

Long cycles in vertex transitive digraphs

Bojan Mohar

Simon Fraser University, Canada and University of Ljubljana, Slovenia

mohar@sfu.ca

This is joint work with Matija Bucić, Kevin Hendrey, Raphael Steiner, and Liana Yepremyan

One of the most well-known conjectures concerning Hamiltonicity in graphs asserts that any sufficiently large connected vertex transitive graph contains a Hamilton cycle. In this form, it was first written down by Thomassen in 1978, inspired by a closely related conjecture due to Lovász from 1969. It has been attributed to several other authors in a survey on the topic by Witte and Gallian in 1984.

The analogous question for vertex transitive digraphs has an even longer history, having been first considered by Rankin in 1946. It is arguably more natural from the group-theoretic perspective underlying this problem in both settings. Trotter and Erdős proved in 1978 that there are infinitely many connected vertex transitive digraphs which are not Hamiltonian. This left open the very natural question of how long a directed cycle one can guarantee in a connected vertex transitive digraph on n vertices.

In 1981, Alspach asked if the maximum perimeter gap (the gap between the circumference and the order of the digraph) is a growing function in n . We answer this question in the affirmative, showing that it grows at least as fast as $(1 - o(1)) \ln n$. On the other hand, we prove that one can always find a directed cycle of length at least $\Omega(n^{1/3})$, establishing the first lower bound growing with n , providing a directed analogue of a famous result of Babai from 1979 in the undirected setting.

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