The Four Color Theorem: Generalizations and Faster Algorithms

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The celebrated Four Color Theorem (4CT) was conjectured by Francis Guthrie in 1852, and remained open for more than 100 years, until Appel and Haken found a proof in 1976. The proof is computer-assisted, and a simplified version was found by Robertson et al. in 1997. It has been known since 1880 (by Tait) that the 4CT is equivalent to stating that every 2-connected cubic planar graph is 3-edge-colorable.

In our ongoing project, we aim to utilize today's machine power to extend the computer-assisted proof (and computer-free extension) of 4CT, giving significantly stronger results and providing a much faster algorithm. Topics include:

- Three-edge cubic graphs on a surface: Here, we completely characterize all non-three-edge colorable cubic graphs (i.e., snarks) that can be embedded in a projective plane or in a torus (while the 4CT has no extension to other surfaces).
- A much faster algorithm to 4-color planar graphs, namely: we can 4-color a planar graph of order n in $O(n \log n)$ time. This generalizes the $O(n^2)$ algorithm by Robertson et al. in 1997.

Based on the work in [**?**].

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References

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