Konstantinova, China Three Gorges University
Roman Nedela, University of West Bohemia
Yaokun Wu, Shanghai Jiao Tong University
Min Yan, Hong Kong University of Science and Technology

Organized by:
HKUST Jockey Club Institute for Advanced Study of Hong Kong University of Science and Technology

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Website: `http://ias.ust.hk/events/2026g2t2/home.php`

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DEPARTMENT OF MATHEMATICS
SCHOOL OF SCIENCE, HKUST

The 11th edition of the G2: Tilings of surfaces

Tilings of surfaces, also called tessellations, have a long history that spans art, architecture, mathematics, and, more recently, computer science. A tiling is a way of covering a surface with shapes (tiles) that fit together without gaps or overlaps. The search for patterns, symmetry, and structure in such coverings has driven developments from ancient craftsmanship to modern geometry and topology. In the ancient world, tilings primarily appeared as decorative patterns. Mesopotamian, Egyptian, Greek, Roman, and Islamic artisans used repeating motifs to cover floors, walls, and domes. The Romans laid intricate mosaic floors from small, regularly shaped tiles, while Islamic art, constrained by aniconic traditions, developed highly sophisticated geometric tilings based on stars, polygons, and interlacing bands. These patterns, seen in sites such as the Alhambra in Spain, often exhibit high degrees of symmetry and foreshadow ideas that only much later became formalized in mathematics.

The first systematic mathematical study of tilings on the Euclidean plane emerged in the 19th century alongside the rise of group theory and crystallography. Mathematicians and crystallographers sought to classify all ways a pattern could repeat by translations, rotations, and reflections. This led to the classification of the 17 wallpaper groups: all possible symmetry types for periodic patterns on the flat plane. At the same time, mathematicians studied regular tilings those made from congruent regular polygons meeting in the same way at every vertex and showed that in the Euclidean plane there are only three: by equilateral triangles, squares, and regular hexagons.

The development of non Euclidean geometry in the 19th century opened a new chapter. On curved surfaces such as the sphere or the hyperbolic plane, the rules for angles and distances differ, and so do the possible tilings. On the sphere, for example, regular tilings correspond to the Platonic solids: arrangements of congruent regular polygons on the spherical surface. In the hyperbolic plane, by contrast, there is an infinite variety of regular tilings, far richer than in Euclidean geometry. The visualizations of hyperbolic tilings later became popular through the work of M. C. Escher, whose art was inspired by mathematical constructions provided by H. S. M. Coxeter.

The 20th century brought deeper and sometimes surprising questions: which sets of shapes can tile a surface, and in what ways? On the Euclidean plane, researchers studied tilings by congruent copies of a single polygon, and the classification of convex pentagons that tile the plane turned out to be especially difficult, requiring results spread throughout the century and into the early 21st. Another striking development was the discovery of aperiodic tilings: sets of tiles that can cover the plane but never do so in a repeating (periodic) way. The most famous of these are Penrose tilings, discovered in the 1970s, which exhibit non repeating but highly ordered patterns with fivefold symmetries that are impossible in periodic tilings.

These ideas soon connected to physics and materials science. In the 1980s, quasicrystals, solids with non periodic but ordered atomic arrangements, were discovered, and Penrose like tiling models helped explain their structure. At the same time, tilings of surfaces other than the plane, such as tori or more complicated topological surfaces, became important in topology and combinatorics. Graph embeddings on surfaces can be viewed as tilings, and counting such embeddings is linked to map enumeration and the study of moduli spaces.

In contemporary mathematics and computer science, tilings play multiple roles. In geometry and topology, they provide concrete models of surfaces and spaces, including those with curvature. In logic and theoretical computer science, tiling problems capture deep issues of computability: the classical "domino problem" asks whether a given set of tiles can tile the infinite plane, and it was shown in the 1960s to be undecidable in general. In computer graphics and architectural design, tilings are used for efficient meshing and for generating patterns on curved surfaces, with algorithms built on centuries of mathematical insight.

From ancient mosaics to undecidability proofs and models of novel materials, the history of tilings of surfaces reflects a recurring theme: the human drive to understand how local rules shapes placed edge to edge give rise to global order, symmetry, and complexity.

Our 11th edition of the G2 offers four short courses by leading experts and invites an active group of experienced researchers and promising students to share their ideas, aiming to inspire people with beautiful mathematics presented from the perspective of Graphs, Groups, Tessellations, and Transformations! As with all events of the G2-series, in the two weeks of G2T2 we wish you not only gain better insight into the interesting mathematical world but also strengthen ties of friendship and mutual understandings with old and new friends. Enjoy the art of mathematics with us!

Below is the list of the previous editions of the G2-series:

G2A2-2025: Groups and Graphs, Algebras and Applications, Novosibirsk, Russia, August 3-17, `https://mca.nsu.ru/g2a2-2025/`.

G2C2-2024: Graphs and Groups, Complexity and Convexity, Shijiazhuang, China, August 11-25, `http://app.hebtu.edu.cn/2024g2c2/`.

G2S2-2021: Groups and Graphs, Semigroups and Synchronization, Sochi, Russia, August 9-13, 2021, `https://siriusmathcenter.ru/en/all-russian-conference` (the website is updating).

G2G2-2021: Graphs and Groups, Geometry and GAP, Portotož-Rogla, Slovenia, June 20 - July 3, 2021, `https://conferences.famnit.upr.si/event/13`.

G2D2-2019: Groups and Graphs, Designs and Dynamics, Yichang, China, August 12-25, 2019, `http://math.sjtu.edu.cn/conference/G2D2`.

G2R2-2018: Graphs and Groups, Representations and Relations, Novosibirsk, Russia, August 06-19, 2018, `http://math.nsc.ru/conference/g2/g2r2`.

G2M2-2017: Groups and Graphs, Metrics and Manifolds, Yekaterinburg, Russia, July 22-30, 2017, `http://g2.imm.uran.ru/g2m2`.

G2S2-2016: Graphs and Groups, Spectra and Symmetries, Novosibirsk, Russia, August 15-28, 2016, `http://math.nsc.ru/conference/g2/g2s2`.

G2A2-2015: Groups and Graphs, Algorithms and Automata, Yekaterinburg, Russia, August 09-15, 2015, `http://g2a2.imm.uran.ru`.

G2C2-2014: Graphs and Groups, Cycles and Coverings, Novosibirsk, Russia, September 23-26, 2014, `http://math.nsc.ru/conference/g2/g2c2`.

Timetable on July, 12-18, 2026

The International Conference and PhD-Master Summer School:
Graphs and Groups, Tessellations and Transformations, G2T2-2026

Sunday July 12	Monday July 13	Tuesday July 14	Wednesday July 15	Thursday July 16	Friday July 17	Saturday July 18
9:00-21:00 Arrivals	7:00-8:30 Breakfast					
	9:30-11:30 Morning sessions					
	8:30-9:00 Registration				8:30-9:00 Registration	
	9:00-9:30 Opening G2T2				9:00-9:30 Opening G2T2 conference	
	9:30-10:20 Thomas Karam Lecture 1	9:30-10:20 Thomas Karam Lecture 3	9:30-10:20 Thomas Karam Lecture 5	9:30-10:20 Thomas Karam Lecture 7	9:30-10:20 Bojan Mohar	9:30-10:20 Jie Ma
	10:20-10:40 Coffee/tea break					
	10:40-11:30 Thomas Karam Lecture 2	10:40-11:30 Thomas Karam Lecture 4	10:40-11:30 Thomas Karam Lecture 6	10:40-11:30 Thomas Karam Lecture 8	10:40-11:30 Martin Skoviera	10:40-11:30 Huiqiu Lin
	11:30-14:00 Lunch				11:30-11:45 Conference Photo	
	14:00-18:30 Afternoon sessions					
	14:00-14:50 Caiheng Li Lecture 1	14:00-14:50 Caiheng Li Lecture 3	14:00-14:50 Caiheng Li Lecture 5	14:00-14:50 Caiheng Li Lecture 7	14:00-14:50 Brendan McKay	14:00-14:50 Andrei Raigorodskii
	14:50-15:10 Coffee/tea break					
	15:10-16:00 Caiheng Li Lecture 2	15:10-16:00 Caiheng Li Lecture 4	15:10-16:00 Caiheng Li Lecture 6	15:10-16:00 Caiheng Li Lecture 8	15:10-16:00 Edita Macajova	15:10-16:00 Jack Koolen
	16:00-16:20 Coffee/tea break					
	Xiaocong He	Meng-Yue Cao	Marie Cris B. Mahinay	Tongyu Nian	16:20-17:10 Chi Hoi Yip	16:20-17:10 Wei-Hsuan Yu
	Binwei Zhao	Wenying Zhu	Shikang Yu	Yilin Xie		
	17:10-17:15 Short break					
		Hao Yu		Jianbing Lu	17:15-18:05 Jin-Xin Zhou	17:15-18:05 Kenichi Kawarabayashi
		Ksenia Zimireva		Yanzhen Xiong		
	Free discussions				18:05-18:30 Huye Chen 18:30-18:55 Xiaoye Liang 18:55-19:20 Peter Zeman	18:05-18:30 Kadyan 18:30-18:55 Somlai 18:55-19:20 Somlai
	18:30-20:30 Dinner					19:45-22:00 Conference dinner
	20:30-21:30 Thomas Karam Problem solving	20:30-21:30 Caiheng Li Problem solving	20:30-21:30 Thomas Karam Problem solving	20:30-21:30 Caiheng Li Problem solving	Free discussions	

Timetable on July, 19-25, 2026

The International Conference and PhD-Master Summer School: Graphs and Groups, Tessellations and Transformations, G2T2-2026						
Sunday July 19	Monday July 20	Tuesday July 21	Wednesday July 22	Thursday July 23	Friday July 24	Saturday July 25
Excursions	7:00-9:00 Breakfast					
	9:30-11:30 Morning sessions					
	9:30-10:20 Roman Nedela	9:30-10:20 Yibo Gao	9:30-10:20 Marston Conder Lecture 1	9:30-10:20 Marston Conder Lecture 3	9:30-10:20 Marston Conder Lecture 5	9:30-10:20 Marston Conder Lecture 7
	10:20-10:40 Coffee/tea break					
	10:40-11:30 Chao Yang	10:40-11:30 Ting-Wei Chao	10:40-11:30 Marston Conder Lecture 2	10:40-11:30 Marston Conder Lecture 4	10:40-11:30 Marston Conder Lecture 6	10:40-11:30 Marston Conder Lecture 8
	11:30-14:00 Lunch					
	14:00-18:30 Afternoon sessions					
	14:00-14:50 Alexander Mednykh	14:00-14:50 Victor Chepoi	14:00-14:50 János Pach/Gábor Tardos Lecture 1	14:00-14:50 János Pach/Gábor Tardos Lecture 3	14:00-14:50 János Pach/Gábor Tardos Lecture 5	14:00-14:50 János Pach/Gábor Tardos Lecture 7
	14:50-15:10 Coffee/tea break					
	15:10-16:00 Tao Zhang	15:10-16:00 Akihiro Munemasa	15:10-16:00 János Pach/Gábor Tardos Lecture 2	15:10-16:00 János Pach/Gábor Tardos Lecture 4	15:10-16:00 János Pach/Gábor Tardos Lecture 6	15:10-16:00 János Pach/Gábor Tardos Lecture 8
	16:00-16:20 Coffee/tea break					
	16:20-17:10 Petr Stehlık	16:20-17:10 Rade Zivaljevic	Julia Q. D. Du Moe Moe Oo	Nikolay Abrosimov Young Soo Kwon	Robert Barish Dmitrii Panasenko	16:20-17:20 János Pach/Gábor Tardos Problem solving
	17:10-17:15 Short break					
	17:15-18:05 Josef Siran	17:15-18:05 Nathan Lindzey	17:15-17:40 Haofang Sun 17:40-18:05 Hoi Ping Luk	17:15-17:40 Yifei Wang 17:40-18:05 Galina Sokolova 18:05-18:30 Min Xie	17:15-17:40 Rhys Evans 17:40-18:05 Anyuan Tian 18:05-18:30 Pengyu Zhu	17:20-17:30 Closing G2T2
	18:05-18:30 Kan Hu 18:30-18:55 Cindy Tsang 18:55-19:20 Ilya Mednykh	Special session: G2-2027	Free discussions			
	18:30-20:30 Dinner					
	Free discussion		20:30-21:30 Marston Conder Problem solving	20:30-21:30 János Pach/Gábor Tardos Problem solving	20:30-21:30 Marston Conder Problem solving	Free discussions

G2T2-Program

Sunday, July 12

09:00 - 21:00	Registration

Monday, July 13

07:00 - 08:30	Breakfast
08:30 - 09:00	**Registration**
09:00 - 09:30	**Opening G2T2**
	Minicourse 1
	Chair: Chi Hoi Yip
09:30 - 10:20	Thomas Karam: *Lecture 1*
10:40 - 11:30	Thomas Karam: *Lecture 2*
11:30 - 14:00	Lunch
	Minicourse 2
	Chair: Min Yan
14:00 - 14:50	Caiheng Li: *Lecture 1*
15:10 - 16:00	Caiheng Li: *Lecture 2*
16:00 - 16:20	Coffee break
	Contributed talks
	Chair: Ting-Wei Chao
16:20 - 16:45	Xiaocong He: *The Turán number of odd prism*
16:45 - 17:10	Binwei Zhao: *On EKR-type problem for hypergraph matchings*
17:10 - 17:15	Break
20:30 - 21:30	**Problem solving: Minicourse 1**
	Chair: Chi Hoi Yip

Tuesday, July 14

07:00 - 09:00	Breakfast
	Minicourse 1
	Chair: Chi Hoi Yip
09:30 - 10:20	Thomas Karam: *Lecture 3*
10:40 - 11:30	Thomas Karam: *Lecture 4*
11:30 - 14:00	Lunch
	Minicourse 2
	Chair: Min Yan
14:00 - 14:50	Caiheng Li: *Lecture 3*
15:10 - 16:00	Caiheng Li: *Lecture 4*
16:00 - 16:20	Coffee break
	Contributed talks
	Chair: Jack Koolen
16:20 - 16:45	Meng-Yue Cao: *On signed graphs with fixed smallest eigenvalue*
16:45 - 17:10	Wenying Zhu: *Digraphs with non-diagonalizable adjacency matrix*
17:10 - 17:15	Break
	Chair: Alexander Mednykh
17:15 - 17:40	Hao Yu: *Cayley maps and product groups*
17:40 - 18:05	Ksenia Zimireva: *Representation of braid-like groups*
18:30 - 20:30	Dinner
20:30 - 21:30	**Problem solving: Minicourse 2**
	Chair: Min Yan

Wednesday, July 15

07:00 - 09:00	Breakfast
	Minicourse 1
	Chair: Chi Hoi Yip
09:30 - 10:20	Thomas Karam: *Lecture 5*
10:40 - 11:30	Thomas Karam: *Lecture 6*
11:30 - 14:00	Lunch
	Minicourse 2
	Chair: Min Yan
14:00 - 14:50	Caiheng Li: *Lecture 5*
15:10 - 16:00	Caiheng Li: *Lecture 6*
16:00 - 16:20	Coffee break
	Contributed talks
	Chair: Elena V. Konstantinova
16:20 - 16:45	Marie Chris B. Mahinay: *Characterizing the set chromatic number of the zero-divisor graph of a finite ring*
16:45 - 17:10	Shikang Yu: *Fractional clique decompositions of dense balanced multipartite graphs*
18:30 - 20:30	Dinner
20:30 - 21:30	**Problem solving: Minicourse 1**
	Chair: Chi Hoi Yip

Thursday, July 16

07:00 - 09:00	Breakfast
	Minicourse 1
	Chair: Chi Hoi Yip
09:30 - 10:20	Thomas Karam: *Lecture 7*
10:40 - 11:30	Thomas Karam: *Lecture 8*
11:30 - 14:00	Lunch
	Minicourse 2
	Chair: Min Yan
14:00 - 14:50	Caiheng Li: *Lecture 7*
15:10 - 16:00	Caiheng Li: *Lecture 8*
16:00 - 16:20	Coffee break
	Contributed talks
	Chair: Wei-Hsuan Yu
16:20 - 16:45	Tongyu Nian: *q-deformation of graphical arrangement and its generalizations*
16:45 - 17:10	Yilin Xie: *On second maximum intersecting families of Ree Unital*
17:10 - 17:15	Break
	Chair: Petr Stehlík
17:15 - 17:40	Jianbing Lu: *Recent progress in the classification of primitive flag-transitive generalized quadrangles*
17:40 - 18:05	Yanzhen Xiong: *Amorphic association schemes and beyond*
18:30 - 20:30	Dinner
20:30 - 21:30	**Problem solving: Minicourse 2**
	Chair: Min Yan

Friday, July 17

08:30 - 09:00	**Registration**
09:00 - 09:30	**Opening G2T2 conference**
	Plenary talks
	Chair: Roman Nedela
09:30 - 10:20	Bojan Mohar: *Long cycles in vertex transitive digraphs*
10:20 - 10:40	Coffee break
10:40 - 11:30	Martin Škoviera: *Bounding Hamilton cycles in polytopal maps*
11:30 - 11:45	**Conference photo**
11:45 - 14:00	Lunch
	Plenary talks
	Chair: János Pach
14:00 - 14:50	Brendan McKay: *Eulerian orientations*
14:50 - 15:10	Coffee break
15:10 - 16:00	Edita Máčajová: *Rich nowhere-zero flows*
16:00 - 16:20	Coffee break
	Plenary talks
	Chair: Andrei Raigorodskii
16:20 - 17:10	Chi Hoi Yip: *Extensions of the Carlitz–McConnel and Blokhuis–Sziklai theorems*
17:10 - 17:15	Break
17:15 - 18:05	Jin-Xin Zhou: *The finite k-set homogeneous graphs*
	Contributed talks
	Chair: Kan Hu
18:05 - 18:30	Huye Chen: *Isomorphic factorizations of the complete graphs into Cayley graphs on CI-groups*
18:30 - 18:55	Xiaoye Liang: *The Dunkl–Watanabe Duality*
18:55 - 19:20	Peter Zeman: *NPA hierarchy for quantum isomorphism and homomorphism indistinguishability*
19:20 - 20:30	Dinner
20:30	**Free discussions**

Saturday, July 18

	Plenary talks
	Chair: Jack Koolen
09:30 - 10:20	Jie Ma: *Spectral supersaturation for color-critical graphs*
10:20 - 10:40	Coffee break
10:40 - 11:30	Huiqiu Lin: *Extremal eigenvalues with respect to graph minors*
11:30 - 14:00	Lunch
	Plenary talks
	Chair: Bojan Mohar
14:00 - 14:50	Andrei M. Raigorodskii: *Graphs and combinatorial geometry*
14:50 - 15:10	Coffee break
15:10 - 16:00	Jack Koolen: *The regular two-graph on 276 vertices, revisited*
16:00 - 16:20	Coffee break
	Plenary talks
	Chair: Brendan McKay
16:20 - 17:10	Wei-Hsuan Yu: *Maximum two-distance sets in Hamming, Johnson and Euclidean space*
17:10 - 17:15	Break
17:15 - 18:05	Ken-ichi Kawarabayashi: *The four color theorem: generalizations and faster algorithms*
	Contributed talks
	Chair: Huye Chen
18:05 - 18:30	Monu Kadyan: *A generalization of Ramanujan's sum over finite groups*
18:30 - 18:55	Gábor Somlai: *Cylindrical set conjecture*
18:55 - 19:20	Gábor Somlai: *Tiles of cyclic group*
19:45 - 22:00	Conference dinner

Sunday, Huly 19

Excursions

Monday, July 20

	Plenary talks
	Chair: Marston Conder
09:30 - 10:20	Roman Nedela: *Tiling the sphere with regular polygons*
10:20 - 10:40	Coffee break
10:40 - 11:30	Chao Yang: *Hilbert's 18th problem and undecidability of translational tiling*
11:30 - 14:00	Lunch
	Plenary talks
	Chair: Tador Gábor
14:00 - 14:50	Alexander Mednykh: *Geometry of knots and links in the spaces of constant curvature*
14:50 - 15:10	Coffee break
15:10 - 16:00	Tao Zhang: *On the structures of tiling sets in finite abelian groups*
16:00 - 16:20	Coffee break
	Plenary talks
	Chair: Rade Živaljiević
16:20 - 17:10	Petr Stehlík: *Graphs, symmetries and orderings in dynamical systems*
17:10 - 17:15	Break
17:15 - 18:05	Jozef Širáň: *Non-orientable regular hypermaps of arbitrary hyperbolic type*
	Contributed talks
	Chair: Young Soo Kwon
18:05 - 18:30	Kan Hu: *Cyclic complementary extensions of groups and skew morphisms*
18:30 - 18:55	Cindy (Sin Yi) Tsang: *Skew braces with no proper left ideals*
18:55 - 19:20	Ilya Mednykh: *Laplace polynomials for the Moebius strip and Klein bottle*
19:30 - 20:30	Dinner
20:30	**Free discussions**

Tuesday, July 21

	Plenary talks
	Chair: Victor Chepoi
09:30 - 10:20	Yibo Gao: *Symmetric structures in the Bruhat order*
10:20 - 10:40	Coffee break
10:40 - 11:30	Ting-Wei Chao: *Counting rainbow triangles via entropy method*
11:30 - 14:00	Lunch
	Plenary talks
	Chair: Yaokun Wu
14:00 - 14:50	Victor Chepoi: *Ample sets in Cartesian products*
14:50 - 15:10	Coffee break
15:10 - 16:00	Akihiro Munemasa: *Equiangular lines, distance-regular covers of complete graphs, and roux schemes*
16:00 - 16:20	Coffee break
	Plenary talks
	Chair: Min Yan
16:20 - 17:10	Rade Živaljiević: *Combinatorics, geometry, and topology of Bier spheres*
17:10 - 17:15	Break
17:15 - 18:05	Natan Lindzey: *An eventown result for permutations*
18:05 - 18:30	Natan Lindzey: *G2-2027*
18:30 - 20:30	Dinner
20:30	**Free discussions**

Wednesday, August July 22

07:00 - 09:00	Breakfast
	Minicourse 3
	Chair: Roman Nedela
09:30 - 10:20	Marston Conder: *Lecture 1*
10:40 - 11:30	Marston Conder: *Lecture 2*
11:30 - 14:00	Lunch
	Minicourse 4
	Chair: Akihiro Munemasa
14:00 - 14:50	János Pach/Gabor Tardos: *Lecture 1*
15:10 - 16:00	János Pach/Gabor Tardos: *Lecture 2*
16:00 - 16:20	Coffee break
	Contributed talks
	Chair: Chao Yang
16:20 - 16:45	Julia Q. D. Du: *On some properties of Archimedean tiling graphs*
16:45 - 17:10	Moe Moe Oo: *Paired dominating sets and domino tilings in fixed-width grid graphs*
17:10 - 17:15	Break
	Chair: Tao Zhang
17:15 - 17:40	Haofang Sun: *Most curvilinear tilings and Hurwitz theorem*
17:40 - 18:05	Hoi Ping Luk: *Is the rank function of a delta-matroid the rank function of a matroid?*
18:30 - 20:30	Dinner
20:30 - 21:30	**Problem solving: Minicourse 3**
	Chair: Hoi Ping Luk

Thursday, July 23

07:00 - 09:00	Breakfast
	Minicourse 3
	Chair: Roman Nedela
09:30 - 10:20	Marston Conder: *Lecture 3*
10:40 - 11:30	Marston Conder: *Lecture 4*
11:30 - 14:00	Lunch
	Minicourse 4
	Chair: Akihiro Munemasa
14:00 - 14:50	János Pach/Gabor Tardos: *Lecture 3*
15:10 - 16:00	János Pach/Gabor Tardos: *Lecture 4*
16:00 - 16:20	Coffee break
	Contributed talks
	Chair: Jozef Širáň
16:20 - 16:45	Nikolay Abrosimov: *Geometric structures on knot complements: construction of fundamental polyhedra*
16:45 - 17:10	Young Soo Kwon: *Several applications of skew-morphisms of groups*
17:10 - 17:15	Break
	Chair: Edita Máčajová
17:15 - 17:40	Yifei Wang: *Converse Casey's theorem in hyperbolic geometry*
17:40 - 18:05	Galina Sokolova: *Plans theorem for a cone over sandwich graphs*
18:05 - 18:30	Min Xie: *Phylogenetic tree reconstruction from parsimony layered filters*
18:30 - 20:30	Dinner
20:30 - 21:30	**Problem solving: Minicourse 4**
	Chair: Robert Barish

Friday, July 24

07:00 - 09:00	Breakfast
	Minicourse 3
	Chair: Roman Nedela
09:30 - 10:20	Marston Conder: *Lecture 5*
10:40 - 11:30	Marston Conder: *Lecture 6*
11:30 - 14:00	Lunch
	Minicourse 4
	Chair: Akihiro Munemasa
14:00 - 14:50	János Pach/Gabor Tardos: *Lecture 5*
15:10 - 16:00	János Pach/Gabor Tardos: *Lecture 6*
16:00 - 16:20	Coffee break
	Contributed talks
	Chair: Martin Škoviera
16:20 - 16:45	Robert Barish: *Falsifying some conjectures in organic chemistry on the structure of perfect matchings in subgraphs of the honeycomb lattice*
16:45 - 17:10	Dmitrii Panasenko: *Computing set upper tolerance for the minimal spanning tree problem*
17:10 - 17:15	Break
	Chair: Ting-Wei Chao
17:15 - 17:40	Rhys J. Evans: *Orderly generation of highly symmetrical discrete objects*
17:40 - 18:05	Anyuan Tian: *A symmetry characterization of t-designs in the Boolean lattice*
18:05 - 18:30	Pengyu Zhu: *New combinatorial problems from the Hermitian Sum-of-Squares conjecture*
18:30 - 20:30	Dinner
20:30 - 21:30	**Problem solving: Minicourse 3**
	Chair: Hoi Ping Luk

Saturday, July 25

07:00 - 09:00	Breakfast
	Minicourse 3
	Chair: Roman Nedela
09:30 - 10:20	Marston Conder: *Lecture 7*
10:40 - 11:30	Marston Conder: *Lecture 8*
11:30 - 14:00	Lunch
	Minicourse 4
	Chair: Akihiro Munemasa
14:00 - 14:50	János Pach/Gabor Tardos: *Lecture 7*
15:10 - 16:00	János Pach/Gabor Tardos: *Lecture 8*
16:00 - 16:20	Coffee break
16:20 - 17:20	**Problem solving: Minicourse 4**
	Chair: Robert Barish
17:20 - 17:30	**Closing G2T2**
	Chair: Elena V. Konstantinova

Sunday, July 26

Departure

Abstracts

Plenary and Contributed talks are listed alphabetically with respect to the Presenting Author

Minicourses

Minicourse I: From lattice covering numbers to the ranks of tensors *(10 hours)*

Lecturer:
Thomas Karam
Shanghai Jiao Tong University, Shanghai, China
thomaskaram(at)sjtu.edu.cn

Tensors are higher-dimensional generalisations of matrices. The matrix rank has been generalised to several competing notions of rank on tensors that we will begin by presenting, paying particular attention to the slice rank and its role in the resolution of the cap-set problem. In a second part, after studying several basic properties of the covering numbers of lattices we will establish a result of Sawin and Tao providing an interpretation of the slice rank in terms of lattice coverings in the special case of "sparse" tensors, which reduces the study of the former to the latter in that special case. In a third part, we will use the linear groups of transformations on tensors to explain why there is already a major gap in difficulty between the study of basic properties of the ranks of tensors between order-2 tensors (matrices) and order-3 tensors, and then discuss which properties can or cannot reasonably be expected to hold for the ranks of tensors. In a final part, we will discuss proofs of some of these properties, highlighting the guidance provided by lattice covering numbers even when the connection of Sawin and Tao does not apply.

Outline of the course is as follows.

Lecture 1. The notions of rank on tensors and their history. Unlike in the case of matrices, there is no single canonical notion of rank of tensors. Rather, the most useful notion of rank depends on the context and application that one has in mind, and over the past decade alone many notions have seen the light of day, such as the slice rank [12] and partition rank [10], and more recently the geometric rank [8] and local rank [9]. This lecture focuses on a history of these notions and of their motivations and applications, emphasizing the major similarities and differences in the definitions.

Lecture 2. The slice rank of tensors and the solution of the cap-set problem. This lecture focuses on the origin of the slice rank. The cap-set problem in number theory asks for the size of the largest subset $A \subset \mathbb{F}_3^n$ such that $A \times A \times A$ contains no solutions to $x + y + z = 0$ aside from the trivial solutions $x = y = z$. A breakthrough from the 2016 papers of Croot-Lev-Pach [1] and Ellenberg-Gijswijt [3] (both published in Ann. Math.) shows that $|A|/3^n$ must be exponentially small in n. We will present their proof reformulated in terms of the slice rank, that Tao defined afterwards [12] in order to write the proof while treating variables more symmetrically.

Lecture 3. Lattice coverings: basic questions, results and proofs. This lecture introduces a second main object which will play an important role throughout the course: the covering number of a lattice subset, defined to be the smallest number of sets of some prescribed type (for instance, slices) that suffice to cover the lattice subset. We will prove some simple properties of lattice covering numbers [5]: for instance, must some small restriction of the lattice have the same covering number as the original lattice, and how much flexibility is there in choosing a lattice covering with minimal size?

Lecture 4. A connection between the slice covering number and the slice rank. This lecture establishes a characterisation of the slice rank of a tensor by Sawin and Tao [11] in the special case where the support of the tensor is contained in an antichain: in this special case, the slice rank of the tensor is equal to the smallest number of slices that suffices to cover its support. From this connection it follows that in this special case, proving basic results on the slice rank reduces to proving the analogous results on the slice covering number.

Lecture 5. The linear group of transformations on tensors, and the subrank. This lecture provides fundamental reasons why the study of rank notions on tensors is much more difficult than the study of the matrix rank. As the size of the group of base change transformations is considerably smaller than the set of order-d tensors on $[n]^d$ for $d \geq 3$, it is then no longer possible to reduce all tensors to "simple" normal forms purely with base change transformations as could be done in the matrix case, although the idea still leads to the definition of another notion of rank on tensors called the subrank.

Lecture 6. Formulations of basic properties of the ranks of tensors. This lecture formulates statements aiming to extend basic properties of matrix rank to the ranks of tensors, for instance the property that every rank-k matrix contains some $k \times k$ rank-k submatrix and the property that every matrix only has one minimal-length rank decomposition up to a change of basis. A theme arises repeatedly: the naive extension to the ranks of tensors of the property that held for matrix rank fails in a major way, but may nonetheless be rectified to a property which (in addition to being true) is not too complicated to state and in a spirit very close to that of the original property.

Lecture 7. Proving the extension of the submatrix property to tensors. This lecture focuses on the proof [4] of the property that if a tensor has high rank then it must necessarily contain a subtensor with bounded size that nonetheless still has high rank, establishing it in some special cases involving ideas that are representative of the proof in the general case. The many analogies with the setting of lattice covering numbers [5] are stressed: even if the ideas from that setting do not by themselves suffice to obtain the proof for the ranks of tensors they lead to several central ideas in that proof, most notably a sufficient condition guaranteeing that a tensor has high rank and a sufficient condition guaranteeing that two notions of rank are "equivalent" on some tensor.

Lecture 8. Proving the boundedness for optimal slice rank decompositions. This lecture focuses on the proof [6] of the property that if an order-d tensor has slice rank k over a finite field $\mathbb{F}$, then up to a natural class of transformations (that is indispensable in such a statement) it may only have a number of minimal-length slice rank decompositions that is bounded in terms of $d, k, |\mathbb{F}|$. This concludes the course.

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Minicourse II: Fimple groups, symmetric graphs, and geometries *(10 hours)*

Lecturer:
Caiheng Li
Southern University of Science and Technology, Shenzhen, China
lich(at)sustech.edu.cn

The theme of this short course is group actions on graphs, geometries and maps, which includes the following topics.

(1) **Finite groups: normal subgroups, quotient groups, simple groups, and Jordan-Holder Theorem.**

(2) **Permutation groups: primitive groups, quasiprimitive groups, and O'Nan-Scott Theorem.**

(3) **Edge transitive graphs, and flag-transitive incidence geometries.**

(4) **A new characterization of the five type arc-transitive maps.**

(5) **Vertex quasiprimitive regular maps.**

(6) **Smooth coverings of surfaces, maps and groups.**

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Minicourse III: Combinatorial methods in group theory and group-theoretic methods in combinatorics *(10 hours)*

Lecturer:
Marston Conder
University of Auckland, New Zealand
m.conder(at)auckland.ac.nz

This mini-course describes a number of helpful methods from combinatorics and group theory that can be applied in those areas, and more especially to the study of discrete objects such as graphs and maps and polytopes with a large degree of symmetry.

Topics include the following.

(1) **Basic applications of counting.**

(2) **Methods for generating random elements of a group.**

(3) **Cayley graphs.**

(4) **Schreier coset graphs and their applications.**

(5) **Backtrack search to find subgroups of small index.**

(6) **Double-coset graphs and some applications.**

(7) **Möbius inversion on lattices and applications.**

Minicourse IV: Geometry and matrices: extremal theory *(10 hours)*

Lecturer:
János Pach & Gábor Tardos
Alfréd Rényi Institute of Mathematics, Budapest, Hungary
pach(at)renyi.hu & trdos(at)renyi.hu

Outline of the course is as follows.

Lecture 1. Clique number vs. independence number: Ramsey's theorem. Erdős-Hajnal conjecture. Intersection graphs of geometric objects, string graphs. Stronger versions of the Erdős-Hajnal property (strong, mighty). Enumeration of segment graphs, string graphs, pseudo-segment graphs.

Lecture 2. Chromatic number vs. clique number. Chi-boundedness. Hadwiger's conjecture, Hajós conjecture.

Lecture 3. Crossing numbers of graphs. Albertson's conjecture. Which crossing number is it, anyway? Three approaches to crossing numbers.

Lecture 4. Unit distances, repeated distances. Incidences, cell decompositions. Erdős-Gyárfás problem. Szemerédi theorem on arithmetic progressions. Unit distances and (a) semialgebraic graphs, (b) rigidity, (c) 0-1 matrices.

Lecture 5. Extremal problems in ordered graphs and and 0-1 matrices. A variant of the Erdős-Stone-Simonovits theorem. Variants of the "almost linear conjecture" and special cases where they hold.

Lecture 6. Edge-ordered graphs and why the "almost linear conjecture" holds for them.

Lecture 7. Lower bound constructions.

Lecture 8. The non-bipartite case: differences between the ordered an unordered settings. Results from (unordered) extremal graph theory that do carry over.

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Plenary Talks

Counting rainbow triangles via entropy method

Ting-Wei Chao
MIT, Cambridge MA, USA
twchao(at)mit.edu

This is joint work with Maya Sankar and Hung-Hsun Hans Yu

Shannon entropy is a powerful tool used in many recent progresses in extremal combinatorics. Recently, new proofs of several classical results such as Turán's theorem and the Kruskal–Katona theorem via the entropy method were found. In this talk, we will talk about a rainbow version of the Kruskal–Katona theorem and possibly talk about its connection with the joints problem in incidence geometry.

Ample sets in Cartesian products

Victor Chepoi
Aix-Marseille Université, Marseille, France
victor.chepoi(at)lis-lab.fr

This is joint work with Matthew Maat

The main goal of the talk is to define ample sets in Cartesian products and to present their main characterizations and properties. First, we will briefly recall the definition and the properties of ample sets in hypercubes. In this case, ample sets were defined by A. Dress (1995) by a sandwich inequality and they turn out to be equivalent to lopsided sets of J. Lawrence (1983), to simple sets of D. Wiedemann (1986), and to extremal sets of B. Bollobàs and A. J. Ratcliffe (1995).

We consider subsets S of Cartesian products $U := U_1 \times \ldots \times U_m$ of nonempty finite sets $U_1, \ldots, U_m$. To define ampleness in this general setting, we apply to Cartesian products the *shattering→strong shattering principle*, which is one of the equivalent definitions of classical ampleness. In hypercubes $\{0,1\}^E$, this principle is asserting that whenever a hypercube $\{0,1\}^Y$ with $Y \subseteq E$ is shattered by S, then S contains a copy of $\{0,1\}^Y$ (i.e., it is strongly shattered by S). In case of Cartesian products, we apply the shattering→strong shattering principle to minor-subproducts of U. Each *minor subproduct* $M = M(\Lambda)$ of U is associated to a generalized partition $\Lambda = (\alpha_1, \ldots, \alpha_m)$, where each α_i is a partition of U_i. Then $M(\Lambda) = M_1 \times \ldots \times M_m$, where each M_i is the minor of U_i obtaining by contracting each block of the partition α_i into a single vertex. The generalized partition $\Lambda = (\alpha_1, \ldots, \alpha_m)$ defines a partition $\mathcal{B}(\Lambda)$ of U into boxes, which are Cartesian products of the blocks from different partitions $\alpha_i, i = 1, \ldots, m$ of Λ. Then, alternatively $M(\Lambda)$ is obtained from U by contracting each box of the partition $\mathcal{B}(\Lambda)$ into a single vertex. A minor-subpriduct $M(\Lambda)$ is *shattered* by S if S intersects each box of $\mathcal{B}(\Lambda)$ and M is *strongly shattered* by S if S contains a copy of $M(\Lambda)$. A set $S \subseteq U$ is called *ample* whenever a minor-subproduct $M(\Lambda)$ is shattered by S, then $M(\Lambda)$ is strongly shattered by S. In an analogous way, we can define the operations of *projection* S_M and *strong projection* of a set $S \subseteq U$ on a minor-subproduct M.

We show that the main characterizations (but not all) of ample sets of hypercubes generalize to ample sets of arbitrary Cartesian products. We will also present some other properties of ample sets: push-downs and corner peelings, Euler characteristic and contractibility of their box/prism complexes. Here is a list of main characterizations of ample sets.

Theorem 1. *For a set $S \subseteq U = U_1 \times \ldots \times U_m$, the following conditions are equivalent:*

1. *S is ample;*
2. *S^M is isometric for all minor-subproducts M (superisometricity);*
3. *S is box-superisometric;*
4. *S^M is connected for all minor-subproducts M (superconnectivity);*
5. *$(S^M)_{M'} = (S_{M'})^M$ for all minor-subproducts M, M' with disjoint supports (commutativity);*
6. *the complement $S^* = U - S$ is ample;*
7. *S is isometric and both S^e and S_e are ample for some elementary minor-subproduct $e = \{a, b\}$;*
8. *S is connected and S^e is ample for every elementary minor-subproduct e;*
9. *$S \cap [u, v]$ is ample in $[u, v]$ for all $u, v \in S$.*

Symmetric structures in the Bruhat order

Yibo Gao
Peking University, Beijing, China
gaoyibo(at)bicmr.pku.edu.cn

This is joint work with Christian Gaetz

The Bruhat order encodes algebraic and topological information of Schubert varieties in the flag manifold and possesses rich combinatorial properties. In this talk, we discuss some interrelated stories regarding the Bruhat order and in particular, regarding the smooth permutations, including self-dual Bruhat intervals, vertex-transitive elements, minimal exponents in the Kazhdan-Lusztig polynomials and Billey-Postnikov decompositions.

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The four color theorem: generalizations and faster algorithms

Ken-ichi Kawarabayashi
National Institute of Informatics, The University of Tokyo, Tokyo, Japan
k_keniti(at)nii.ac.jp

This is joint work with Yuta Inoue, Atsuyuki Miyashita, Bojan Mohar, Carsten Thomassen, Mikkel Thorup

The celebrated Four Color Theorem (4CT) was conjectured by Francis Guthrie in 1852, and remained open for more than 100 years, until Appel and Haken found a proof in 1976. The proof is computer-assisted, and a simplified version was found by Robertson et al. in 1997. It has been known since 1880 (by Tait) that the 4CT is equivalent to stating that every 2-connected cubic planar graph is 3-edge-colorable.

In our ongoing project, we aim to utilize today's machine power to extend the computer-assisted proof (and computer-free extension) of 4CT, giving significantly stronger results and providing a much faster algorithm. Topics include:

- Three-edge cubic graphs on a surface: Here, we completely characterize all non-three-edge colorable cubic graphs (i.e., snarks) that can be embedded in a projective plane or in a torus (while the 4CT has no extension to other surfaces).
- A much faster algorithm to 4-color planar graphs, namely: we can 4-color a planar graph of order n in $O(n \log n)$ time. This generalizes the $O(n^2)$ algorithm by Robertson et al. in 1997.

Based on the work in [1].

Acknowledgments. The work has been supported by JSPS Kakenhi 26K21777 and JP25K24465, and by JST ASPIRE JPMJAP2302.

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The regular two-graph on 276 vertices, revisited

Jack Koolen
University of Science and Technology of China, Hefei, China
koolen(at)ustc.edu.cn

This is based on ongoing joint work with Akihiro Munemasa

In this talk, I will discuss the regular two-graph on 276 vertices. This is a beautiful mathematical object which has automorphism group $Co_3.2$. We will classify certain graphs in the switching class of this two-graph, and show how you can use this classification to obtain a Euclidean 2-distance set of order 277 in the Euclidean space of dimension 23, which was found in 2025 by Hong-Jun Ge, Akihiro Munemasa and me. There are many Euclidean 2-distance sets of order 276 in dimension 23.

Extremal eigenvalues with respect to graph minors

Huiqiu Lin
East China University of Science and Technology, Shanghai, China
huiqiulin(at)126.com

Minors play a crucial role in various branches of graph theory, including structural graph theory, extremal graph theory, and topological graph theory, and have garnered significant interest in these areas.

In this talk we explore the maximal spectral radius, denoted $spex(n, \mathbb{H}_{minor})$, of n-vertex graphs that exclude any graph from a fixed family $\mathbb{H}$ as a minor.

We derive the asymptotic value for $spex(n, \mathbb{H}_{minor})$ and establish a unified stable structure for extremal graphs by introducing a novel application of the absorbing method to eigenvalue analysis, along with models and partitions with respect to a general H minor. In particular, we prove three central theorems, the most fundamental of which asserts that every graph with spectral radius $\rho \geq spex(n, \{H\}_{minor})$ contains either an H minor or a spanning book $B_{\gamma_H, n-\gamma_H}$, where $\gamma_H = |H| - \alpha_H - 1$ and α_H is the independence number of H.

These three theorems, combined with detailed combinatorial analysis, enable us to determine $spex(n, \{H\}_{minor})$ for every complete r-partite graph H. This extends the result of Tait for $spex(n, \{K_r\}_{minor})$ and provides a stronger solution to his conjecture for $spex(n, \{K_{s,t}\}_{minor})$ [J. Combin. Theory Ser. A 166, 2019]. Additionally, these theorems imply or strengthen other existing eigenvalue-extremal results on minors, such as planar graphs by Tait and Tobin, $K_r - E(H)$ minors by Chen, Liu and Zhang, and friendship graph minors by He, Li and Feng.

An eventown result for permutations

Nathan Lindzey
University of Memphis, Memphis, USA
nathan.lindzey(at)memphis.edu

A family of permutations $\mathcal{F} \subseteq S_n$ is *even-cycle-intersecting* if $\sigma\pi^{-1}$ has an even cycle for all $\sigma, \pi \in \mathcal{F}$. We show that if $\mathcal{F} \subseteq S_n$ is an even-cycle-intersecting family of permutations, then $|\mathcal{F}| \leq 2^{n-1}$, and that equality holds when n is a power of 2 and $\mathcal{F}$ is a double-translate of a Sylow 2-subgroup of S_n. This result can be seen as an analogue of the classical eventown problem for subsets and it confirms a conjecture of János Körner on maximum reversing families of the symmetric group. Along the way, we show that the canonically intersecting families of S_n are also the extremal *odd-cycle-intersecting* families of S_n for all even n. While the latter result has less combinatorial significance, its proof uses an interesting new character-theoretic identity that might be of independent interest in algebraic combinatorics.

Spectral supersaturation for color-critical graphs

Jie Ma
University of Science and Technology of China, Hefei, China
Tsinghua University, Beijing, China
jiema(at)ustc.edu.cn

This is joint work with Longfei Fang, Yongtao Li and Huiqiu Lin

A graph is *color-critical* if it contains an edge whose deletion reduces its chromatic number. This class of graphs, including cliques and odd cycles, plays a central role in extremal graph theory. In this paper, following an influential line of research initiated by Bollobás–Nikiforov, we study the spectral supersaturation problem for color-critical graphs. Let $T_{n,r}$ be the r-partite Turán graph, let $\mathcal{T}_{n,r,q}$ denote the family of graphs obtained from $T_{n,r}$ by adding q edges, and let $\lambda(G)$ be the spectral radius of a graph G. We first prove that for any color-critical graph F with chromatic number $r+1$, there exists $\delta_F > 0$ such that for sufficiently large n and all $1 \leq q \leq \delta_F\sqrt{n}$, any n-vertex graph G with $\lambda(G) \geq \min_{T\in\mathcal{T}_{n,r,q}} \lambda(T)$ contains at least $q \cdot c(n,F)$ copies of F, where $c(n,F)$ denotes the minimum number of copies of F created by adding a single edge to $T_{n,r}$; moreover, any extremal graph G must belong to $\mathcal{T}_{n,r,q}$. Next, we prove a spectral supersaturation result for the analogous condition $\lambda(G) \geq \max_{T\in\mathcal{T}_{n,r,q}} \lambda(T)$, valid for all $1 \leq q \leq \delta_F n$. Together, these results provide a complete resolution to a problem proposed by Ning–Zhai, and establish a spectral counterpart to the well-known results of Mubayi and Pikhurko–Yilma in the extremal supersaturation setting. A notable feature of our first result is that the restriction $q = O(\sqrt{n})$ is tight up to a constant factor, in contrast to the linear bounds provided by other settings discussed above. As applications, we extend a result of Liu–Mubayi, and solve a related conjecture by Li–Lu–Peng. Our proof is based on a novel spectral incremental technique for graphs close to the Turán graph $T_{n,r}$, which may be of independent interest.

Rich nowhere-zero flows

Edita Máčajová
Comenius University, Bratislava, Slovakia
macajova(at)dcs.fmph.uniba.sk

A graph admits a nowhere-zero k-flow if its edges can be oriented and assigned values from the set $\{1, 2, \ldots, k-1\}$ in such a way that, at every vertex, the sum of the incoming values equals the sum of the outgoing values. The concept of a nowhere-zero flow is one of the most important concepts in graph theory and has been studied for more than half a century.

Recently a notion of a *rich* nowhere-zero k-flow, introduced by the speaker, attracted much attention: one where the values at every vertex are pairwise distinct in absolute value. We review known results and open problems of various authors in this area and connections to other problems. Among other things, we show that every bridgeless cubic graph admits a rich nowhere-zero 11-flow and conjecture that 6 in place of 11 will do. Also a linear upper bound for the rich flow number of a graph with bounded maximal degree will be discussed.

Eulerian orientations

Brendan McKay
Australian National University, Canberra, Australia
brendan.mckay(at)anu.edu.au

This is joint work with Mikhail Isaev, Tejas Iyer, and Rui-Ray Zhang

An Eulerian orientation of a graph is an orientation of the edges such that, at each vertex, the number of incoming edges and the number of outgoing edges are equal. This implies that the degree of each vertex is even, which is easily seen to be also a sufficient condition.

This talk concerns the number $\mathrm{EO}(G)$ of Eulerian orientations. The quantity $\mathrm{eo}(G) = \frac{1}{n} \log \mathrm{EO}(G)$ is known as the residual entropy and can also be defined by a limiting process for infinite graphs with a periodic structure. A famous lower bound on $\mathrm{eo}(G)$ was introduced in 1967 by the physicist Elliot Lieb in his study of the behaviour of water ice.

We will consider three issues.

1. We will obtain precise estimates of $\mathrm{EO}(G)$ for graphs that are sufficiently dense and have sufficient expansion. This reveals an unexpected inverse relationship to the number of spanning trees that so far does not have a heuristic combinatorial explanation.

2. We will show that Lieb's bound is low by at most a constant and find an estimate for $\mathrm{eo}(G)$ in the case of regular graphs of low degree that outperforms previous estimates.

3. We will show that under weak conditions on the number of short cycles Lieb's estimate is asymptotically precise for regular graphs of increasing degree.

Geometry of knots and links in the spaces of constant curvature

Alexander Mednykh
Sobolev Instiitute of Mathematics, Novosibirsk State University, Novosibirsk, Russia
smedn(at)mail.ru

We investigate the existence of hyperbolic, spherical or Euclidean structure on cone-manifolds whose underlying space is the three-dimensional sphere and singular set is a given knot or link. We present trigonometrical identities which involve the lengths of singular geodesics and cone angles of such cone-manifolds. Then these identities are used to produce exact integral formulae for the volume of the corresponding cone-manifold modelled in the hyperbolic, spherical and Euclidean geometries.

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The results are published in papers [1–3]:

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Long cycles in vertex transitive digraphs

Bojan Mohar
Simon Fraser University, Canada and University of Ljubljana, Slovenia
mohar(at)sfu.ca

This is joint work with Matija Bucić, Kevin Hendrey, Raphael Steiner, and Liana Yepremyan

One of the most well-known conjectures concerning Hamiltonicity in graphs asserts that any sufficiently large connected vertex transitive graph contains a Hamilton cycle. In this form, it was first written down by Thomassen in 1978, inspired by a closely related conjecture due to Lovász from 1969. It has been attributed to several other authors in a survey on the topic by Witte and Gallian in 1984.

The analogous question for vertex transitive digraphs has an even longer history, having been first considered by Rankin in 1946. It is arguably more natural from the group-theoretic perspective underlying this problem in both settings. Trotter and Erdős proved in 1978 that there are infinitely many connected vertex transitive digraphs which are not Hamiltonian. This left open the very natural question of how long a directed cycle one can guarantee in a connected vertex transitive digraph on n vertices.

In 1981, Alspach asked if the maximum perimeter gap (the gap between the circumference and the order of the digraph) is a growing function in n. We answer this question in the affirmative, showing that it grows at least as fast as $(1 - o(1)) \ln n$. On the other hand, we prove that one can always find a directed cycle of length at least $\Omega(n^{1/3})$, establishing the first lower bound growing with n, providing a directed analogue of a famous result of Babai from 1979 in the undirected setting.

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Equiangular lines, distance-regular covers of complete graphs, and roux schemes

Akihiro Munemasa
China University of Geosciences, Beijing, China
munemasa(at)tohoku.ac.jp

This is joint work with Jesse Lansdown, Alexander Gavrilyuk, Sho Suda

The concept of voltage assignment on graphs has been used to construct regular coverings of graphs. In particular, certain voltage assignments on the set of arcs of a complete graph give rise to antipodal distance-regular covers of complete graphs, as shown by Godsil and Hensel [1]. Klin and Pech [3] regarded voltage assignments on a complete graph on n vertices as an $n \times n$ matrix with entries in the group algebra in 2011, and it was axiomatized using the theory of association schemes by Iverson and Mixon [4]. The terms roux matrix and roux scheme, introduced by Iverson and Mixon, capture not only antipodal distance-regular covers of complete graphs, but also real and complex equiangular lines. In this talk, I will introduce the so-called local construction of roux schemes, and how the remarkable set of 64 equiangular lines discovered by Hoggar [2] can be characterized as an association scheme.

Our main theorem in [5] is as follows.

Theorem 1. *There exists a rank* 8 *roux scheme on* 256 *points representing* 64 *equiangular lines in* $\mathbb{C}^8$ *constructed by Hoggar, and this roux scheme is locally constructed from the rank* 5 *association scheme obtained from the action of* $PSU(3,3)$ *on* 63 *orthogonal bases. Moreover, these two association schemes are uniquely characterized by their parameters.*

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Tiling the sphere with regular polygons

Roman Nedela
University of West Bohemia, Pilsen, Czech republic
nedela(at)kma.zcu.cz

This is joint work with Hoi Ping Luk and Christopher Purcell

We give a complete classification of edge-to-edge tilings of the sphere by regular polygons under a unified framework. Without assuming convexity of the tiles or polyhedrality of the underlying graph, our proof is independent of the Johnson-Zalgaller classification of solids with regular faces (1967) [1, 2], which took over 200 pages. We apply a blend of trigonometric, algebraic and combinatorial tools of independent interest.

Our main theorem is as follows.

Theorem 1. *The edge-to-edge spherical tilings by regular polygons are*

- *the five Platonic tilings,*
- *the thirteen Archimedean tilings,*
- *the twenty-five tilings corresponding to circumscribable Johnson-Zalgaller solids:*

$$J_1, J_2, J_3, J_4, J_5, J_6, J_{11}, J_{19}, J_{27}, J_{34}, J_{37}, J_{62}, J_{63},$$
$$J_{72}, J_{73}, J_{74}, J_{75}, J_{76}, J_{77}, J_{78}, J_{79}, J_{80}, J_{81}, J_{82}, J_{83},$$

- *the infinite families of prisms and antiprisms,*
- *the infinite families of hosohedra and dihedra.*

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Graphs and combinatorial geometry

Andrei M. Raigorodskii
Moscow Institute of Physics and Technology, Moscow, Russia
mraigor(at)yandex.ru

In my talk, I suppose to speak about several problems of combinatorial geometry which can be deeply treated with the help of graph theory. For example, by a graph of diameters of a (finite) set V in a metric space I mean the graph, whose vertex set coincides with V and edges are formed by all pairs of vertices at the maximum distance (diameter) in V. This graph is related to classical Borsuk's problem on partitioning sets into parts of smaller diameter, and here I will reach quite recent and somehow unexpected results in l_p-norms. Another example of geometric graphs is provided by distance graphs, which are closely connected with Nelson–Hadwiger's problem on coloring metric spaces avoiding some distances between monochromatic points. These graphs also have vertex sets lying in metric spaces, but here the edges are given by pairs of vertices at some distance apart (not necessarily the maximum one). Among others, I will speak about Johnson's graphs and their generalizations, once again reaching very recent results in the area.

Non-orientable regular hypermaps of arbitrary hyperbolic type

Jozef Širáň
Slovak University of Technology
siran(at)lux.svf.stuba.sk

This is joint work with Gareth A. Jones and Martin Mačaj

One of the consequences of residual finiteness of triangle groups is that for any given hyperbolic triple (ℓ, m, n) there exist infinitely many regular hypermaps of type (ℓ, m, n) on compact orientable surfaces. The same conclusion also follows from a classification of those finite quotients of hyperbolic triangle groups that are isomorphic to linear fractional groups over finite fields. A non-orientable analogue of this, that is, existence of regular hypermaps of a given hyperbolic type on *non-orientable* compact surfaces, appears to have been proved only for *maps*, which arise when one of the parameters ℓ, m, n is equal to 2.

In this paper we establish a non-orientable version of the above statement in full generality by proving the following much stronger assertion: for every hyperbolic triple (ℓ, m, n) there exists an infinite set of primes p of positive Dirichlet density, such that (i) there exists a regular hypermap $\mathcal{H}$ of type (ℓ, m, n) on a compact non-orientable surface such that the automorphism group of $\mathcal{H}$ is isomorphic to $\mathrm{PSL}(2, p)$, and, moreover, (ii) the carrier compact surface of *every* regular hypermap of type (ℓ, m, n) with rotation group isomorphic to $\mathrm{PSL}(2, p)$ is necessarily non-orientable.

In this talk I will report on recent progress in distance-regular graphs.

Bounding Hamilton cycles in polytopal maps

Martin Škoviera
Comenius University, Bratislava, Slovakia
skoviera(at)dcs.fmph.uniba.sk

This is joint work with Roman Nedela

In this talk we develop the idea that embedding a cubic graph into a closed surface can serve as a convenient tool for finding a Hamilton cycle in it. We establish a necessary and sufficient condition for a cubic graph embedded in a closed surface, orientable or not, to have a bounding Hamilton cycle. With this characterisation and its consequences we can guarantee Hamilton cycles in wide classes of cubic graphs. Among others, we provide a unified and relatively short proof of a result due to Glover, Marušič, Kutnar, and others proved in a series of four papers [1–4] published over the years 1996-2012 that cubic Cayley graphs of finite quotients of the modular group have a Hamilton path and, except in one special case, they also have a Hamilton cycle. We also show that in the remaining case these Cayley graphs have no bounding Hamilton cycle, which indicates that identifying a Hamilton cycle in this case is likely to be very difficult.

Acknowledgments. The work has been supported by the grants APVV-23-0076 of the Slovak Research and Development Agency and by VEGA 1/0613/26.

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Graphs, symmetries and orderings in dynamical systems

Petr Stehlík
University of West Bohemia, Pilsen, Czech Republic
pstehlik(at)kma.zcu.cz

This is joint work with M. Jukić, L. Morelli, H. J. Hupkes (Leiden),
Vladimír Švígler, and Jonáš Volek (Pilsen)

Modern dynamical systems provide many connections to discrete structures and combinatorics. In this talk, we will outline why these relationships are natural both from the mathematical as well as from the applied point of view. We focus on two perspectives. First, we discuss the role of graph characteristics on the properties of specific discrete-space dynamical systems. Next, we investigate the intricate symmetries and orderings of solutions of complex and chaotic dynamical systems. The interplay between traditionally separate mathematical fields implies that both areas offer large number of open problems at basic level of understanding. The talk is intended for audience without detailed knowledge of advanced and complex dynamical systems.

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Hilbert's 18th problem and undecidability of translational tiling

Chao Yang
Guangdong University of Foreign Studies, Guangzhou, China
yangchao(at)gdufs.edu.cn

Hilbert's 18th problem shows that, at the dawn of the twentieth century, researchers had not yet grasped the possibility of aperiodicity and undecidability in the plane-tiling problem. The first breakthroughs in these directions came with Hao Wang's 1961 formulation of the domino problem. In this talk, I will survey the ensuing history of undecidability in translational tiling, including some recent results of our own.

Extensions of the Carlitz–McConnel and Blokhuis–Sziklai theorems

Chi Hoi Yip
Georgia Institute of Technology, Atlanta, United States
cyip30(at)gatech.edu

This is joint work with Shamil Asgarli and Maosheng Xiong.

In this talk, I will discuss two beautiful theorems, one due to Carlitz–McConnel and the other due to Blokhuis–Sziklai. I will discuss their rich connections with algebraic graph theory, arithmetic combinatorics, finite geometry, group theory, and number theory. I will also some highlight some recent extensions of these theorems.

Maximum two-distance sets in Hamming, Johnson and Euclidean space

Wei-Hsuan Yu
National Central University, Taoyuan, Taiwan
whyu(at)math.ncu.edu.tw

This is joint work with Barg, Glazyrin, Kao, Lai, Tseng

We address the maximum size of binary codes and binary constant weight codes with few distances. Previous works established a number of bounds for these quantities as well as the exact values for a range of small code lengths. As our main results, we determine the exact size of maximal binary codes with two distances for all lengths. We also determine the maximum size of two-distance sets in R^n for n between 9 to 14.

Theorem 1. *The maximum size of two-distance sets in binary Hamming space is* $1 + \frac{n(n-1)}{2}$ *for all codewords length n at least 6.*

On the structures of tiling sets in finite abelian groups

Tao Zhang
Xidian University, Xi'an, China
zhangtao03(at)xidian.edu.cn

The Fuglede conjecture establishes the relationship between spectral and tilings. Spectral is one of the core concepts in harmonic analysis. Since tilings are relatively easy to verify while spectral are more difficult, researchers are particularly interested in what kind of tiling sets are spectral sets (T-S for short). We mainly focus on the structure of tiling sets in finite abelian groups. Many researchers have considered tiling sets with special structures in finite abelian groups, such as groups with Hajós property, groups with Redei property, and quasi-periodic groups. However, there are only a few groups with Hajós property and groups with Redei property. Additionally, there are groups with quasi-periodic properties that do not have T-S, and there are groups with T-S that do not have quasi-periodic properties. Therefore, it is necessary to propose a more suitable property that is easy to verify, has a good structure, and satisfies T-S.

In this talk, we introduce the periodic tiling (PT) property for finite abelian groups. A finite abelian group is said to have the PT property if every non-periodic set that tiles the group by translation admits a periodic tiling complement. This notion extends the scope beyond groups with the Hajós property. We give a complete classification of cyclic groups possessing the PT property and identify certain non-cyclic groups that enjoy the PT property but fail to satisfy the Hajós property. As a byproduct, we obtain new families of groups for which the implication "Tile $\Longrightarrow$ Spectral" holds. Furthermore, for elementary p-groups with the PT property, by analyzing the structure of tiles, we prove that every tile is a complete set of representatives of the cosets of some subgroup.

The finite k-set homogeneous graphs

Jin-Xin Zhou
Beijing Jiaotong University, Beijing, China
jxzhou(at)bjtu.edu.cn

This is joint work with Cai Heng Li and Fu-Gang Yin

A classification is given of finite k-set-homogeneous graphs for $k \geqslant 2$, leading to a striking result that each finite k-set-homogeneous graph is k-homogeneous. It shows that 3-set-homogeneous graphs are rare, consisting of the following graphs and their complements: C_5, $K_n \Box K_n$, nK_m, the Schläfli graph of order 27, the Higman-Sims graph, the MaLaughlin graph, affine polar graphs, and elliptic orthogonal graphs. As an ingredient for the proof, it is shown that all orbitals in a primitive permutation group of rank 4 are self-paired, except for $PSU_3(3)$ acting on 36 points.

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Combinatorics, geometry, and topology of Bier spheres

Rade Živaljević
Mathematical Institute SASA, Belgrade, Serbia
rade(at)turing.mi.sanu.ac.rs

This is joint work with Filip Jevtić, Marija Jelić Milutinović, Ivan Limonchenko, and Marinko Timotijević

Each simplicial complex K (or a simple game $2^{[n]} \setminus K$) with n vertices (n players) is associated an $(n-2)$-dimensional, combinatorial sphere on (at most) $2n$-vertices. This is the so called Bier sphere $Bier(K)$ (named after Thomas Bier [1]), formally defined as the deleted join $Bier(K) := K *_\delta K^\circ$ of K with its (combinatorial) Alexander dual K°. Bier spheres have been studied from the viewpoint of combinatorics (simplicial complexes), topology (polyhedral products, toric manifolds), convex polytopes (generalized permutahedra, algorithmic Steinitz problem), game theory (cooperative games), experimental mathematics (nonpolytopal spheres), combinatorial optimization (submodular functions), etc.

We present an overview of selected results from the multidisciplinary project *Bier-2016-2026*, initiated at ICERM in (2016), `https://icerm.brown.edu/program/semester_program/sp-f16`. One of the cornerstone results is the following theorem, which links Bier spheres with *braid arrangements, polymatroids, and generalized permutahedra.* This result and its ramifications, [6,7,8,9,10], pave the way for many applications, from polyhedral combinatorics to toric geometry and topology.

Theorem 1. ([6,7,10]) *Each Bier sphere admits a canonical star-shaped geometric realization whose radial fan coarsens the braid arrangement fan. Moreover, this fan is unimodular in the lattice A_n^*, dual to the root lattice A_n.*

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Contributed talks

Geometric structures on knot complements: construction of fundamental polyhedra

Nikolay Abrosimov
Sobolev Institute of Mathematics, Novosibirsk, Russia
abrosimov(at)math.nsc.ru

The talk is based on our joint work with L. Grunwald, A. Mednykh, A. Qutbaev and B. Vuong

We present a general approach how to construct a canonical fundamental polyhedron for a cone-manifold over some knot in 3-sphere. Then studying the relations of this polyhedron we can establish necessary and sufficient conditions for the existence of such manifolds in both Euclidean and hyperbolic geometries, and derive explicit volume formulas in each case.

Consider cone-manifolds $3_1(\alpha, \gamma)$ whose underlying space is 3-dimensional sphere and whose singular set is the trefoil knot with a bridge with conical angles α and γ along the knot and the bridge respectively. By Wirtinger algorithm we get a presentation of the fundamental group of trefoil knot. Then we find a martix representation of the holonomy group of $3_1(\alpha, \gamma)$ as the group, generated by two rotations by angle α with respect to singular component of the corresponding fundamental set in the Euclidean and in the hyperbolic space. This representation allows us to construct explicitly a canonical fundamental polyhedron for $3_1(\alpha, \gamma)$ in $\mathbb{E}^3$ and $\mathbb{H}^3$. Then we obtain relations between the dihedral angles and the edge lengths of the fundamental polyhedron, that is the relations between the conical angles and the lengths of singular geodesics of $3_1(\alpha, \gamma)$. We establish necessary and sufficient conditions for the existence of $3_1(\alpha, \gamma)$ in $\mathbb{E}^3$ and $\mathbb{H}^3$, and derive explicit volume formulas in each case. Namely, in the hyperbolic case we obtain the following two theorems (see [1]).

Theorem 1. *Let $\frac{\pi}{3} < \alpha < \pi$ and let γ satisfy the following equation*

$$\cos^2 \frac{\gamma}{2} = \frac{(C^3 - A\,B)^2}{C^6},$$

where $A = (4\,\mathcal{X}^2 + 3\,\mathcal{X} + 1)\,\mathcal{Y} - 3\,\mathcal{X}^2 - \mathcal{X}$, $B = (4\,\mathcal{X} + 3)\,\mathcal{Y}^2 + (-3\,\mathcal{X} - 1)\,\mathcal{Y} + \mathcal{X}$, $C = \mathcal{Y} - \mathcal{X} + 2\,\mathcal{X}\,\mathcal{Y}$, $\mathcal{X} = \cosh a$, $\mathcal{Y} = \cos\theta$. *Then the cone-manifold $3_1(\alpha, \gamma)$ admits a hyperbolic structure.*

Moreover, there exists a 12-faced polyhedron in $\mathbb{H}^3$ which is the canonical fundamental set of the cone-manifold $3_1(\alpha, \gamma)$.

Theorem 2. *Let $\mathcal{X} = \cosh a$ and $\mathcal{Y} = \cos\theta$. Then the hyperbolic volume of the cone-manifold $3_1(\alpha, \gamma)$ can be calculated by the formula*

$$Vol(3_1(\alpha, \gamma)) = -\frac{1}{2}\int_1^{\mathcal{X}_0} L_\gamma(\mathcal{X})d(\gamma(\mathcal{X})), \quad \textit{where}$$

$$\cosh L_\gamma = \frac{(\mathcal{X}+1)\mathcal{Y}^2 + (-\mathcal{X}^2 + \mathcal{X} + 1)\mathcal{Y} + \mathcal{X}^2 - \mathcal{X} - 1}{(\mathcal{X}+1)\mathcal{Y}^2 + \mathcal{X}^2\mathcal{Y} - \mathcal{X}^2},$$

and $\mathcal{X}_0$ is the only solution of the system of equations

$$\begin{cases} \dfrac{\sinh^2 L_\gamma}{\tan^2 \frac{\gamma}{4}} \dfrac{\operatorname{csch}^2 L_\alpha}{\tan^2 \frac{\alpha}{2}} = \dfrac{(\mathcal{X} - \mathcal{Y})^2}{4}, \\ \cot^2 \dfrac{\alpha}{2} = (2\mathcal{X}+1)(2\mathcal{Y}-1). \end{cases}$$

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Falsifying some conjectures in organic chemistry on the structure of perfect matchings in subgraphs of the honeycomb lattice

Robert D. Barish
University of Tokyo, Japan
rdbarish(at)gmail.com

This is joint work with Tetsuo Shibuya

In mathematical chemistry, we abstract the *carbon skeleton* of a molecular structure as an undirected graph, where vertices represent carbon atoms, edges represent single carbon-carbon bonds, and we ignore the presence of other atom types (e.g., hydrogen). Here, a *polyhex* is a connected carbon skeleton that is isomorphic to an induced subgraph of the honeycomb lattice with minimum degree 2. If a polyhex has at most one face (respectively, more than one face) of length ≥ 7, corresponding to the outer boundary (respectively, outer boundary or a hole), then we may refer to it as a *benzenoid* (respectively, a *coronoid*). Letting G be a polyhex, we refer to a perfect matching for G as a *Kekulé structure*, and for a particular Kekulé structure $\mathcal{M}$, we refer to any 6-sided face in G with 3 edges in $\mathcal{M}$ as a *resonant benzene face.* We can now define the *Fries number* $F(G)$ [5] for G as the maximum number of resonant benzene faces that can be realized by any Kekulé structure. Additionally, we can define the *Clar number* $C(G)$ [2] for G as the maximum number of mutually vertex-disjoint resonant benzene faces that can be realized by any Kekulé structure. Finally, we can define the *Fries set* $\mathcal{F}$ (respectively, *Clar set* $\mathcal{C}$) for G as the set of all Kekulé structures inducing $F(G)$ resonant benzene faces (respectively, $C(G)$ mutually vertex-disjoint resonant benzene faces).

In this talk we elaborate on coronoid counterexamples we have discovered for the well-known *Clar–Fries conjecture* [1, 3, 4] for polyhexes (or bipartite planar carbon skeletons more generally [1]) that at least one Clar set is a subset of a Fries set. It was previously only known that the Clar–Fries conjecture was false for fullerenes (i.e., necessarily non-bipartite 3-regular planar graphs with pentagonal and hexagonal faces), and true for special classes of fullerenes and benzenoids.

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On signed graphs with fixed smallest eigenvalue

Meng-Yue Cao
University of Science and Technology of China, HeFei, China
caomengyue(at)ustc.edu.cn

This is joint work with Jack H. Koolen, Jing-Yuan Liu and Qianqian Yang

Let G be a graph with smallest eigenvalue $\lambda_{\min}(G)$. In 1973, Hoffman [1] showed that: (i) for any real number $\lambda \leq -1$, if $\lambda_{\min}(G) \geq \lambda$, then there exists a positive integer $t = t_\lambda$, such that G is $\{K_{1,t}, \widetilde{K_{2t}}\}$-free; (ii) for any integer t, if G is $\{K_{1,t}, \widetilde{K_{2t}}\}$-free, then there exists a positive integer $\lambda = \lambda_t$, such that $\lambda_{\min}(G) \geq \lambda$. In 2016, Kim, Koolen and Yang [2] gave a structure theory for graphs with fixed smallest eigenvalue.

In this talk, I will present a generalization of these results to signed graphs and conclude by discussing an application to signed graphs with smallest eigenvalue greater than $-1-\sqrt{2}$ [3].

Our results are as follows.

Theorem 1. *Let (G,σ) be a signed graph with smallest eigenvalue $\lambda_{\min}(G,\sigma)$. The following hold.*

1. *For any real number $\lambda \leq -1$, there exists a positive integer $t = t(\lambda)$, such that if $\lambda_{\min}(G,\sigma) \geq \lambda$, then (G,σ) is $\{\widetilde{K}_{2t}^{(0)}, \widetilde{K}_{2t}^{(-)}, (K_{t+1},-), (K_{1,t},+)\}$-switching-free.*
2. *For any positive integer t, there exists a non-positive real number $\lambda = \lambda(t)$, such that if (G,σ) is $\{\widetilde{K}_{2t}^{(0)}, \widetilde{K}_{2t}^{(-)}, (K_{t+1},-), (K_{1,t},+)\}$-switching-free, then $\lambda_{\min}(G,\sigma) \geq \lambda$.*

Theorem 2. *Let $\lambda \leq -1$ be a real number. There exists a positive integer d_λ such that if (G,σ) is a signed graph with smallest eigenvalue at least λ and minimum valency at least d_λ, then there exists a set of induced subgraphs $N_1, N_2, \ldots, N_r$ of (G,σ), where r is a positive integer, satisfying the following conditions.*

1. *Each vertex of (G,σ) lies in at least one and at most $\lfloor -\lambda \rfloor$ N_i's.*
2. *The induced subgraph N_i is switching equivalent to a signed graph whose positive graph is a $(\lfloor \lambda^2 + 2\lambda + 2 \rfloor)$-plex, for $i = 1, 2, \ldots, r$.*
3. *The intersection $V(N_i) \cap V(N_j)$ contains at most $4\lfloor -\lambda \rfloor - 4$ vertices for $1 \leq i < j \leq r$.*
4. *the subgraph (G', σ') has maximum valency at most $d_\lambda - 1$, where $G' = (V(G), E(G) \setminus \bigcup_{i=1}^{r} E(N_i))$ and $\sigma' = \sigma|_{E(G')}$.*

Let s be a positive integer. A signed graph (G,σ) with smallest eigenvalue $\lambda_{\min}$ is *s-integrable*, if there exists an integer-valued matrix N such that $s(A + \lceil -\lambda_{\min} \rceil \mathbf{I}) = N^T N$, where A is the adjacency matrix of G. Note that (G,σ) is s-integrable if and only if the integral lattice generated by the columns of $\frac{1}{\sqrt{s}}N$ is s-integrable.

Theorem 3. *Let λ be a real number in $(-1-\sqrt{2}, -1]$. There exists a positive integer d'_λ such that if a connected signed graph (G,σ) has smallest eigenvalue $\lambda_{\min}(G,\sigma) \geq \lambda$ and minimum valency at least d'_λ, then $\lambda_{\min}(G,\sigma) \geq -2$ and (G,σ) is 1-integrable.*

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Isomorphic factorizations of the complete graphs into Cayley graphs on CI-groups

Huye Chen
Guangxi University Nanning, China
chenhy280(at)gxu.edu.cn

This is a joint work with Jingjian Li, Hao Yu and Zitong Yu

Isomorphic factorizations of complete graphs originate from the seminal work of Frank Harary and collaborators spanning [1] to [2], who initiated the systematic study of decompositions of complete graphs into pairwise isomorphic spanning subgraphs. In this paper, we investigate isomorphic factorizations of complete graphs into Cayley graphs on CI-groups.

Let $\Gamma = \mathrm{Cay}(G, S)$ denote the Cayley graph of finite group G. We obtain a necessary and sufficient condition on CI-group G so that the complete graph on $|G|$ vertices can be edge-partitioned into k-copies of Cayley graph of the same CI-group G each isomorphic to $\mathrm{Cay}(G, S)$ for some inverse-closed subset $S \subset G \setminus \{1\}$. Further we give a construction of isomorphic factorizations of the complete graphs into Cayley graphs on CI-group.

Let Γ be a graph, and let $\mathcal{P} := \{P_0, P_1, \cdots, P_{k-1}\}$ be a partition of edge set $E(\Gamma)$ with $k \geq 2$. Then $(\Gamma, \mathcal{P})$ is called a *decomposition* of the graph Γ. Moreover, if the subgraph induced by each of the set P_i are all spanning subgraph of Γ, then the decomposition $(\Gamma, \mathcal{P})$ is called a *factorization* of the graph Γ, and each spanning subgraph Γ_i is called a *factor* of Γ, where $0 \leq i \leq k-1$. A factorization $(\Gamma, \mathcal{P})$ is termed *isomorphic factorization* if all its factors are pairwise isomorphic. For such a factorization, we denote the common factor (up to isomorphism) by Γ/k. In the specific case of a complete graph K_n, each factor K_n/k in an isomorphic factorization is called a k-if graph (where 'if' abbreviates isomorphic factorization). A finite group G is said to have *k-if property* if there exists a Cayley graph $X = \mathrm{Cay}(G, S)$ of G is a k-if Cayley graph.

Further, we get the following theorem.

Theorem 1. *Let G be a finite CI-group. Then G has k-if property if and only if G is the direct product of elementary abelian group, and the order of each Sylow subgroup G_p of G satisfy $2k \mid |G_p| - 1$ when p is odd; $k \mid |G_p| - 1$ when $p = 2$.*

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On some properties of Archimedean tiling graphs

Julia Q. D. Du
Hebei Normal University, Shijiazhuang, China
qddu(at)hebtu.edu.cn

This is joint work with Liping Yuan

A plane tiling $\mathcal{T}$ is a countable family of closed sets with non-empty interiors $\{T_1, T_2, \cdots\}$, which cover the plane without gaps or overlaps. Every closed set $T_i \in \mathcal{T}$ is called a tile of $\mathcal{T}$. We consider the special case in which each tile is a polygon. If the corners and sides of a polygon coincide with the vertices and edges of the tiling, we call the tiling edge-to-edge. A so-called type describes the neighbourhood of any vertex of the tiling. We consider plane edge-to-edge tilings in which all tiles are regular polygons, and all vertices are of the same type. There exist exactly eleven such tilings, which are called Archimedean tilings. A graph formed by an Archimedean tiling is called an Archimedean tiling graph.

Pick's theorem gives the area of a simple lattice polygon P using the number of lattice points in the interior and boundary of P. We generalize Pick's theorem to some lattice polygons in each of Archimedean tilings. For the (3.3.4.3.4) tiling, we obtain that if the boundary of a lattice polygon P is made up of the lattice segments which are parallel to the edges of the tiling, or of the symmetrical lattice segments, then the area of P is given by

$$A(P) = \frac{1}{8}[(2+\sqrt{3})b + (4+2\sqrt{3})i + (2-\sqrt{3})c + 8\sqrt{3} - 24],$$

where b is the number of lattice points on its boundary and i is the number of lattice points in its interior, c is the boundary characteristic of P. Moreover, we obtain a unified Pick-type formula of some special lattice polygons for all of the 11 Archimedean tilings.

For a graph $G = (V, E)$, a labeling $\partial \colon V \cup E \to \{1, 2, \dots, k\}$ is called an edge irregular total k-labeling of G if the weights of any two different edges are distinct, where the weight of the edge xy under ∂ is defined to be $wt(xy) = \partial(x) + \partial(xy) + \partial(y)$. The total edge irregularity strength $\text{tes}(G)$ of G is the minimum k for which G has an edge irregular total k-labeling. In this talk, we will determine the exact value of the total edge irregularity strength for the hexagonal grid graph H_n^m by giving an edge irregular total $\left\lceil \frac{3mn+2(m+n)+1}{3} \right\rceil$-labeling.

Orderly generation of highly symmetrical discrete objects

Rhys J. Evans
University of Sydney, Australia
rhysjohn.evans(at)sydney.edu.au

This is based on joint work with Primož Potočnik and Kolja Knauer

Collections of small mathematical objects play an important role in forming and testing conjectures, and often provide counterexamples to open problems. Their enumeration is usually computationally hard, but additional structure and symmetry can make isomorph-free exhaustive generation feasible. Many highly symmetrical discrete objects, including regular maps, maniplexes and Cayley graphs, can be described in terms of a finite group together with a generating set satisfying certain properties.

In this talk, I describe an orderly generation algorithm for enumerating minimal generating sets of a given group. Simple group-theoretical observations lead to significant improvements over the basic algorithm, extending previous enumerations to groups of much larger order. This approach has been used to generate a complete list of minimal Cayley graphs on up to 511 vertices.

I also discuss applications to other highly symmetrical discrete objects, as well as the databases and software we are developing to make these collections available to a wider audience.

The Turán number of odd prism

Xiaocong He
Hebei Normal University, Shijiazhuang, China
hexc2018(at)qq.com

This is joint work with Lihua feng and Yongtao Li

The Turán number of a graph H, $ex(n, H)$, is the maximum number of edges in a graph on n vertices which does not have H as a subgraph. The odd prism $C_{2k+1}\square K_2$ is the cartesian product of C_{2k+1} and K_2. The powerful theorem of Erdős, Stone and Simonovits determines the asymptotic behavior of $ex(n, C_{2k+1}\square K_2)$. In the present paper, we determine the exact value of $ex(n, C_3\square K_2)$. Applying a deep theorem of Simonovits, we characterize the extremal graphs of $ex(n, C_{2k+1}\square K_2)$ for sufficiently large n.

Theorem 1. *The maximum number of edges in an n-vertex $C_3\square K_2$-free graph ($n \neq 5$) is as follows:*

$$ex(n, C_3\square K_2) = \begin{cases} \left\lfloor \frac{n^2}{4} \right\rfloor + \left\lfloor \frac{n-1}{2} \right\rfloor, & n \equiv 1, 2, 3 \pmod 6, \\ \left\lfloor \frac{n^2}{4} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil, & \textit{otherwise}. \end{cases}$$

Theorem 2. *Let n be sufficiently large. Then we have:*

$$\mathrm{ex}(n, C_{2k+1}\square K_2) = \max\left\{ n_a(1+n_b) + \frac{1}{2}(j^2 - 3j) : n_a + n_b = n, n_a \equiv j \pmod 3 \right\}.$$

Moreover, all extremal graphs are of the form of a complete bipartite graph with an extremal graph for P_4 added to one of the parts.

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Cyclic complementary extensions of groups and skew morphisms

Kan Hu
Zhejiang Ocean University, Zhoushan, China
hukan(at)zjou.edu.cn

Based on joint works with Robert Jajcay, Istvan Kovacs and Young Soo Kwon

A finite group G is said to be a cyclic complementary extension of a subgroup A if G has a cyclic subgroup C such that $G = AC$ and $A \cap C = 1$. A chosen generator c of C determines a permutation φ on A and an integer-valued function $\Pi : A \to \mathbb{Z}_{|c|}$ via the identity $cx = \varphi(x)c^{\Pi(x)}$ $(x \in A)$ with the following properties:

(a) $\varphi(1_A) = 1_A$ and $\varphi(xy) = \varphi(x)\varphi^{\Pi(x)}(y)$ for all $x, y \in A$.

(b) $\Pi(1_A) = 1$ and $\Pi(xy) = \sum_{i=1}^{\Pi(x)} \Pi(\varphi^{i-1}(y))$ for all $x, y \in A$.

The permutation φ and the function Π are known as a skew morphism of A and an extended power function of φ, respectively. It turns out that every cyclic complementary extension of a given finite group A can be characterized by the corresponding skew morphism of A and the associated extended power function, which allows us to investigate certain combinatorial objects such as regular Cayley maps as well as generalized regular Cayley maps.

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A generalization of Ramanujan's sum over finite groups

Monu Kadyan
Shenzhen Institutes of Advanced Technology, Chinese Academy of Sciences, Shenzhen, China
monu4math(at)gmail.com

This is joint work with Priya Dhankhar and Sanjay Kumar Singh

Let G be a finite group, and let $x \in G$. Define $[x^G] := \{y \in G : \langle x^G \rangle = \langle y^G \rangle\}$, where $\langle x^G \rangle$ denotes the normal subgroup of G generated by the conjugacy class of x. In this paper, we determine an explicit formula for the eigenvalues of the normal Cayley graph $\mathrm{Cay}(G, [x^G])$. These eigenvalues can be viewed as a generalization of classical Ramanujan sums in the setting of finite groups. Surprisingly, the formula we derive for the eigenvalues of $\mathrm{Cay}(G, [x^G])$ extends the known formula of classical Ramanujan sums to the context of finite groups. This generalization not only enriches the theory of Ramanujan sums but also provides new tools in spectral graph theory, representation theory, and algebraic number theory.

Several applications of skew-morphisms of groups

Young Soo Kwon
Yeungnam University, Kyungsan, South Korea
ysookwonl(at)ynu.ac.kr

A skew morphism of a finite group G is a permutation φ of G fixing the identity element and for which there is an integer-valued function π on G such that $\varphi(gh) = \varphi(g)\varphi^{\pi(g)}(h)$ for all $g, h \in G$. For any complementary product $\Gamma = GC$ of a group G and a cyclic group C, one can get a skew-morphism of G by a commutting rule. So skew-morphisms of a group G can be applied to classification of a complementary product of a group and a cyclic group, classifications of regular Cayley maps on a group G, classifications of regular Cayley hypermaps on a group G and so on. In this talk, we will consider the classification of regular Cayley hypermaps on dihedral groups and other applications of skew-morphisms.

In 2018, by using Ramsey and Hoffman theory, Koolen et al. gave a structural result on graphs with smallest eigenvalue at least -3 and large minimum degree. Recently, without using Ramsey theory, we combine Bose-Laskar type argument and Hoffman theory to show structural results about μ-bounded graph with smallest eigenvalue at least -3. In my talk, I will introduce these results and give some applications to distance-regular graphs with classical parameter (D, b, α, β).

The Dunkl–Watanabe Duality

Xiaoye Liang
Anhui Jianzhu University, Hefei, China
liangxy0105(at)ahjzu.edu.cn

This is joint work with Tatsuro Ito and Ying-Ying Tan

Let $\widetilde{X}$ denote the set of all subspaces of the N-dimensional vector space $\mathbb{F}_q^N$ over the finite field $\mathbb{F}_q$ of q elements. The general linear group $\mathrm{GL}(N,q)$ acts on $\widetilde{X}$ naturally. Fix a D-dimensional subspace x_0 from $\widetilde{X}$ and let H be the stabilizer in $\mathrm{GL}(N,q)$ of x_0.

Let $M_{\widetilde{X}}(\mathbb{C})$ denote the full matrix algebra over the complex number field $\mathbb{C}$ whose rows and columns are indexed by elements of $\widetilde{X}$. Let $M^H_{\widetilde{X}}(\mathbb{C})$ denote the centralizer algebra of H, i.e., the set of elements of $M_{\widetilde{X}}(\mathbb{C})$ that commute with the action of H on $\widetilde{X}$.

In this talk, we show $M^H_{\widetilde{X}}(\mathbb{C}) = \mathcal{H}$, where $\mathcal{H}$ is the algebra introduced by Watanabe in [3]. We call the algebra $\mathcal{H}$ the Watanabe algebra, and the relation $M^H_{\widetilde{X}}(\mathbb{C}) = \mathcal{H}$ the Dunkl-Watanabe duality, because it bridges the papers [1], [3].

The Watanabe algebra is defined as follows. For $0 \le i \le D,\ 0 \le j \le N-D$, set

$$X_{i,j} := \{x \in \widetilde{X} \mid \dim(x) = i+j,\ \dim(x \cap x_0) = i\}.$$

Then we have a disjoint sum $\widetilde{X} = \bigcup X_{i,j}$ over $0 \le i \le D$, $0 \le j \le N-D$. Let $\widetilde{V} := \mathbb{C}\widetilde{X}$ denote the vector space over $\mathbb{C}$ with $\widetilde{X}$ an orthonormal basis. Then we have a direct sum $\widetilde{V} = \bigoplus V_{i,j}$ over the same ranges of i and j, where $V_{i,j}$ is the subspace of $\widetilde{V}$ spanned by $X_{i,j}$. The orthogonal projection from $\widetilde{V}$ to $V_{i,j}$ is denoted by $E^*_{i,j}$. By identifying the endomorphism algebra $\mathrm{End}(\widetilde{V})$ with $M_{\widetilde{X}}(\mathbb{C})$, we understand $E^*_{i,j}$ is a diagonal matrix of $M_{\widetilde{X}}(\mathbb{C})$.

Define matrices $L_1, L_2, R_1, R_2 \in M_{\widetilde{X}}(\mathbb{C})$ by

$$L_1(x,y) := \begin{cases} 1 & \text{if } x \in X_{i-1,j},\ y \in X_{i,j},\ x \subset y, \\ 0 & \text{otherwise}, \end{cases}$$

$$L_2(x,y) := \begin{cases} 1 & \text{if } x \in X_{i,j-1},\ y \in X_{i,j},\ x \subset y, \\ 0 & \text{otherwise}, \end{cases}$$

and $R_1 = L_1^t$, $R_2 = L_2^t$, where t denotes the transpose. The Watanabe algebra $\mathcal{H}$ is the subalgebra of $M_{\widetilde{X}}(\mathbb{C})$ generated by L_1, L_2, R_1, R_2 together with all $E^*_{i,j}$ $(0 \le i \le D, 0 \le j \le N-D)$.

In [2], irreducible representations of the Terwilliger algebra $T = T(x_0)$ of the q-Johnson scheme $J_q(N,D)$ are determined up to isomorphism using [3]. We plan to use the Dunkl-Watanabe duality as a structural tool to find how far T is from the centralizer algebra for the stabilizer of x_0 in the full automorphism group of $J_q(N,D)$, measured in terms of the coherent length in the framework of the Weisfeiler-Leman stabilization.

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Recent progress in the classification of primitive flag-transitive generalized quadrangles

Jianbing Lu
National University of Defense Technology, Changsha, China
jianbinglu(at)nudt.edu.cn

Finite generalized quadrangles (GQs) are point-line incidence geometries whose bipartite incidence graphs have diameter 4 and girth 8. Classical examples arise from polar spaces of rank 2. Their automorphism groups are closely related to finite simple groups of Lie type. Specifically, for these classical GQs, the full automorphism group is almost simple, with simple socle a classical group of Lie type, and it acts primitively on both points and lines, and transitively on flags.

Bamberg et al. (2012) proved that if a finite thick GQ admits an automorphism group acting primitively on both points and lines, then the group must be almost simple. Their work handled the cases where the socle is a sporadic or an alternating simple group. By the Classification of Finite Simple Groups, the only remaining possibility is that the socle is a simple group of Lie type.

In this talk, we introduce new methods to classify the low-dimensional Lie type socles for point-primitive and line-primitive actions. We completely determine the possibilities when the socle is $\mathrm{PSL}_2(q)$(which forces $q = 9$ and the GQ to be $W(3,2)$), $\mathrm{PSU}_3(q)$ (no such GQ exists), and $\mathrm{PSp}_4(q)$ (the GQ is the classical symplectic quadrangle $W(3,q)$). We then additionally assume flag-transitivity while retaining point-primitivity and line-primitivity, and consider general classical socles. Using Aschbacher's classification of maximal subgroups of classical groups, we prove that under these hypotheses the only possibilities are the classical generalized quadrangles.

Is the rank function of a delta-matroid the rank function of a matroid?

Hoi Ping Luk
Západočeská univerzita v Plzni, Czech Republic
hoi@connect.ust.hk

This is joint work with Relinde Jurrius, Dmitry Mineev and Lauren Nowak

In this talk, we report a result on delta-matroids in the context of enveloping matroids. Matroid theory was independently established by Hassler Whitney and Takeo Nakasawa. The motivation was to provide an abstraction of "independence" which is common to both graphs and matrices. Delta-matroids are a generalisation of matroids. Delta-matroids were introduced by André Bouchet to study combinatorial objects for which the greedy algorithm returns an optimal solution. They were later applied to study many other problems, including Eulerian tours in graphs and graph embeddings. A recent development has shed new light on their connection in geometry. Through the lens of covering of set systems or polytopes, it is natural to investigate whether a delta-matroid admits an enveloping matroid. However, given a delta-matroid it is not unknown whether it admits an enveloping matroid. In the literature, there was only one counterexample hidden in the context of multimatroid. We obtain an infinite family of delta-matroids which do not admit an enveloping matroid.

Characterizing the set chromatic number of the zero-divisor graph of a finite ring

Marie Cris B. Mahinay
University of the Philippines Cebu, Cebu City, Philippines
mabulay-og(at)up.edu.ph

This is joint work with Maria Czarina T. Lagura-Arreza and Reginaldo M. Marcelo

Let $c : V(G) \to \mathbb{N}$ be a vertex coloring of a simple nontrivial connected graph G, where adjacent vertices may be assigned the same color. For each vertex $v \in V(G)$, the *neighborhood color set* of v, denoted by $NC(v)$, is defined as the set of colors assigned to the neighbors of v. A coloring c is called a *set coloring* if for every pair of adjacent vertices $u, v \in V(G)$, the neighborhood color sets of u and v are distinct, that is, $NC(u) \neq NC(v)$. The *set chromatic number* of G, denoted by $\chi_s(G)$, is the minimum number of colors required in a set coloring of G. In this paper, we focus on the set coloring problem for the zero-divisor graph of the ring of integers modulo n, denoted by $\Gamma(\mathbb{Z}_n)$. The vertices of this graph correspond to the nonzero zero-divisors of $\mathbb{Z}_n$, and there is an edge between two vertices a and b if and only if $ab \equiv 0 \bmod n$. Our objective is to determine the set chromatic number $\chi_s(\Gamma(\mathbb{Z}_n))$ for all values of $n \in \mathbb{N}$.

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Laplace polynomials for the Moebius strip and Klein bottle

Ilya Mednykh
Sobolev Instiitute of Mathematics, Novosibirsk State University, Novosibirsk, Russia,
ilyamednykh(at)mail.ru

Riemann surfaces endowed with Euclidean geometry have special properties and are unique exceptions in the theory of Riemann surfaces. The list of these surfaces is finite and well known. These include the torus, cylinder, Moebius strip, and Klein bottle. The fundamental groups of these surfaces are non-Hopfian and admit nontrivial endomorphisms. This circumstance leads to the possibility of nontrivial coverings of the surface over itself and other surfaces.

This talk is devoted to the properties of the Laplace operator defined on discrete versions of the above-mentioned Euclidean surfaces. Discrete analogues of these surfaces are presented as rectangular lattices, with suitable identification of the sides. The goal is to study the structure of the characteristic polynomial of the discrete Laplace operator for such graphs.

Consider the discrete Moebius strip $M_{m,n}$ obtained by identification of the sides of $m \times n$ rectangular lattice. Then the following theorem holds.

Theorem 1. *The characteristic polynomial of discrete Moebius strip $M_{m,n}$ is given by the formula*

$$\chi_{M_{m,n}}(\lambda) = \prod_{j=1}^{n} (2T_m(2 - \frac{\lambda}{2} - \cos \frac{\pi\,(j-1)}{n}) + (-1)^j 2)$$

where $T_n(w) = \cos(n \arccos(w))$ is the Chebyshev polynomial of the first kind.

Similar theorems proved for the discrete Klein bottle and other surfaces. Non-Hopfian property of Euclidean surfaces translates into curious property of theirs respective discrete variants. In particular, $\chi_{M_{m,n}}(\lambda)$ divides $\chi_{M_{pm,qn}}(\lambda)$ if $p+q$ is even.

Generally, for any discrete Euclidean surface obtained from $m \times n$ rectangular lattice, its characteristic polynomial $\chi_{m,n}(\lambda)$ divides polynomial $\chi_{pm,qn}(\lambda)$ for natural numbers p and q.

The related results are partially published in [1, 2].

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q-deformation of graphical arrangement and its generalizations

Tongyu Nian
The University of Osaka, Toyonaka, Japan
nianty2000(at)gmail.com
u487971i(at)ecs.osaka-u.ac.jp

We introduce two new classes of hyperplane arrangements inspired by graphic arrangements and their q-deformations. Building on the analogy between the braid arrangement and the arrangement of all hyperplanes over a finite field, we extend the notion of q-deformation from graphs to simplicial complexes by replacing cliques with faces. We also study a special 1-dimensional setting and discuss characteristic polynomials, freeness, and extensions to fields containing primitive roots. This talk is based on [1].

In [2], the authors introduce a kind of arrangement on graphs called q-deformation, defined as follows.

Definition 1. *Let $G = ([\ell], E)$ be a simple graph on ℓ vertices, $C(G)$ denotes the set of cliques of the graph. The q-deformation of graphic arrangement $\mathcal{A}_G$ is defined by*

$$\mathcal{A}_G^q := \bigcup_{\{i_1,\ldots,i_r\}\in C(G)} \{\ker(a_{i_1}x_{i_1} + a_{i_2}x_{i_2} + \cdots + a_{i_r}x_{i_r}) \mid a_{i_j} \in \mathbb{F}_q^\times \text{ for all } j\}.$$

And a certain limit $q \to 1$ is expected to recover the chromatic polynomial as follows.

Conjecture 2. *The characteristic polynomial of the q-deformation $\chi(\mathcal{A}_G^q, t)$ is a polynomial in q and t, such that*

$$\lim_{q\to 1} \frac{\chi(\mathcal{A}_G^q, q^s)}{(q-1)^\ell} = \chi(G, s),$$

where $\chi(G, t)$ denotes the chromatic polynomial of the graph G.

In [1], we prove that triangle-free graphs satisfy this conjecture by generalizing q-deformations to simplicial complexes and extend the conjecture to the case of simplicial complexes. By restricting simplicial complexes to 1-dimensional case, we prove that all 1-dimensional simplicial complexes satisfy the generalized conjecture.

Finally we extend the field from $\mathbb{F}_q$ to all fields with primitive r-th root of unity and call this kind of arrangement **graphic monomial arrangement** since it shows close relationship with monomial arrangements based on reflection groups.

Definition 3. *Let $G = ([\ell], E)$ be a graph, z be a primitive r-th root of unity in the field. Define the* ***graphic monomial arrangement*** *$\mathcal{M}(G, r)$ by*

$$\mathcal{M}(G, r) = \{\{x_i - z^k x_j = 0\} \mid (i, j) \in E, 1 \le k \le r\} \cup \{\{x_i = 0\} \mid i = 1, \ldots, \ell\}.$$

We show that graphic monomial arrangements satisfy the generalized conjecture and show the equivalence between chordality of the underlying graph and freeness of the corresponding graphic monomial arrangement, following the proof of q-deformations and graphic arrangements. We give an explicit free basis of this arrangement using combinations of determinants of Vandermonde matrices.

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Paired dominating sets and domino tilings in fixed-width grid graphs

Moe Moe Oo
King Mongkut's University of Technology Thonburi, Bangkok, Thailand
moemoe.oo01(at)mail.kmutt.ac.th

This is joint work with Pawaton Kaemawichanurat

In this paper, the number of paired dominating sets and domino tilings on fixed-width grid graphs $P_m \square P_n$, $m \in \{2, 3, 4\}$, are analysed, and their asymptotic growth rates as n increases are determined. Explicit rational generating functions are derived from the linear recurrences, and their asymptotic behaviour is obtained using singularity analysis together with the Cauchy integral formula, where the dominant singularity governs the exponential growth and the residue determines the leading constant.

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Computing set upper tolerance for the minimal spanning tree problem

Dmitrii Panasenko
Umeå University, Umeå, Sweden
dmitrii.panasenko(at)umu.se

This is joint work with Gerold Jäger

Sensitivity analysis in combinatorial optimization examines how changes in the costs of elements affect the optimality of solutions. In this work, we focus on the concept of upper tolerance. In particular, the *single upper tolerance* gives how much the cost of a given element can be increased while ensuring that all current optimal solutions remain optimal and the *set upper tolerance* gives how much the sum of the costs of the elements of a given set can be increased while still maintaining optimality of all current solutions [1].

In this work, we study upper tolerances for the *Minimum Spanning Tree Problem* (MSTP), which seeks a minimum-weight spanning tree in a weighted undirected graph. While efficient algorithms for computing single upper tolerances for the MSTP are well known (see, e.g., [2, 3]), set upper tolerances for this problem have not been previously investigated, to the best of our knowledge.

We present two polynomial-time algorithms for computing set upper tolerances for the MSTP. Whereas the first algorithm is based on a linear programming formulation, the second one is a purely combinatorial algorithm independent of linear programming. We investigate the complexities of both algorithms and compare them also experimentally. Finally, we discuss possible directions for improving the efficiency of the combinatorial algorithm.

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Plans theorem for a cone over sandwich graphs

Galina Sokolova
Novosibirsk State University, Novosibirsk, Russia
Sobolev Institute of Mathematics, Novosibirsk, Russia
g.sokolova(at)g.nsu.ru

This talk considers a family of graphs called sandwich graphs. The structure of the Jacobian for a cone over a sandwich graph is studied. Namely, a result analogous to Plans theorem for knots is established for this mathematical object. Classical Plans theorem states that the first homology group of an odd-fold cyclic covering of the sphere $\mathbb{S}^3$, branched over a knot, is the direct sum of two copies of some Abelian group. A similar result holds for the first homology group of an even-fold cyclic covering factored by the reduced homology group of a two-fold covering. Special cases of sandwich graphs are the I-graph and the generalized Petersen graph (see, for example, [2, 3, 5]), as well as the generalized prism [1].

Consider a graph consisting of a single vertex v and having no edges. A *cone over a graph* G is a graph $\widehat{G} = G \star \{v\}$ whose set of vertices and edges is $V(\widehat{G}) = V(G) \cup v$ and $E(\widehat{G}) = E(G) \cup \{\{w, v\}, w \in V(G)\}$. The cone $\widehat{G} = G \star \{v\}$ over G is a discrete analogue of n-fold cyclic branched covering over the knot from Plans theorem. Let us introduce the concept of a sandwich graph.

Let $n \in \mathbb{N}$, consider a finite set of integers $1 \leqslant s_1 < s_2 < \ldots < s_k \leqslant \frac{n}{2}$. A graph $G = C_n(s_1, s_2, \ldots, s_k)$ on n vertices is called *a circulant graph* if each of its i-th vertices is adjacent to vertices $i \pm s_1, \ldots, i \pm s_k$ modulo n. Let P_m be a path-graph with vertices v_j. Consider circulant graphs $G_j = C_n(s_{j1}, s_{j2}, \ldots, s_{jk_j})$ with n vertices, $j = 1, 2, \ldots, m$, and define the *sandwich graph* $SG_n = SG_n(G_1, G_2, \ldots, G_m)$ as a graph with vertex set $V(SG_n) = \{(\ell, v_j) \,|\, \ell = 1, 2, \ldots n,\, j = 1, 2, \ldots, m\}$ in which each vertex (ℓ, v_j) is adjacent to the vertices $(\ell \pm s_{ij1}, v_i), (\ell \pm s_{j2}, v_j), \ldots, (\ell \pm s_{jk_j}, v_j)$ modulo n, and for all $\ell = 1, 2, \ldots n$ the vertices $(\ell, v_1), (\ell, v_2), \ldots, (\ell, v_m)$ form a path graph P_m.

Denote a cone over SG_n as $\widehat{SG_n}$. For each graph $\widehat{SG_n}$, a monic Laurent polynomial $P(t)$ is defined. The structure of $P(t)$ and its coefficients are described in the article [4]. Note $P(t) \in \mathbb{Z}[t, t^{-1}]$ is palindromic, hence it can be transformed using the Chebyshev transform into a composition of functions

$$P(t) = Q \circ h(t) = a_0 + 2\sum_{k=0}^{m} a_k \mathcal{T}_k\left(\frac{h(t)}{2}\right), \quad \text{where} \quad Q(t) = a_0 + 2\sum_{k=0}^{m} a_k \mathcal{T}_k\left(\frac{t}{2}\right),$$

and $h(t) = t + t^{-1}$. The companion matrices C_P and C_Q are well defined. Here $\mathcal{T}_s(t)$, $\mathcal{U}_s(t)$ and $\mathcal{W}_s(t)$ are Chebyshev polynomials of the first, second and fourth kinds, respectively.

Theorem. *For the Jacobian of the cone* $\widehat{SG_n}$ *the following statements are valid.*

1. *If n is odd, then there exists an epimorphism $\varphi : Jac(\widehat{SG}_{2r+1}) \to Jac(\widehat{SG}_1)$ whose kernel splits into direct sum of two copies of the Abelian group represented by the matrix $\mathcal{W}_r\,(C_Q/2)$.*
2. *If n is even, then there exists an epimorphism $\varphi : Jac(\widehat{SG}_{2r}) \to Jac(\widehat{SG}_2)$ whose kernel splits into direct sum of two copies of the Abelian group represented by the matrix $\mathcal{U}_{r-1}\,(C_Q/2)$.*

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Cylindrical set conjecture

Gábor Somlai
The University of Melbourne, Melbourne, Australia
gabor.somlai(at)unimelb.edu.au

This is joint work with Gergely Kiss, Ádám Markó and Zoltán Lóránt Nagy

Rédei conjectured that for any tile of an elementary abelian p-group of rank 3 ($A \bigoplus B = \mathbb{F}_p^3$) either A or B are contained in a plane. Simeon Ball outlined the following strategy of proving the conjecture, see also [2]. Prove the cylindrical set conjecture and prove that a subset of $\mathbb{F}_p^3$ of size p is contained in a plane or determines at least p directions.

A set of points $S \subseteq \mathbb{F}_p^n$ is called *p-divisible* if every affine hyperplane in $\mathbb{F}_p^n$ intersects S in 0 (mod p) points. The cylindrical set conjecture of Ball asserts that if S is a p-divisible set of p^2 points in $\mathbb{F}_p^3$, then S is a cylinder. We proved [1] that every p-divisible multiset S is both a $\mathbb{F}_p$-linear and $\mathbb{Z}$-linear combination of characteristic functions of cylinders. In addition, the multisets of size p^2 are $\mathbb{Z}$-linear combinations of a plane and weighted differences of parallel lines.

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Tiles of cyclic groups

Gábor Somlai
The University of Melbourne, Melbourne, Australia
gabor.somlai(at)unimelb.edu.au

This is joint work with Gergely Kiss, Izabella Laba and Caleb Marshall

We say that (A, B) is a tiling pair of the abelian group G if every element g of G can uniquely be expressed as $a + b = g$, where $a \in A$ and $b \in B$. On his blog, Tao suggested to prove that every tile of a cyclic group of square-free order is a transversal. This statement follows from an old result of Tijdeman's as it was pointed out by Łaba and Meyerowitz. This question arose in connection with the article by Coven and Meyerowitz [1] on subsets of cycle groups, in which they formulated sufficient conditions that ensure for subsets to be tiles, and following Tao's blogpost the necessity of these conditions is what we call the Coven-Meyerowitz conjecture. The conjecture was confirmed when the order of the group has at most 2 different prime divisors and in a series of long papers written by Łaba and Londner [2] it was proved for slightly larger class of integers.

We propose [3] a new approach to either prove the conjecture or disprove it using our new constructions of tiles. In the meantime we propose a weaker conjecture saying that if A and B are tiling pairs of the cyclic group $\mathbb{Z}_M$ then either both of them or none of the satisfies the conditions of Coven and Meyerowitz.

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Most curvilinear tilings and Hurwitz theorem

Haofang Sun
Hong Kong University of Science and Technology, China
hsunav(at)connect.ust.hk

This is joint work with Min Yan

The classical Hurwitz realisation problem was largely solved by Edmond, Kulkarni, Stong in 1980's. Some of their work was done by relating to tilings of surfaces. In this talk, we revisit the problem from the fresh angle of edge-to-edge tilings of surfaces by congruent curvilinear polygons, where by curvilinear we mean the edges are not necessarily straight. We concentrate on the most extreme curvilinear case, where the polygon is equivalent to a planar diagram D.

Let $V_1, V_2, \ldots, V_k$ be vertices in the surface S_D determined by the planar diagram D. Then the vertices in a tiling by D are $V_1^{k_1}, V_2^{k_2}, \ldots, V_p^{k_p}$. The tiling gives a branched cover over S_D, with k_i as the branching data. Our first result is the necessary and sufficient condition for a branched data to come from a tiling.

The classification of tilings of all surfaces by the given planar diagram D is the same as finding "minimal tilings", which are tilings that do not cover other tilings. We have the following results.

Theorem 1. *Given natural numbers $k_1, \ldots, k_p \geq 2$ with $p \geq 2$, suppose D is a planar diagram with p vertices such that $\chi(S_D) \leq 0$. Then for s big enough prime number, there exists minimal tilings by S_D with sd' tiles. (Let $\lambda = LCM(k_1, \ldots, k_p)$, define $d' = 2\lambda$ if λ is even, but λ/k_i is odd for exactly an odd number of k_i; otherwise define $d' = \lambda$.)*

Theorem 2. *For natural numbers $k_1 \geq 3$, suppose D is a planar diagram with 1 vertex such that $\chi(S_D) \leq 0$. Then for any integer $s \geq 2$, there exists minimal tilings by S_D with vertices V^{k_1} and sd' tiles. For $k_1 = 2$, then for any integer $s = 2^m$, $m \geq 1$ is a natural number, there exists minimal tilings by S_D with vertices V^{k_1} and sd' tiles.*

So, for D satisfying $\chi(S_D) \leq 0$, the number of minimal tilings with the given vertices $V_1^{k_1}, V_2^{k_2}, \ldots, V_p^{k_p}$ is always infinite. This shows the classification problem can be hard to solve for all surfaces. We also discuss minimal surfaces for surfaces of fixed genus.

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A symmetry characterization of t-designs in the Boolean lattice

Anyuan Tian
Shanghai Jiao Tong University, Shanghai, China
tiananyuan(at)sjtu.edu.cn

This is joint work with Yaokun Wu

Let n, a, b be positive integers with $a \leq b < n/2$, and write $[n] \doteq \{1, \ldots, n\}$. For a family $\mathcal{F} \subseteq \binom{[n]}{b}$, we prove that $\mathcal{F}$ is a $\min\{2a-1, b\}$-design if and only if the following symmetry condition holds: for any $S_1, S_2 \in \binom{[n]}{a}$, the number of members of $\mathcal{F}$ containing S_1 and disjoint from S_2 equals the number of members of $\mathcal{F}$ containing S_2 and disjoint from S_1. We are led to this result in our investigation of a forbidden subposet problem in the Boolean lattice, and intend to explore possible extension/application of this symmetry principle in a broader algebraic combinatorics framework.

Skew braces with no proper left ideals

Cindy (Sin Yi) Tsang
Ochanomizu University, Tokyo, Japan
tsang.sin.yi(at)ocha.ac.jp

Skew brace is a group-like and ring-like algebraic structure that was introduced in [2, 4] as a tool to study set-theoretic solutions to the Yang–Baxter equation. Recall that a *skew brace* is a set $A = (A, \cdot, \circ)$ equipped with two group operations $\cdot$ and $\circ$ such that left-distributive-like relation

$$a \circ (b \cdot c) = (a \circ b) \cdot a^{-1} \cdot (a \circ c)$$

holds for all $a, b, c \in A$, where a^{-1} denotes the inverse of a in $(A, \cdot)$. It is easy to see that $(A, \cdot)$ and $(A, \circ)$ share the same identity element 1. Moreover, it is not difficult to show that

$$\lambda_a : (A, \cdot) \longrightarrow (A, \cdot); \quad \lambda_a(b) = a^{-1} \cdot (a \circ b)$$

is an isomorphism for every $a \in A$, and the map

$$\lambda : (A, \circ) \longrightarrow \mathrm{Aut}(A, \cdot); \quad a \mapsto \lambda_a$$

is a homomorphism. A subgroup I of $(A, \cdot)$ is said to be a *left ideal* of A if $\lambda_a(I) \subseteq I$ holds for all $a \in A$. For example, the characteristic subgroups of $(A, \cdot)$ are clearly left ideals of A. Left ideals are ubiquitous in the study of skew braces and play a very important role. Let us say that A is *left-simple* if $A \neq 1$ and A has no left ideals other than 1 and A. If A is left-simple, then clearly $(A, \cdot)$ has to be characteristically simple, so we have $(A, \cdot) \simeq T^n$ for some $n \in \mathbb{N}$ and simple group T when A is finite.

Our main result is the following partial classification of finite left-simple skew braces.

Theorem 1. *Let $A = (A, \cdot, \circ)$ be a finite skew brace such that $(A, \cdot) \simeq \mathbb{F}_p^n$ for some prime p and $n \in \mathbb{N}$. Then A is left-simple if and only if $n = 1$, in which case A is trivial, i.e. $a \circ b = a \cdot b$ for all $a, b \in A$.*

Theorem 2. *Let $A = (A, \cdot, \circ)$ be a finite skew brace such that $(A, \cdot) \simeq T^n$ for some non-abelian simple group T and $n \in \mathbb{N}$. It is well-known that $\mathrm{Aut}(T^n) = \mathrm{Aut}(T) \wr S_n = \mathrm{Aut}(T)^n \rtimes S_n$, where S_n denotes the symmetric group on n letters.*

(*a*) *For $n = 1$, we have A is left-simple if and only if A is almost trivial, i.e. $a \circ b = b \cdot a$ for all $a, b \in A$.*

(*b*) *For $n \geq 2$, if A is left-simple, then $(A, \circ)$ is isomorphic to some subgroup of $\mathrm{Aut}(T^n)$ whose projection onto S_n is a transitive subgroup, and $\mathrm{Im}(\lambda)$ intersects trivially with $\mathrm{Inn}(A, \cdot)$.*

The proof of Theorem 1 uses modular representation theory and is fairly simple. On the other hand, the proof of Theorem 2(a) relies on the classification of factorizations of finite almost simple groups with a solvable factor that is given in [3].

Remark. The motivation of this research originated from Hopf–Galois theory on finite Galois extensions. More specifically, it follows from [5] that finite left-simple skew braces correspond to minimal Hopf–Galois structures on finite Galois extensions, where *minimal* is defined as in [1].

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Converse Casey's theorem in hyperbolic geometry

Yifei Wang
Novosibirsk State University, Novosibirsk, Russia
691275014(at)qq.com

This is joint work with Nikolay Abrosimov

In 1881 Irish mathematician John Casey generalized Ptolemy's theorem in the following way (see [1], p. 103).

Casey's theorem *Let circles O_1, O_2, O_3, O_4 on a plane touch given circle O in vertices p_1, p_2, p_3, p_4 of a convex quadrilateral. Denote by t_{ij} the length of a common tangent of the circles O_i and O_j. If O separates O_i and O_j then the internal tangent should be taken as t_{ij} else the external tangent should be taken. In both cases we assume that the tangents are exist. Then*

$$t_{12}t_{34} + t_{23}t_{14} = t_{13}t_{24}.$$

The converse to Casey's theorem in $\mathbb{E}^2$ was proved much later by different authors, and with different kinds of restrictions (see [2]).

Casey's converse theorem *Given four circles O_1, O_2, O_3, O_4 on a plane and t_{ij} denotes a length of common tangent of circles O_i and O_j. If the orientations of O_i and O_j coincide then t_{ij} is external tangent, otherwise t_{ij} is internal one. Then if one of three numbers $t_{12} \cdot t_{34}, t_{23} \cdot t_{14}, t_{13} \cdot t_{24}$ is equal to the sum of two others, then the circles O_1, O_2, O_3, O_4 either pass through one point or touch a circle O or touch one straight line P.*

In our paper [3] we obtained a hyperbolic version of Casey's direct theorem, while the converse theorem remained unproven in $\mathbb{H}^2$. During the talk we will give the proof of Casey's converse theorem on the Lobachevsky plane.

Casey's converse theorem in $\mathbb{H}^2$ *Given four circles O_1, O_2, O_3, O_4 on the hyperbolic plane and T_{ij} denotes a length of common tangent of circles O_i and O_j. If the orientations of O_i and O_j coincide then T_{ij} is external tangent, otherwise T_{ij} is internal one. Then if one of three numbers*

$$\sinh t_{12} \cdot \sinh t_{34}, \quad \sinh t_{23} \cdot \sinh t_{14}, \quad \sinh t_{13} \cdot \sinh t_{24}$$

is equal to the sum of two others, then the circles O_1, O_2, O_3, O_4 either pass through one point or touch a curve O of constant geodesic curvature (that is a circle, horocycle or one branch of equidistant.

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Phylogenetic tree reconstruction from parsimony layered filters

Min Xie
Shanghai Jiao Tong University, Shanghai, China
xieminsjtu(at)sjtu.edu.cn

This is joint work with Chengyang Qian and Yaokun Wu

Let X be a finite set. A phylogenetic X-tree is a finite tree with leaf set X that contains no degree-2 vertex. Two phylogenetic X-trees are X-isomorphic if there is a graph isomorphism f between them fixing X pointwise. An unordered subset pair $\{A, B\}$ of X is called an X-split if $A \cap B = \emptyset$ and $A \cup B = X$. Let T be a phylogenetic X-tree. For any X-split $\{A, B\}$, its parsimony score on T is the maximum number of edge-disjoint paths in T with one endpoint in A and the other in B. For a nonnegative integer ℓ, let $\mathcal{P}_{T,\ell}$ be the set of all X-splits with parsimony score ℓ on T, and define $\mathcal{P}_{T,\leq \ell} \doteq \bigcup_{i=0}^{\ell} \mathcal{P}_{T,i}$.

Let $\mathbb{N}$ denote the set of all positive integers. For any $n \in \mathbb{N}$, let $\mathcal{T}_n$ be the set of all phylogenetic $[n]$-trees (up to $[n]$-isomorphism) and let $\mathcal{S}_n$ represent the set of all $[n]$-splits. A map f from $(\bigcup_{n\in\mathbb{N}} \mathcal{T}_n) \times \mathbb{N}$ to $\bigcup_{n\in\mathbb{N}} \mathcal{S}_n$ is called a parsimony layered filter if for every $(T, \ell) \in (\bigcup_{n\in\mathbb{N}} \mathcal{T}_n) \times \mathbb{N}$ we have $\mathcal{P}_{T,\ell} \subseteq f(T, \ell) \subseteq \mathcal{P}_{T,\leq \ell}$. Fix positive integers ℓ and n with $n \geq 2\ell + 5$, and let T and T' be phylogenetic $[n]$-trees. For any parsimony layered filter f, we show that the following three statements are equivalent: 1) T and T' are $[n]$-isomorphic; 2) $f(T, \ell) = f(T', \ell)$; 3) $\mathcal{P}_{T,\ell} \cup \mathcal{P}_{T',\ell} \subseteq \mathcal{P}_{T,\leq \ell} \cap \mathcal{P}_{T',\leq \ell}$.

On second maximum intersecting families of Ree Unital

Yilin Xie
Hebei Normal University, Shijiazhuang, China
xyl_crazystu(at)hebtu.edu.cn

This is joint work with Sergey Goryainov and Rhys Evans

In combinatorics, a unital is a set of $n^3 + 1$ points arranged into subsets of size $n + 1$ so that every pair of distinct points of the set are contained in exactly one subset. This is equivalent to saying that a unital is a $2 - (n^3 + 1, n + 1, 1)$ block design. One of the most important infinite families of unitals is the family of Ree unitals. Let $\Gamma = \mathrm{Ree}(q)$ be the Ree group of type ${}^2G_2$ of order $(q^3 + 1)q^3(q - 1)$ where $q = 3^{2m+1}$. The Ree unital is a unital with points set the Sylow 3-subgroups of Γ and its blocks set those Sylow 3-subgroups normalized by an involution of Γ. It is a $2-(q^3+1, q+1, 1)$ design, with its automorphism group the full automorphism group of Γ.

An family of sets $\mathcal{F}$ is called intersecting if any two sets of $\mathcal{F}$ intersects non-trivially. The canonical intersecting family given by Erdős-Ko-Rado is all the sets containing a given point. Hilton-Milner construct an intersecting family, which is given by a single set S outside an canonical intersecting family $\mathcal{F}$, together with all sets in $\mathcal{F}$ intersect with S, this family is usually known as the second largest intersecting family of different objects.

De Boeck proved in 2015 that the maximum intersecting family of blocks of Unitals are the canonical ones. We proved that Hilton-Milner type intersecting families of blocks of Ree Unitals are maximal intersecting families, and we wonder if they are second maximum intersecting families.

Amorphic association schemes and beyond

Yanzhen Xiong
National University of Defense Technology, Changsha, China
xiongyanzhen(at)nudt.edu.cn

This is joint work with Edwin van Dam and Jack Koolen

Let $\mathcal{R}$ be a d-class association scheme with relations $A_0(= I), A_1, \ldots, A_d$. Let π be a partition of $\{1, \ldots, d\}$ with blocks $\pi(1), \ldots, \pi(d')$. We say that π gives rise to a fusion scheme $\mathcal{R}_\pi$ of $\mathcal{R}$ if the configuration $\mathcal{R}_\pi$ with relations $A_0, A_{\pi(1)}, \ldots, A_{\pi(d')}$ — where $A_{\pi(j)} = \sum_{i\in\pi(j)} A_i$ — is an association scheme. We call $\mathcal{R}$ *amorphic* if every partition π of $\{1, \ldots, d\}$ gives rise to a fusion scheme of $\mathcal{R}$.

Ivanov showed that all nontrivial relations (relations other than A_0) in an amorphic d-class association scheme with $d \geq 3$ are strongly regular of (negative) Latin square type. He conjectured that an association scheme is amorphic if every relation is strongly regular. This conjecture was later disproved, leading to many interesting constructions of amorphic or nonamorphic association schemes.

In this talk, I will present our characterizations of amorphic association schemes and our constructions of almost amorphic association schemes. Our main results are as follows.

In [1], in terms of fusing pairs of relations, we give three equivalent conditions for $\mathcal{R}$ to be amorphic:

- Every pair $\{A_i, A_j\}$ $(1 \leq i < j \leq d)$ fuses;
- For $d \geq 3$, the fusing-relations graph of $\mathcal{R}$ is connected but not a path;
- At most one nontrivial relation fails to be a strongly regular graph of (negative) Latin square type.

The *fusing-relations graph* of $\mathcal{R}$ is defined as the graph on the vertex set $\{A_1, \ldots, A_d\}$ where A_i and A_j are joined by an edge if fusing them yields a fusion scheme.

In [2], in terms of fusing triples of relations, we give two equivalent conditions for $\mathcal{R}$ to be amorphic:

- For $d \geq 5$, every triple $\{A_i, A_j, A_k\}$ $(1 \leq i < j < k \leq d)$ fuses;
- For $d \geq 5$, the fusing-relations 3-hypergraph of $\mathcal{R}$ contains two 3-sunflowers.

The *fusing-relations* 3-*hypergraph* of $\mathcal{R}$ is defined as the 3-uniform hypergraph on the vertex set $\{A_1, \ldots, A_d\}$ where $\{A_i, A_j, A_k\}$ is an edge if and only if fusing them yields a fusion scheme. A 3-*sunflower* is a 3-hypergraph such that its edges cover all vertices and have exactly two vertices in common.

In [3], we prove that for any $d \geq 2$, there exist nonamorphic d-class association schemes in which exactly $d - 2$ of the nontrivial relations are strongly regular of Latin square type.

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Cayley maps and product groups

Hao Yu
Guangxi University, Nanning, China
haoyu(at)gxu.edu.cn

A (generalized) regular Cayley map $\mathcal{CM}$ is a special 2-cell embedding of a Cayley graph onto orientable (or any) compact connected surface. Let $X = \mathrm{Aut}(\mathcal{CM})$ be the automorphism group of $\mathcal{CM}$ on the group G. It is well known that either $X = G \cdot \mathbb{Z}_n$ or $X = G \cdot D_{2n}$. This is closely related to problems about group factorizations or product groups. Generally, a group X is said to be properly factorizable if $X = GH$ for two proper subgroups G and H of X. In this case, X is the product group of G and H. In this talk, some recent results on (generalized) regular Cayley maps and product groups will be introduced.

Fractional clique decompositions of dense balanced multipartite graphs

Shikang Yu
Beijing Jiaotong University, Beijing, China
healthyu(at)bjtu.edu.cn

This is joint work with Tao Feng and Hengrui Liu

This talk concerns fractional K_s-decompositions of multipartite graphs. For integers $r \geq s \geq 3$, we consider balanced r-partite graphs G on rn vertices. We establish necessary conditions for G to admit a fractional K_s-decomposition, extending the notion of s-admissibility from the case $r = s$ to $r > s$. Using an association scheme on the edge set of a complete r-partite graph, we prove that if $r \geq s + 2$ and the partite minimum degree of G is at least $(1 - c)n$ with $c \leq 1/((s - 2)(s + 1)(s - 1)^4)$, then G has a fractional K_s-decomposition. For $r = s + 1$, we show that under the condition $c \leq 1/(3s^3(s - 2)^2)$, every s-admissible balanced $(s + 1)$-partite graph with partite minimum degree at least $(1-c)n$ admits a fractional K_s-decomposition. These results provide new degree thresholds for fractional K_s-decompositions of multipartite graphs with more than s parts.

NPA hierarchy for quantum isomorphism and homomorphism indistinguishability

Peter Zeman
Charles University, Prague, Czechia
peter.zeman(at)matfyz.cuni.cz

This is joint work with Prem Nigam Kar, David E. Roberson, Tim Seppelt

Mančinska and Roberson in 2020 showed that two graphs are quantum isomorphic if and only if they admit the same number of homomorphisms from any planar graph. Atserias et al. in 2029 proved that quantum isomorphism is undecidable in general, which motivates the study of its relaxations. In the classical setting, Roberson and Seppelt in 2023 characterized the feasibility of each level of the Lasserre hierarchy of semidefinite programming relaxations of graph isomorphism in terms of equality of homomorphism counts from an appropriate graph class. The NPA hierarchy, a noncommutative generalization of the Lasserre hierarchy, provides a sequence of semidefinite programming relaxations for quantum isomorphism. In the quantum setting, we show that the feasibility of each level of the NPA hierarchy for quantum isomorphism is equivalent to equality of homomorphism counts from an appropriate class of planar graphs. Combining this characterization with the convergence of the NPA hierarchy, and noting that the union of these classes is the set of all planar graphs, we obtain a new proof of the result of Mančinska and Roberson that avoids the use of quantum groups. Moreover, this homomorphism indistinguishability characterization also yields a randomized polynomial-time algorithm deciding exact feasibility of each fixed level of the NPA hierarchy of SDP relaxations for quantum isomorphism.

On EKR-type problem for hypergraph matchings

Binwei Zhao
Beijing Jiaotong University, Beijing, China
bwzhao(at)bjtu.edu.cn

This is joint work with Tao Feng, Xiaomiao Wang and Menglong Zhang

Given integers $1 \leq t \leq k$, a family of k-matchings in a complete r-partite r-uniform hypergraph is said to be t-intersecting if any two of its members share at least t common edges. This concept unifies several well-studied classes of intersecting families, including classical intersecting families, intersecting families of permutations, partial permutations, and generalized permutations, as well as intersecting families of injections. In this talk, we will introduce the method of t-covers to determine the maximum size of t-intersecting families of k-matchings and to characterize the extremal families that attain this bound. Our results extend a number of existing theorems in the literature.

New combinatorial problems from the Hermitian Sum-of-Squares conjecture

Pengyu Zhu
Shanghai Jiao Tong University, Shanghai, China
zhupengyu(at)sjtu.edu.cn

This is joint work with Anyuan Tian and Yaokun Wu

Let $n, s \geq 2$ be integers, and let $(\mathbb{Z}^n_{\geq 0})_s$ be the set of n-tuples of nonnegative integers summing to s. For each $i = 1, \ldots, n$, denote by $\mathbf{e}_i$ the length-n $(0, 1)$-vector with a 1 in the i-th coordinate and 0 elsewhere. For any subset $Z \subseteq (\mathbb{Z}^n_{\geq 0})_s$, define $\partial Z = \{z + \mathbf{e}_i : z \in Z, 1 \leq i \leq n\}$. Set $\Gamma_{n,s} = \{(X, Y) : X, Y \subseteq (\mathbb{Z}^n_{\geq 0})_s, X \cap Y = \emptyset, \partial X \supseteq \partial Y, Y \neq \emptyset\}$ and $\mathrm{p}_n(s) = \min\{|\partial X \setminus \partial Y| : (X, Y) \in \Gamma_{n,s}\}$. We prove that

$$\mathrm{p}_n(s) = \begin{cases} 8, & \text{if } n = 4, \\ \frac{(n-1)(n+2)}{2}, & \text{if } n \neq 4. \end{cases}$$

This result is motivated by the Hermitian Sum-of-Squares (SOS) conjecture in several complex variables proposed by Ebenfelt [1, Conjecture 1.2]. Consequences of our combinatorial result include that the SOS conjecture holds in the monomial case, and that a related conjecture raised by Wang, Yue, and Zhou [2, p. 9] is valid. If time permits, we will also report some combinatorial investigations of the general SOS conjecture.

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Digraphs with non-diagonalizable adjacency matrix

Wenying Zhu
Hebei Normal University, Shijiazhuang, China
zfwenying(at)hebtu.edu.cn

This is joint work with Yuxuan Li, Binzhou Xia and Sanming Zhou

The fact that the adjacency matrix of every finite graph is diagonalizable plays a fundamental role in spectral graph theory. Since this fact does not hold in general for digraphs, it is natural to ask whether it holds for digraphs with certain level of symmetry. Interest on this question dates back to early 1980s, when P. J. Cameron [1] asked for the existence of arc-transitive digraphs with non-diagonalizable adjacency matrix. This was answered in the affirmative by L. Babai [2] in 1985. Then Babai [2] posed the open problem of constructing a 2-arc-transitive digraph and a vertex-primitive digraph whose adjacency matrices are not diagonalizable. In this talk, we solve Babai's problem by constructing infinite families of digraphs with the required properties.

To build an infinite family of digraphs from an existing one, we use the *tensor product* $\Gamma \times \Sigma$ of digraphs Γ and Σ, where $\Gamma \times \Sigma$ is the digraph with vertex set $V(\Gamma) \times V(\Sigma)$ such that $(u_1, v_1) \to (u_2, v_2)$ if and only if $u_1 \to u_2$ in Γ and $v_1 \to v_2$ in Σ. For an integer $n \geq 1$, denote by $\Gamma^{\times n}$ the tensor product of n copies of digraph Γ. Our main result gives infinite families of non-diagonalizable s-arc-transitive digraphs and non-diagonalizable vertex-primitive digraphs. The basic digraphs in these two families are as follows.

Construction 1. *For each integer $s \geq 2$, let $a_s = (2s-1, 2s)(4s-1, 4s) \in \mathrm{Sym}_{4s}$, let $b_s = (1, 3, 5, \dots, 4s-1, 2, 4, 6, \dots, 4s) \in \mathrm{Sym}_{4s}$, let $R_s = \langle a_s, b_s \rangle$ be the group generated by a_s and b_s, and let $\Gamma_s = \mathrm{Cay}(R_s, \{a_s b_s, b_s\})$.*

Construction 2. *Let $R = \langle a, b \mid a^7 = b^3 = 1, b^{-1}ab = a^2 \rangle \times \langle c, d \mid c^7 = d^3 = 1, d^{-1}cd = c^2 \rangle$, let γ be the automorphism of R interchanging a with c and b with d, let*

$$S = (S_1 \cup S_1^{-1})(S_3 \cup S_3^{-1})^\gamma \cup (S_3 \cup S_3^{-1})(S_1 \cup S_1^{-1})^\gamma \cup S_1 S_2^\gamma \cup S_2 S_1^\gamma \cup S_1^{-1} S_4^\gamma \cup S_4 (S_1^{-1})^\gamma,$$

where

$$S_1 = \{a,\, a^5,\, a^6 b,\, a^6 b^2\}, \quad S_2 = \{ab,\, (ab)^{-1}\},$$
$$S_3 = \{a^3,\, b,\, ab^2,\, a^4 b^2\}, \quad S_4 = \{a^2 b,\, (a^2 b)^{-1}\},$$

and let $\Sigma = \mathrm{Cay}(R, S)$.

Our main result is as follows.

Theorem 3. *For all positive integers n and $s \geq 2$, for the digraph Γ_s in Construction 1 and for the digraph Σ in Construction 2, the digraphs $\Gamma_s^{\times n}$ and $\Sigma^{\times n}$ satisfy the following:*

(a) $\Gamma_s^{\times n}$ *is s-arc-transitive;*

(b) $\Sigma^{\times n}$ *is vertex-primitive;*

(c) $\Gamma_s^{\times n}$ *and* $\Sigma^{\times n}$ *are non–diagonalizable.*

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Representation of braid-like groups

Ksenia Zimireva
Novosibirsk State University, Novosibirsk, Russia
kseniazimireva(at)mail.ru

This is joint work with Maxim Dudar

Elements of the cactus group have a geometric representation by strands on a plane or in space. Braid group has a similar representation, therefore cactus groups can be considered as some analogue of braid groups.

The cactus group was appeared in the works of S. L. Devadoss [1] and M. Davis, T. Januszkiewicz [2].

A cactus group J_n is generated by a_i, $2 \leqslant i \leqslant n$, with defining relations:

$$a_i^2 = 1,\ 2 \leqslant i \leqslant n, \quad (a_i a_k a_j a_k)^2 = 1,\ i \leqslant j,\ i + j \leqslant k,$$
$$a_i a_k a_j a_k = a_{i+j-k} a_j a_{i+j-k} a_i,$$
$$4 \leqslant j + 2 \leqslant i \leqslant n; \quad j < k < i; \quad 2 \leqslant i + j - k \leqslant n; \quad 2k \leqslant i + j.$$

We construct a linear representation of J_n which is based on the work [3], where faithful linear representation of generalized cactus groups was constructed. The general construction of this linear representation does not give an explicit linear representation of the cactus group J_n.

Let $\mathbb{S} = \{\langle (i_1, i_2), (i_2, i_3) \ldots, (i_k, i_{k+1}) \rangle \mid i_j < i_{j+1},\ 1 \leqslant j \leqslant k \leqslant n-1,\ (i_j, i_{j+1}) \in S_n\}$. Let us define mappings $f_i : \mathbb{S} \to \mathbb{S}$ for $2 \leqslant i \leqslant n$, which acts on $K \in \mathbb{S}$ according to the rule $f_i(K) = \langle (j_1, j_2), \ldots, (j_k, j_{k+1}) \rangle$, where $i_l \leqslant i$ for some maximal l and

$$j_s = \begin{cases} i - i_{l-s+1} + 1, & 1 \leqslant s \leqslant l, \\ i_s, & l < s \leqslant k+1. \end{cases}$$

Theorem 1. *From the general construction of the R. Yu's linear representation for the group J_n we obtain a representation $\Pi : J_n \to \mathrm{Aut}(V)$ by automorphisms of the free $\mathbb{Z}[t]$-module V with basis e_K, $K \in \mathbb{S}$, in which the images of the generators a_i are the automorphisms φ_i acting as follows:*

$$\phi_i(e_K) = \begin{cases} -e_K, & K = W_i, \\ e_K, & i < i_1, \\ e_{f_i(K)} + 2te_{W_i}, & W_i \not\subset f_i(K) \text{ and } i_1 \leqslant i < i_{k+1}, \\ e_{f_i(K)}, & W_i \subset f_i(K) \text{ or } i_{k+1} \leqslant i. \end{cases}$$

It was proven that this linear representation is reducible. In the case $n = 3$, the resulting reduced representation is reducible for all $t \in \mathbb{R}$.

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List of Participants

Nikolay Abrosimov, Sobolev Institute of Mathematics
Mackenzie Bookamer, Washington University in St. Louis
Meng-Yue Cao, University of Science and Technology of China
Tai Chan, Hong Kong University
Yu Hao Chang, National Central University
Ting-Wei Chao, Massachusetts Institute of Technology
Huye Chen, Guangxi University Nanning
Jianfu Chen, Wuyi University
Qi Chen, East China University of Science and Technology
Yuanyuan Chen, Xinjiang University
Victor Chepoi, Aix-Marseille Université
Ka Hei Choy, Hong Kong University of Science and Technology
Marston Conder, University of Auckland
Xiao Kai Ding, University of Science and Technology of China
Julia Q. D. Du, Hebei Normal University
Rhys John Evans, University of Sydney
Xiang Gao, Hubei University
Yibo Gao, Peking University
Zihao Geng, East China University of Science and Technology
Ipek Görgel, Middle East Technical University
Leio Zhi Lei Guo, Asymto Global
Jiawei He, Nanchang Hangkong University
Xiaocong He, Hebei Normal University
Kan Hu, Zhejiang Ocean University
Yanling Hu, East China University of Science and Technology
Xin Qi Huang, University of Science and Technology of China
Keyi Jiang, East China University of Science and Technology
Vladimir Janković, University of Primorska
Seoung Dal Jung, Jeju National University
Wazeen Akram Kabrachi, Ecole Normale Supérieure Paris-Saclay Shanghai Jiao Tong University
Monu Kadyan, Shenzhen Institutes of Advanced Technology
Thomas Karam, Shanghai Jiao Tong University
Ken-ichi Kawarabayashi, National Institute of Informatics, The University of Tokyo
Umme Nasreen Khanam, Jaintapur Tayob Ali Technical College
Faiyaz Shahriar Khandooker Mohammed, North South University
Jack Koolen, University of Science and Technology of China
Elena V. Konstantinova, China Three Gorges University & Sobolev Institute of Mathematics & Novosibirsk State University
Young Soo Kwon, Yeungnam University
Ming Fei Lau, Hong Kong University and Technology
Caiheng Li, Southern University of Science and Technology
Chunlin Li, ZheJiang Normal University
Dan Li, Xinjiang University
Ivy Li, N.I.C.(International) Holding Ltd
Ke Li, The University of Hong Kong
Zhen-Qi Liao, National Central University
Xiaoye Liang, Anhui Jianzhu University
Xiao Tong Liang, Shenzhen Senior High School Group Sihui High School
Huiqiu Lin, East China University of Science and Technology
Yupeng Lin, University of Science and Technology of China
Nathan Lindzey, University of Memphis
Jiayin Liu, Tsinghua University
Jingyuan Liu, University of Science and Technology of China
Juejun Liu, ZheJiang Normal University
Lianping Liu, East China University of Science and Technology
Jianbing Lu, National University of Defense Technology
Hoi Ping Luk, Západočeská univerzita v Plzni
Jie Ma, University of Science and Technology of China
Edita Máčajová, Comenius University

Giuseppe Mariani, King's College London
Brendan McKay, Australian National University
Marie Cris B. Mahinay, University of the Philippines Cebu
Alexander Mednykh, Sobolev Instiitute of Mathematics & Novosibirsk State University
Ilya Mednykh, Sobolev Instiitute of Mathematics & Novosibirsk State University
Bojan Mohar, Simon Fraser University & University of Ljubljana
Akihiro Munemasa, China University of Geosciences
Roman Nedela, University of West Bohemia
Tongyu Nian, Osaka University
Moe Moe Oo, King Mongkut's University of Technology Thonburi
János Pach, Alfréd Rényi Institute of Mathematics
Jiangmin Pan, Yunnan University of Finance and Economics
Dmitrii Panasenko, Umeå University
Venessa Perpetua, University of the Philippines
Tao Qiu, Zhejiang Ocean University
Andrei M. Raigorodskii, Moscow Institute of Physics and Technology
Jiaxin Shen, Wuyi University
Penghui Shi, East China University of Science and Technology
Jozef Širáň, Slovak University of Technology
Martin Škoviera, Comenius University
Galina Sokolova, Novosibirsk State University & Sobolev Institute of Mathematics
Petr Stehlík, University of West Bohemia
Gábor Somlai, The University of Melbourne
Yimin Song, Anhui Jianzhu University
Haofang Sun, Hong Kong University of Science and Technology
Zhentian Sun, ZheJiang Normal University
Ying Ying Tan, Anhui Jianzhu University
Zhong Wen Tang, Hebei Normal University
Gábor Tardos, Alfréd Rényi Institute of Mathematics
Anyuan Tian, Shanghai Jiao Tong University, Shanghai, China
Ying Zhi Tian, Xinjiang University
Cindy (Sin Yi) Tsang, Ochanomizu University
Dajie Wang, North China University of Water Resources and Electric Power
Yifei Wang, Novosibirsk State University
Jie Wu, ZheJiang Normal University
Jui Che Wu, National Central University
Yaokun Wu, shanghai Jiao Tong University
Jiawei Xie, East China University of Science and Technology
Min Xie, Shanghai Jiao Tong University
Yilin Xie, Hebei Normal University
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Jin-Xin Zhou, Beijing Jiaotong University
Jie Zhu, ZheJiang Normal University
Pengyu Zhu, Shanghai Jiao Tong University
Wenying Zhu, Hebei Normal University

Yinfen Zhu, East China University of Science and Technology
Rade Živaljiević, Mathematical Institute SASA
Ksenia Zimireva, Novosibirsk State University

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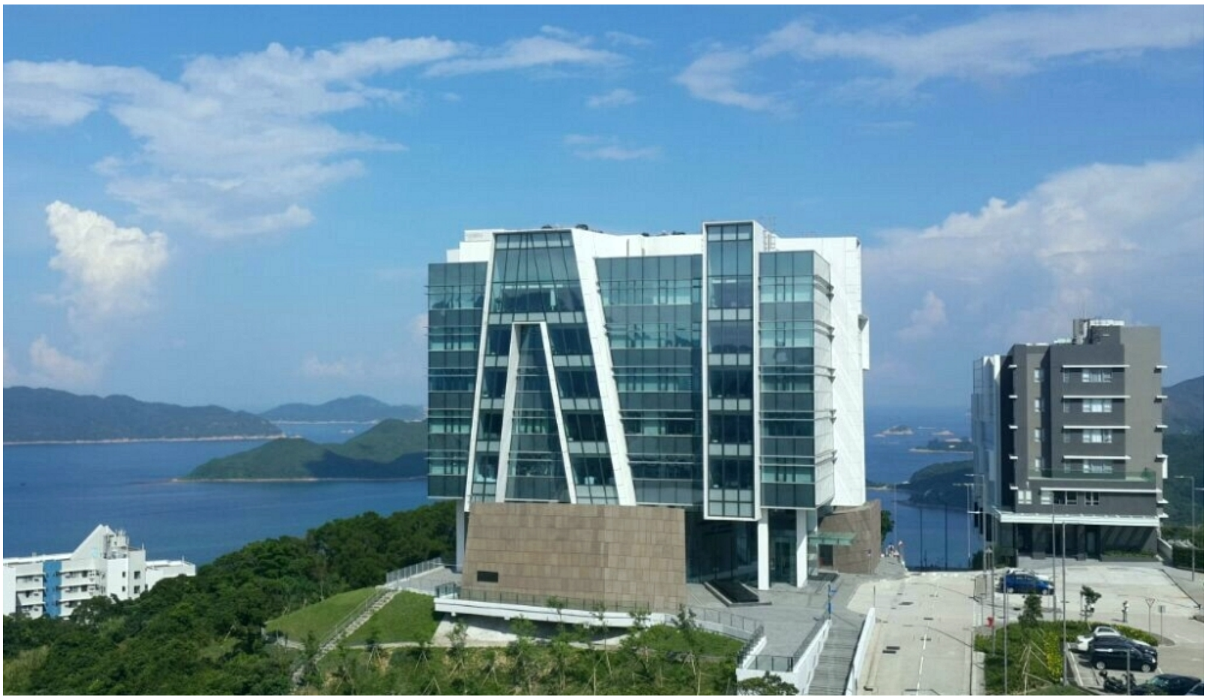

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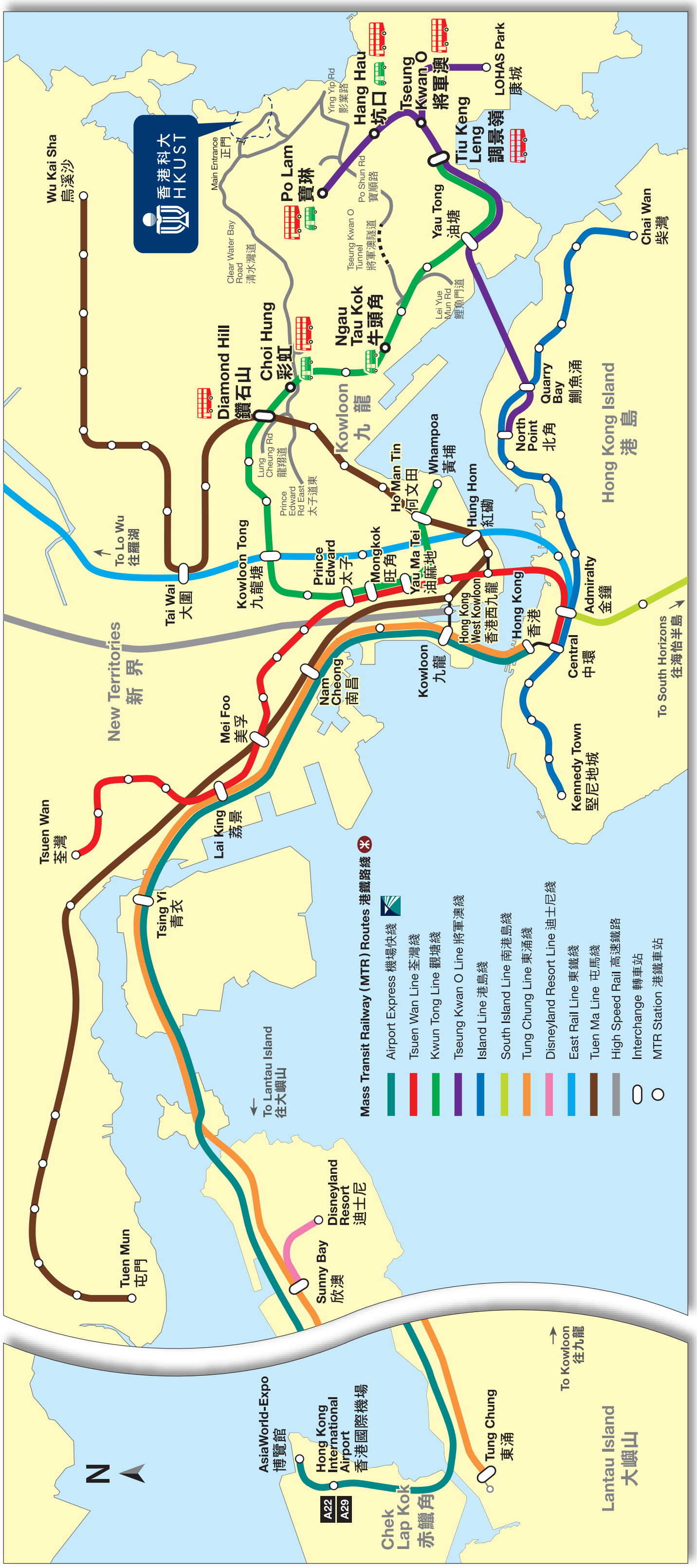

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