# On the structures of tiling sets in finite abelian groups

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**Abstract**

The Fuglede conjecture establishes the relationship between spectral and tilings. Spectral is one of the core concepts in harmonic analysis. Since tilings are relatively easy to verify while spectral are more difficult, researchers are particularly interested in what kind of tiling sets are spectral sets (T-S for short). We mainly focus on the structure of tiling sets in finite abelian groups. Many researchers have considered tiling sets with special structures in finite abelian groups, such as groups with Hajós property, groups with Redei property, and quasi-periodic groups. However, there are only a few groups with Hajós property and groups with Redei property. Additionally, there are groups with quasi-periodic properties that do not have T-S, and there are groups with T-S that do not have quasi-periodic properties. Therefore, it is necessary to propose a more suitable property that is easy to verify, has a good structure, and satisfies T-S.

In this talk, we introduce the periodic tiling (PT) property for finite abelian groups. A finite abelian group is said to have the PT property if every non-periodic set that tiles the group by translation admits a periodic tiling complement. This notion extends the scope beyond groups with the Hajós property. We give a complete classification of cyclic groups possessing the PT property and identify certain non-cyclic groups that enjoy the PT property but fail to satisfy the Hajós property. As a byproduct, we obtain new families of groups for which the implication "Tile $\Longrightarrow$ Spectral" holds. Furthermore, for elementary $p$-groups with the PT property, by analyzing the structure of tiles, we prove that every tile is a complete set of representatives of the cosets of some subgroup.