

Bounding Hamilton Cycles in Polytopal Maps

Martin Škoviera

Comenius University, Bratislava, Slovakia

skoviera@dcs.fmph.uniba.sk

This is joint work with Roman Nedela

In this talk we develop the idea that embedding a cubic graph into a closed surface can serve as a convenient tool for finding a Hamilton cycle in it. We establish a necessary and sufficient condition for a cubic graph embedded in a closed surface, orientable or not, to have a bounding Hamilton cycle. With this characterisation and its consequences we can guarantee Hamilton cycles in wide classes of cubic graphs. Among others, we provide a unified and relatively short proof of a result due to Glover, Marušič, Kutnar, and others proved in a series of four papers [1–4] published over the years 1996–2012 that cubic Cayley graphs of finite quotients of the modular group have a Hamilton path and, except in one special case, they also have a Hamilton cycle. We also show that in the remaining case these Cayley graphs have no bounding Hamilton cycle, which indicates that identifying a Hamilton cycle in this case is likely to be very difficult.

Acknowledgments. The work has been supported by the grants APVV-23-0076 of the Slovak Research and Development Agency and by VEGA 1/0613/26.

References

- [1] H. H. Glover, T. Y. Yang, A Hamilton cycle in the Cayley graph of the $(2, p, 3)$ -presentation of $\mathrm{PSL}_2(p)$. *Discrete Math.* **160** (1996) 149–163.
- [2] H. H. Glover, D. Marušič, Hamiltonicity of cubic Cayley graphs. *J. Eur. Math. Soc.* **9** (2007) 775–787.
- [3] H. H. Glover, K. Kutnar, D. Marušič, Hamiltonian cycles in cubic Cayley graphs: the $(2, 4k, 3)$ case. *J. Algebraic Combin.* **30** (2009) 447–475.
- [4] H. H. Glover, K. Kutnar, A. Malnič, D. Marušič, Hamilton cycles in $(2, \text{odd}, 3)$ -Cayley graphs. *Proc. London Math. Soc.* **104** (2012) 1171–1197.