

Non-orientable regular hypermaps of arbitrary hyperbolic type

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Abstract

One of the consequences of residual finiteness of triangle groups is that for any given hyperbolic triple (ℓ, m, n) there exist infinitely many regular hypermaps of type (ℓ, m, n) on compact orientable surfaces. The same conclusion also follows from a classification of those finite quotients of hyperbolic triangle groups that are isomorphic to linear fractional groups over finite fields. A non-orientable analogue of this, that is, existence of regular hypermaps of a given hyperbolic type on *non-orientable* compact surfaces, appears to have been proved only for *maps*, which arise when one of the parameters ℓ, m, n is equal to 2.

In this paper we establish a non-orientable version of the above statement in full generality by proving the following much stronger assertion: for every hyperbolic triple (ℓ, m, n) there exists an infinite set of primes p of positive Dirichlet density, such that (i) there exists a regular hypermap \mathcal{H} of type (ℓ, m, n) on a compact non-orientable surface such that the automorphism group of \mathcal{H} is isomorphic to $\mathrm{PSL}(2, p)$, and, moreover, (ii) the carrier compact surface of *every* regular hypermap of type (ℓ, m, n) with rotation group isomorphic to $\mathrm{PSL}(2, p)$ is necessarily non-orientable.