Equiangular lines, distance-regular covers of complete graphs, and roux schemes

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The concept of voltage assignment on graphs has been used to construct regular coverings of graphs. In particular, certain voltage assignments on the set of arcs of a complete graph give rise to antipodal distance-regular covers of complete graphs, as shown by Godsil and Hensel [?]. Klin and Pech [?] regarded voltage assignments on a complete graph on n vertices as an $n \times n$ matrix with entries in the group algebra in 2011, and it was axiomatized using the theory of association schemes by Iverson and Mixon [?]. The terms roux matrix and roux scheme, introduced by Iverson and Mixon capture not only antipodal distance-regular covers of complete graphs, but also real and complex equiangular lines. In this talk, I will introduce the so-called local construction of roux schemes, and how the remarkable set of 64 equiangular lines discovered by Hoggar [?] can be characterized as an association scheme. Our main theorem in [?] is as follows:

Theorem 1. *There exists a rank* 8 *roux scheme on* 256 *points representing* 64 *equiangular lines in* $\mathbb{C}^8$ *constructed by Hoggar, and this roux scheme is locally constructed from the rank* 5 *association scheme obtained from the action of* $PSU(3,3)$ *on* 63 *orthogonal bases. Moreover, these two association schemes are uniquely characterized by their parameters.*

References

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