Finite Groups, Symmetric Graphs, and Geometries

Cai Heng Li
Southern University of Science and Technology, Shenzhen, China
lich@sustech.edu.cn

The theme of this short course is group actions on graphs, geometries and maps, which includes the following topics:

(1) Finite groups: normal subgroups, quotient groups, simple groups, and Jordan-Holder Theorem.

(2) Permutation groups: primitive groups, quasiprimitive groups, and O'Nan-Scott Theorem.

(3) Edge transitive graphs, and flag-transitive incidence geometries.

(4) A new characterization of the five type arc-transitive maps.

(5) Vertex quasiprimitive regular maps.

(6) Smooth coverings of surfaces, maps and groups.

References

[1] W.M. Kantor, Primitive permutation groups of odd degree, and an application to finite projective planes. *J. Algebra* **106** (1987), no. 1, 15-45.

[2] J.D. Dixon, B. Mortimer, *Permutation groups*, in: Graduate Texts in Mathematics, vol. 163, Springer-Verlag, New York, 1996.

[3] C.E. Praeger, An O'Nan-Scott theorem for finite quasiprimitive permutation groups and an application to 2-arc transitive graphs, *J. Lond. Math. Soc. (2)* **47 (2)** (1993) 227-239.

[4] J.E. Graver, M.E. Watkins, Locally finite, planar, edge-transitive graphs. *Mem. Amer. Math. Soc.* **126** (1997), no. 601, vi+75 pp.

[5] A. Gardiner, R. Nedela, J. Širáň, M. Škoviera, Characterisation of graphs which underlie regular maps on closed surfaces. *J. London Math. Soc. (2)* **59** (1999), 100-108.

[6] R. Jajcay, C.H. Li, J. Širáň, Y. Wang, Regular and orientably-regular maps with quasiprimitive automorphism groups on vertices. *Geom. Dedicata* **203** (2019), 389-418.

[7] J. Bamberg, C.H. Li, E. Swartz, A classification of finite locally 2-transitive generalized quadrangles. *Trans. Amer. Math. Soc.***374** (2021), 1535-1578.

[8] C.H. Li, C.E. Praeger, S.J. Song, Locally finite vertex-rotary maps and coset graphs with finite valency and finite edge multiplicity. *J. Combin. Theory Ser. B* **169** (2024), 1-44.

[9] J. Chen, W. Fan, C.H. Li, Y.Z. Zhu, Coverings of Groups, Regular Dessins, and Surfaces, arXiv:2409.01979